

LISA and γ -ray telescopes as multi-messenger probes of a first-order cosmological phase transition

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ABSTRACT

Aims. We study two possible cosmological consequences of a first-order phase transition in the temperature range of 1 GeV to 10^3 TeV: the generation of a stochastic gravitational wave background (SGWB) within the sensitivity of the Laser Interferometer Space Antenna (LISA) and, simultaneously, primordial magnetic fields that would evolve through the Universe's history and could be compatible with the lower bound from γ -ray telescopes on intergalactic magnetic fields (IGMF) at present time.

Methods. The SGWB spectrum is evaluated adopting semi-analytical models, accounting for both the contributions from sound waves and magnetohydrodynamic (MHD) turbulence in the aftermath of the first-order phase transition. Turbulence is assumed to arise only after an initial period of sound waves, and the magnetic field is assumed to be in equipartition with the turbulent kinetic energy. Several paths are considered for the magnetic field evolution throughout the radiation-dominated era, in order to predict the amplitude and correlation length scale of the resulting IGMF today. Comparing the SGWB level with the sensitivity of LISA and the IGMF parameters with the sensitivity reach of the CTA γ -ray telescope, we identify a range of first-order phase transition parameters providing observable signatures at both detectors.

Results. If even a small fraction of the kinetic energy in sound waves is converted into MHD turbulence, a first-order phase transition occurring at a temperature between 1 and 10^6 GeV can give rise to an observable SGWB signal in LISA and, at the same time, an IGMF compatible with the lower bound from the γ -ray telescope MAGIC, for all proposed evolutionary paths of the magnetic fields throughout the radiation-dominated era (i.e., for both helical and non-helical magnetic fields). For the following fractions of the energy density converted into turbulence, $\varepsilon_{\text{turb}} = 0.1$ and 1, we provide the range of first-order phase transition parameters (strength α , duration β^{-1} , bubble wall speed v_w , and temperature T_*), together with the corresponding range of magnetic field strength B and correlation length λ , which would lead to the SGWB and IGMF being observable with LISA and MAGIC. The resulting magnetic field strength at recombination can also correspond to the one that has been proposed to induce baryon clumping, previously suggested as a possible way to ease the Hubble tension. In the limiting case $\varepsilon_{\text{turb}} \ll 1$, the SGWB is only sourced by sound waves, but an IGMF is still generated with a magnetic field amplitude proportional to $\sqrt{\varepsilon_{\text{turb}}}$. We find that for values as small as $\varepsilon_{\text{turb}} \sim \mathcal{O}(10^{-13})$ or $\mathcal{O}(10^{-9})$, respectively, helical or non-helical primordial magnetic fields provide IGMF compatible with MAGIC's lower bound.

Key words. magnetohydrodynamics (MHD) – cosmology: observations – cosmology: theory – early Universe – large-scale structure of Universe

1. Introduction

Magnetic fields are ubiquitous at all scales in the Universe. In particular, observations from TeV blazars suggest the existence of intergalactic magnetic fields (IGMF) in the cosmic voids of the large scale structure (LSS) with a lower bound in their strength of $B > 1.8 \times 10^{-17}$ gauss at scales $\lambda_B \gtrsim 0.2$ Mpc (Neronov & Vovk 2010; Acciari et al. 2023). Although the origin of these IGMF is still unclear, a seed of cosmological origin might be the most plausible explanation, since astrophysical mechanisms (e.g., via Biermann battery) seem to lack the potential to generate IGMF in cosmic voids with a large volume filling factor (Dolag et al. 2011).

The presence of a primordial magnetic field in the early Universe is, on the other hand, constrained by the Big Bang nucleosynthesis (BBN) limit, first analysed in Shvartsman (1969), as well as by the cosmic microwave background (CMB) experi-

ments (Ade et al. 2016; Galli et al. 2022). Several mechanisms have been proposed that would lead to the generation of magnetic fields from cosmological phase transitions, especially if the phase transition is of the first order; see Durrer & Neronov (2013) and Vachaspati (2021) for reviews and references therein. According to the Standard Model (SM), both the electroweak and the quantum chromodynamics (QCD) transitions have occurred as crossovers in the early Universe (Kajantie et al. 1996; Stephanov 2006). However, extensions of the SM can provide the required conditions for baryogenesis at the electroweak scale and would produce, at the same time, a first-order phase transition; see Caprini et al. (2020) for a review.

First-order phase transitions, as well as turbulent motion, occurring around the electroweak scale, can be responsible for the production of a stochastic gravitational wave background (SGWB); see, for example, Kamionkowski et al. (1994), detectable by the Laser Interferometer Space Antenna (LISA; Amaro-Seoane et al. 2017; Colpi et al. 2024). Due to the high conductivity of the primordial plasma in the early Universe,

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a magnetic field would interact with it, inevitably leading to the development of magnetohydrodynamic (MHD) turbulence (Brandenburg et al. 1996). The production of gravitational waves (GWs) from (M)HD turbulence has been extensively studied (see Caprini & Figueroa 2018 for a review and references therein), and numerical simulations (Roper Pol et al. 2020) have provided the tools to compute and validate an analytical template for the SGWB (Roper Pol et al. 2022a).

Primordial magnetic fields generated at a phase transition, either non-helical (Vachaspati 1991) or helical (Vachaspati 2001), have a characteristic length scale that is limited by the size of the horizon, for example, $\tilde{\lambda}_H \sim 10^{-9}$ Mpc (redshifted to present time due to the expansion of the Universe) for a temperature scale of $T_* \sim 100$ GeV. The magnetic field follows a dynamical evolution driven by decaying turbulence from the time of its generation until the end of the radiation-dominated era. Later, the characteristic turbulent power-law evolution of the magnetic field strength and length scale became logarithmic functions of cosmic time, such that the field evolution at large scales is virtually frozen (Banerjee & Jedamzik 2004; Subramanian 2016).

