Euclid preparation

CV. CosmoPostProcess: A simulation calibrated framework for weak lensing selection bias in richness-selected galaxy clusters

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ABSTRACT

We present `CosmoPostProcess`, a simulation-based forward model algorithm calibrated to reproduce optical cluster observables in line with measurements taken with the *Euclid* space telescope. The main deliverable of `CosmoPostProcess` is a correction for stacked surface-density profiles, binned in terms of richness and redshift, that accounts for selection-related systematic effects. These corrections take into account the modification to the stacked weak lensing signal from richness-selected samples of clusters identified in the photometric *Euclid* survey compared with an unbiased reference sample. In this work, we focus on the *Euclid* richness definition currently foreseen for the cosmological analysis, which does not apply a colour selection. *Euclid* also provides an alternative richness estimate based on a red-sequence galaxy selection, which is not considered here. The algorithm processes N-body simulations by painting galaxies with a halo-occupation model and emulating the survey's detection and richness-assignment algorithms. We implemented a novel algorithm to estimate the optical cluster centres from galaxy-projected densities and validated it against the official *Euclid* pipelines. The effect of baryonic physics on the halo density profiles is incorporated through a correction calibrated on hydrodynamical simulations, yielding total-matter profiles consistent with those measured in the reference hydrodynamical simulations. In validation against hydrodynamical simulations, the baryon-corrected excess surface density is in agreement to within 2 % for cluster-centric radii, $r \in [0.1, 5]\,h^{-1}$ Mpc. To assess the impact of the different contributions to the selection bias, we performed dedicated tests that included variations of both cosmological parameters and the parameters of the mass-richness relation. Across all these tests, the selection bias induced by projection alone follows a robust pattern: a large-scale structure that is physically correlated with the main halo and projected along the line of sight enhances the stacked surface density profile near the transition from one-halo to two-halo dominance, with a peak radius of about $1\,h^{-1}$ Mpc and an amplitude of about 20 – 40 %, with a dependence on richness and redshift. This behaviour is mild at low and intermediate redshifts ($z \lesssim 0.7$), where the impact remains at the level of a few per cent, but it becomes increasingly relevant at higher redshift ($z \gtrsim 0.7$), with a more pronounced enhancement of the transition-scale peak. Finally, baryonic modifications remain sub-dominant outside the core, with an impact of about 2 % beyond $r \gtrsim 0.3\,h^{-1}$ Mpc. As an outcome of this analysis, the `CosmoPostProcess` framework delivers radial profile corrections with associated uncertainties, combining the selection bias from projection effects with the impact of baryonic physics and miscentring. These corrections will be a key ingredient to ensure controlled systematics in the *Euclid* DR1 galaxy cluster cosmological analysis.

Key words. Cosmology: large-scale structure of Universe - Cosmology: dark matter - Galaxies: clusters: general - Gravitational lensing: weak - Surveys

1. Introduction

Galaxy clusters represent the most massive gravitationally bound systems formed through hierarchical gravitational collapse. They anchor the nodes of the cosmic web and trace the growth of structure, providing a powerful probe of cosmology (e.g. Borgani & Guzzo 2001; Allen et al. 2011; Kravtsov & Borgani 2012). The comoving number density of clusters as a function of mass and redshift is sensitive to the total matter density parameter, Ω_m, and to σ_8, which sets the normalisation of the linear matter power spectrum at the scale of $8\,h^{-1}$ Mpc (e.g. Sheth et al. 2001; Tinker et al. 2008; Watson et al. 2013; Despali et al. 2016; Euclid Collaboration: Castro et al. 2023). Furthermore, modern surveys demonstrated the capability of clusters to yield competitive constraints on dark energy and structure growth by comparing observed counts with halo mass function predictions (e.g. Mantz et al. 2015; Costanzi et al. 2019; Giocoli et al. 2021; Lesci et al. 2022; Ghirardini et al. 2024; Bocquet et al. 2025).

To exploit this sensitivity, cluster cosmology requires robust selection functions and precise mass calibration (Weinberg et al. 2013; Pratt et al. 2019). Therefore, cluster samples ought to be detected efficiently over wide areas, with well-characterised selection observables (e.g. optical richness, X-ray flux, or a Sunyaev–Zeldovich signal) and an accurately calibrated mapping between these observables and halo mass. Forthcoming wide-field surveys will sharpen these requirements.

Euclid (Laureijs et al. 2011; Euclid Collaboration: Mellier et al. 2025) will survey the extragalactic sky with two complementary instruments: the Visible Imager (VIS), providing high-resolution optical imaging for shape measurements (Cropper et al. 2016; Euclid Collaboration: Cropper et al. 2025), and the Near-Infrared Spectrometer and Photometer (NISP), delivering photometry in the Y_E, J_E, and H_E bands (Euclid Collaboration: Jahnke et al. 2025; Euclid Collaboration: Hormuth et al. 2025). Together, they will cover about $14\,000\,\mathrm{deg}^2$. The wide survey is expected to detect of order $O(10^5)$ galaxy clusters and protoclusters out to $z \gtrsim 2$ (Sartoris et al. 2016). These data will provide large homogeneous cluster samples with high-quality lensing and photometric information. Over such an area, statistical uncertainties become sub-dominant and the constraining power

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is set increasingly by the control of measurement and selection effects, particularly in the cluster mass calibration.

Achieving this level of precision requires an accurate calibration of the relation between a survey mass proxy, such as the optical richness, λ, and the true halo mass. Weak lensing measurements of cluster density profiles are the primary tool because they probe the total matter distribution in this context (e.g. von der Linden et al. 2014; McClintock et al. 2019; Euclid Collaboration: Paltani et al. 2024; Giocoli et al. 2025), without having to rely on assumptions of hydrostatic or dynamical equilibrium, and they are only weakly sensitive to the dynamical state of individual clusters. They can also be obtained homogeneously over wide areas and broad redshift ranges, and combined in stacked analyses to reduce the statistical noise for low-mass or high-redshift systems. Results from the Dark Energy Survey (DES, DES Collaboration: Abbott, T. et al. 2005) illustrate both the power of this approach and its sensitivity to analysis choices.[1] The DES Year 1 analysis of optically selected clusters found a strong tension with *Planck* cosmic microwave background cosmology (Costanzi et al. 2019; Sunayama et al. 2020; Planck Collaboration: Aghanim et al. 2020), later traced to biases in the weak lensing mass calibration induced by selection effects. A forward-modelling treatment of selection, consistently accounting for line-of-sight (LoS) projections, brought DES Year 1 into agreement with *Planck* (Salcedo et al. 2024; Abbott et al. 2025). These results highlight the need to control three main sources of systematic uncertainty in weak lensing cluster mass calibration: LoS projections, miscentring, and baryonic physics.

Projection effects originate from structures located along the same LoS that contaminate the observables of photometrically selected galaxy clusters; this confusion is primarily induced by the uncertainty in photometric redshift estimates. As a result, both richness measurements and weak lensing signals can be biased (Meneghetti et al. 2014; Euclid Collaboration: Giocoli et al. 2024; Euclid Collaboration: Ragagnin et al. 2025). These superposed structures may artificially increase stacked density profiles and enhance the inferred clustering amplitude (Wu et al. 2022; Sunayama 2023; Zhou et al. 2024). The effect is strongly scale dependent, with the largest impact generally found around the cluster boundary (Nde et al. 2026; Lee et al. 2025).

Miscentring constitutes a second major source of uncertainty. In optical catalogues, the cluster centre is typically assigned to the brightest cluster galaxy (BCG) or to the peak of the galaxy density map. However, these proxies can be offset from the assumed halo centre, defined in simulations as the minimum of the gravitational potential or the maximum of the mass density field. Furthermore other observational tracers such as X-ray or Sunyaev–Zeldovich (SZ) peaks can provide a pathway to calibrate this systematic (Saro et al. 2015; Hollowood et al. 2019; Giocoli et al. 2021; Ding et al. 2025). Offsets between assigned and true centres bias the inner lensing profile and, if uncorrected, lead to underestimated halo masses. From observations, miscentring affects 20 % to 40 % of clusters, depending on redshift and selection method. This is commonly accounted by assuming a radial offset distribution often calibrated with ICM data (Johnston et al. 2007; Rozo et al. 2014; Chiu et al. 2022; Sommer et al. 2024).

Finally, baryonic effects from gas cooling, star formation, and active galactic nucleus (AGN) feedback modify halo density profiles and therefore the corresponding lensing signal. Hydrodynamical simulations provide detailed predictions for these effects, but are computationally expensive. As an alternative, semi-analytic baryonification methods (Cui et al. 2014; Schneider & Teyssier 2015; Schneider et al. 2019; Chisari et al. 2019; Castro et al. 2021; Aricò & Angulo 2024; Euclid Collaboration: Castro et al. 2024) allow us to model the redistribution of stars, cold gas, and hot gas, while displacing dark matter particles accordingly, with parameters calibrated on hydrodynamical simulations. This provides a realistic correction to dark matter-only predictions while avoiding the cost of full hydrodynamical runs.

As we demonstrate in this work, these systematics are not independent, as projection, miscentring, and baryonic effects can couple in nontrivial ways, impacting both selection and mass calibration. Therefore, a coherent forward modeling approach is essential to meeting the requirements for the control on systematic errors in *Euclid*.

In this paper, we describe `CosmoPostProcess`, a forward-modelling pipeline developed to generate realistic mock observables for *Euclid* optical cluster analyses. The code takes as input large-volume N-body simulations and produces cluster catalogues with galaxy-based richness and stacked weak lensing signals, designed to mirror as closely as possible the measurement choices adopted within the *Euclid* cluster cosmology framework.

The pipeline paints galaxies onto the simulated matter distribution with calibrated halo-occupation prescriptions and assigns richness with a neural-network emulator that reproduces the behaviour of the official *Euclid* richness algorithm. Weak lensing observables are then constructed from the same realisation, including two key effects that influence both richness and lensing: 1) offsets between the adopted and true centres; and 2) baryonic modifications of the matter profiles. These ingredients are implemented directly at the particle level, using flexible prescriptions adapted to *Euclid*.

With these mocks, we have the ability to quantify the lensing-selection bias by comparing a richness-selected sample with a mass-matched reference sample. Here, we study how the effect varies with scale, redshift, and richness. The same framework also allows us to test the stability of the inferred trends against changes in the forward-model assumptions. The results presented here provide a baseline for the treatment of these systematics in stacked-lensing measurements for *Euclid* Data Release 1 (DR1).

The paper is organised as follows: Sect. 2 summarises the *Euclid* cluster finders and the richness estimator, which define the behaviour we emulate in `CosmoPostProcess`. Section 3 describes the simulations used as input to the mock generation and for calibration. Section 4 details the forward model, including galaxy painting and richness assignment, the computation of projected density and lensing profiles, miscentring, and the implementation of baryonic effects, which are discussed in depth in Sect. 4.5. Next, Sect. 5 presents the selection bias, separating the contributions from projections, miscentring, and baryons, while quantifying their variation with halo-occupation assumptions and cosmology. In Sect. 6, we discuss the results and draw our conclusions.

2. Galaxy clusters detection in *Euclid*

In this section, we introduce the main *Euclid* algorithms used for the optical cluster detection and probabilistic richness assignment. These are the components of the official *Euclid* pipeline that `CosmoPostProcess` emulates to construct realistic mock cluster catalogues (see Sect. 4.3.2). *Euclid* adopts two complementary cluster finders (Euclid Collaboration: Adam et al. 2019;

[1] See also `https://www.darkenergysurvey.org/` for an updated overview of the survey.

Maturi et al. 2019): AMICO is an optimal matched-filter algorithm that produces minimum-variance estimates of cluster amplitude and assigns probabilistic memberships through an iterative cleaning of the filtered maps (Bellagamba et al. 2018); PZWav (Gonzalez 2014; Werner et al. 2023; Doubrawa et al. 2024) is an adaptive tomographic wavelet method that constructs probability-weighted density maps in redshift slices and identifies significant peaks with minimal model assumptions (Euclid Collaboration: Adam et al. 2019). These two cluster-finding algorithms have demonstrated robust performance on *Euclid* data, as shown by Euclid Collaboration: Bhargava et al. (2025), where they successfully recovered hundreds of high-significance systems with richness estimates that trace external mass proxies using the first *Euclid* Quick Data Release (Q1). Besides *Euclid*, AMICO has been validated in wide weak lensing imaging (Maturi et al. 2025; Lesci et al. 2025) and in deep fields such as COSMOS, a two square degree multi-wavelength survey, and COSMOS-Web, a deep JWST NIRCam survey that probes structure at earlier cosmic times (Toni et al. 2024, 2025; Thongkham et al. 2026). As for PZWav, it has been tested in realistic multiband datasets and in J-PAS, a wide photometric survey that observes galaxies through many narrow optical filters to obtain accurate information on photometric redshifts, where it displays a close agreement with AMICO as shown in Doubrawa et al. (2024). Taken together, these results support the adoption of AMICO and PZWav as the baseline cluster finders for *Euclid*. Although AMICO also provides a richness estimate, the richness definition adopted for the *Euclid* cosmological analysis can be obtained with the dedicated RICH-CL algorithm; further details are given in Sect. 4.3.1. Their application to real data demonstrates also well-quantified selection functions, competitive purity and completeness, and consistent agreement with external mass proxies (Euclid Collaboration: Adam et al. 2019; Toni et al. 2024; Euclid Collaboration: Bhargava et al. 2025). We employed runs using both finders on official *Euclid* galaxy mock light cones based on the *Euclid* Flagship2 simulation (Euclid Collaboration: Castander et al. 2025) to keep the emulator tied to pipeline observables and to the operational definitions of the algorithms (see Sects. 3, 4.2, and 4.3.2).

As a case study, in the following we present results based on the PZWav cluster catalogue. However, given the similar performance of the two cluster-finding algorithms, we expect that the main results presented in this paper can be easily extended to AMICO.

3. Simulations

This section describes the numerical simulations used to calibrate and apply CosmoPostProcess. The halo occupation model and the probabilistic richness emulator are calibrated on the forward-modelled *Euclid* mock galaxy catalogue Flagship2 (Sect. 3.2). The baryonic modification of halo density profiles is calibrated using the hydrodynamical Magneticum simulations (Sect. 3.3). The fully calibrated pipeline is then applied to large-volume dark matter simulations from the PICCOLO suite (Sect. 3.1), which provide the underlying matter distribution used to quantify selection biases.

3.1. PICCOLO

The dark matter large-scale structure we applied the forward model to was based on simulations from the PICCOLO suite, originally presented by Euclid Collaboration: Castro et al. (2023). That work introduced a set of 15 ΛCDM cosmologies designed to sample parameter combinations consistent with cluster abundance constraints from the joint DES and South Pole Telescope analysis (Costanzi et al. 2021).

In the present analysis, we extended that original set by including six additional cosmologies and by re-running both the original and the newly added models at higher mass resolution. These additional simulations were produced in view of the future *Euclid* DR1 selection bias analysis, for which a broader sampling of cosmological parameter space will be required. The simulations retain the same box size and numerical framework as in Euclid Collaboration: Castro et al. (2023), while increasing the particle number to improve the modelling of small-scale density profiles and projected observables relevant for cluster lensing.

Each simulation follows the evolution of 4×2560^3 particles in a periodic box of side length $1290\,h^{-1}$ Mpc. For the fiducial cosmology this corresponds to a particle mass of $2.8 \times 10^9\,h^{-1}\,M_\odot$. Initial conditions are generated at $z_{\rm init} = 24$ using third-order Lagrangian perturbation theory with the monofonIC code (Hahn et al. 2020), and the evolution is performed with the TreePM gravity solver implemented in OpenGadget3 (Groth et al. 2023; Damiano et al. 2024; Dolag et al. 2025). For each cosmology, we stored 16 snapshots spanning the redshift range $0 \le z \le 2$.

Among the available cosmologies, we focus here on two representative models, labelled C0 and C1. The reference cosmology C0 is consistent with *Planck* results (Planck Collaboration: Aghanim et al. 2020) and corresponds to the fiducial model of the original PICCOLO suite. It adopts $\Omega_{\rm m} = 0.3158$ and $\sigma_8 = 0.8102$, yielding $S_8 \equiv \sigma_8\,\sqrt{\Omega_{\rm m}/0.3} = 0.831$ (Table 1).

The second cosmology C1 is characterised by a lower matter density, $\Omega_{\rm m} = 0.1986$, and a higher fluctuation amplitude, $\sigma_8 = 0.8590$, corresponding to $S_8 = 0.699$. The contrast between C0 and C1 probes variations both along and across the $\Omega_{\rm m}$–σ_8 degeneracy direction that affects many large-scale structure observables. Differences between these models primarily impact the amplitude of the two-halo contribution in cluster lensing profiles and therefore provide a useful test of the sensitivity of the selection bias signal to cosmological parameters. For the scope of this paper, we restricted the analysis to the fiducial C0 run and to C1, which together serve to illustrate the capability of the pipeline to capture the cosmological dependence of the bias. A comprehensive exploration of the full cosmological grid, motivated by the future DR1 analysis, is deferred to future work, for the currently used sets of cosmological parameters are summarised in Table 1.

Table 1: PICCOLO high-resolution simulations.

Run	$\Omega_{\rm m}$	h	$n_{\rm s}$	σ_8
C0	0.3158	0.6732	0.9661	0.8102
C1	0.1986	0.7267	0.9775	0.8590

Notes. Columns list the present-day matter density, $\Omega_{\rm m}$, reduced Hubble parameter, $h \equiv H_0/(100\,{\rm km\,s^{-1}\,Mpc^{-1}})$, scalar spectral index, $n_{\rm s}$, and linear-theory fluctuation amplitude, σ_8.

3.2. Euclid Flagship2

Flagship2 is the reference mock galaxy catalogue for *Euclid*, built from a $(3600\,h^{-1}\,{\rm Mpc})^3$ dark matter simulation evolved with $16\,000^3$ particles using the PKDGRAV3 code (Potter et al. 2017) and adopting a flat ΛCDM cosmology consistent with

Planck.[2] A light cone covering the full sky out to $z = 3$ is produced on the fly; an octant of approximately $5200\,\mathrm{deg}^2$ is released with galaxies painted using a hybrid halo occupation and abundance-matching model to reproduce the *Euclid* photometric and spectroscopic selection functions (Euclid Collaboration: Castander et al. 2025). In this work, we used the `Flagship2` catalogues retrieved through `CosmoHub` (Carretero et al. 2017; Tallada et al. 2020) and restricted the analysis to an eight-tile $400\,\mathrm{deg}^2$ portion of the light cone, corresponding to the same subset used in Appendix C to calibrate the halo occupation model and to train the neural-network emulator of P_{mem}. This choice ensures that the catalogue used in the main analysis is fully consistent with the reference sample underlying both the mass–richness calibration and the richness-emulation framework. It also reflects a pragmatic compromise: for the specific calibration target considered here, the adopted $400\,\mathrm{deg}^2$ subset already provides sufficient statistics, while retaining a calibration scheme that is computationally light and can be readily rerun and retuned as the modelling choices are updated. Although using the full octant would further reduce the statistical noise, Appendix C shows that the emulator trained on these eight tiles preserves the mass–richness relation with residuals consistent with zero within the uncertainty band, and that the resulting selection bias signal is consistent with that measured directly on `Flagship2`. Optical cluster catalogues are generated using the `AMICO` and `PZWav` detection pipelines, and cluster richness is assigned with `RICH-CL` using the probabilistic scheme described in Sect. 4.3.1. The resulting catalogue provides the reference membership probabilities and mass–richness relation used to calibrate and validate the halo occupation model and the membership emulator implemented in `CosmoPostProcess`. For this purpose, detected clusters are matched to `Flagship2` halos using a galaxy-membership-based scheme, associating each cluster with the halo contributing the largest fraction of its observed richness. We refer to quantities obtained with this procedure as membership-matched. This matching strategy is intended to anchor the calibration to the per-galaxy membership information extracted from `Flagship2`; namely, the relation between the membership probability and the observables entering the richness assignment. In this sense, the `Flagship2`-based calibration is not meant to define the final DR1 galaxy population model, but rather to demonstrate that `CosmoPostProcess` can reproduce the relevant cluster observables in a controlled way for a given HOD prescription. The residual sensitivity to the assumed galaxy population is expected to enter primarily through the HOD modelling, and will be assessed through the suite of HOD variations adopted in the production of the final DR1 mocks.