The details of the field evolution during the radiation-dominated era are not fully understood. If the turbulent decay of non-helical magnetic fields from the electroweak epoch proceeds through the “selective decay” of short-range modes (Banerjee & Jedamzik 2004), its present-day relic strength is expected to be below the lower bound from the γ -ray observations when the redshifted size of the horizon is around 10^{-9} Mpc or smaller (for $T_* \gtrsim 100$ GeV) and the comoving initial field strength is limited by the BBN bound $\tilde{B} \lesssim 10^{-6}$ G (Kahniashvili et al. 2011). In contrast, helicity conservation of decaying helical magnetic fields leads to an inverse cascade that amplifies the magnetic field length scale and the field amplitude at large scales, while the magnetic energy density decays (Banerjee & Jedamzik 2004). Hence, helical fields are compatible with the IGMF inferred from the γ -ray observations for a larger set of initial conditions. However, it has been argued from simulations that a similar large-scale field amplification, although less effective than for fully helical fields, can also occur for non-helical fields (Brandenburg et al. 2015). This has also later been derived using theoretical arguments: Hosking & Schekochihin (2023) propose that the conservation of local helicity fluctuations, characterised by the so-called Hosking integral (Zhou et al. 2022), can lead to an inverse cascade even when the net helicity is zero. Therefore, fields produced around the electroweak scale with any fractional helicity could, in principle, provide a seed field compatible with the γ -ray lower bounds of the IGMF.

In addition, Jedamzik & Pogosian (2020) proposed that a primordial magnetic field present at recombination, with sufficient amplitude, could reduce the sound horizon by inducing baryon clumping, which would affect the estimate of the present-time Hubble constant from CMB data in a way that would relax the “Hubble tension”. Neronov et al. (2023) and Roper Pol et al. (2022a) showed that a non-helical field originating in the QCD phase transition could satisfy the γ -ray lower bound of the IGMF, induce enough baryon clumping (Jedamzik & Pogosian 2020; Galli et al. 2022), and generate the SGWB that has been detected by several pulsar timing array collaborations (Agazie et al. 2023; Antoniadis et al. 2023; Reardon et al. 2023; Xu et al. 2023; see, in particular, Antoniadis et al. 2024, where (M)HD turbulence was analysed as a possible source of the SGWB, and Afzal et al. 2023, where sound waves and bubble collisions were analysed instead).

In the present work, we provide a framework for studying a first-order phase transition with a “multi-messenger” approach. On the one hand, we analyze the detectability by LISA of a SGWB produced around the electroweak scale, from both sound waves (Hindmarsh et al. 2014) and (M)HD turbulence. On the other hand, we analyze the evolution down to recombination of a magnetic field also generated around the electroweak scale, compatible with the aforementioned SGWB, and determine whether it can explain the blazars observations in the cosmic voids of the LSS, the CMB and BBN constraints, as well as the possibility that it can alleviate the Hubble tension. All the results and models presented can be reproduced using the COSMOGW package that is publicly available on GitHub (Roper Pol 2024).

In the following, the notation is such that all characteristic scales and time intervals are physical and, therefore, time-dependent. They are understood to be redshifted when compared with quantities at the present time, for example, with the GW frequency f or with the conformal Hubble factor at the phase transition time, $\mathcal{H}_* \equiv (a_*/a_0)H_*$, H_* being the Hubble rate and a_*/a_0 the ratio of the scale factor at the time of GW production with that at the present time.

2. Models for the stochastic gravitational wave background

We study the potential detectability of an SGWB from a first-order phase transition around the electroweak scale, incorporating updated results on the contributions to the SGWB from sound waves (Hindmarsh & Hijazi 2019; Jinno et al. 2023) and (M)HD turbulence (Roper Pol et al. 2022a). Since it has been found that the electroweak phase transition bubbles hardly run away (Bodeker & Moore 2017), we ignore the contribution to the SGWB from broken-phase bubble collisions.

Following Caprini et al. (2016, 2020), we characterise the phase transition by four parameters.

1. T_* is the phase transition temperature (interpreted as the percolation temperature for a weak to moderately strong thermal phase transition).
 2. β is the rate of change in the nucleation rate of broken-phase bubbles. Its inverse value at the moment of the phase transition can be considered as a characteristic timescale of the phase transition, $t_\beta = \beta^{-1} \lesssim H_*^{-1}$, where H_* is the expansion rate at the epoch of the phase transition.
 3. α characterises the strength of the phase transition: it represents the vacuum energy density in the symmetric phase released in the broken-phase bubbles, in units of the radiation energy density¹ $\alpha = \rho_{\text{vac}}/\rho_{\text{rad}}^*$.
 4. v_w is the velocity of the broken-phase bubbles.
- Apart from these dynamical parameters, the inclusion of (M)HD turbulence in the calculation of the SGWB spectrum adds another parameter:
5. $\varepsilon_{\text{turb}}$ is the fraction of the kinetic energy density induced in the primordial plasma by the expansion and collisions of bubbles that is converted from sound waves into vortical turbulent motion.

In the analysis of possible SGWB spectra from a cosmological first-order phase transition, we scan the four-dimensional parameter space $(T_*, \beta, \alpha, v_w)$ and consider two possible levels

¹ Note that this parameter has also been generalised beyond the bag equation of state using, for example, the trace anomaly of the energy momentum tensor or the difference among the energies in the two phases (Caprini et al. 2020). In practice, for the purpose of this work, all three definitions would be equivalent.

of $\varepsilon_{\text{turb}} = 0.1$ and 1. The case $\varepsilon_{\text{turb}} = 0$, where the SGWB is dominated by sound waves, is analysed by the LISA Cosmology Working Group in [Caprini et al. \(2024\)](#).

The resulting SGWB spectrum is the result of the addition of two contributions: sound waves and (M)HD turbulent plasma motion. However, in the limiting case $\varepsilon_{\text{turb}} = 0$, the contribution of (M)HD turbulence is absent. The interplay between these two contributions is uncertain, and it is an open, active topic of research ([Caprini et al. 2025](#); [Correia et al. 2025](#)). For example, a mixed compressional-vortical mode would appear in the GW spectrum when both sound waves and turbulence are present in the fluid perturbations, which has not yet been studied in detail (see [Correia et al. 2025](#) for a recent numerical study of this term). In this work, we assume that sound waves (compressional motion) produce GWs from the time of bubble collision until the development of nonlinearities at $\tau_{\text{nl}} \sim \lambda_*/v_f$, where v_f is the enthalpy-weighted rms fluid velocity ([Caprini et al. 2024](#)) and λ_* is the characteristic length scale of the fluid motion (see Sect. 2.1). After this time, vortical (M)HD turbulence is produced due to the development of nonlinearities from the initial sound waves, yielding an additional contribution to the SGWB, and the remaining compressional motion is assumed to have dissipated. Therefore, values of $\varepsilon_{\text{turb}}$ smaller than unity would take into account the decay of sound waves from their time of formation to the time of the development of nonlinearities and the potential additional energy dissipation due to the development of turbulence ([Caprini et al. 2025](#)). We assume that the turbulent sourcing of GWs operates only in the decaying phase, which amounts to assuming that the development of turbulence is instantaneous. Since the details of the turbulence generation from the nonlinearities and shocks after the phase dominated by sound waves are unknown, we remove this potential source of uncertainties in the determination of the resulting SGWB ([Roper Pol et al. 2020, 2022b](#); [Brandenburg et al. 2021a](#); [Roper Pol et al. 2022a](#)). Furthermore, we omit the effect of reheated droplets that can be produced when bubbles collide in strong deflagrations, as found in [Cutting et al. \(2020\)](#). This effect can significantly impact the GW amplitude, but it is still uncertain how it depends on the phase transition parameters.