3.3. `Magneticum`

`Magneticum` is a suite of cosmological hydrodynamical simulations run with the `P-Gadget3` code, an evolution of the public `P-Gadget2` code (Springel 2005). It includes sub-resolution models for radiative cooling, star formation, chemical enrichment, supernova-driven winds, black-hole growth, and AGN feedback (Hirschmann et al. 2014). The background cosmology corresponds to WMAP7 (Komatsu et al. 2011).

We used two configurations: the high-resolution Box 3 run and a multi-cosmology variant spanning 15 cosmologies obtained by varying Ω_{m}, Ω_{b}, σ_8, and h around the WMAP7 fiducial values. Subgrid physics is held fixed across all runs (Ragagnin et al. 2023; Singh et al. 2020). These simulations were used to calibrate the baryonic modification of halo density profiles implemented in the forward model. In Table 2, we summarise the relevant configurations.

4. Methodology

Here, we describe the methodology used to self-consistently simulate projected density profiles (Sect. 4.1) and richness values, which are two key observables of the *Euclid* cluster catalogue. The assignment of richness proceeds by first painting galaxies on the simulations (Sect. 4.2) and then assigning the membership to each cluster (Sect. 4.3.2). Finally, we assess the effect of miscentring on the observables (Sect. 4.4).

4.1. Density profiles

We computed 3D and 2D density profiles selecting and projecting dark matter particles by applying a tree-search based on the `KDTree` implementation in `SciPy` (Virtanen et al. 2020). This choice provides good scaling with the number N of particles, $O(N \log N)$, which is convenient given the large number of particles per box (see Sect. 3).

For the 3D profiles, we searched for dark matter particles inside spheres centred on `SubFind` halos (Springel et al. 2001; Dolag et al. 2009). We used thirty log-equispaced radial bins from $0.01\,h^{-1}\,\mathrm{Mpc}$ to $5\,h^{-1}\,\mathrm{Mpc}$. By summing all particles enclosed in a sphere, we can compute the cumulative mass as

$$M(<r_{\mathrm{i}}) = N_i^{\mathrm{part}} m_{\mathrm{part}}\,, \tag{1}$$

where m_{part} is the mass of a DM particle in our simulations, and N_i^{part} is the number of particles in the i-th sphere around the target halo centre. We apply this to all halos with $M_{\mathrm{vir}} \geq 10^{13}\,h^{-1}M_{\odot}$. These profiles serve as input for the baryonic correction model (see Sect. 4.5).

To compute surface density profiles, we counted particles inside a cylinder (see e.g. Wu et al. 2022; Salcedo et al. 2024). The cylinder has a depth of $\pm 50\,h^{-1}\,\mathrm{Mpc}$ around the target cluster. The default value of the base radius is $R_{\max} = 10\,h^{-1}\,\mathrm{Mpc}$ and can be customised. The cylinder is then split using 20 log-spaced annuli from $R_{\min} = 0.01\,h^{-1}\mathrm{Mpc}$ to $R_{\max}$. We projected particles along the chosen axis and computed

$$\Sigma(r_i^{\mathrm{cen}}) = \frac{\left(N_i^{\mathrm{part}} - N_{i-1}^{\mathrm{part}}\right) m_{\mathrm{part}}}{\pi\,(R_i^2 - R_{i-1}^2)}\,, \tag{2}$$

where R_{i-1} and R_i are the bin edges and $r_i^{\mathrm{cen}} = \sqrt{R_i\,R_{i-1}}$.

We then applied baryonic corrections (see Sect. 4.5) inside a spherical region covering $5\,h^{-1}\,\mathrm{Mpc}$ in radius. This region encompasses the full one-halo term and extends slightly beyond the transition to the two-halo regime, allowing us to trace the full baryonic density profile measured in `Magneticum` (see Appendix B). An actual split in one-halo and two-halo component of the profiles is applied afterward, during cylinder projection, to disentangle the effects of correlated structures from that of the main halo. More specifically, this decomposition is defined geometrically in configuration space rather than through a separate halo-model fit. During the projection step, particles inside the cylinder are assigned to the one-halo component if their three-dimensional (3D) distance from the halo centre satisfies $R^2 + z^2 \leq r_{1\mathrm{h}}^2$, where R is the projected separation, z is the

[2] For reference, `Flagship2` assumes $\Omega_{\mathrm{m}} = 0.319$, $\Omega_{\mathrm{b}} = 0.049$, $h = 0.67$, $n_{\mathrm{s}} = 0.96$, and $\sigma_8 = 0.83$ (Euclid Collaboration: Castander et al. 2025).

Table 2: `Magneticum` configurations used in this work.

Run	$L_{\rm box}$ $[h^{-1}\,{\rm Mpc}]$	Number of particles	$m_{\rm part}$ $[h^{-1}\,M_\odot]$
Box 3/uhr	128	2×1536^3	3.6×10^7
Multi-cosmology (M1–M15)	896	2×1536^3	1.3×10^{10}

Notes. Cosmologies M1–M15 vary $\Omega_{\rm m}$, $\Omega_{\rm b}$, σ_8, and h around the WMAP7 fiducial cosmology (Ragagnin et al. 2023; Singh et al. 2020).

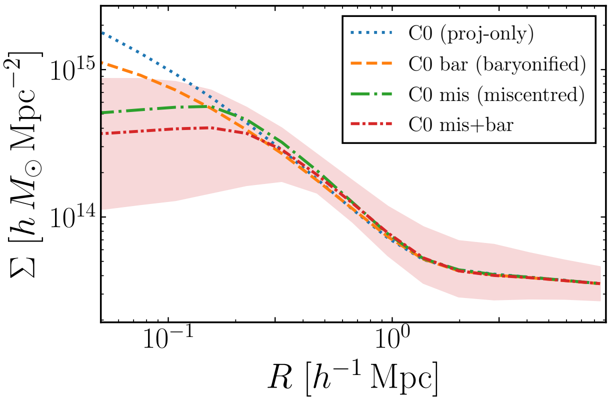

Fig. 1: Median surface density profiles, Σ, and relative scatter for about 3000 objects within a narrow mass bin centred on $M = 10^{14}\,h^{-1}\,M_\odot$. The blue dotted curve is for the original average profile, while dashed orange, dot-long dashed green, and dot-short dashed red curves refer to the effect of including baryonic correction, miscentring, and both effects together, respectively. For the latter, the shaded area indicates the $1\,\sigma$ statistical uncertainty.

LoS separation, and $r_{\rm 1h} = r_{\rm vir}$ is taken as the reference halo radius used in the simulation catalogue, optionally rescaled by a user-provided factor. Particles outside this radius, but still within the projection cylinder, contribute to the two-halo component. The two terms are then accumulated separately in the same projected radial bins. For illustrative purposes, in Fig. 1 we show the median projected profile (blue dashed line) measured in the `PICCOLO` C0 simulation, measured in a narrow mass bin (using the virial mass definition) around $M = 10^{14}\,h^{-1}\,M_\odot$ together with the modifications introduced by the various systematics (see Sects. 4.4 and 4.5). To increase the statistics of our sample, we extract three profile catalogues by projecting along the three Cartesian axes for each box analysed. As an additional feature for studying orientation biases, we also measured the triaxial shape parameters and the orientation angle between the main axis of the inertia momentum tensor of the halo and the LoS (see Appendix A for further details).

4.2. Galaxy painting

Selection bias arises from the correlation, at fixed halo mass, between the up-scatter of the observed richness used for sample selection and the up-scatter of the weak lensing signal. To model this correlation consistently, halos must be populated with galaxies using the same dark matter structures employed to measure the projected density profiles. We therefore implemented a halo occupation distribution (HOD) model calibrated on the `Flagship2` mock galaxy catalogue and subsequently applied to the `PICCOLO` simulations.

The HOD parameters were determined by fitting the occupation statistics to the `Flagship2` cluster catalogue, ensuring that the simulated galaxy population reproduced the reference mass–richness relation described in Appendix C. Once calibrated, the same parametrisation was used to populate halos in `PICCOLO`, thereby isolating selection and projection effects from changes in the underlying galaxy model.

The model assigns central galaxies at the `SubFind` centre of each halo by drawing from a Bernoulli distribution with a probability of

$$P_{\rm cen}(M_{\rm vir}) = \frac{1}{2}\left\{1+{\rm erf}\left[\frac{1}{\sigma_{\log M,{\rm cen}}}\log_{10}\left(\frac{M_{\rm vir}}{M_{\rm min,cen}}\right)\right]\right\}\,. \tag{3}$$

The parameter $M_{\rm min,cen}$ sets the characteristic minimum halo mass required to host a central galaxy, while $\sigma_{\log M,{\rm cen}}$ controls the width of the transition between halos with negligible central probability and those almost certainly hosting one. The function increases smoothly with halo mass, capturing the intrinsic scatter in the central–halo mass relation.

For halos hosting a central galaxy, the number, $N_{\rm sat}$, of satellite galaxies is drawn from a Poisson distribution with an expectation value of

$$N_{\rm sat}(M_{\rm vir}, z_{\rm cl}) = N_{\rm cen}(M_{\rm vir})\left(\frac{M_{\rm vir}-M_{\rm min,cen}}{M_{\rm 1,sat}-M_{\rm min,cen}}\right)^{\alpha}\left(\frac{1+z_{\rm cl}}{1+z_0}\right)^{\epsilon}\,. \tag{4}$$

Here, $M_{\rm 1,sat}$ defines the mass scale at which halos host on average one satellite galaxy, and α governs the growth of satellite occupation with halo mass. The parameter ϵ introduces a redshift dependence to account for evolutionary effects. Variations in $M_{\rm 1,sat}$ and α modify the satellite abundance at fixed $M_{\rm vir}$, thereby affecting both the richness distribution and its correlation with projected mass density.

The HOD described by Eqs. (3) and (4) was calibrated using the mass–richness relation measured from a set of optical cluster detections produced with `PZWav` on the `Flagship2` light cone. Detected clusters were associated to `Flagship2` halos through a membership-based matching scheme, whereby each cluster was matched to the halo contributing the largest fraction of its assigned richness. This choice suppresses, by construction, the contribution of chance projections to the calibration sample, yielding a reference relation that traces the intrinsic halo occupation at fixed mass.

The HOD parameters can then be obtained by fitting this matched mass–richness relation. Calibrating on a projection-minimised reference ensures that when `CosmoPostProcess` is applied to `PICCOLO`, the resulting richness boosts and scatter are driven by the `PICCOLO` density field. In this way, the projection-induced corrections inferred from `PICCOLO` quantify the selection effects native to those simulations, rather than residual contamination inherited from the calibration step. To apply the painting in the `PICCOLO` simulation, we passed the virial mass derived with `SubFind` and the redshift for all halos with

$M_{\rm vir} \geq 10^{13}\,h^{-1}\,M_\odot$ to the model. The redshift of the target halos are derived from the simulation box redshift $z_{\rm box}$ and the LoS comoving coordinate, $x_{\rm proj}$ in h^{-1} Mpc, via

$$z_{\rm cl} = z_{\rm box} + D_{\rm c}^{-1}(x_{\rm proj})\,, \tag{5}$$

where $D_{\rm c}^{-1}$ denotes the inverse comoving distance–redshift relation.

Satellite galaxies were placed by uniformly selecting $N_{\rm sat}$ dark matter particles from the host halo. For the painted galaxy catalogue to be consistent with the richness definition adopted later in the pipeline, this selection was restricted to the projected aperture entering the probabilistic RICH-CL estimator, namely, $R_{\rm pmem}$. Since the PICCOLO halo catalogues provide virial radii $R_{\rm vir}$ rather than $R_{\rm pmem}$, we constructed a proxy mapping between the two scales. This mapping can be obtained by interpolating the empirical relation between $R_{\rm vir}$ and $R_{\rm pmem}$ measured in the membership-matched Flagship2 cluster catalogue, including its intrinsic scatter. The resulting correspondence is shown in Fig. 2.

This choice ensures that the spatial extent of the painted satellite population is tied to the same effective aperture used to estimate richness in the probabilistic framework described in Sect. 4.3.1. The uniform selection of dark matter particles preserves the triaxial structure of halos and maintains a direct connection between galaxy positions and the underlying mass distribution. The resulting galaxy catalogue provides the input for the membership assignment described in the following section.

4.3. Membership assignation

Optical richness is an observational mass proxy designed to trace the underlying cluster galaxy population. In the *Euclid* cluster cosmology pipeline, cluster masses are inferred from stacked tangential shear profiles using the COMB-CL pipeline, which constructs individual and stacked lensing profiles under homogeneous source selection and lensing modelling. This approach has been validated on Stage-III data and simulations (Euclid Collaboration: Sereno et al. 2024; Euclid Collaboration: Ingoglia et al. 2025). The optical richness entering the mass–richness calibration is estimated by the RICH-CL algorithm.

The RICH-CL code includes two alternative richness estimators: a red-sequence-based implementation and a probabilistic membership implementation. In this work, we adopted the latter as reference, since it is the estimator currently foreseen for the official *Euclid* DR1 cosmological analysis. In contrast to the red-sequence-based definition, the probabilistic estimator does not apply a colour selection and is therefore expected to be more affected by projection effects from structures aligned along the LoS. Throughout this section, the RICH-CL richness refers to this probabilistic definition. Within CosmoPostProcess, we reproduce it through a neural-network emulator trained to match the RICH-CL output on Flagship2. We first summarise the theoretical framework underlying this estimator and then describe its implementation in the forward model.

4.3.1. Theoretical framework

In optical cluster cosmology, richness estimators do not correspond to a direct count of member galaxies; rather, they are defined as weighted sums over galaxies in the cluster region, where the weights encode the probability of membership and account for projection effects and background contamination (e.g. Rozo et al. 2009; Rykoff et al. 2014; Rozo et al. 2015; Andreon 2015). Our description follows the methodology presented by Castignani & Benoist (2016, hereafter CB16), which provides a procedure closely related to that implemented in RICH-CL and serves as a practical reference for defining the inputs and validation steps of a *Euclid*-like richness estimator. We focus, in particular, on the no-threshold variant of the probabilistic richness introduced by CB16. This discussion is intended as a guide to the structure of a probabilistic membership estimator and to the meaning of the relevant ingredients. It should not be interpreted as a one-to-one description of the implementation adopted in CosmoPostProcess, for which the operative photometric-redshift treatment is described below in Sect. 4.3.2.

The richness λ is computed as a weighted sum over galaxies within a projected aperture, $R_{\rm pmem}$,

$$\lambda = \sum_{R\leq R_{\rm pmem}} \frac{P_{\rm mem}}{(1-f_{\rm bkg})\,C(m,z)}\,, \tag{6}$$

where the sum runs over all galaxies whose projected separation from the cluster centre satisfies $R \leq R_{\rm pmem}$. In practice, $R_{\rm pmem}$ defines the characteristic aperture within which the probabilistic membership assignment, and hence the richness estimate, is evaluated. This is the same aperture scale that we use in the galaxy-painting step to set the projected region from which satellite tracers are drawn. The factor $1-f_{\rm bkg}$ accounts for the correction for survey geometry and masking, where $f_{\rm bkg}$ is the fraction of the aperture lost to survey masking. The completeness correction $C(m,z)$ accounts for the fraction of galaxies missing from the catalogue due to magnitude limits. In CB16, richness is evaluated within a cluster-size aperture, taken as $r_{\rm vir}$ in their tests.[3]

The membership probability, $P_{\rm mem}$, combines redshift information from both the galaxy and the cluster,

$$P_{\rm mem} = \frac{(1-\beta)\sum_i P_{\rm gal}(z_i)P_{\rm cl}(z_i)}{\mathcal{N}(z_{\rm cl},\sigma_0,\delta z)}\,, \tag{7}$$

where $P_{\rm gal}(z)$ and $P_{\rm cl}(z)$ are the redshift probability distributions of the galaxy and the cluster. The sum runs over a redshift grid of spacing δz, and $\mathcal{N}$ ensures proper normalisation of the discrete redshift-overlap term. The galaxy redshift probability distribution is modelled as

$$P_{\rm gal}(z) = \frac{1}{\sqrt{2\pi}\,\sigma_z}\exp\left[-\frac{(z-z_{\rm gal}^{\rm photo})^2}{2\,\sigma_z^2}\right], \quad \sigma_z = \sigma_0(1+z)\,, \tag{8}$$

where σ_0 sets the width of the illustrative Gaussian photometric-redshift kernel in this reference formulation. In the mock implementation described below, the effective photometric-redshift scatter is instead modelled with Eq. (11).

The factor β quantifies contamination from background galaxies and is defined as

$$\beta = \frac{\langle N_{\rm bkg}(m_{\rm gal},z_{\rm cl})\rangle}{\langle N_{\rm tot}(m_{\rm gal},z_{\rm cl},R)\rangle}\,, \tag{9}$$

where $N_{\rm bkg}$ and $N_{\rm tot}$ denote the background and total galaxy counts at magnitude $m_{\rm gal}$ and redshift $z_{\rm cl}$. In the CB16 framework, $N_{\rm tot}$ is evaluated locally at the projected cluster-centric

[3] The virial radius $r_{\rm vir}$ is defined following the spherical-overdensity criterion adopted in the SubFind group catalogues, that is as the radius enclosing a mean interior density $\bar{\rho}(<r_{\rm vir}) = \Delta_{\rm vir}(z)\,\rho_{\rm c}(z)$, where $\rho_{\rm c}(z) = 3H^2(z)/(8\pi G)$ is the critical density of the Universe at redshift z, and $\Delta_{\rm vir}(z)$ is the redshift-dependent overdensity predicted by the spherical-collapse model (Bryan & Norman 1998). The corresponding virial mass is $M_{\rm vir} \equiv M(<r_{\rm vir})$.

separation $R \equiv R_{\rm c,g}$, while $N_{\rm bkg}$ is estimated from an outer annulus around the cluster. Consequently, $1-\beta$ carries an explicit radial dependence: near the cluster centre, $N_{\rm tot} \gg N_{\rm bkg}$ and $\beta \ll 1$, while at large projected radii $\beta \to 1$. As a simple approximation,

$$1-\beta(R) \approx \frac{\Sigma_{\rm mod}(R) - \Sigma_{\rm bkg}}{\Sigma_{\rm mod}(R)}\,, \tag{10}$$

where $\Sigma_{\rm mod}(R)$ is the projected surface density predicted by a reference model profile (Navarro et al. 1997), and $\Sigma_{\rm bkg}$ is the projected background surface density estimated at large radii. In CB16, no specific reference surface density profile is assumed, whereas more recent versions of `RICH-CL` constrain $R_{\rm pmem}$ iteratively using a power-law profile. This probabilistic formulation defines richness as a sum over membership probabilities and provides the observable that we reproduce within the forward model. Further details on the `RICH-CL` probabilistic membership scheme in its definitive configuration for the Euclid DR1 cosmological analysis will be presented in Euclid Collaboration: Benoist et al. (in prep.).

4.3.2. Membership emulator

The probabilistic richness defined in Eq. (6) depends on the membership probability, $P_{\rm mem}$, given by Eqs. (7) and (9). In `CosmoPostProcess`, we do not emulate the richness directly as a black-box mass–richness relation. Instead, for each galaxy-cluster pair entering the richness aperture, we emulate the corresponding membership probability and denote it by $P_{\rm mem}^{\rm emu}$, see Appendix C for extra details. This quantity is intended as a fast surrogate for the probabilistic `RICH-CL` membership weight, so that the observed richness can still be constructed as a sum over per-galaxy membership probabilities, consistently with Eq. (6). This preserves the structure and functional dependencies of the `RICH-CL` estimator while allowing an efficient evaluation for large simulated catalogues.

We emulate $P_{\rm mem}^{\rm emu}$ with a fully connected neural network comprising three hidden layers with Leaky ReLU activations and a sigmoid output, ensuring $0 \leq P_{\rm mem}^{\rm emu} \leq 1$. The implementation uses `PyTorch` (Paszke et al. 2019). Training and validation are performed on the `Flagship2` simulation, where the reference `RICH-CL` membership probabilities are available. The trained network is subsequently applied to the painted `PICCOLO` catalogues. Besides reproducing the operational `RICH-CL` probabilistic estimator, this emulation provides a computationally efficient evaluation of $P_{\rm mem}^{\rm emu}$ for large simulated catalogues. This is particularly important for `CosmoPostProcess`, whose main goal is to trace how cosmology affects the amount of projection-induced contamination entering the richness selection, and therefore the resulting weak lensing selection bias, by processing many independent realisations and multiple cosmological parameter choices. In this framework, the $P_{\rm mem}$ emulator is not intended to encode a direct dependence on cosmological parameters. Rather, the same calibrated membership prescription is applied to cosmology-dependent galaxy and halo populations, so that the cosmology dependence of the bias arises from the projected structure entering the richness estimate. A direct evaluation of the full membership model for all simulated catalogues would be prohibitively expensive. The physical information is nevertheless preserved, since the input variables reflect directly the quantities entering the probabilistic formulation: cluster redshift, projected galaxy-cluster separation normalised by $R_{\rm pmem}$, background-area fraction, $f_{\rm bkg}$, and galaxy-cluster redshift separation, Δz.

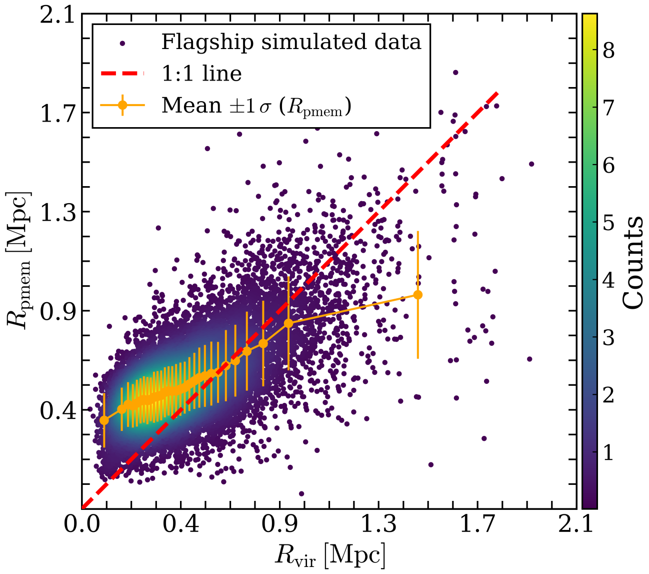

Fig. 2: Relation between $R_{\rm vir}$ and $R_{\rm pmem}$ in proper Mpc units from `Flagship2`. Orange points show the binned mean and the standard deviation as a function of $R_{\rm vir}$.

To compute Δz, we simulate photometric redshift estimates for both galaxies and clusters. Starting from the true galaxy redshift, we added a Gaussian scatter with standard deviation,

$$\sigma_{\text{photo-}z}(z) = z_0 + b\,z + a\,z^2\,, \tag{11}$$

with default values $a = 0.041$, $b = -0.046$, and $z_0 = 0.034$, derived from a `Phosphoros` DR1-like run on the same `Flagship2` sample. An analogous scatter is applied to the cluster redshift, reduced by a factor $1/\sqrt{N_{\rm mem}}$, consistently with the `RICH-CL` probabilistic definition. This choice should be regarded as an educated baseline configuration adopted to demonstrate the capabilities of the mock-generation framework, rather than as a final description of the DR1 photometric-redshift uncertainties. As the understanding of the DR1 data improves, both the galaxy and cluster photometric-redshift uncertainties will likely require a broader effective description than the one assumed here. The observed richness is then computed by summing the emulated per-galaxy membership probabilities within $R_{\rm pmem}$,

$$\lambda_{\rm obs} = \sum_{R \leq R_{\rm pmem}} P_{\rm mem}^{\rm emu}\,. \tag{12}$$

As previously described for $R_{\rm pmem}$, we implemented a prescription to model an $f_{\rm bkg}$ proxy in `CosmoPostProcess`. For each cluster included in the richness catalogue, by default all galaxy-painted-halos with $M_{\rm vir} > 10^{13}\,h^{-1}\,M_\odot$, we consider an annulus centred on the projected cluster position spanning $R_{\rm min}^{\rm area} = 3\,h^{-1}$ Mpc to $R_{\rm max}^{\rm area} = 5\,h^{-1}$ Mpc, and compute the fraction of this annular area obscured by neighbouring projected clusters. In practice, we evaluate the area covered by circles of radius $R_{\rm pmem}$ centred on the projected positions of clusters overlapping the annulus; the remaining unobscured fraction defines our estimate of $f_{\rm bkg}$. Figure 3 compares the overall distribution of $f_{\rm bkg}$ for objects above $10^{13}\,h^{-1}\,M_\odot$ measured in `Flagship2` and in `CosmoPostProcess` applied to `PICCOLO`. The agreement is only qualitative, but this is sufficient for the present purpose. In particular, the main role of the $f_{\rm bkg}$ model is to capture the contribution from neighbouring foreground clusters, while we do not attempt

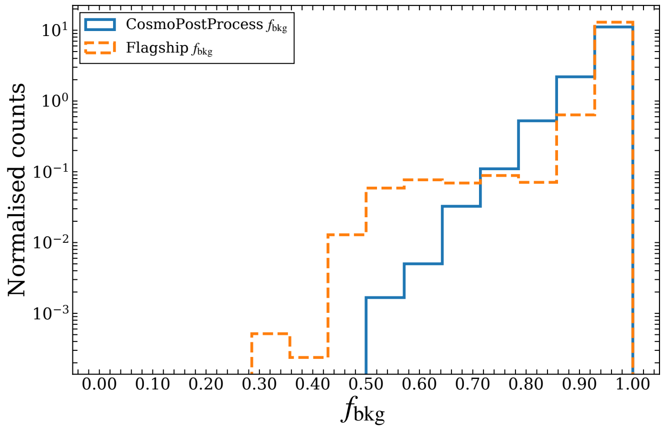

Fig. 3: Normalised counts as a function of $f_{\rm bkg}$, as predicted by the `CosmoPostProcess` model (solid blue line) and by the `Flagship2` model (orange dashed line).

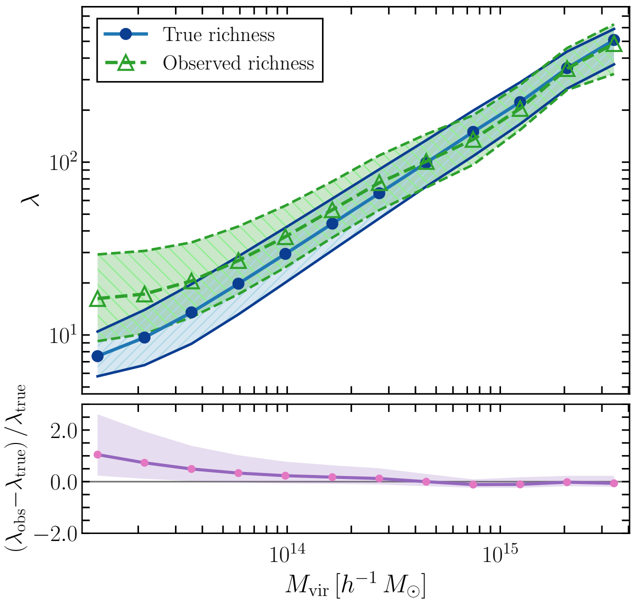

Fig. 4: *Upper panel:* Median mass–richness relation and 1σ scatter envelope. The blue curve shows the relation for the true richness calibrated on `Flagship2`, while the green curve shows the observed richness obtained by recomputing the richness with the `CosmoPostProcess` membership-probability emulator. Here, observed richness denotes the final richness returned by the mock pipeline after the emulation and post-processing steps, in contrast to the true richness defined at the HOD level. *Lower panel:* Relative variation of the observed richness with respect to the true richness, $(\lambda_{\rm obs} - \lambda_{\rm true})/\lambda_{\rm true}$. The offset is not introduced through a separate correction term, but arises because projected galaxies and galaxies overlapping in photometric redshift can receive non-zero emulated membership probabilities, $P_{\rm mem}^{\rm emu}$, and therefore contribute to $\lambda_{\rm obs}$. This naturally produces an upward shift and an increased scatter in the reconstructed richness, especially at low halo masses, with projection effects acting on top of the usual scatter induced by Eddington bias.

here to reproduce the full survey masking pattern, which is more complex and not yet in a form that can be injected straightforwardly into the simulations for DR1. As shown in Appendix C, $f_{\rm bkg}$ is the least informative input for $P_{\rm mem}$, and tests in which it is omitted altogether produce no appreciable change in the calibration. For this reason, we regard the simplified treatment adopted here as sufficient for the current implementation.

In Fig. 4, we show the true and observed mass–richness relations measured in `PICCOLO` via `CosmoPostProcess` using the emulator calibrated as described in Appendix C, where the observed richness is constructed by summing the emulated per-galaxy membership probabilities $P_{\rm mem}^{\rm emu}$. Projection effects are already visible at this stage, since they boost the observed richness and increase the scatter at fixed mass compared to the intrinsic relation. This effect has been empirically confirmed with SZ (Grandis et al. 2025) and spectroscopic (Myles et al. 2021, 2025) follow-up observation of optically selected clusters. Furthermore, to illustrate the emulator behaviour in a representative case, we show the predicted membership probabilities $P_{\rm mem}^{\rm emu}$ for a cluster of mass close to $3\times10^{14}\,h^{-1}\,M_\odot$ in Fig. 5. The top panel displays the transverse distribution of galaxies within $R_{\rm pmem}$, colour-coded by $P_{\rm mem}^{\rm emu}$, while the bottom panel shows the same system along the LoS within a redshift slice of width $\sigma_{\text{photo-}z}(z_{\rm cl})$, as defined in Eq. (11). This visualisation highlights how the emulator combines projected separation and redshift proximity to down-weight background structures and to assign higher membership probabilities to galaxies that are both spatially concentrated and compatible in photometric-redshift space.

4.4. Miscentring

`CosmoPostProcess` injects miscentring by shifting the projected friends-of-friends (FoF) centre, identified via the halo-finding algorithm `SubFind` (Dolag et al. 2009), in polar coordinates within the plane perpendicular to the LoS. Miscentring is a known systematic effect, affecting both cluster lensing and richness estimates.

We implemented a substructure-based method to inject miscentring on top of the FoF centres of our halos. Around each target halo, we selected a substructures inside a cylinder centred on the FoF position with an aperture, $n\,R_{\rm pmem}$. The factor $n = 1.5$ is tuned to match a chosen radial miscentring reference, here the one measured from the `Flagship2` cluster catalogue. More specifically, the reference distribution is the radial offset between the centre of the membership-matched halo, taken as the true centre, and the centre returned by the cluster finder in the `Flagship2` catalogue. In the present case study, the calibration is performed against the `PZWav` centres, consistently with the calibration catalogue used in this work. The cylinder depth follows the photometric redshift dispersion in Eq. (11), so we included structures satisfying $z_{\rm cl} - \sigma_z \le z \le z_{\rm cl} + \sigma_z$. We convolved the projected substructure positions with a Gaussian kernel with standard deviation of $\sigma_{\rm mc} = 4$ px. We discretised the map on a grid with 100 by 100 pixels. The new centre is the weighted mean of the pixels with counts above the median.

We validated this substructure method on the `Flagship2` galaxy catalogue adopting as true centre of the detection the centre of the matched halo. We used galaxies with z values within σ_z of the cluster redshift (Fig. 6). We also applied a cut on the membership probability of $P_{\rm mem} > 0.5$ that empirically reproduces the coarser spatial distribution of substructures used in the `PICCOLO` miscentring calculation. We selected the 1000 most massive clusters and then restrict the validation sample to systems with $z_{\rm cl} < 0.45$, yielding 358 objects. This cut is adopted conservatively to validate the miscentring prescription on the same calibration sample used in the rest of the analysis while restricting the comparison to a statistically robust `Flagship2` regime, where the detection behaviour is more stable. It also matches the depth of the first `PICCOLO` C0 snapshot used in the calibration. With this choice, we avoid over-interpreting the higher-redshift `Flagship2` behaviour in the val-

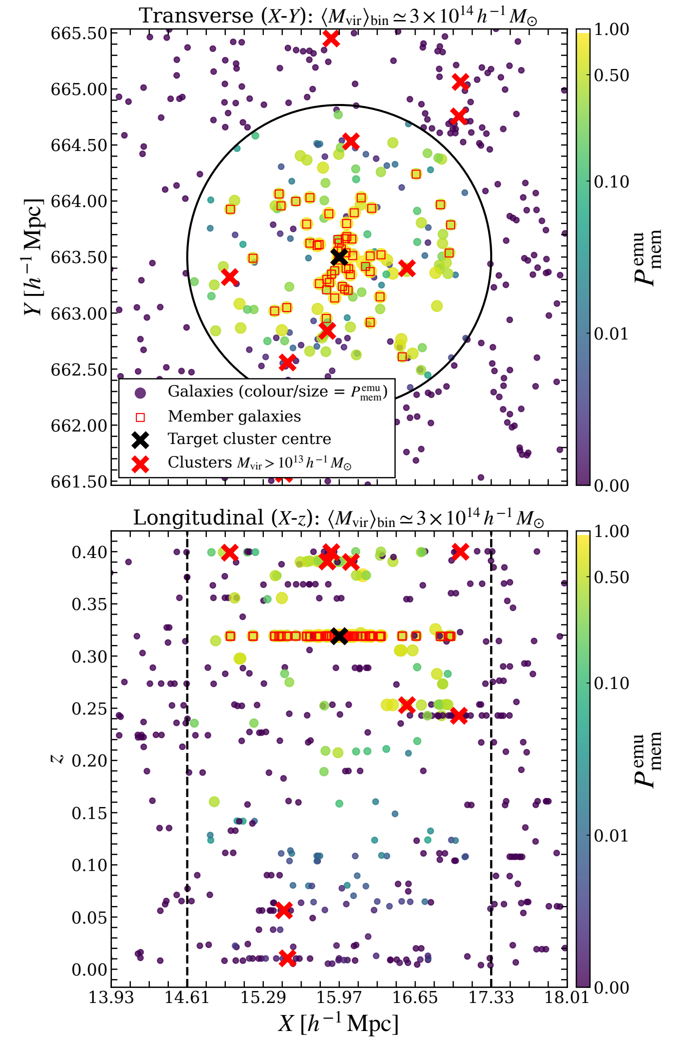

Fig. 5: *Top panel:* Transverse section of a cluster with $M_{\rm vir} \simeq 3 \times 10^{14}\,h^{-1}\,M_\odot$ from the PICCOLO C0 simulation processed with CosmoPostProcess. The black circle marks the $R_{\rm pmem}$ projected radial cut, with galaxies colour-coded according to their $P^{\rm emu}_{\rm mem}$ values. True member galaxies are highlighted by red boxes, while red crosses indicate all objects above $M_{\rm vir} = 10^{14}\,h^{-1}\,M_\odot$. *Bottom panel:* LoS view of the same object restricted to the redshift interval covered by the first PICCOLO snapshot.

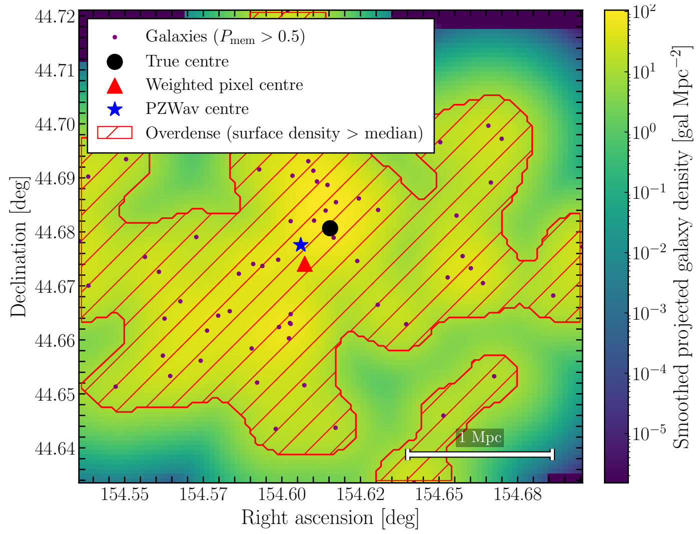

Fig. 6: Illustration of our miscentring scheme, applied to a $M_{\rm vir} = 1.62 \times 10^{14}\,h^{-1}\,M_\odot$ cluster in the Flagship2 simulation. Purple points show galaxies with $P_{\rm mem} > 0.5$ and $z_{\rm cl} - \sigma_z \leq z_{\rm gal} \leq z_{\rm cl} + \sigma_z$. The colour map shows the Gaussian-smoothed projected galaxy number density of the same galaxies, in units of galaxies per Mpc^{-2}. The red hatched region marks pixels with surface density above the median value of the map. The red triangle, blue star, and black dot mark the weighted pixel centre, the PZWav centre, and the true centre, respectively. The scale bar shown in the panel corresponds to 1 Mpc at the cluster redshift. For a full validation of the miscentring model, including the agreement at the level of the miscentring distribution, we refer the reader to Fig. 7.

idation step, while allowing the redshift dependence in the mock prescription to be driven by the adopted photometric-redshift uncertainties. Figure 7 shows the recovered radial miscentring distribution (left panel) and the angular offset distribution between our pixel weighted centre and the PZWav centre (right panel). Both distributions are displayed as function of the physical separation of the projected centres, $R_{\rm mis}$. These results demonstrate that our miscentring scheme successfully reproduces both the magnitude and the direction of the centre displacement. In particular, a Kolmogorov–Smirnov (KS) test comparing our radial separation distribution to that measured relative to the PZWav centres shows excellent agreement with KS statistic $D = 0.046$ and p-value $= 0.95$. By contrast, a KS test of the measured angular separations against a uniform distribution strongly rejects uniformity producing $D = 0.73$ which translates in p-value $\simeq 0$.