Alternatively, both compressional and vortical motion (i.e., the development of turbulence) could occur directly together at the time of bubble collisions ([Caprini et al. 2025](#); [Correia et al. 2025](#)). In this scenario, the energy density of each contribution would directly come from the energy budget available from the first-order phase transition (for example, a fraction γ of the total kinetic energy transferred to the fluid by the bubble expansion would go into compressional or acoustic modes, and a fraction $1 - \gamma$ into vortical modes). The limiting case $\gamma = 0$, corresponding to only turbulence being produced at the phase transition, was considered in [Roper Pol et al. \(2022a\)](#). This case could model a scenario where the magnetic field is produced by an alternative mechanism, instead of being amplified by the turbulent motion following an initial development of sound waves, for example, when the magnetic field is directly produced by bubble collisions ([Zhang et al. 2019](#)) or driven by a scalar axion-like field ([Miniati et al. 2018](#)). We compare the results of our analysis, in which we always account for the initial compression phase characterised by sound waves, with the results of [Roper Pol et al. \(2022a\)](#) in Sect. 3.

For the sound-wave component, we considered two templates. The first is based on the original sound shell model (SSM) of [Hindmarsh \(2018\)](#) and [Hindmarsh & Hijazi \(2019\)](#), including the correction for short shock times ([Caprini et al. 2020](#)). In this

model, the spectrum has two breaks, one at the frequency corresponding to the inverse mean size of the broken-phase bubbles, λ_*^{-1} , and one at the scale corresponding to the inverse sound shell thickness, $\delta\lambda_*^{-1} \geq \lambda_*^{-1}$. To ensure the correct hierarchy of scales proposed within the SSM, we fixed $v_w > 0.3$, since lower v_w would yield $\delta\lambda_*^{-1} \leq \lambda_*^{-1}$; see, in comparison Eq. (2). The spectral shape is a double broken power law, proportional to f^9 at small $f \lesssim \lambda_*^{-1}$, proportional to f at intermediate $\lambda_*^{-1} \lesssim f \lesssim \delta\lambda_*^{-1}$, and proportional to f^{-4} at large f . Since we rely on analytical templates, in this work we do not account for the fact that the SSM shape, in particular the intermediate slope, likely has a more complicated dependence on v_w , as shown in [Gowling & Hindmarsh \(2021\)](#), [Roper Pol et al. \(2024\)](#).

After a revision of the sound shell model in [Roper Pol et al. \(2024\)](#), the f^9 part of the spectrum has been shown to appear at $f \lesssim \lambda_*^{-1}$ only when the duration of the sound-wave sourcing is long, $\tau_{\text{nl}}/\lambda_* \sim 1/v_f \gg 1$. The authors also show that this steep spectrum is followed by a linear and then a cubic scaling at smaller frequencies. The finding of a cubic scaling below λ_*^{-1} is compatible with numerical studies of phase transitions following the Higgsless (HL) approach ([Jinno et al. 2023](#); [Caprini et al. 2025](#)), with the numerical study of sound waves using the PENCIL CODE ([Brandenburg et al. 2021b](#)) by [Sharma et al. \(2023\)](#), and with the analytical analysis by [Cai et al. \(2023\)](#). Hence, we considered a second template based on these updated results: a spectrum proportional to f^3 is considered at small frequencies, with f and f^{-3} scaling at intermediate and large f , respectively. For simplicity, we omitted the steep spectrum below the peak, since its prominence is dependent on the duration of the sound-wave regime and it is expected to be mostly relevant for weak phase transitions, which are of less importance for a potential detection with LISA. The behavior at large frequencies is not in contradiction with that of [Hindmarsh & Hijazi \(2019\)](#) since their fit asymptotically tends to f^{-4} but their numerical results show a f^{-3} spectrum, their fit being accurate at small and moderate f . The differences between the two templates, which are described in Sect. 2.1, are illustrated in Fig. 1.

The SGWB of the turbulent component is described in Sect. 2.2. It is based on the model presented and validated against simulations of MHD turbulence in [Roper Pol et al. \(2022a\)](#). This model has also been validated for purely kinetic turbulence in [Auclair et al. \(2022\)](#). Although this model has been tested for non-helical fields, the numerical simulations in [Roper Pol et al. \(2022b\)](#) indicate that the SGWB sourced by decaying turbulence does not significantly depend on the fractional helicity. Therefore, we applied this template to the case of helical magnetic fields as well, in particular when we considered the time evolution of the field (cf., Fig. 4).

The turbulent spectrum also presents two breaks. In this case, the first break corresponds to the inverse effective duration of the turbulence $\delta t_{\text{fin}}^{-1}$, and the second is $f_{\text{turb}} \sim O(\lambda_*^{-1})$. The first break in the GW spectrum is from f^3 to f , and the second break (the peak of the spectrum) is from f to $f^{-8/3}$, where $8/3$ is characteristic of Kolmogorov turbulence ([Roper Pol et al. 2020](#)). Note that this template applies under the assumption that turbulence is generated instantaneously. The resulting sound wave and turbulence templates are shown in Fig. 1 for a specific choice of the parameters describing the phase transition. The templates described in this section, both for sound waves and for (M)HD turbulence, have been included in the publicly available Python package COSMOGW ([Roper Pol 2024](#)), where a tutorial is also available to follow the results presented in our work.

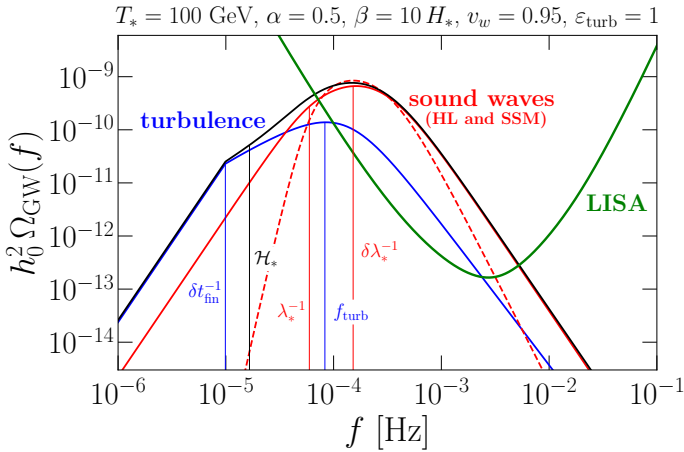


Fig. 1. Spectra of the different components of the SGWB for $T_* = 100$ GeV, $\alpha = 0.5$, $\beta = 10H_*$, $v_w = 0.95$, and $\epsilon_{\text{turb}} = 1$, compared to LISA’s power law sensitivity from Caprini et al. (2020) with an S/N threshold of 10. The SGWB spectra from sound waves are based on the SSM of Hindmarsh & Hijazi (2019, dashed red) and on the fit from HL simulations of Jinno et al. (2023) and Caprini et al. (2025, solid red). The spectrum of the turbulence is based on the model developed and numerically validated in Roper Pol et al. (2022a). The vertical lines indicate the relevant frequencies of the turbulence template ($\delta t_{\text{fin}}^{-1}$ and f_{turb}) in blue, of the sound-wave template (λ_*^{-1} and $\delta\lambda_*^{-1}$) in red, and the conformal Hubble frequency \mathcal{H}_* at the time of GW generation in black.

2.1. Gravitational wave background from sound waves

The template of the spectrum of GWs from sound waves relies on the updated analysis of Hindmarsh & Hijazi (2019), within the framework of the SSM (Hindmarsh 2018). Within this model, the GW power spectrum has two characteristic scales. The first one is the mean size of the broken-phase bubbles at the moment of percolation

$$\lambda_* = \frac{(8\pi)^{1/3} \max(v_w, c_s)}{\beta}, \quad (1)$$

where the sound speed is taken to be $c_s = 1/\sqrt{3}$ during the radiation-dominated era. We added the correction at low v_w , following Caprini et al. (2020) and Ellis et al. (2020). The second scale is the sound shell thickness (Hindmarsh & Hijazi 2019)

$$\delta\lambda_* = \Delta_w \lambda_*, \quad \text{with } \Delta_w = \frac{|v_w - c_s|}{v_w}. \quad (2)$$

We adopted the following analytic expression to approximate the SSM SGWB spectrum (Hindmarsh & Hijazi 2019; Hindmarsh et al. 2021)

$$\Omega_{\text{GW}}^{\text{sw}}(f) = 3 \mathcal{B}(\Delta_w) \frac{(K\lambda_*\mathcal{H}_*)^2}{\sqrt{K} + \lambda_*\mathcal{H}_*} F_{\text{GW}}^0 S_{\text{sw}}(s_1, s_2, \Delta_w), \quad (3)$$

where S_{sw} is the spectral shape, given in Eq. (6) for the SSM and in Eq. (7) for the fit based on HL simulations; $s_1 = \lambda_* f$ and $s_2 = \delta\lambda_* f$ are the frequencies expressed in units of each of the relevant scales. The GW efficiency is $\mathcal{B} = \tilde{\Omega}_{\text{gw}}/\mu(\Delta_w)$, where $\tilde{\Omega}_{\text{gw}} \sim \mathcal{O}(10^{-2})$ is the integrated value of the spectrum, based on numerical simulations (Hindmarsh et al. 2017). The normalisation variable μ is defined to ensure that the integral of S_{sw}/μ over $\ln f$ is unity. It is a function of the sound shell thickness

$$\mu(\Delta_w) = \int_0^\infty S_{\text{sw}}(f, \lambda_*, \Delta_w) d \ln f. \quad (4)$$

K denotes the amount of sound-wave kinetic energy density as a fraction of the total energy density

$$K = \frac{\rho_K}{\rho_{\text{vac}} + \rho_{\text{rad}}^*} = \frac{\kappa_v \alpha}{1 + \alpha}, \quad (5)$$

where $\kappa_v \equiv \rho_K/\rho_{\text{vac}}^*$ is the fraction of the vacuum energy released that is converted to the kinetic energy density of the plasma ρ_K (Espinosa et al. 2010). Therefore, in Eq. (5) we make the implicit assumption that the totality of the kinetic energy available from the phase transition is in the form of sound waves. In the simulations of Hindmarsh et al. (2017), it has been demonstrated that this is a good assumption for small α . We used the results of Espinosa et al. (2010) to calculate κ_v as a function of α and v_w .

The spectral shape S_{sw} in the SSM is a function of the frequency s_2 , expressed in units of the global peak frequency $\delta\lambda_*^{-1}$, i.e., the inverse sound shell thickness

$$S_{\text{sw}}^{\text{SSM}}(s_2, \Delta_w) = s_2^9 \left(\frac{\Delta_w^4 + 1}{\Delta_w^4 + s_2^4} \right)^2 \left(\frac{5}{5 - m + m s_2^2} \right)^{\frac{5}{2}}, \quad (6)$$

where $m = (9\Delta_w^4 + 1)/(\Delta_w^4 + 1)$ is defined so that $S_{\text{sw}}^{\text{SSM}} = 1$ at the peak $s_2 = 1$.

On the other hand, following Jinno et al. (2023), we also considered a double broken power law with slopes 3, 1, and -3 at small, intermediate, and large frequencies, respectively, again expressed in terms of the frequencies s_1 and s_2

$$S_{\text{sw}}^{\text{HL}}(s_1, s_2, \Delta_w) = 16 s_2^3 \frac{(1 + \Delta_w^{-3})^{\frac{2}{3}}}{(1 + s_1^3)^{\frac{2}{3}} (3 + s_2^2)^2}. \quad (7)$$

The spectral shape of this fit is based on the results of the HL simulations of Jinno et al. (2023) and Caprini et al. (2025, see also Stomberg & Roper Pol 2025 for a summary of their results), while the frequency breaks and the amplitude are taken from the SSM. The spectral shape is normalised so that $S_{\text{sw}}^{\text{HL}} = 1$ at the peak $s_2 = 1$.