4.5. Baryonification

Our aim is to map the dark matter-only (DMO) density profiles measured from the PICCOLO suite into a baryonified dark matter profile that reproduces hydrodynamical simulation results, while retaining speed and flexibility. While we based our baryonification on the model originally proposed by Schneider et al. (2019), we introduced two modifications to their method to trace the effect of baryons out to the cluster edge and to capture its redshift dependence. First, the hot gas was described by a unified core-tail model whose parameters are calibrated snapshot by snapshot, with the introduction of a tail component to trace the profile up to $5\,h^{-1}$ Mpc. Second, we applied a single, redshift-dependent adiabatic contraction (AC) mapping directly on the DMO profiles, following Schneider et al. (2025). The parameter values used in this work are calibrated against the Magneticum Box 3 data (see Sect. 3.3). Here, we provide only a summary of the model, while the full specification of the profiles and mappings, including all relevant equations, is collected in Appendix B.

4.5.1. DMO profiles and baryonified reconstruction

We started from the DMO profiles measured in Sect. 4.1 for the C0 and C1 PICCOLO runs. The code reads cumulative DMO shell masses built on the same radial edges $\{r_i\}$ used in the 3D density profiles, converts them to shell densities $\rho_{\rm DMO}(r)$, and applies a Savitzky–Golay smoothing in $\log_{10}\rho_{\rm DMO}$ to suppress high-frequency particle noise while keeping the large-scale slope intact. Onto these smoothed DMO profiles we add the baryonic response associated with AC, namely the increase of the inner dark matter density induced by the slow condensation of baryons, which deepens the potential and approximately preserves orbital adiabatic invariants. In the standard spherical/circular-orbit pic-

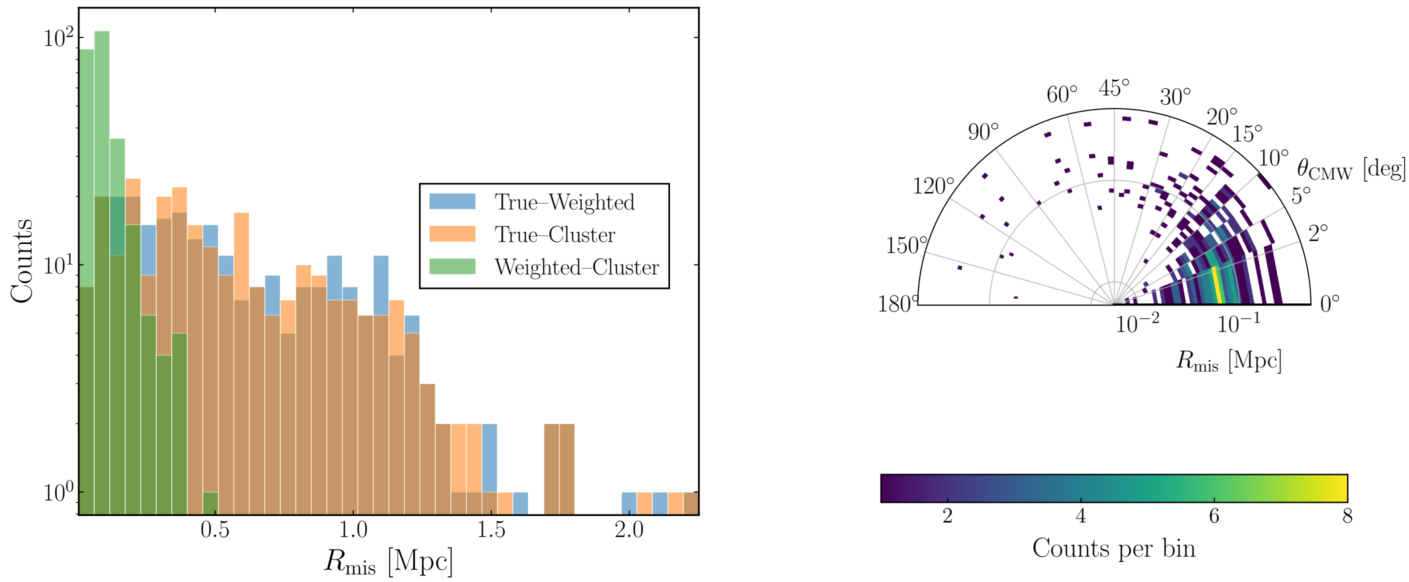

Fig. 7: *Left panel:* Radial miscentring distributions. Blue compares the membership centre, which is the truth, with the pixel weighted centre. Orange compares the membership centre with the cluster finder centre. Green compares the cluster finder centre with the pixel weighted centre. The overlap of the blue and orange histograms and the peak at low $R_{\rm mis}$ (in proper Mpc) in the green histogram indicate that the weighted pixel centre tracks the cluster finder centre closely. *Right panel:* 2D histogram of the projected separation, $R_{\rm mis}$, and angular aperture with respect to the true centre, $\theta_{\widehat{\rm CMW}}$, between the pixel-weighted and cluster-finder centres. The pixel colour encodes the counts in log-scale. The distribution concentrates near zero degrees at small $R_{\rm mis}$ demonstrating the ability of our miscentring scheme to reproduce the actual direction of the centre displacement.

ture this is often expressed as $r\,M(<r) \approx$ const for mass shells, with improved prescriptions accounting for eccentric orbits via orbit-averaged radii (Blumenthal et al. 1984; Gnedin et al. 2004). We model AC in two steps: (i) we predict the hot-gas and central-stellar components as a function of mass and redshift; (ii) we displace the dark matter particle position with a single global mapping $\xi_{\rm ac}(r\,|\,z)$ evaluated at the density level. This procedure yielded a contracted dark matter density profile, $\rho^{\rm ac}_{\rm dm}(r)$, which was then converted, under the assumption of mass conservation, into the baryonified density profile $\rho_{\rm dmb}(r)$ used in our analysis.

The validation was performed by first establishing a one-to-one (biunivocal) halo correspondence between the hydrodynamical and DMO simulations. For each matched halo pair, profiles are computed and subsequently stacked within mass bins. With this approach, the model reproduces the stacked $\Delta\Sigma$ profiles of the hydrodynamical simulation to within 1–2 % over the range 0.1–5 h^{-1}Mpc across four mass bins (see Fig. 8).

4.5.2. Hot-gas model and redshift evolution

To capture the redshift evolution of the hot component, we calibrated the parameters of our model at four simulation snapshots, $z_{\rm snap} \in \{0.00, 0.25, 0.47, 1.17\}$, and describe its evolution by interpolating the best-fit parameters between these epochs. To apply the baryonic correction, we selected the snapshot with redshift nearest to that of the target halo. The parameter vector governs the mass dependence of the hot-gas model through: a core pivot parameter and mixing index controlling the inner-slope steepening, an ejection scale, a tail-onset location, and a broken-slope description for the outer-tail decay. Parameter calibrated values are given in Table B.1.

The hot gas was modelled as the sum of a pressure-supported, normalised one-halo core and a diffuse tail that transitions smoothly to the cosmic mean density, thereby avoiding artificial truncations. The resulting model combines a tunable β-model (Cavaliere & Fusco-Femiano 1976) for the core region and a power-law tail, with a sigmoid convolution to ensure a smooth transition between the two (see Eqs. B.7, B.9, and B.10). The core region is tuned via a characteristic core size and the slope of the β-model and must obey a normalisation bound to enforce consistency with the gas fraction measured in `Magneticum`. The tail is controlled by an onset radius and an outer slope that vary with halo mass and redshift. Numerically, the core normalisation uses cached integrals to improve computational speed, while the tail employs a smooth logistic-to-power-law transition; stabilisation by parameter floors ensures robust behaviour for all halos.

4.5.3. Stellar and gas mass components

Stellar mass was split into a compact central component and a diffuse satellite term. The central galaxy (CGA) was modelled with a fixed scale radius $r_{\rm h} = 0.075\,r_{\rm vir}$. The CGA contributes in the AC mapping, where the ratio between the cumulative mass of the central galaxy and the dark matter modulates the feedback of the latter. It is then used to compute the final total density in the validation tests. The satellite galaxy (SGA) term follows the local dark matter slope and contributes only implicitly to the quasi-AC, while hot gas and CGA components are regulated by a feedback strength parameter, which determines the modification they induce on the DMO profile.

Stellar and gas mass fractions were modelled as smooth functions of halo mass, with parameters that evolve with redshift and depend on the cosmic baryon fraction $f_{\rm b}$. These fractions were used to normalise the corresponding stellar and gas density profiles that enter the baryonification scheme. At each redshift, we enforce global baryon conservation by requiring $f_\star + f_{\rm gas} = f_{\rm b}$, while the stellar mass associated with the central

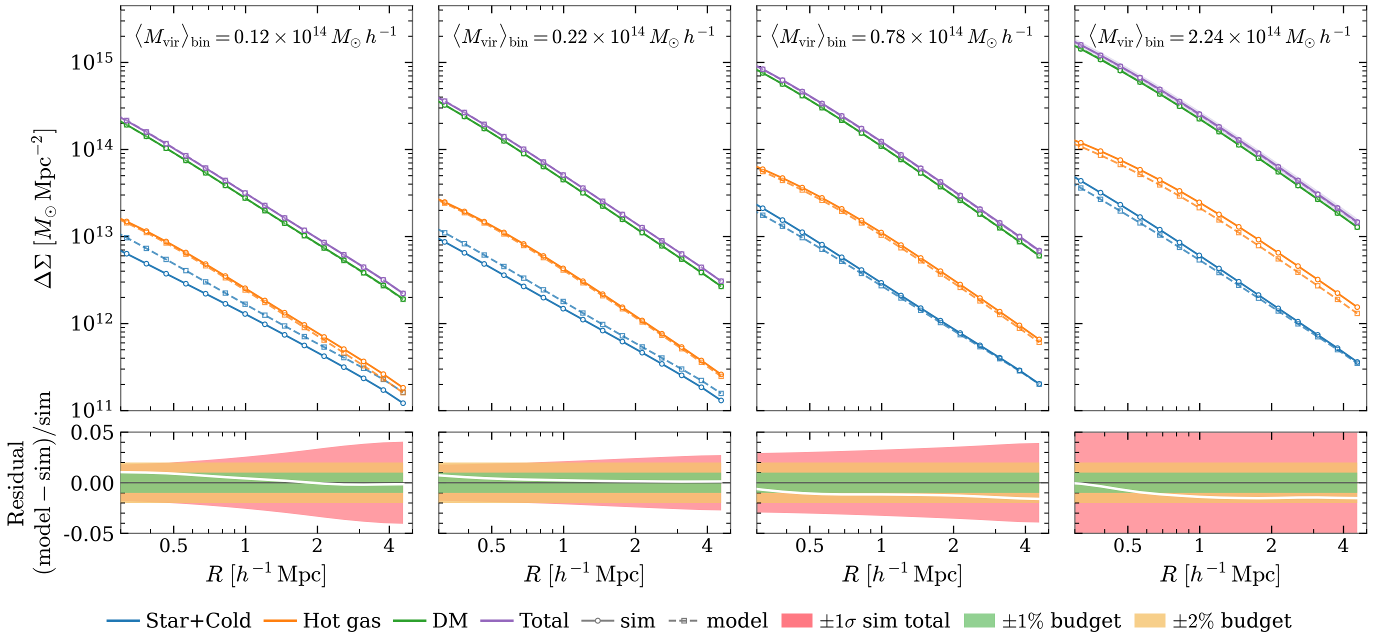

Fig. 8: Excess surface density $\Delta\Sigma$ in four mass bins for the halos identified in `Magneticum` Box 3 at high resolution. The colour code indicates the species of the components with stars and cold gas in light blue, hot gas in orange, and the dark matter density after the quasi-AC in green. Solid lines denote the simulation data used for calibration, while the dashed lines are used for the model. In the *bottom panels*, the coloured bands in the residuals panel indicate: the jackknife $1\,\sigma$ uncertainty on the total density profile (red), the 1 % relative error on the total (green), and the 2 % relative error (orange).

galaxy is constrained to satisfy, $f_{\rm cga} \leq f_\star$. Unless otherwise specified, the baryon fraction is fixed to the cosmological value of the simulation, $f_{\rm b} = \Omega_{\rm b}/\Omega_{\rm m}$. The functional forms of the stellar and gas components, together with the normalisation prescriptions based on $f_\star$, $f_{\rm cga}$, and $f_{\rm gas}$, are defined in Appendix B (see Eqs. B.2, B.3, B.7, and B.10).

4.5.4. Quasi-AC mapping and particle remapping

The impact of baryons on the dark matter distribution is incorporated through a single, global quasi-AC mapping applied at the level of the density profiles. Starting from the DMO profile $\rho_{\rm DMO}(r)$, we computed the enclosed mass profiles of the baryonic components; namely, hot gas, CGA, and SGA, using the density models introduced in Appendix B. These enclosed masses enter the definition of a redshift-dependent contraction response function $\xi_{\rm ac}(r)$, whose amplitude depends on: (i) a set of normalisation and slope parameters that encode its redshift evolution; (ii) the enclosed mass fractions of central stars and hot gas; and (iii) the halo peak height $\nu(M,z)$. The contraction mapping $\xi_{\rm ac}(r)$ is applied to the cumulative dark matter mass profile, which is subsequently differentiated with respect to radius to obtain the contracted dark matter density profile $\rho^{\rm ac}_{\rm dm}(r)$ – see Appendix B and Sect. 4.

To implement the contracted dark matter distribution consistently at the particle level, we apply a quantile-based remapping of particle radii. In practice, we construct the cumulative mass profile corresponding to $\rho^{\rm ac}_{\rm dm}(r)$ and define a monotonic mapping that associates each quantile of the original particle-radius distribution to the corresponding quantile of the contracted model. Particle positions are then rescaled radially with respect to the halo centre, while preserving their angular coordinates, and wrapped back into the simulation box when necessary. Since selection effects and projected density profiles are computed using particle masses, we also rescale the mass of each dark matter particle by a factor $f_{\rm CDM} = 1 - f_{\rm b}$ (where CDM stands for cold dark matter). This ensures that the total projected mass associated with the contracted particle distribution is consistent with the baryonified dark matter profile and that correlations between richness and lensing are preserved throughout the forward model.

4.5.5. Validation in `Magneticum`

The baryonification scheme was validated by applying the quasi-AC to halos in the DMO counterpart of the `Magneticum` Box 3 simulation. For each halo, the contracted dark matter profile is combined with the associated baryonic components, and the corresponding 2D surface density profile is computed via an Abel transform. Finally, the surface density, $\Sigma(R)$, is converted into excess surface density, $\Delta\Sigma(R)$, according to

$$\Delta\Sigma(R) = \bar{\Sigma}(<R) - \Sigma(R)\,, \quad \bar{\Sigma}(<R) = \frac{2}{R^2}\int_0^R \Sigma(R')\,R'\,{\rm d}R'\,, \quad (13)$$

which is the conversion we adopted throughout this work to convert from surface to excess surface density. It is worth noticing that in Sect. 5, we present the results of the selection bias in terms of $\Sigma(R)$, since in this case the features in the bias radial profile are easier to visualise.

We show in Fig. 8 a comparison between the $\Delta\Sigma(R)$ profiles predicted by our calibrated model of baryonification and those actually measured from the `Magneticum` simulations. Curves of different colours refer to the different components that make the total density profiles. With our baryonic correction model, we reproduced the excess surface density profiles from `Magneticum` with an accuracy of 1–2 %. We point out that, to correctly propagate baryonic effects through our pipeline, the baryonic correction was applied to the particles before painting the galaxies. The characteristic suppression effects of baryonic correction (orange line) and miscentring (green line) can be seen in Fig. 1, together with their combined effect (red line).

5. Results

Having established the structure and validation of the pipeline, we now quantify the systematics affecting the weak lensing signal of the optical cluster catalogue. Throughout this section we characterise these effects through the selection bias estimator,

$$b(R\,|\,\lambda) \;\equiv\; \frac{\Sigma_\lambda^{\rm sel}(R)}{\Sigma_\lambda^{\rm ref}(R)}\,, \tag{14}$$

which measures the scale-dependent modification of the projected density profile at fixed observed richness λ. The numerator $\Sigma_\lambda^{\rm sel}(R)$ denotes the stacked surface-density profile of richness-selected clusters, while the denominator $\Sigma_\lambda^{\rm ref}(R)$ is the corresponding mass-selected reference constructed as described in Sect. 4. Here, the individual object profiles that enter the stacking are computed using Eq. (2) in both the numerator and the denominator. The ratio therefore isolates the impact of richness selection on the inferred lensing signal.

Unless stated otherwise, we adopt the C0 cosmology and the HOD calibrated on the `Flagship2` cluster catalogue as our fiducial configuration. Uncertainties correspond to 68 per cent bootstrap intervals obtained by resampling within the $(\ln M, z)$ cells used to build the reference stacks.

5.1. One-halo and two-halo contributions

We first examined a representative bin, $60 < \lambda \leq 90$, $0.70 < z \leq 1.00$, shown in Fig. 9. In the fiducial configuration including miscentring and baryonification, the bias exhibits a clear scale dependence, with a pronounced enhancement near the transition between the one-halo and two-halo regimes at $R \simeq 1\,h^{-1}$Mpc. The peak reaches $\sim$ 20–40 per cent relative to the mass-selected reference.

The decomposition demonstrates that the two-halo contribution dominates the bias beyond $R \simeq 0.5\,h^{-1}$Mpc, indicating that the transition-scale enhancement is primarily sourced by correlated large-scale structure projected along the LoS. This behaviour is consistent with previous studies of projection-induced biases in optically selected cluster samples and forward-modelling analyses of richness–lensing coupling (e.g. Sunayama et al. 2020; Wu et al. 2022; Zhou et al. 2024; Sunayama 2023; Euclid Collaboration: Giocoli et al. 2024; Euclid Collaboration: Ragagnin et al. 2025).

5.2. Redshift and richness dependence

The full redshift and richness dependence of the bias in the C0 cosmology is shown in Fig. 10. In the projection-only configuration, the bias is positive and peaks near the one-halo to two-halo transition. The amplitude increases with redshift, reaching its largest values in the $0.70 < z \leq 1.50$ bins.

The bias amplitude exhibits a non-monotonic dependence on observed richness. At intermediate richness, $60 < \lambda \leq 150$, the transition-scale enhancement is most pronounced. At the highest richness, $\lambda > 150$, the peak amplitude decreases and the curves become flatter.

The reduction of projection contamination at high $\lambda_{\rm obs}$ is physically expected. At large observed richness, the selected sample is increasingly dominated by intrinsically massive halos whose internal density profile overwhelms typical LoS fluctuations, reducing the fractional contribution of projected structures. In our framework this behaviour is further modulated by halo geometry. Orientation and triaxiality can enhance projected densities and weak lensing signals at fixed mass, an effect that we discuss in Appendix A and that has been investigated in the context of halo structure studies (e.g. Giocoli et al. 2012). For the specific trend with observed richness, Giocoli et al. (2025) and Euclid Collaboration: Giocoli et al. (2024) find in hydrodynamical simulations that projection-induced richness contamination decreases toward high $\lambda_{\rm obs}$ as intrinsically massive systems dominate the selected sample, qualitatively corroborating the behaviour seen in Fig. 10.