Finally, the transfer function F_{GW}^0 that considers the redshift of the GW energy density is

$$F_{\text{GW}}^0 = \left(\frac{a_*}{a_0} \right)^4 \left(\frac{H_*}{H_0} \right)^2 \simeq 1.64 h_0^{-2} \times 10^{-5} \left(\frac{100}{g_*} \right)^{\frac{1}{3}}, \quad (8)$$

where h_0 is the current Hubble rate in units of 100 km/s/Mpc.

2.2. Gravitational wave background from (M)HD turbulence

To model the production of GWs from (M)HD vortical turbulence, we followed the results presented in Roper Pol et al. (2022a): we assumed that the onset of turbulence is instantaneous and that, afterward, the GW sourcing stresses are constant in time during the GW production, i.e., for a time that we denoted δt_{fin} (more details of this quantity are given below). This assumption gives a good approximation to the SGWB signal evaluated by numerical simulations in which the turbulence is inserted in the initial conditions, as demonstrated in Roper Pol et al. (2022a) for MHD turbulence and in Auclair et al. (2022) for purely vortical kinetic turbulence. The SGWB spectrum reads

$$\Omega_{\text{GW}}^{\text{turb}}(f) = 3 \mathcal{A} \Omega_*^2 (\lambda_* \mathcal{H}_*)^2 F_{\text{GW}}^0 S_{\text{turb}}(s_1, s_3). \quad (9)$$

The amplitude of the SGWB spectrum scales with the square of the fraction of the total turbulent energy density Ω_* , given

by the sum of the vortical velocity component and the magnetic field. We assumed Ω_* to be a fraction $\varepsilon_{\text{turb}}$ of K ,

$$\Omega_* = \Omega_{B^*} + \Omega_{v^*}^\perp = \frac{\rho_{B^*} + \rho_{v^*}^\perp}{\rho_{\text{vac}} + \rho_{\text{rad}}^*} = \varepsilon_{\text{turb}} K, \quad (10)$$

where ρ_{B^*} is the magnetic energy density, $\rho_{v^*}^\perp$ is the vortical component of the kinetic energy density, and Ω_{B^*} and $\Omega_{v^*}^\perp$ are their ratios to the total energy density. Note that $\Omega_{v^*}^\perp$ refers to the kinetic vortical motion while K , defined in Eq. (5), refers to the kinetic compressional motion in the form of sound waves.

Following [Banerjee & Jedamzik \(2004\)](#), we assumed that the fields are in equipartition, so that $\Omega_{v^*}^\perp \simeq \Omega_{B^*} \simeq \frac{1}{2}\Omega_*$. We defined the characteristic velocity u_* as the maximum of the enthalpy-weighted rms fluid velocity v_f and the Alfvén speed v_A . For a radiation-dominated Universe with pressure $p = c_s^2 \rho_{\text{rad}} = \frac{1}{3}\rho_{\text{rad}}$, the subrelativistic Alfvén speed v_A is ([Brandenburg et al. 1996](#); [Roper Pol & Midiri 2025](#))

$$v_A^2 = \frac{\langle B^2 \rangle}{\langle \rho_{\text{rad}} + p \rangle} = \frac{2\Omega_B}{(1 + c_s^2)} = \frac{3}{2}\Omega_B, \quad (11)$$

and v_f , considering only the vortical motion v_\perp , is

$$v_f^2 = \frac{\langle w v_\perp^2 \rangle}{\langle w \rangle} = \Omega_v^\perp, \quad (12)$$

where $w = \rho_{\text{rad}} + p$ is the enthalpy of the fluid. From Eqs. (11) and (12), we find that the characteristic velocity u_* , assuming equipartition, is

$$u_* = \sqrt{\max\left(\Omega_{v^*}^\perp, \frac{3}{2}\Omega_{B^*}\right)} \simeq \sqrt{\frac{3}{4}\Omega_*}. \quad (13)$$

The parameter δt_{fin} represents the effective turbulence duration, and we express it in terms of a multiple \mathcal{N} of the eddy turnover time $\delta t_{\text{fin}} = \mathcal{N}\lambda_*/u_*$. According to the numerical simulations in [Roper Pol et al. \(2022a\)](#), one can set $\mathcal{N} \simeq 2$. However, note that the value of \mathcal{N} is a subject of ongoing study. The effective turbulence duration determines the position of the first break in the SGWB spectrum, occurring at $f \sim \delta t_{\text{fin}}^{-1}$.

The SGWB spectral shape in Eq. (9) is given by $S_{\text{turb}}(s_1, s_3)$, where $s_3 = \delta t_{\text{fin}} f$ denotes the frequency in units of the inverse effective duration of the turbulence. We recall that $s_1 = \lambda_* f$: as usual, we assume that the initial correlation length of the turbulence corresponds to the mean size of the broken-phase bubbles at the onset of their collisions, λ_* , given in Eq. (1). This determines the position of the global peak of the spectrum, located at $s_{\text{turb}} \equiv \lambda_* f_{\text{turb}} \gtrsim 1$. The exact value of s_{turb} depends on the spectral shape of the anisotropic stresses $p_\Pi(s_1)$; see Eqs. (17)–(19).

The spectral shape S_{turb} is determined both by the anisotropic stress power spectrum itself, and by how the latter sources the GWs in time. Assuming that the sourcing process is constant in time over the duration δt_{fin} provides the following SGWB spectral shape

$$S_{\text{turb}}(s_1, s_3) = \frac{4\pi^2 s_1^3 \mathcal{T}_{\text{GW}}(s_1, s_3)}{(\lambda_* \mathcal{H}_*)^2} \frac{p_\Pi(s_1)}{s_{\text{turb}} p_\Pi(s_{\text{turb}})}, \quad (14)$$

with \mathcal{T}_{GW} being ([Roper Pol et al. 2022a](#))

$$\mathcal{T}_{\text{GW}}(s_1, s_3) = \begin{cases} \ln^2[1 + \mathcal{H}_* \delta t_{\text{fin}} / (2\pi)], & \text{if } s_3 < 1, \\ \ln^2[1 + \lambda_* \mathcal{H}_* / (2\pi s_1)], & \text{if } s_3 \geq 1. \end{cases} \quad (15)$$

S_{turb} is normalised so that its maximum is approximately one. Its exact value at the peak f_{turb} , where $s_3 = \delta t_{\text{fin}} f_{\text{turb}} > 1$, is

$$S_{\text{turb}}(s_{\text{turb}}) = 4\pi^2 \left(\frac{s_{\text{turb}}}{\lambda_* \mathcal{H}_*} \right)^2 \ln^2 \left[1 + \frac{\lambda_* \mathcal{H}_*}{2\pi s_{\text{turb}}} \right], \quad (16)$$

which asymptotically tends to 1 for $\lambda_* \mathcal{H}_* \ll 2\pi s_{\text{turb}}$. Since $s_{\text{turb}} \gtrsim 1$ and $\lambda_* \mathcal{H}_* \leq 1$, one can compute the value of $S_{\text{turb}}(s_{\text{turb}})$ at $\lambda_* \mathcal{H}_* = 1$, to get $S_{\text{turb}}(s_{\text{turb}}) \gtrsim S_{\text{turb}}(1) \simeq 0.86$. As $\lambda_* \mathcal{H}_*$ decreases, $S_{\text{turb}}(1)$ quickly tends to 1, with a relative error below 1.6% when $\lambda_* \mathcal{H}_* < 0.1$. Hence, the assumption that S_{turb} is one at the frequency peak is accurate for most relevant cases.

The power spectrum of the anisotropic stresses, $p_\Pi(s_1)$, depends on the power spectrum of the kinetic turbulence and the magnetic field. Here we assume that they follow a von Kármán spectral shape² ([von Kármán 1948](#)), characterised by a Batchelor spectrum f^4 at low f , expected from causality ([Durrer & Caprini 2003](#)), and a turbulent Kolmogorov $f^{-5/3}$ spectrum at large f . Within this assumption, the normalised (i.e., $p_\Pi(0) = 1$) anisotropic stress power spectrum can be approximated as

$$p_\Pi(s_1) \simeq \left[1 + \left(\frac{s_1}{s_\Pi} \right)^{\alpha_\Pi} \right]^{-\frac{11}{3\alpha_\Pi}}, \quad (17)$$

where $\alpha_\Pi \simeq 2.15$ and $s_\Pi = 2.2$. Compared to the numerical computations in [Roper Pol et al. \(2022a\)](#) and [Auclair et al. \(2022\)](#), the maximum relative error of the analytical fit of Eq. (17) is less than 6% for all frequencies $s_1 \leq 10$. In larger s_1 , the relative error of the fit increases. However, since $p_\Pi \sim s_1^{-11/3}$, at higher frequencies $p_\Pi(s_1 > 10)$ has decayed below $\mathcal{O}(10^{-3})$.

Having normalised both S_{turb} and p_Π , the GW efficiency from (M)HD turbulence, \mathcal{A} in Eq. (9), becomes ([Roper Pol et al. 2022a](#))

$$\mathcal{A} = \frac{112}{15} \sqrt{\frac{3}{5\pi^5}} \frac{\Gamma[\frac{13}{6}]\Gamma^2[\frac{17}{6}]}{\Gamma[\frac{17}{3}]\Gamma^2[\frac{1}{3}]} s_{\text{turb}} p_\Pi(s_{\text{turb}}) \simeq 2 \times 10^{-3} s_{\text{turb}} p_\Pi(s_{\text{turb}}), \quad (18)$$

where s_{turb} , the peak frequency of the GW spectrum, is located where $s_1 p_\Pi(s_1)$ is maximal

$$s_{\text{turb}} = \lambda_* f_{\text{turb}} = s_\Pi \left(\frac{3}{8} \right)^{\frac{1}{\alpha_\Pi}} \simeq 1.4, \quad (19a)$$

$$p_\Pi(s_{\text{turb}}) = \left(\frac{11}{8} \right)^{-\frac{11}{3\alpha_\Pi}} \simeq 0.6. \quad (19b)$$

Substituting these values into Eq. (18), the GW efficiency becomes

$$\mathcal{A} \simeq 1.75 \times 10^{-3}. \quad (20)$$

Note that [Roper Pol et al. \(2022a\)](#) uses a different notation, where $s_{\text{turb}} \equiv k_{\text{GW}}/k_*$, and our parameter \mathcal{A} corresponds to $s_{\text{turb}} p_\Pi(s_{\text{turb}}) C / (8\pi^2 \mathcal{A}^2)$ in their Eq. (24).

3. The SGWB and the magnetic field, relics of a first-order phase transition

In this section, we investigate which regions in the parameter space of a first-order phase transition would lead to a SGWB

² The von Kármán spectrum can be expressed using, for example, Eq. (6) in [Roper Pol et al. \(2022a\)](#) with $\alpha = 6/17$.