5.3. Miscentring and baryonic effects

Miscentring produces the expected suppression of the inner profile and enhancement around the typical offset scale. The suppression is strongest at small radii and becomes more pronounced at low richness. Because the displaced centre is often shifted toward locally overdense regions, richness can increase while the central surface density is suppressed. This redshift dependence arises naturally in our implementation because the miscentring prescription is coupled to the richness reconstruction through the substructure distribution used to define the displaced centre. In turn, the substructures entering this procedure are selected within a redshift slice whose width is set by the adopted photometric-redshift uncertainty. As this uncertainty broadens with redshift, the set of structures contributing to the effective centre assignment changes, which induces a corresponding redshift dependence in the impact of miscentring on the weak lensing bias.

Baryonic modifications alone produce comparatively modest changes in $b(R\,|\,\lambda)$. Although the 3D density profile can be altered at the $\sim$ 20 per cent level in the inner regions, projection dilutes these modifications and confines their effect to small radii (e.g. Schneider et al. 2019; Euclid Collaboration: Ragagnin et al. 2025). When miscentring and baryonic effects are combined, their impact is amplified at high redshift, where baryon-induced contraction steepens the inner density profile and deepens the central deficit relative to the reference stack.

5.4. Cosmological dependence

To assess the robustness of the inferred selection bias to cosmological assumptions, we compared the fiducial simulation (C0) with the alternative cosmological configuration C1. The two runs share a similar fluctuation amplitude but differ primarily in the matter density parameter, with C1 characterised by a lower value of $\Omega_{\rm m}$. This change modifies both the abundance of dark matter halos and their large-scale clustering, which in turn affects the amount of correlated structure projected along the LoS.

Figure 11 shows the resulting bias profile for clusters in the richness interval $60 < \lambda \leq 90$. The overall radial structure of the bias remains qualitatively unchanged between the two cosmologies, confirming that the characteristic peak at the transition between the one-halo and two-halo regimes is a robust feature of richness-selected samples. However, the amplitude of the effect decreases in the lower-density cosmology. In particular, the peak of the selection bias is reduced by up to $\sim$ 9 per cent relative to the fiducial configuration.

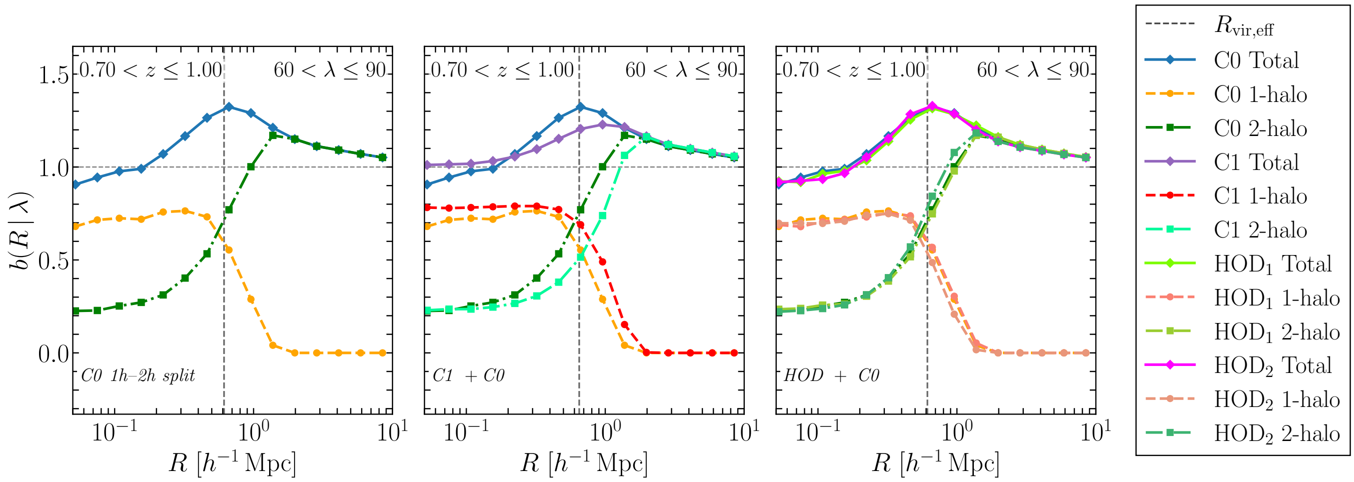

Fig. 9: Contributions of the one-halo and two-halo terms to the selection bias of the projected density profile in the $60 < \lambda \leq 90$, $0.70 < z \leq 1.00$ bin. The bias is defined as $b(R\,|\,\lambda) = \Sigma^{\rm sel}_{\lambda}(R)/\Sigma^{\rm ref}_{\lambda}(R)$. In each panel, solid lines show the total bias, dashed lines the one-halo contribution, and dash-dotted lines the two-halo contribution; the vertical dashed line marks the median effective virial radius $R_{\rm vir,eff}$. *Left:* C0 with miscentring and baryonification (total in blue; one-halo in orange; two-halo in green). *Middle:* C1 (total in purple; one-halo in red; two-halo in turquoise), with the C0 curves overplotted for reference. *Right:* two HOD variants overlaid on C0 mis+bar, with $\mathrm{HOD_1}$ (total in lawngreen) defined by $(\alpha, \log_{10}[M_{1,\rm sat}\,h/M_\odot]) = (0.839,\ 12.205)$ and $\mathrm{HOD_2}$ (total in fuchsia) by $(\alpha, \log_{10}[M_{1,\rm sat}\,h/M_\odot]) = (0.918,\ 12.176)$; for each HOD, the corresponding one-halo and two-halo terms are shown with the same line styles and lighter companion colours. The dashed vertical line marks the median $R_{\rm vir}$ of the mass-selected sample for the C0 fiducial case.

The location of the peak also shifts slightly towards larger radii, by approximately 0.1–0.8 h^{-1} Mpc. This behaviour reflects the change in the halo population associated with a fixed richness selection. In a cosmology with lower $\Omega_{\rm m}$ the abundance of massive halos is reduced, so clusters populating a given richness bin correspond on average to slightly lower masses and therefore somewhat larger relative virial radii when expressed in physical units. The dashed–dotted line in Fig. 11 marks the median virial radius in the C1 configuration, illustrating the small displacement between the peak of the bias profile and the typical halo scale.

These results indicate that the detailed amplitude of the selection bias depends on the cosmological model through its impact on halo abundance and clustering. Nevertheless, the qualitative behaviour and radial structure of the effect remain stable across the explored cosmologies. At the same time, the non-negligible variation in the bias amplitude indicates that this ingredient should ultimately be treated self-consistently in the DR1 cosmological analysis. In practice, although the mock-generation framework is sufficiently modular to be retuned efficiently, it is not designed to be recomputed on the fly within the likelihood analysis. The current strategy is therefore to use the set of `PICCOLO` cosmologies to build an emulator that interpolates the weak lensing bias across cosmological parameter space. This would allow the correction to be matched to the cosmological region explored by the inference, while retaining the flexibility of the present framework and avoiding the need for a full rerun of the mocks at each likelihood step.

5.5. HOD variations

We also investigated the sensitivity of the selection bias to the galaxy population model used to populate dark matter halos. This can be achieved by varying the parameters of the halo occupation distribution (HOD), which determines the abundance of satellite galaxies at fixed halo mass. In particular, we vary the satellite normalisation scale $\log_{10}(M_{1,\rm sat}\,h/M_\odot)$ and the slope α of the satellite occupation relation. The explored variations follow the direction orthogonal to the main $\log_{10}(M_{1,\rm sat}\,h/M_\odot)$–$\alpha$ degeneracy and correspond approximately to a 2σ excursion around the fiducial HOD calibration.

Figure 12 shows the resulting bias profiles for clusters in the richness range $60 < \lambda \leq 90$. Increasing the satellite abundance, obtained by lowering $\log_{10}(M_{1,\rm sat}\,h/M_\odot)$ and steepening α, enhances the probability that galaxies associated with neighbouring halos are counted as members of the target cluster. This increases the level of projection contamination and produces a modest amplification of the bias profile, at the level of a few per cent relative to the fiducial case. Conversely, decreasing the satellite fraction reduces the probability of such projections and slightly suppresses the bias amplitude.

Therefore, the main effect of these HOD variations is a change in the overall amplitude of the bias, while the radial structure of the profile remains largely unchanged. The redshift dependence of this response is generally weak, which is why only two representative redshift bins are shown in Fig. 12. An exception occurs near the peak scale at the highest redshift, where the trend partially reverses. In this regime the larger intrinsic scatter in the mass–richness relation modifies how clusters scatter across richness thresholds, producing a mild inversion of the HOD dependence near the transition between the one-halo and two-halo regimes. Overall, the sensitivity of the selection bias to realistic HOD variations remains modest compared to the dominant contribution from projection effects.

5.6. Mass calibration implications

The scale-dependent distortions of the stacked density profiles discussed above translate directly into biases in weak lensing mass calibration. To quantify the net impact on inferred cluster masses, we fit the stacked excess surface density profiles with a standard Navarro–Frenk–White (NFW) model (Navarro et al. 1997) and compare the recovered masses to the true halo masses in the simulations. This allows us to separate the effects of pro-

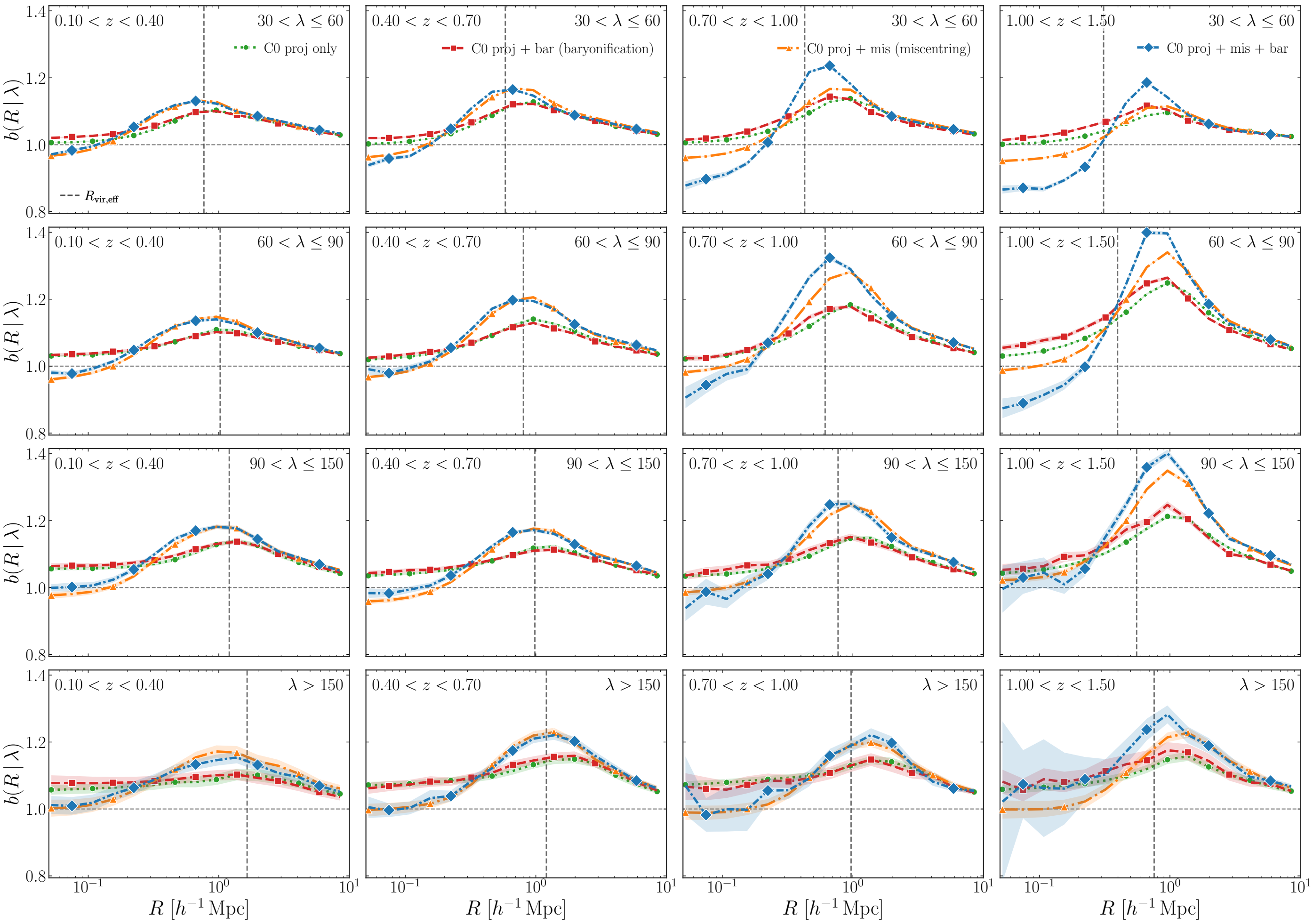

Fig. 10: Bias, estimated through Eq. (14), in stacked richness bins for cosmology C0. Curves show the full case (blue), projections with miscentring (orange), projections with baryons (red), and projection-only (green). Shaded regions indicate 68 per cent bootstrap intervals; vertical dashed lines show an indicative separation scale for the one-halo and two-halo terms, at the median virial radius $R_{\rm vir,eff}$ in the bin. Baryons produce only a few-percent change in the inner profile at $z > 0.7$, while miscentring dominates the bias by suppressing the central signal and enhancing the transition scale. When combined, baryons further amplify this inner suppression. The vertical dashed line display the median $R_{\rm vir}$ in the richness-selected bin.

jection, miscentring, and baryonic physics on the mass calibration, as well as to isolate the contribution arising purely from the richness-based selection,

$$b_{\rm mass} \equiv \left\langle \frac{M_{\rm fit}}{M_{\rm true}} \right\rangle . \tag{15}$$

Projection-only stacks yield $b_{\rm mass} = 1.22$. Including miscentring and baryons shifts masses to $b_{\rm mass} = 0.86$. To isolate the bias induced purely by the richness selection, we compare the masses obtained from the richness-selected stacks to those derived from the mass-selected reference sample,

$$b_{\rm sel} \equiv \frac{M_{\rm fit}^{\rm (rich)}}{M_{\rm fit}^{\rm (mass\text{-}sel)}} . \tag{16}$$

We find $\langle b_{\rm sel} \rangle = 1.17$ across the 16 (z, λ) bins. This amplitude is consistent and the corresponding $\sim$ 5 per cent mitigation is consistent with recent forward-modelling and observational analyses of optical cluster samples, which assumed miscentred NFW profile (Grandis et al. 2021, 2024). In simulations, Giocoli et al. (2025) and Euclid Collaboration: Ragagnin et al. (2025) report richness-dependent lensing mass biases at the $\sim$ 10–20 per cent level once projection and centring systematics are included. Although the precise value depends on modelling choices, our inferred $\langle b_{\rm sel} \rangle = 1.17$ lies well within the range reported in these studies, supporting the interpretation that correlated large-scale structure is the dominant driver of positive selection bias in richness-selected samples.

5.7. Richness-density coupling

Figure 13 shows a clear positive correlation between richness and projected density fluctuations, with Pearson $r = 0.40$ and Spearman $\rho = 0.34$. Upward fluctuations in $\lambda_{\rm obs}$ are systematically associated with enhanced projected densities, confirming that the transition-scale peak in $b(R \,|\, \lambda)$ originates from correlated projection effects.

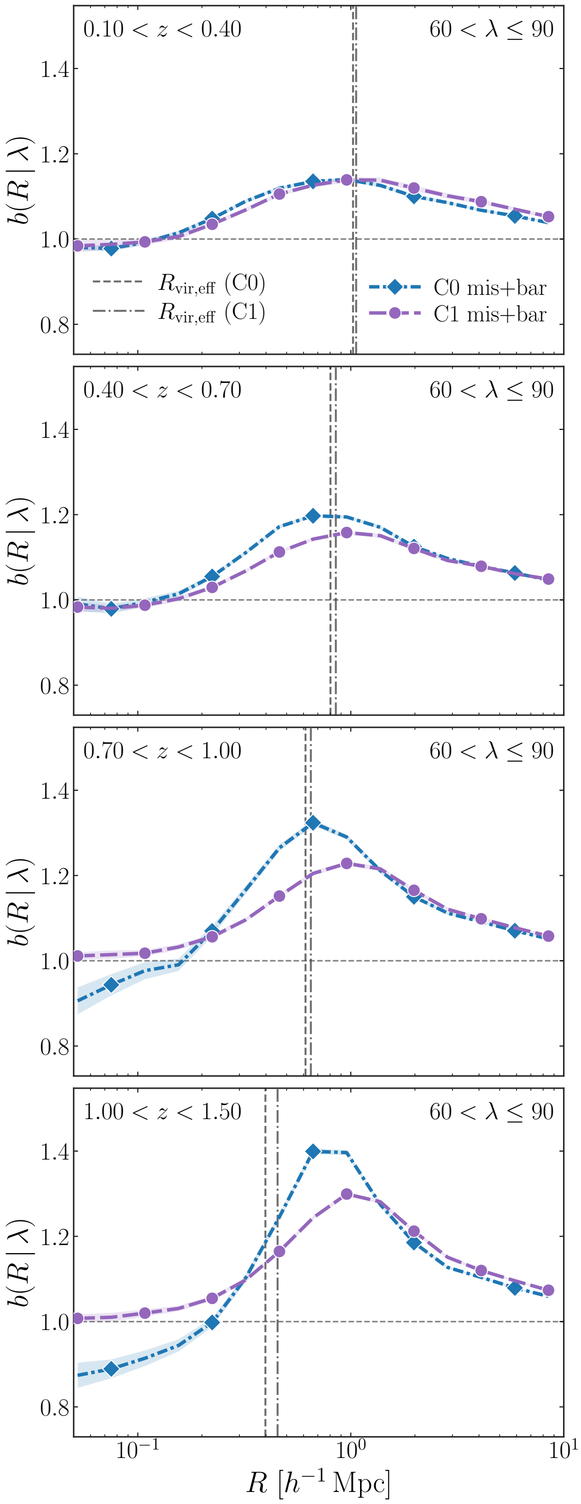

Fig. 11: Cosmological dependence of the selection bias, shown for the C1 cosmological parameter set including all configurations (baryonification and miscentring). The comparison is performed at fixed richness ($60 < \lambda \leq 90$). Differences with respect to the fiducial C0 case reflect the reduced matter density in C1, which leads to a lower bias amplitude and a modest outward shift of the transition between the one-halo and two-halo regimes. The dashed–dotted line indicates the median virial radius $R_{\rm vir}$ for the C1 configuration, highlighting the displacement of the peak relative to the characteristic halo scale at which the transition occurs.

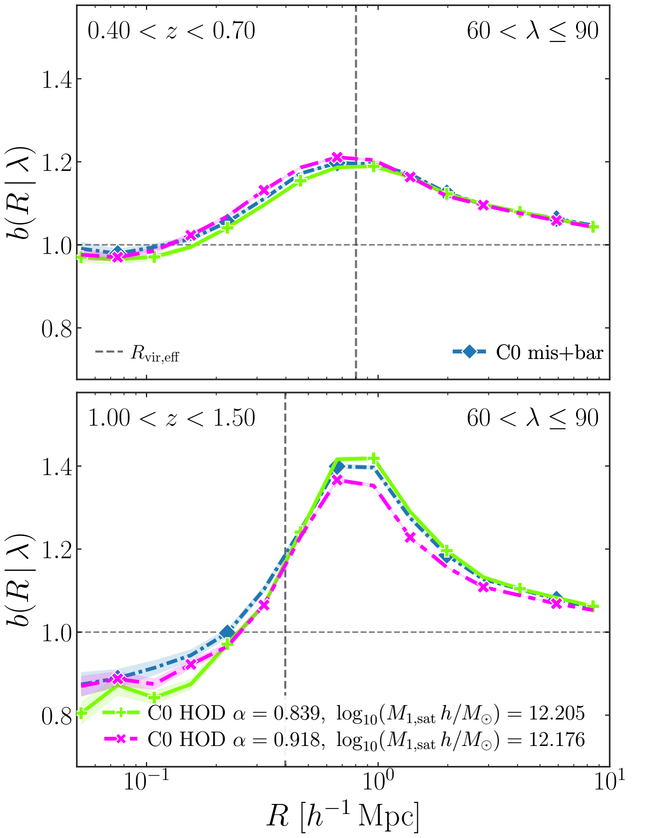

Fig. 12: Response of the selection bias to variations in the halo occupation distribution (HOD) parameters for the $60 < \lambda \leq 90$ richness bin. We vary the satellite pivot mass $\log_{10}(M_{1,\rm sat}\, h/M_\odot)$ and the slope α of the satellite occupation relation. Lower $\log_{10}(M_{1,\rm sat}\, h/M_\odot)$ and higher α increase the satellite contribution and the associated projection contamination, producing a bias that is a few per cent larger (magenta curve) than the fiducial case (blue curve). Conversely, higher $\log_{10}(M_{1,\rm sat}\, h/M_\odot)$ and shallower slopes result in a slightly reduced bias (green curve). Since the redshift dependence of the HOD-induced variations is generally mild, we show two representative redshift bins. At the highest redshift the trend partially reverses near the transition scale and at larger radii, while the inner region remains suppressed relative to the fiducial configuration for both HOD variants.

6. Conclusions and outlook

We introduce `CosmoPostProcess`, a fast and modular forward model that ingests large-volume N-body simulations, populates halos with an HOD calibrated on the `Flagship2` *Euclid* simulation, emulates the `RICH-CL` procedure developed for *Euclid* to obtain a survey-like richness, applies a substructure-informed miscentring prescription validated against membership-based centres, and incorporates baryonic physics through a mass- and redshift-dependent correction calibrated on hydrodynamical simulations. In this work, the `RICH-CL` richness refers to the probabilistic membership definition, which is currently foreseen for the official *Euclid* DR1 cosmological analysis. *Euclid* also provides an alternative red-sequence-based richness definition, which is not considered here and is expected to be less affected by projection effects owing to its colour selection. The miscentring prescription adopted here improves upon the isotropic models commonly employed in the literature by capturing the richness-lensing correlation induced by miscentring in a physically motivated manner. Together, these components provide

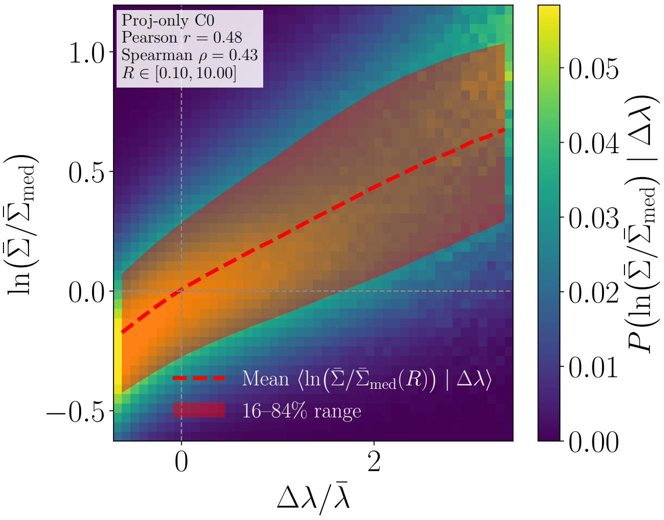

Fig. 13: Correlation between fluctuations in projected galaxy density and richness for the projection-only C0 sample. The colour map shows the conditional probability density $P\left[\ln(\bar{\Sigma}/\bar{\Sigma}_{\rm med}) \,\middle|\, \Delta\lambda\right]$, where $\Delta\lambda = \lambda - \lambda_{\rm med}$ is the deviation from the median richness, and $\ln(\bar{\Sigma}/\bar{\Sigma}_{\rm med})$ is the logarithmic deviation of the projected surface-density profile from its median. The overbar denotes an average over radii, $R \in [0.1,\ 10]\,h^{-1}$Mpc. The red dashed line indicates the mean relation $\left\langle \ln(\bar{\Sigma}/\bar{\Sigma}_{\rm med}) \mid \Delta\lambda \right\rangle$, while the shaded band encloses the central 16–84 percentile range. A clear positive correlation is observed, with Pearson $r = 0.40$ and Spearman $\rho = 0.34$, indicating that clusters with upward richness fluctuations systematically exhibit enhanced projected density profiles.

per-bin, selection-aware corrections to stacked cluster-lensing profiles that are compatible with *Euclid* pipeline definitions.

Our results reveal a consistent physical picture for the optical selection bias on galaxy cluster lensing profiles across richness and redshift. The correlated LoS structure enhances the lensing signal near the one-two halo transition, producing a peak of $20-40\,\%$ around $R \simeq 1\,h^{-1}$Mpc. The anti-correlation between richness and small-scale density profile induced by miscentring suppresses the inner signal at $r \lesssim 0.1$–$0.5\,h^{-1}$ Mpc, while leaving the transition-scale enhancement largely unchanged.

Baryonic physics modifies the projected density profile primarily at small radii and when considered in isolation, it is shown to affect the selection bias of a richness-selected sample at the percent level. However, baryons play an important role once miscentring is included. While the anti-correlation in the inner bias profile is driven primarily by miscentring, baryonic effects further enhance this suppression at the smallest scales and simultaneously amplify the bias peak at the transition between the one-halo and two-halo regimes. This behaviour demonstrates that baryonic physics non-negligibly contributes to shaping the radial dependence of the selection bias through its interplay with miscentring.

Variations in the underlying cosmological parameters primarily impact the selection bias by altering the abundance and large-scale clustering of massive halos. In particular, a lower matter density leads to a reduction in the overall bias amplitude and to an outward shift of the bias peak towards larger radii, reflecting the displacement of the transition between the one-halo and two-halo regimes. These effects are most pronounced at high redshift and high richness, where the bias is largest. This sensitivity motivates, for DR1, an emulator-based treatment of the correction, where the weak lensing bias is interpolated across the set of cosmologies sampled by the `PICCOLO` simulations, rather than recomputed on the fly within the likelihood analysis. Our tests with alternative HOD parameter choices show that reasonable variations of the mass–richness relation induce only modest changes in the bias, typically around 1–2 %, with a mild inversion of the trend at high redshift due to the interplay between redshift evolution in the mass–richness relation and the increased scatter in the observed richness.

These findings highlight that a single set of corrections calibrated at fixed cosmology and HOD cannot capture the full response of the bias across parameter space. For *Euclid* DR1, a practical approach is to provide corrections derived at the fiducial cosmology and HOD, accompanied by an uncertainty estimate reflecting the residual sensitivity to model variations. For *Euclid* DR1, our baseline strategy is therefore to combine corrections derived from the fiducial calibration with an emulator-based description of their cosmological dependence, trained on the set of `PICCOLO` runs. Beyond DR1, we plan to extend the framework by generating increasingly survey-realistic mocks and by developing a more flexible emulator-based description of the selection bias that can accommodate a broader range of cosmological and halo-occupation variations. In the present implementation, we therefore treat the uncertainty budget primarily through variations of the physical ingredients that dominate the bias prediction; namely, the HOD, baryonic, and miscentring prescriptions, while the residual statistical uncertainty of the individual $P_{\rm mem}$ emulator is assumed to be subdominant.

A remaining ingredient for a fully end-to-end correction at the catalogue level is the survey selection function. In the present work, the calibration and validation were performed on the basis of simulations where the richness mapping can be sampled down to arbitrarily low intrinsic and observed values. Thus, the forward model can effectively capture the full $P(\lambda_{\rm obs} \,|\, \lambda_{\rm true})$ without the additional truncations that (in real data) would translate to purity and completeness factors. For *Euclid* DR1, the same forward-modelling approach will be combined with the empirically inferred selection function of the cluster finder to propagate these catalogue-level effects consistently in the cosmological analysis. A closely related strategy, in which the cluster selection enters explicitly in the inference from optically selected samples, has been adopted in recent cosmological applications of AMICO cluster catalogues (Lesci et al. 2025).

The code architecture is naturally suited for simulation-based inference (SBI), enabling future analyses in which cosmological and HOD parameters can be constrained directly from forward-simulated catalogues and lensing observables. A key advantage of the framework is that richness, lensing, miscentring, and baryonic corrections are derived consistently from the same simulated realisation, with the relevant modifications implemented directly at the particle level. This makes it possible to track how changes in the forward-modelling ingredients propagate jointly to the galaxy distribution and to the resulting observables. Planned extensions of this work include expanding the training set of cosmological hydrodynamical simulations used for the baryonic correction, sampling multiple feedback models to ensure a robust and agnostic calibration, and incorporating more complete treatments of photometric-redshift and source-selection systematics. Our long-term goal is to lower modelling systematics below the statistical precision at the transition scales most sensitive to selection effects, thereby enabling high-fidelity mass calibration for cluster cosmology in forthcoming *Euclid* data.

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Appendix A: Triaxiality and orientation

The code features a module to measure halo triaxiality inside $r_{\rm vir}$ and reports the inclination angle θ with respect to a chosen LoS, along with the axis ratios $q = b/a$ and $s = c/a$, where $a \geq b \geq c$ are the semi–axes of the best-fitting ellipsoid.

By default we adopt principal component analysis (PCA) scheme, PCA identifies the orthogonal axes of maximum variance in the particle distribution and is used to infer the halo shape and orientation. Starting from particle positions relative to the halo centre, we first apply a spherical cut at $r_{\rm vir}$ to avoid box-alignment artefacts and then, if necessary, draw an unbiased subsample capped at 4096 particles. Shapes are not attempted when fewer than 100 particles survive this preselection and in any case to avoid bottleneck in the galaxy painting loop we limit this part of the analysis to halos above the mass threshold of $10^{13}\,h^{-1}\,M_\odot$, used also for the surface density profiles.

Initialisation comes from a 3D PCA of the centred positions (implemented with `scikit-learn`; Pedregosa et al. 2011). The principal directions define a rotation matrix R; the associated spreads along those directions set provisional semi-axis lengths (a, b, c), which immediately give the q and s shape parameters.

Refinement proceeds iteratively. At each pass we rotate into the current principal frame, keeping only particles inside the triaxial envelope

$$r_{\rm tri} = \sqrt{d_1^2 + \left(\frac{d_2}{q}\right)^2 + \left(\frac{d_3}{s}\right)^2} < r_{\rm vir}\,, \tag{A.1}$$

where d_i are the particle Cartesian coordinates in the reference frame of the halo. We then recompute the PCA on the retained set to update our set of geometrical variables: the rotation matrix and axes (R, a, b, c) and therefore (q, s). The process stops once both ratios stabilise,

$$\left|1 - \frac{q_{\rm new}}{q}\right| < 10^{-6} \quad \text{and} \quad \left|1 - \frac{s_{\rm new}}{s}\right| < 10^{-6}\,, \tag{A.2}$$

or after 100 iterations, in which case the halo is flagged as not converged. Finally, letting $\hat{\mathbf{A}}$ be the unit vector along the major axis (first row of R) and $\mathbf{e}_n$ the unit vector of the selected Cartesian LoS, the inclination angle can be written as

$$\theta = \cos^{-1}(\,|\hat{\mathbf{A}} \cdot \mathbf{e}_n|\,) \in [0^\circ, 90^\circ]\,. \tag{A.3}$$

For completeness, a reduced tensor-of-inertia variant is available that follows the same iterative envelope but replaces the PCA step with reduced tensor-of-inertia principal axes. This option is retained for cross-checks and diagnostics; in practice, the approach based on the tensor-of-inertia is disfavoured because it is noticeably slower than the PCA path. In Fig. A.1 we study the selection bias on density profiles as a function of inclination angle θ. Binning in orientation we isolate the orientation bias whereby systems whose richness is artificially enhanced by projection effects are over-represented at small θ that is, preferentially aligned with the LoS. This is even more evident in Fig. A.2, where we focus on up-scattered objects exhibiting a positive richness boost, $(\lambda_{\rm obs} - \lambda_{\rm true})/\lambda_{\rm true} \equiv \Delta\lambda/\lambda$, and analyse how this quantity correlates with both the LoS inclination of the halo and its triaxiality parameter $T = (1-q^2)/(1-s^2)$, which provides a measure of the halo shape by quantifying deviations from spherical symmetry in terms of the intermediate-to-major (q) and minor-to-major (s) axis ratios. From this analysis it emerges that prolate ($T > 2/3$) objects show the largest boost when aligned to the LoS due to the larger amount of contaminants along the LoS. Oblate objects ($T < 1/3$) are associated to lower boost values, with exception of few sets of object at $\theta > 45^\circ$ close to $T = 0$. These classes of object are probably enhanced by contaminants with small radial separation entering the richness computation due to the object being relative spread on the plane perpendicular to the LoS.

Appendix B: Definition and calibration for the baryonification scheme

This appendix presents a single, end-to-end account of the baryonification model and its calibration, written to be implementation-ready but also to clarify how each modelling choice is tied to the `Magneticum` hydrodynamical simulations (see Sect. 3.3). Throughout we use the same symbols and centring conventions as in the main text. All the distances hereafter are comoving.

Stellar components and baryon fractions (multi-cosmology; interpolated inputs): We start from the multi-cosmology `Magneticum` suite by fitting the total stellar and central-galaxy stellar mass fractions as functions of host-halo mass and redshift. To produce a baryonic correction for our sample in the `PICCOLO` simulations we need calibrated relations for $z < 2$. However given the availability of `Magneticum` simulations we extended the sampling of the redshift evolution from $z \simeq 0$ to $z \simeq 6$ using all 14 snapshots in the multi-cosmology suite. These fits are used to build interpolators that provide the stellar fractions used in the subsequent calibration steps.

The central-galaxy and satellite-associated density profiles used in the baryonification model are

$$\rho_{\rm cga}(r) = \frac{f_{\rm cga}\,M_{\rm vir}}{4\pi^{3/2}\,r_{\rm h}\,r^2}\,\exp\!\left[-\frac{1}{2}\left(\frac{r}{r_{\rm h}}\right)^3\right], \quad \rho_{\rm sga}(r) = \frac{f_{\rm sga}}{4\pi r^2}\,\frac{{\rm d}M_{\rm dm}(r)}{{\rm d}r}\,, \tag{B.1}$$

with $f_{\rm sga} = f_\star - f_{\rm cga}$ and $f_{\rm gas} = f_{\rm b} - f_\star$. The stellar and central-galaxy mass fractions are fitted as smooth functions of mass and redshift,

$$f_\star(M, z) = A_\star \left(\frac{M}{10^{13}\,h^{-1}\,M_\odot}\right)^{\eta_\star(z, f_{\rm b})} + \Delta_\star(z, f_{\rm b})\,, \tag{B.2}$$

$$f_{\rm cga}(M, z) = A_{\rm cga} \left(\frac{M}{10^{13}\,h^{-1}\,M_\odot}\right)^{\eta_{\rm cga}(z, f_{\rm b})} + \Delta_{\rm cga}(z, f_{\rm b})\,. \tag{B.3}$$

We place safeguard checks in the interpolator for the cosmological and redshift dependence of the fractions to ensure they are always in the interval $[0, f_{\rm b}]$ and their sum adds up to 1.

To propagate the fitted $\eta_\star, \eta_{\rm cga}, \Delta_\star, \Delta_{\rm cga}$ to arbitrary $(z, f_{\rm b})$, we build smooth 2D interpolants over the sampled $(z, f_{\rm b})$ points. We compute the Delaunay triangulation and evaluate values inside the convex hull via piecewise-linear (barycentric) interpolation on each triangle; any remaining undefined locations (e.g. thin gaps) are filled using a nearest-neighbour fallback. This approach preserves the local trends of the discrete fits and avoids spurious oscillations.

In practice, we first fit Eqs. (B.2), and (B.3) on each simulation snapshot, thereby capturing the redshift evolution across the full snapshot grid. We then fix the mass-fraction normalisations to the median of the snapshot best-fit values and perform a second fit in which only the mass-trend parameters are left free, that is the power-law slopes and additive offsets. This yields $A_\star = 0.04119$ and $A_{\rm cga} = 0.01084$. The posterior means

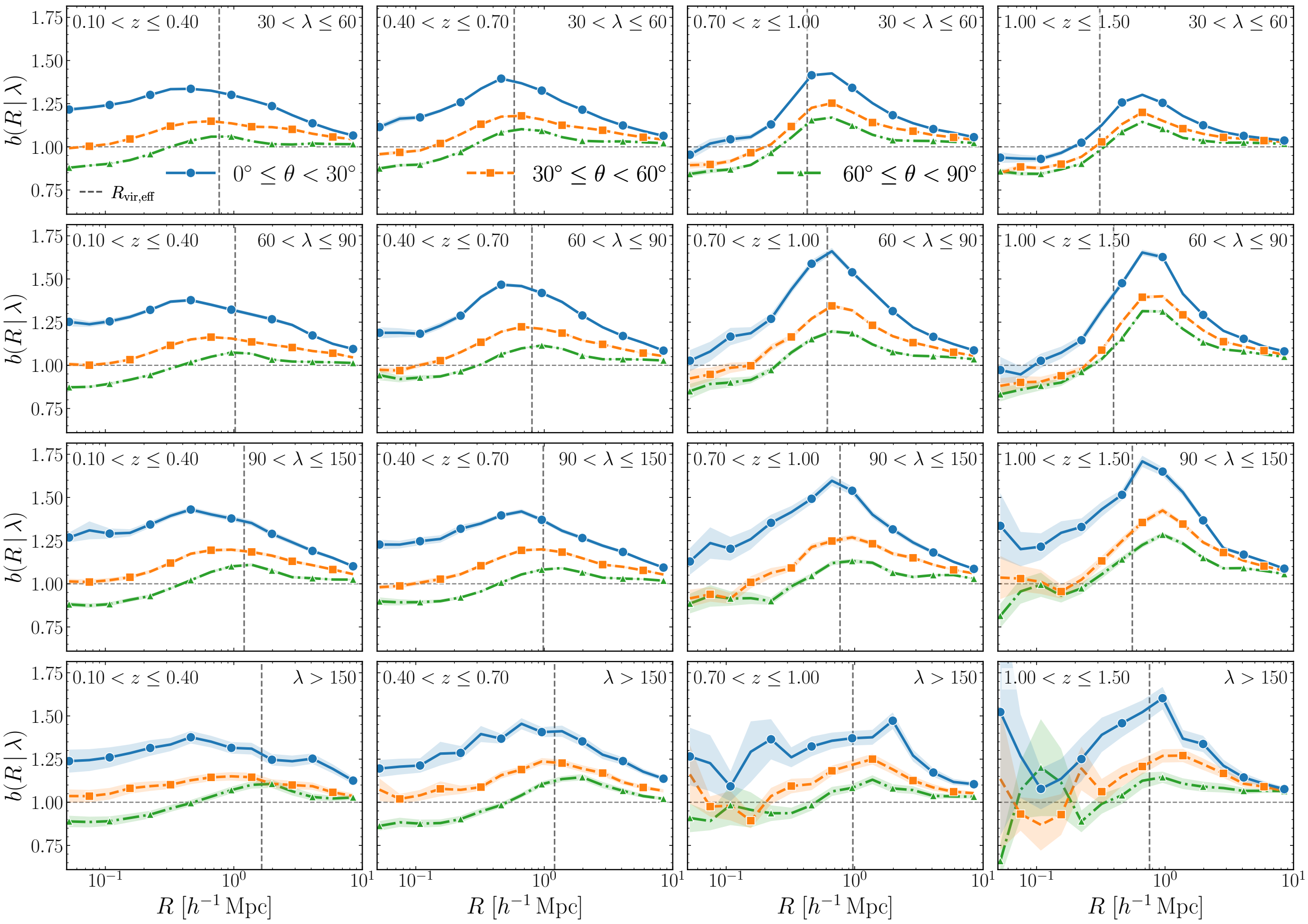

Fig. A.1: Selection bias in Σ profiles, for the miscentring and baryonified case. In each panel, the three curves with different colours and line types corresponds to different intervals of values of the orientation angle relative to the LoS: $0° \leq \theta < 30°$ in blue, $30° \leq \theta < 60°$ in orange, and $60° \leq \theta < 90°$ green. For lower values of θ the projection bias is boosted due to the objects being more likely oblate and aligned with contaminants in projection. At the same time, these objects have a larger probability of being well-collimated, thus reducing the miscentring anti-correlation trend in the inner region of the profile.