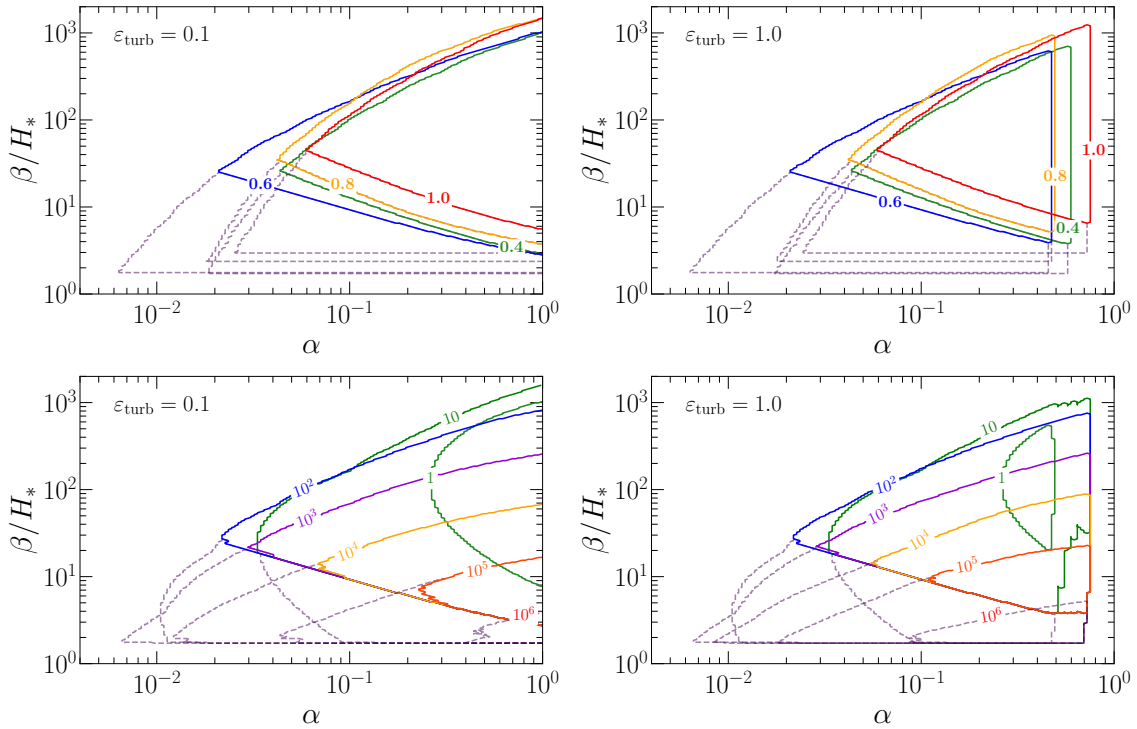


Fig. 2. Contours of the parameters α and β/H_* that lead to a signal above LISA’s power law sensitivity with a S/N threshold of 10 (see Fig. 1). The top panels take a fixed value of the wall velocity $v_w = \{0.4, 0.6, 0.8, 1\}$ (indicated in different colors) and scan over temperatures to construct the contours, while the lower panels fix the temperature in GeV $T_* = \{10, 10^2, 10^3, 10^4, 10^5, 10^6\}$ and scan over wall velocities to construct the contours. Left and right panels correspond to $\varepsilon_{\text{turb}} = 0.1$ and 1, respectively. The results shown correspond to the sound-wave template based on the fit from the Higgsless simulations (cf., Sect. 2.1), as this choice does not affect the magnetic field parameters in Fig. 3. Colored contours correspond to regions of the parameter space where $H_*\tau_{\text{nl}} \lesssim 1$, while their continuation by dashed contours corresponds to regions where $H_*\tau_{\text{nl}} \gtrsim 1$.

detectable with LISA. We also compute the strength and correlation length scales of the corresponding magnetic field, which would result from the amplification of a seed field by the MHD turbulence in the aftermath of the phase transition. We are ultimately interested in assessing whether a first-order phase transition could produce a SGWB detectable with LISA and, at the same time, an IGMF consistent with existing observational bounds. We therefore perform a systematic scan over the first-order phase transition parameter space. The SGWB is considered “detectable” by LISA if its spectrum exceeds the power law sensitivity (PLS) of LISA with a S/N threshold of $S/N = 10$ (Caprini et al. 2020) at any frequency $10^{-5} \text{ Hz} < f < 0.1 \text{ Hz}$. An illustrative example of the SGWB spectrum from turbulence and sound waves with LISA’s sensitivity for the choice of parameters $T_* = 100 \text{ GeV}$, $\alpha = 0.5$, $\beta = 10H_*$, $v_w = 0.95$, and $\varepsilon_{\text{turb}} = 1$ is shown in Fig. 1. In this work, we focus on multi-messenger probes of a first-order phase transition and primordial magnetic fields, and we provide tools for the analysis that have been added to the COSMOGW code (Roper Pol 2024). For a more elaborate parameter reconstruction of the SGWB from first-order phase transitions with LISA, we refer to Caprini et al. (2024).

We performed the analysis for two values of $\varepsilon_{\text{turb}}$, the parameter characterising the relative amount of kinetic energy in (M)HD turbulence; see, for comparison, Eq. (10): $\varepsilon_{\text{turb}} = 0.1$ and 1. The ranges of the phase transition parameters are defined as follows. The broken-phase bubble wall speed is in the range $0.3 \leq v_w \leq 1$, where the lower limit ensures that $\lambda_* > \delta\lambda_*$, and the hierarchy of breaks in the SSM SGWB spectral shape follows, as described in Sect. 2. The upper limits on α are computed such that the ratio of magnetic to total energy density, Ω_{B*} , does not exceed 10%, as imposed by BBN (Shvartsman

1969; Kahniashvili et al. 2011; Kawasaki & Kusakabe 2012)³. The lower limits on β/H_* are set so that the resulting length scale is causal $\lambda_*\mathcal{H}_* \lesssim 1$. Furthermore, T_* , α , and β/H_* satisfy $1 \text{ GeV} \lesssim T_* \lesssim 3 \times 10^6 \text{ GeV}$, $\alpha \gtrsim 5 \times 10^{-3}$, and $\beta/H_* \lesssim 2 \times 10^3$, as inferred by the results of the analysis of the detectability of the SGWB signal.

For each point in the 4D parameter space, we reconstructed the SGWB signal and compared it with the sensitivity of LISA. Figure 2 shows the detectable regions in the $(\alpha, \beta/H_*)$ plane in two ways: i) for different wall velocities v_w , by combining the results for all temperatures T_* in the detectable range (upper panels), and ii) for different temperatures (in GeV), by combining the results for all v_w (bottom panels). The results shown in Fig. 2 are derived using the HL fit for the sound-wave template of Eq. (7), since using the SSM template, given in Eq. (6), does not lead to any appreciable difference when we compute the resulting magnetic field parameters.

It is usually argued that turbulence is not produced if the timescale to develop nonlinearities from sound waves is larger than one Hubble time $H_*\tau_{\text{nl}} \sim H_*\lambda_*/v_f \gtrsim 1$ (cf., Caprini et al. 2020). According to this criterion, we show in Fig. 2 the contours of the regions where turbulence is expected to develop by colored lines, while the dashed lines correspond to the contours of the regions in the parameter space that would be, in principle, excluded by this condition.

³ Relaxing the BBN limit has been recently proposed in Kahniashvili et al. (2022) by considering the turbulence decay from the magnetic field generation scale to BBN. However, we still restrict Ω_{B*} in our analysis to avoid relativistic Alfvén and velocity speeds (Roper Pol & Midiri 2025), a regime in which the SGWB templates have not been validated.