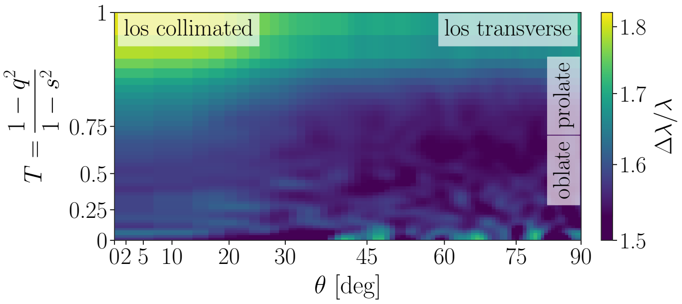

Fig. A.2: 2D histogram, as a function of inclination angle, θ, and triaxiality parameter, T, colour-coded by the richness boost, $\Delta\lambda$. The sample corresponds to the full fiducial analysis and includes only up-scattered systems with $\Delta\lambda/\lambda > 0$.

from this second fitting stage are used to build the interpolators, which are subsequently employed in the profile calibrations. For the gas calibration, the corresponding dependence on the stellar mass fraction enters through $f_{\rm gas} = f_{\rm b} - f_\star$ (in the fitting implementation this relation is used in a direct, explicit form).

Hot gas profiles fitting: In four simulation snapshots, corresponding to the redshifts $z \in \{1.17, 0.47, 0.25, 0.00\}$, we selected halos with $M_{\rm vir} \geq 10^{13}\,h^{-1}M_\odot$ and built spherically averaged density profiles, $\rho^{\rm data}$, for the hot gas ($T > 10^6$ K), stars, dark matter, and for their total matter distribution. Halos are grouped in logarithmic mass bins (4 bins per snapshot) and stacked on a common radial grid. Radial uncertainties are estimated from the stacking procedure and propagated into the fits.

For each stack (i.e. at fixed snapshot z and mass bin b) we fit the hot-gas density profile of Eqs. (B.7), (B.9), and (B.10) using a Gaussian least-squares objective. Defining

$$\chi^2_b(\boldsymbol{\vartheta}) = \sum_j \left[\frac{\rho^{\rm data}_{b,j} - \rho^{\rm model}_{b,j}(\boldsymbol{\vartheta})}{\sigma_{b,j}}\right]^2 , \tag{B.4}$$

where j indexes the available radial points and $\sigma_{b,j}$ is the corresponding uncertainty, we use the equivalent log-likelihood

$$\ln \widetilde{\mathcal{L}}_b(\boldsymbol{\vartheta}) = -\frac{1}{2}\chi_b^2(\boldsymbol{\vartheta}) \,. \tag{B.5}$$

Here $\widetilde{\mathcal{L}}_b$ denotes an unnormalised Gaussian likelihood: all parameter-independent normalisation terms, including the usual factors involving 2π and the data covariance, are omitted. Since the uncertainties are fixed and are not part of the parameter model, these terms do not affect the calibration.

At fixed snapshot, the combined calibration objective is implemented as a mass-weighted sum over bins,

$$\ln \widetilde{\mathcal{L}}_{\rm snap}(\boldsymbol{\vartheta}) = \sum_{b=1}^{B} w(M_b)\, \ln \widetilde{\mathcal{L}}_b(\boldsymbol{\vartheta}), \qquad w(M_b) = \frac{M_b}{10^{14}\, h^{-1}\, M_\odot} \,, \tag{B.6}$$

with M_b as the median mass within each bin. The mass weights $w(M_b)$ are introduced as a calibration choice in the construction of the snapshot-level objective, not as a Bayesian prior. The resulting quantity should therefore be interpreted as a weighted fitting objective, or equivalently as an unnormalised pseudo-likelihood, rather than as a fully normalised probability density. The parametric form of the hot-gas profile is physically motivated and has been validated primarily over a limited mass regime; therefore, we deliberately anchor the global calibration to the bins where the underlying assumptions are expected to hold best. In practice, in hydrodynamical simulations higher-mass halos yield a cleaner and more stable characterisation of the hot component, so up-weighting their contribution prevents the fit from being dominated by low-mass bins where the model is not intended to be equally accurate.

We adopt uninformative priors and sample the posterior using the affine-invariant ensemble MCMC sampler implemented in `emcee` (Foreman-Mackey et al. 2013). Convergence is monitored using the integrated auto-correlation time, τ, estimated from the chains: we require the total chain length to satisfy $N_{\rm step} \gtrsim 50\, \max(\tau)$ and the τ estimates to be stable with fractional changes $\lesssim 1\,\%$ between successive checks. We discard an initial burn-in of twice $\max(\tau)$ and verify trace-plot stability by eye. As a basic sampler health check, we require the mean acceptance fraction to lie in the range ~ 0.2–0.5. Posterior means are reported in Tables B.1 and B.2.

The hot gas is modelled as the sum of a compact core ("one-halo") component and a diffuse tail,

$$\rho_{\rm gas}(r) = \rho_{\rm core}(r) + \rho_{\rm tail}(r) \,. \tag{B.7}$$

The core controls the inner distribution and fixes the gas mass budget, while the tail sets the onset of the diffuse regime and the large-radius decay. In the fitting implementation, radial scalings are expressed relative to the halo size provided by each stack (denoted as $R_{\rm vir}$ in the code; we keep $R_{\rm vir}$ below for consistency).

The core uses two characteristic radii,

$$r_{\rm ej} = \theta_{\rm ej}\, r_{\rm vir} \,, \qquad r_{\rm co} = \theta_{\rm core}\, r_{\rm vir} \,, \tag{B.8}$$

and the core profile is written as

$$\rho_{\rm core}(r) = \frac{\rho_0}{(1 + r/r_{\rm co})^{\beta} \left[1 + (r/r_{\rm ej})^2\right]^{(7-\beta)/2}} \,. \tag{B.9}$$

The parameter β controls the slope for core profile in inner and intermediate region; its value is allowed to vary smoothly with halo mass and to remain bounded between 0 and 3. The central density ρ_0 is fixed by the gas mass budget as described below.

The tail component of the gas density profile is implemented as a smooth rise to an outer normalisation followed by a softened power-law decay,

$$\rho_{\rm tail}(r) = \left[f_{\rm in} + \frac{\rho_{\rm norm} - f_{\rm in}}{1 + {\rm e}^{-s\,(r - x_{\rm t})}}\right] \left[1 + \left(\frac{r}{x_{\rm cut}}\right)^{k}\right]^{-\gamma/k} \,, \tag{B.10}$$

with fixed $(f_{\rm in}, s, k) = (10^7, 10, 5)$. The first factor in Eq. (B.10) describes a smooth transition from an inner floor to an outer normalisation. In the implementation, $f_{\rm in}$ is a fixed, dimensionless baseline value that prevents the tail component from vanishing at small radii, $x_{\rm t}$ sets the radial location of the transition, and s controls its sharpness through the width of the logistic rise. The second factor governs the large-radius behaviour: $x_{\rm cut}$ defines the characteristic cut-off scale, k controls how gradual the turnover is around $x_{\rm cut}$, and γ fixes the asymptotic outer slope, $\rho_{\rm tail} \propto r^{-\gamma}$ for $r \gg x_{\rm cut}$.

For a fixed snapshot, the gas-profile parameters fitted to the stacked profiles are collected in the vector

$$\boldsymbol{\Theta} = \left[\mathcal{M}_{c,0}, \mathcal{M}_{c,1}, \mu, \theta_0, \theta_1, \theta_2, \theta_{\rm ej,br}, x_{t,0}, \gamma_0, \gamma_1, \gamma_2, \gamma_{\rm br}\right], \tag{B.11}$$

where $\mathcal{M}_{c,0}$ and $\mathcal{M}_{c,1}$ control the mass dependence of the core-pivot scale (defined below), μ sets the sharpness of the transition in the core slope, $(\theta_0, \theta_1, \theta_2, \theta_{\rm ej,br})$ parametrise the broken-line mass dependence of the ejection scale $\theta_{\rm ej}$, $x_{t,0}$ fixes the transition location of the tail component, and $(\gamma_0, \gamma_1, \gamma_2, \gamma_{\rm br})$ define the corresponding broken-line dependence of the outer slope γ.

The mass dependence is expressed in terms of the dimensionless variable

$$x_M \equiv \log_{10}\!\left(\frac{M}{10^{14}\, h^{-1}\, M_\odot}\right) , \tag{B.12}$$

and we introduce a characteristic pivot mass $M_c(M)$ through its logarithm (in units of $h^{-1}\, M_\odot$),

$$\log_{10}\!\left(\frac{M_c(M)}{h^{-1}\, M_\odot}\right) \equiv \mathcal{M}_c(M) = \mathcal{M}_{c,0} + \mathcal{M}_{c,1}\, x_M \,. \tag{B.13}$$

With this definition, $\mathcal{M}_c$ is dimensionless and the corresponding mass scale is

$$M_c(M) = 10^{\mathcal{M}_c(M)}\, h^{-1}\, M_\odot \,. \tag{B.14}$$

The inner-slope parameter β in Eq. (B.9) is then written as a smooth function of the ratio between the halo mass and the pivot mass,

$$x \equiv \frac{M_c(M)}{M} \,, \qquad \beta(M) = \frac{3\, x^{\mu}}{1 + x^{\mu}} \,, \tag{B.15}$$

so that x is dimensionless and $0 \le \beta(M) \le 3$. Varying $\mathcal{M}_c$ shifts the characteristic mass scale at which the core steepens, while μ controls how rapidly the transition in $\beta(M)$ occurs with halo mass.

The ejection scale $\theta_{\rm ej}$ and the outer slope γ follow a continuous broken-line dependence in x_M,

$$\mathcal{B}(x_M; p_0, p_1, p_2, x_{\rm br}) = \begin{cases} p_0 + p_1\,(x_M - x_{\rm br}), & x_M < x_{\rm br} \,, \\ p_0 + p_2\,(x_M - x_{\rm br}), & x_M \ge x_{\rm br} \,, \end{cases} \tag{B.16}$$

so that $\mathcal{B}(x_{\rm br}) = p_0$ and the dependence is continuous at the break. In the fit,

$$\theta_{\rm ej}(M) = \mathcal{B}\Big(x_M; \theta_0, \theta_1, \theta_2, \theta_{\rm ej,br}\Big) \,, \tag{B.17}$$

$$\gamma(M) = \mathcal{B}(x_M; \gamma_0, \gamma_1, \gamma_2, \gamma_{\rm br}) \,, \tag{B.18}$$

with the additional implementation safeguard $\gamma \geq 0.2$ when evaluating the model. The tail transition location is kept mass-independent and set to $x_{\rm t} = x_{t,0}$.

The mass-dependence of the core-size parameter is set by a piecewise prescription,

$$\theta_{\rm core}(M,z) = \begin{cases} 0.07\,(1+z)^{1/2}, & M/(h^{-1}\,M_\odot) < 1.2\times10^{13}\,, \\ 0.10\,(1+z)^{1/2}, & 1.2\times10^{13} \leq M/(h^{-1}\,M_\odot) < 7\times10^{13}\,, \\ 0.025\,(1+z)^{1/2}, & M/(h^{-1}\,M_\odot) \geq 7\times10^{13}\,, \end{cases} \tag{B.19}$$

which fixes $r_{\rm co} = \theta_{\rm core}\, r_{\rm vir}$. In the shown fitting implementation, this prescription is evaluated at fixed z within a snapshot fit.

The core amplitude is fixed by enforcing that the total core mass equals the gas mass fraction times the halo mass. In the fitting implementation, the gas fraction entering this normalisation is supplied by an explicit fit $f_{\rm gas}(M)$, and we impose

$$f_{\rm gas}(M)\, M = 4\pi \int_0^\infty \rho_{\rm core}(r)\, r^2\, {\rm d}r\,, \tag{B.20}$$

where the integral is evaluated numerically.

For the tail component we set the cut-off scale proportional to the ejection radius,

$$x_{\rm cut} = 1.25\, \theta_{\rm ej}\, r_{\rm vir}\,, \tag{B.21}$$

which places the transition between the intermediate and asymptotic regimes slightly beyond the gas ejection scale.

The outer normalisation is defined relative to a reference density scale,

$$\rho_{\rm norm} = \Omega_{\rm m}\, \rho_{\rm ref} \left[f_{\rm gas}(M) + 1.25\, \frac{\rho_0\, r_{\rm vir}^3}{M} \right] , \tag{B.22}$$

where $\rho_{\rm ref}$ denotes the reference density used in the implementation (e.g. the critical or mean matter density, depending on the adopted convention). The second term accounts for the residual contribution of the core component to the outer gas distribution and is written in terms of the dimensionless ratio $\rho_0 r_{\rm vir}^3/M$, ensuring that the bracketed quantity remains dimensionless.

The fitted model entering the likelihood is therefore

$$\rho^{\rm model}(r) = \rho_{\rm core}(r) + \rho_{\rm tail}(r)\,, \tag{B.23}$$

evaluated on the same radial grid used for the stacked profiles.

3. Stellar profiles fits: For the stellar profiles we keep the functional forms fixed and use the interpolated fractions from step (1), with $f_{\rm sga} = f_\star - f_{\rm cga}$ and $f_{\rm gas} = f_{\rm b} - f_\star$. To account for the `Magneticum` brightest central galaxies (BCG) being more massive than expected (e.g. Gozaliasl et al. 2019; Anbajagane et al. 2020; Marini et al. 2025), we use a central-galaxy scale radius *rh* five times larger than the fiducial choice:

$$\rho_{\rm cga}(r) = \frac{f_{\rm cga}\, M_{\rm vir}}{4\pi^{3/2}\, r_{\rm h}\, r^2}\, \exp[-\tfrac{1}{2}\,(r/r_{\rm h})^3], \quad r_{\rm h} = 0.075\, r_{\rm vir}\,, \tag{B.24}$$

and the satellite-associated component follows the local dark matter slope,

$$\rho_{\rm sga}(r) = \frac{f_{\rm sga}}{4\pi r^2}\, \frac{{\rm d}M_{\rm dm}(r)}{{\rm d}r}\,. \tag{B.25}$$

The total baryonic density and enclosed mass are then expressed as

$$\rho_{\rm b}(r) = \rho_{\rm gas}(r) + \rho_{\rm cga}(r) + \rho_{\rm sga}(r)\,, \tag{B.26}$$

$$M_{\rm b}(<r) = 4\pi \int_0^r r'^2\, \rho_{\rm b}(r')\, {\rm d}r'\,. \tag{B.27}$$

Adiabatic contraction: Contraction is calibrated by comparing the dark matter density profile in the hydrodynamical run to the DMO profile obtained by applying the Schneider et al. (2025) AC mapping to the matched gravity-only run, while using the simulated baryonic components as inputs. We define: (i) $\rho_{\rm DMO}$ and $M_{\rm DMO}$ as the density and enclosed mass of the gravity-only dark matter; (ii) $\rho_{\rm dm}^{\rm hydro}$ and $M_{\rm dm}^{\rm hydro}$ as the corresponding quantities for the DM component in the hydrodynamical simulations; (iii) a "central" baryonic component $\rho_{\rm cga}$ given by stars plus cold gas from the simulation, and a hot-gas component $\rho_{\rm hga}$ given by the simulated hot gas. We denote by $M_{\rm cga}$ and $M_{\rm hga}$ the corresponding enclosed masses. The cold dark matter (CDM) fraction is fixed to $f_{\rm CDM} = 1 - f_{\rm b}$.

We model the mapping between initial and final radii through $\xi \equiv r_{\rm i}/r_{\rm f}$. Numerically, we first construct a provisional mapping on the native radial grid by defining

$$\xi(r) = 1 + K(r), \qquad K(r) = K_0(r) + K_1(r) + K_2(r)\,, \tag{B.28}$$

with

$$K_0(r) = \frac{Q_0(z)}{1 + (r/r_{\rm step})^{3/2}}\,, \tag{B.29}$$

$$K_1(r) = Q_1(z)\, \frac{M_{\rm cga}(<r)}{M_{\rm dm}^{\rm hydro}(<r)}\,, \tag{B.30}$$

$$K_2(r) = Q_2(z)\, \frac{M_{\rm hga}(<r)}{M_{\rm dm}^{\rm hydro}(<r)}\,, \tag{B.31}$$

and redshift evolution

$$Q_i(z) = q_i (1+z)^{p_i}, \qquad i = 0, 1, 2\,. \tag{B.32}$$

The radial pivot in K_0 is linked to the virial scale and to the peak height ν, which provides a dimensionless measure of halo mass relative to the characteristic fluctuation amplitude of the linear density field at redshift z. We define

$$\nu(M,z) \equiv \frac{\delta_{\rm c}(z)}{\sigma(M,z)}\,, \tag{B.33}$$

where $\sigma(M,z)$ is the root-mean-square fluctuation of the matter density field smoothed with a top-hat filter enclosing mass M (in Lagrangian space) and evaluated at redshift z. The quantity $\delta_{\rm c}(z)$ is the linear overdensity threshold for spherical collapse; in ΛCDM it has a weak redshift dependence and is often approximated as $\delta_{\rm c} \simeq 1.686$. In this notation, the step radius is written as

$$r_{\rm step} = r_{\rm vir}\, \frac{\varepsilon_0 + \varepsilon_1\, \nu}{\varepsilon_0}\,, \tag{B.34}$$

so that, at fixed $r_{\rm vir}$, the pivot shifts to larger radii for higher peak height.

Table B.1: Hot-gas best-fit parameters.

z	$\mathcal{M}_{c,0}$	$\mathcal{M}_{c,1}$	μ	θ_0	θ_1	θ_2	$\theta_{\rm br}$	$x_{\rm t}$	γ_0	γ_1	γ_2	$\gamma_{\rm br}$
0.00	$11.1^{+1.7}_{-2.2}$	$1.71^{+0.62}_{-0.43}$	$0.11^{+0.15}_{-0.05}$	$1.157^{+0.016}_{-0.016}$	$-0.080^{+0.019}_{-0.020}$	$-1.3^{+0.8}_{-1.9}$	$0.36^{+0.04}_{-0.14}$	$0.363^{+0.042}_{-0.051}$	$1.642^{+0.080}_{-0.079}$	$-0.18^{+0.14}_{-0.13}$	$-4.50^{+0.54}_{-0.35}$	$0.072^{+0.043}_{-0.033}$
0.25	$10.9^{+1.9}_{-2.0}$	$2.19^{+0.86}_{-0.67}$	$0.08^{+0.13}_{-0.03}$	$1.196^{+0.015}_{-0.017}$	$-0.015^{+0.022}_{-0.024}$	$-1.2^{+0.8}_{-2.0}$	$0.25^{+0.05}_{-0.15}$	$0.296^{+0.054}_{-0.072}$	$1.79^{+0.11}_{-0.10}$	$0.00^{+0.16}_{-0.17}$	$-3.75^{+0.72}_{-0.78}$	$0.042^{+0.052}_{-0.064}$
0.47	$10.9^{+1.7}_{-1.9}$	$0.94^{+0.24}_{-0.28}$	$0.09^{+0.11}_{-0.04}$	$1.152^{+0.016}_{-0.016}$	$-0.053^{+0.022}_{-0.022}$	$-1.1^{+0.6}_{-1.9}$	$0.23^{+0.07}_{-0.19}$	$0.317^{+0.050}_{-0.073}$	$1.72^{+0.10}_{-0.11}$	$0.04^{+0.14}_{-0.17}$	$-3.1^{+0.9}_{-1.2}$	$0.08^{+0.08}_{-0.10}$
1.17	$11.7^{+1.0}_{-1.7}$	$-0.09^{+0.62}_{-0.89}$	$0.078^{+0.073}_{-0.035}$	$0.942^{+0.040}_{-0.043}$	$-0.29^{+0.20}_{-0.06}$	$1.1^{+1.4}_{-0.7}$	$-0.1^{+1.5}_{-0.2}$	$0.14^{+0.11}_{-0.09}$	$1.47^{+0.17}_{-0.15}$	$-1.53^{+0.42}_{-0.42}$	$2.8^{+1.4}_{-1.9}$	$-0.28^{+0.09}_{-0.11}$

Notes. Parameters are defined in Eqs. (B.7), (B.9), and (B.10). $\mathcal{M}_{c,0}$ and $\mathcal{M}_{c,1}$ define the mass pivot, μ controls the sharpness of the transition, $(\theta_0, \theta_1, \theta_2, \theta_{\rm br})$ govern the mass trend of the ejection scale, $x_{\rm t}$ marks the onset of the diffuse tail, and $(\gamma_0, \gamma_1, \gamma_2, \gamma_{\rm br})$ determine the outskirts slope. Units follow the definitions in Appendix B.

Table B.2: Best-fitting global adiabatic-contraction parameters.

q_0	p_0	q_1	p_1
$0.0225^{+0.0126}_{-0.0095}$	$0.68^{+0.91}_{-0.97}$	$0.158^{+0.073}_{-0.056}$	$1.05^{+0.66}_{-0.69}$
q_2	p_2	ε_0	ε_1
$-0.215^{+0.088}_{-0.098}$	$0.23^{+0.85}_{-1.03}$	$4.09^{+0.84}_{-0.82}$	$0.48^{+0.56}_{-0.54}$

Notes. These are the global parameters $\boldsymbol{\phi}_{\rm AC}$ of the Schneider et al. (2025) AC model, calibrated on matched `Magneticum` halos. The calibration matches the hydrodynamical dark matter density profile to the contracted DMO profile obtained by applying the AC model to the corresponding gravity-only run. It is performed jointly across snapshots and mass bins using a Gaussian likelihood. Values are posterior medians with 1σ (16–84 %) credible intervals.

The provisional relation above defines $r_{\rm f}(r) = r/\xi(r)$. We then re-express the mapping as a function of final radius by inverting this relation (using a monotonic interpolation), yielding $\xi(r_{\rm f})$ on the same radial grid. This $\xi(r_{\rm f})$ is used to compute the contracted dark matter density profile predicted from the gravity-only run,

$$\rho_{\rm dm}^{\rm AC}(r_{\rm f}) = \frac{1}{4\pi r_{\rm f}^2}\frac{\rm d}{{\rm d}r_{\rm f}}\Big[f_{\rm CDM}\, M_{\rm DMO}(\xi(r_{\rm f}), r_{\rm f})\Big]. \tag{B.35}$$

We fit $\rho_{\rm dm}^{\rm AC}(r)$ to the hydrodynamical dark matter density profile $\rho_{\rm dm}^{\rm hydro}(r)$ in $\log_{10}$-space on the same radial grid as the gas fits. The parameters $\boldsymbol{\phi}_{\rm AC} = [q_0, p_0, q_1, p_1, q_2, p_2, \varepsilon_0, \varepsilon_1]$ are inferred jointly across snapshots; the resulting posterior means are reported in Table B.2. Together with $\boldsymbol{\Theta}$ they determine the contracted dark matter component and hence the total density,

$$\rho_{\rm tot}(r) = \rho_{\rm dm}^{\rm AC}(r) + \rho_{\rm gas}(r) + \rho_{\rm cga}(r)\,. \tag{B.36}$$

5. Model validation: Model predictions are compared to surface-density observables using the standard 3D-to-2D explained in text (in the context of surface quantities such as Σ and $\Delta\Sigma$ the r variable must be interpreted as projected radius perpendicular to the LoS). As shown in the main text in Fig. 8, after the conversion to $\Delta\Sigma(r)$ we retain an agreement within 1–2 % between simulation and calibrated model. Furthermore residuals are examined as a function of mass, redshift, and radius. The adopted settings were selected to mitigate residuals mass-dependent trends in the stacks while retaining numerical stability and physical monotonicity across the full radial range.

Appendix C: `Flagship2` calibrations

Here we detail the calibrations of the HOD model and of the neural-network membership emulator used in the main analysis.

HOD calibration: We use an eight-tile subsample covering a total of 400 deg^2 of the `Flagship2` 5200 deg^2 catalogue as our reference set. To produce the final catalogue used to fit the model described in Sect. 4.2 we match halos to detected clusters according to their major contribution to the richness. We then apply a conservative mass threshold of $10^{13}\,h^{-1}\,M_\odot$ and fix the pivot redshift to $z_{\rm piv} = 1.25$. The mean relation between richness, mass, and redshift is as defined in Sect. 4.2. To capture extra dispersion beyond counting noise we adopt a gamma-Poisson (negative binomial) model, with latent rate Λ drawn from a gamma distribution of shape κ and scale θ, and observed richness N drawn from a Poisson distribution conditional on Λ,

$$\begin{aligned} &\Lambda \sim \mathrm{Gamma}(\kappa, \theta)\,, \\ &N \mid \Lambda \sim \mathrm{Poisson}(\Lambda)\,. \end{aligned} \tag{C.1}$$

We choose θ and κ so that $\mathbb{E}[\Lambda] = \bar{\lambda}(M,z)$ and $\mathrm{Var}[\Lambda] = [\bar{\lambda}(M,z)\, s]^2$, implying

$$\theta = \frac{\bar{\lambda}(M,z)}{\kappa}\,, \qquad \kappa = \frac{1}{s^2}\,. \tag{C.2}$$

After marginalisation over Λ the richness follows a negative binomial distribution with probability mass function

$$P_\lambda\left(N = n \mid \bar{\lambda}, \kappa\right) = \frac{\Gamma(n+\kappa)}{\Gamma(\kappa)\, n!}\left(\frac{\kappa}{\kappa+\bar{\lambda}}\right)^{\kappa}\left(\frac{\bar{\lambda}}{\kappa+\bar{\lambda}}\right)^{n}, \tag{C.3}$$

with mean and variance

$$\mathbb{E}[N] = \bar{\lambda}\,, \qquad \mathrm{Var}[N] = \bar{\lambda} + (\bar{\lambda}\, s)^2\,. \tag{C.4}$$

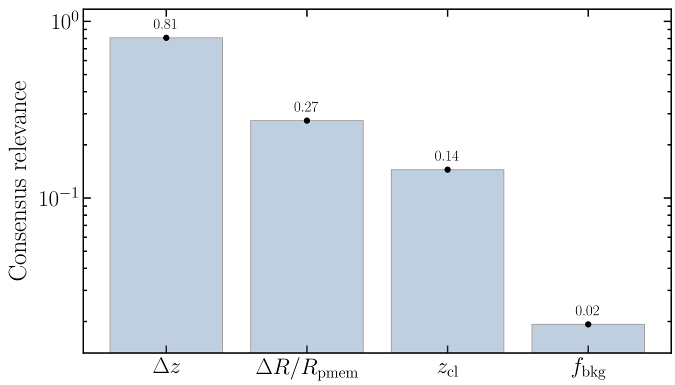

Fig. C.1: Consensus relevance of the four inputs (Δz, $\Delta r/r_{\rm cl}$, $z_{\rm cl}$, $f_{\rm bkg}$) obtained by combining saliency, integrated gradients, and conditional permutation importance.

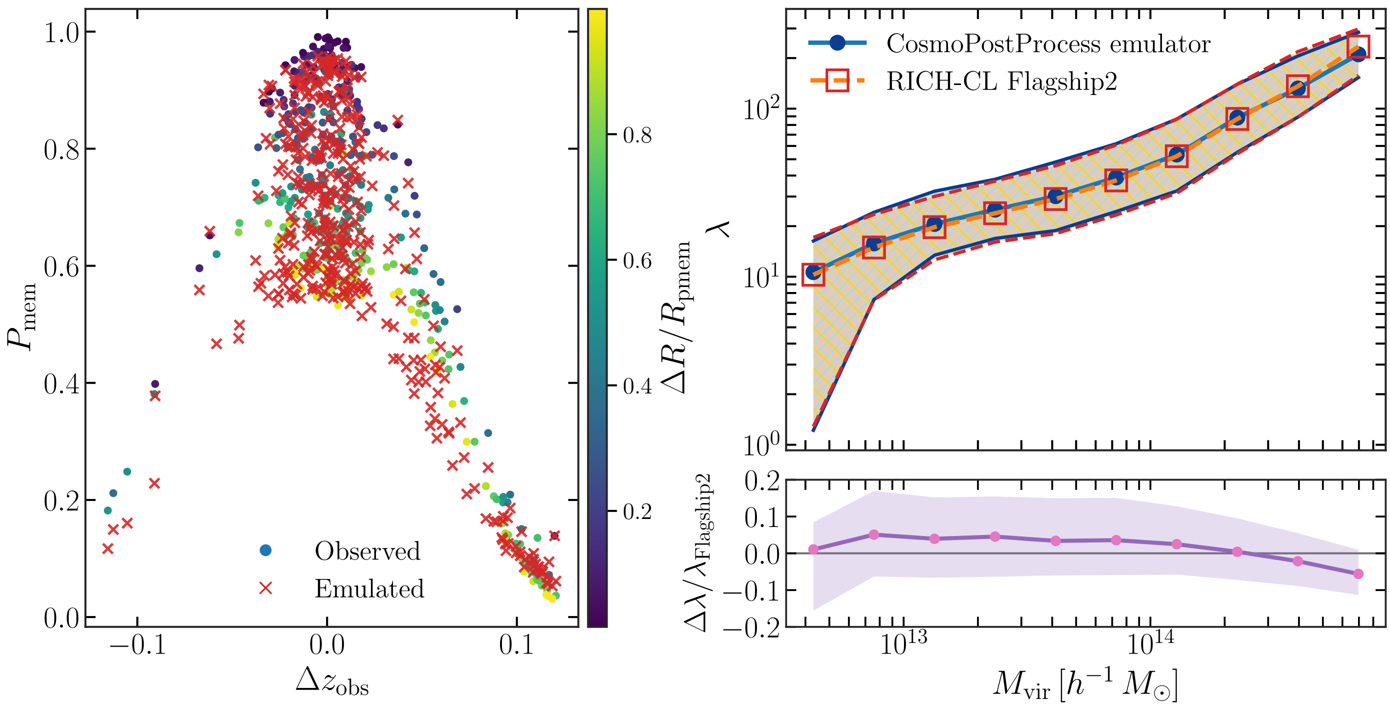

Fig. C.2: *Left panel:* Comparison between observed and emulated $P_{\rm mem}$ as a function of Δz. *Right, top:* Richness versus $M_{\rm vir}$ for the reference pipeline (squares) and for the emulator (circles) with the filled regions marking the 1σ scatter in the relation. *Right, bottom:* Fractional residuals relative to the reference calibration.

The total log-likelihood over the calibration set is

$$\begin{aligned}\ln\mathcal{L} &= \sum_i \ln P_\lambda\big(N_i \mid \bar\lambda_i, \kappa_i\big)\\ &= \sum_i \Bigg[\ln\Gamma(N_i+\kappa_i) - \ln\Gamma(\kappa_i) - \ln(N_i!)\\ &\qquad + \kappa_i \ln\!\left(\frac{\kappa_i}{\kappa_i+\bar\lambda_i}\right) + N_i \ln\!\left(\frac{\bar\lambda_i}{\kappa_i+\bar\lambda_i}\right)\Bigg],\end{aligned} \tag{C.5}$$

with $\bar\lambda_i = \bar\lambda(M_i, z_i)$ and $\kappa_i = 1/s^2$. The posterior for the mass–richness relation used to calibrate the HOD is obtained with the `Nautilus` nested sampler (Lange 2023). The adopted priors and resulting constraints are listed in Table C.1. The HOD variations displayed in Fig. 12 are chosen starting from the DES+SPT joint posterior for the mass–richness relation. Specifically, the HOD parameters are selected by mapping the original model in Costanzi et al. (2021) to ours, and selecting the $2\,\sigma$ extremal points along the direction orthogonal to the parameters $\log_{10}(M_{1,\rm sat}\,h/M_\odot)$ and α degeneracy.

2. Membership emulator: We train a compact fully connected emulator for $P_{\rm mem}$ on the same eight tiles. The inputs are Δz, $\Delta r/r_{\rm cl}$, $z_{\rm cl}$, and $f_{\rm bkg}$ (the same quantities used in Sect. 4.3.2); the architecture has three dense layers with Leaky ReLU and dropout, followed by a sigmoid output. Targets and training tuples are built by matching halos to detections on the eight tiles with the standard membership-based association.

Hyperparameters are selected with `Optuna` (Akiba et al. 2019). To interpret the tuning results, we apply functional ANOVA, which decomposes the variance of a surrogate model of the loss over the hyperparameter space into additive contributions from each hyperparameter. A functional ANOVA (Hutter et al. 2014; van Rijn & Hutter 2017) indicates that the hidden size contributes $< 1\,\%$ of the loss variance; we therefore reduce it from 756 to 300 to accelerate training with negligible impact. We also verify that $P_{\rm mem}$ is insensitive to the hidden size provided it is at least 100 (see Fig. C.1 for input relevance).

Table C.2: Training hyperparameters from `Optuna`

Hyperparameter	Value
Batch size	128
Learning rate	2×10^{-4}
Drop rate	14 %
Number of layers	3
Hidden size	300[*]
Max number of epochs	191

[*] Largest value that keeps training under one hour.

In Fig. C.2, we show that the emulator reproduces the dependence of $P_{\rm mem}$ on Δz and projected separation for an individual object and, at the catalogue level, preserves the mass-richness relation with residuals consistent with zero within the uncertainty band.

To assess which inputs are most informative, we carry out a consensus relevance test that combines saliency-based importance, integrated gradients, and conditional permutation importance. The scores are normalised and aggregated to produce a single ranking, evaluated both globally and in bins of $z_{\rm cl}$ and $\Delta r/r_{\rm cl}$. As anticipated in Sect. 4.3.2, Fig. C.1 identifies Δz, $\Delta R/R_{\rm cl}$, and $z_{\rm cl}$ as the leading predictors, with $f_{\rm bkg}$ contributing little. This supports the simplified proxy used for $f_{\rm bkg}$ and explains why the emulator retains the calibrated mass–richness relation. Nevertheless, runs where $f_{\rm bkg}$ was omitted showed larger residuals compared to Fig. C.2, so we kept it in the input vector. As a final consistency test, Fig. C.3 compares the selection bias computed with `CosmoPostProcess` on stacked $\Delta\Sigma$

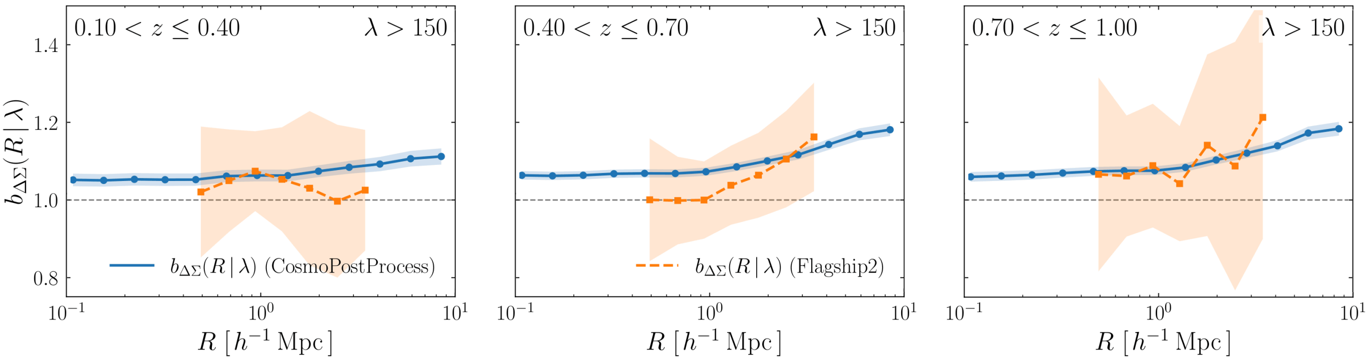

Fig. C.3: Comparison of the selection bias on $\Delta\Sigma$ profiles for the richest objects. In blue the bias calculated with `CosmoPostProcess` using the fiducial `PICCOLO` boxes as input for all redshift bins, in orange the results from `Flagship2` stacking the profiles obtained with `COMB-CL` according to `RICH-CL` richness, and the bands around making the 1σ uncertainty estimated via jackknife.

Table C.1: HOD priors and posterior constraints.

Parameter	Meaning	Prior range	Posterior (68 % CL)
$\log_{10}(M_{1,\mathrm{sat}}\,h/M_{\odot})$	Characteristic mass scale for satellites	[10, 16]	$12.191^{+0.008}_{-0.008}$
α	Slope of the satellite term	[0.5, 1.5]	$0.879^{+0.004}_{-0.004}$
ϵ	Index of the redshift evolution	[−2, 5]	$0.955^{+0.013}_{-0.013}$
σ_{intr}	Intrinsic fractional scatter	[0, 0.3]	$0.208^{+0.003}_{-0.003}$
$\log_{10}(M_{\mathrm{min,sat}}\,h/M_{\odot})$	Minimum mass scale	[11, 13]	$11.073^{+0.062}_{-0.056}$

Notes. Mass-related parameters are sampled in base-ten logarithmic space, with $M_{1,\mathrm{sat}}$ and $M_{\mathrm{min,sat}}$ expressed in units of $h^{-1}\,M_{\odot}$. The physical constraint $10 \leq M_{1,\mathrm{sat}}/M_{\mathrm{min,sat}} \leq 100$ is imposed during sampling as a likelihood veto to avoid unphysical regimes.

profiles to that extracted from the `RICH-CL` and `COMB-CL` runs on `Flagship2`. The $\Delta\Sigma(r)$ profiles are obtained from $\Sigma(r)$ via Eq. (13), and the bias is then computed using Eq. (14).