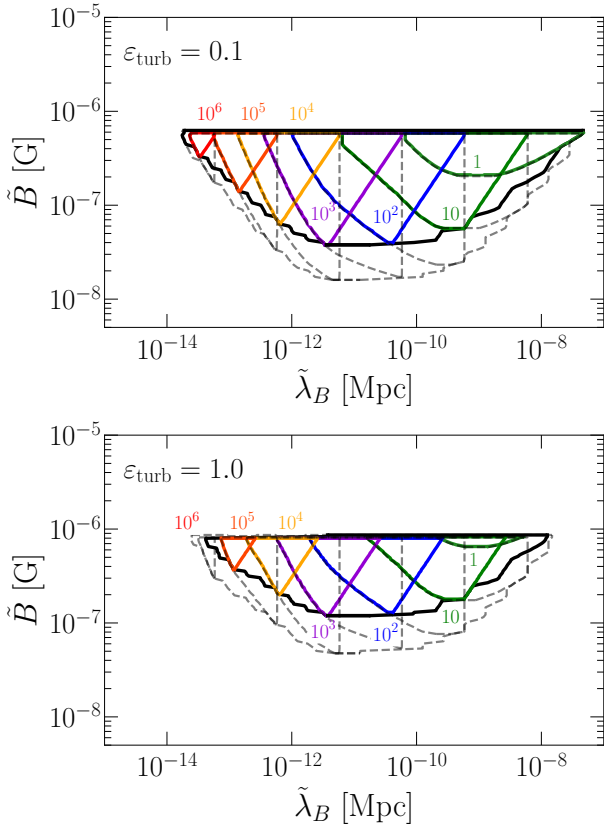


Fig. 3. Range of the comoving magnetic field strength \tilde{B} and correlation length $\tilde{\lambda}_B$ corresponding to the contours in the lower panels of Fig. 2, for $\varepsilon_{\text{turb}} = 0.1$ (top panel) and 1 (bottom panel). The colored numbers indicate the value of the temperature T_* in GeV. For values of $\varepsilon_{\text{turb}}$ smaller than 0.1, the range of observable phase transition parameters is unaffected, and the contour of magnetic field parameters just shifts to smaller amplitudes proportional to $\sqrt{\varepsilon_{\text{turb}}}$.

Plasma motion induced by the first-order phase transition can excite (M)HD turbulence, which could amplify any preexisting seed magnetic fields, for example, generated by charge separation at the bubble walls (Sigl et al. 1997). Turbulence is expected to drive the magnetic field energy density to equipartition with the plasma kinetic energy density (Banerjee & Jedamzik 2004; Durrer & Neronov 2013). The equipartition argument suggests that the initial magnetic field configuration at the end of the phase transition is $B_* \simeq \sqrt{2} \Omega_{B*} \rho_{\text{rad}}^* \simeq \sqrt{\Omega_* \rho_{\text{rad}}^*}$ [see Eq. (11)], where we assume that the total energy density is $\rho_{\text{vac}} + \rho_{\text{rad}} \simeq \rho_{\text{rad}}^*$ for weak and moderately strong phase transitions. After fixing the value of $\varepsilon_{\text{turb}}$, using Eqs. (5) and (10) we can relate the magnetic field strength B_* at the phase transition time, assuming that both the turbulence generation and the establishment of equipartition are instantaneous, to the fraction of vacuum energy that is converted to kinetic energy κ_w , and ultimately to α using the prescription of Espinosa et al. (2010). Furthermore, the initial magnetic field correlation length would correspond to the characteristic fluid scale λ_* , which is related to v_w , $c_s = 1/\sqrt{3}$, and β/H_* via Eq. (1).

Therefore, for each combination of the phase transition parameters analysed above, we can infer the magnetic field parameters B_* and λ_* . As explained in Sect. 4, it is meaningful to express B_* and λ_* in terms of comoving quantities, \tilde{B} and $\tilde{\lambda}_B$, as these are the initial conditions for the magnetic field evolution, i.e., the starting points in the magnetic field evolutionary paths.

In Fig. 3 we therefore show the ranges of the magnetic field parameters \tilde{B} and $\tilde{\lambda}_B$ that correspond to the ranges of the phase transition parameters given in the bottom panels of Fig. 2, i.e., for different values of T_* . Since $K < 1$ and due to equipartition, the values of Ω_{B*} are limited to $0.5 \varepsilon_{\text{turb}}$. In addition, we have limited its value to 10% of the total energy density because of the BBN bound and the condition of non-relativistic turbulence. Therefore, in all the analyses presented $\Omega_{B*} \leq \min(0.1, 0.5 \varepsilon_{\text{turb}})$.

It appears from Fig. 3 that a phase transition with parameters providing a SGWB accessible to LISA, and $\varepsilon_{\text{turb}} = 1$ (upper panel), can lead to fairly strong magnetic fields with strengths up to $\tilde{B} \sim 10^{-6}$ G, corresponding to the limit $\Omega_{B*} \sim 0.1$. For $\varepsilon_{\text{turb}} = 0.1$ (bottom panel), the upper limit $\Omega_{B*} \sim 0.05$ yields instead a maximal field strength of $\tilde{B} \sim 6 \times 10^{-7}$ G. The initial correlation length of the field can vary over a wide range, between 10^{-14} and 10^{-7} Mpc, depending on the temperature of the phase transition T_* , its duration β^{-1} , and the velocity of the bubble wall v_w . The region of parameter space where $H_* \tau_{\text{nl}} \gtrsim 1$, indicated in Fig. 2 by the dashed contours at small α and β/H_* , corresponds in Fig. 3 to the region where \tilde{B} is small and/or $\tilde{\lambda}_B$ is large. As in Fig. 2, the contours in Fig. 3 correspond to the sound-wave template based on the fit from the Higgsless simulations (cf., Sect. 2.1). Using the SSM template would not lead to appreciable differences in the magnetic field parameters.

4. The magnetic field evolution and the IGMF today

After its generation at the phase transition, the magnetic field evolves following the laws of turbulent decay up to the recombination epoch (cf., Durrer & Neronov 2013). Depending on the initial strength B_* , the initial correlation length λ_* , and the initial fractional helicity h , different turbulent evolutionary paths will lead to different IGMF today. In Fig. 4 we report, as initial conditions for the turbulent evolution, the regions in the parameter space given by the comoving initial strength \tilde{B} and correlation length $\tilde{\lambda}_B$ that are compatible with a SGWB detectable by LISA, according to the analyses presented in Sect. 3 (Figs. 2 and 3). Since the resulting SGWB signal is expected to depend very weakly on the fractional helicity (Roper Pol et al. 2022b), the initial condition regions are considered to be valid both for non-helical and helical magnetic fields. The black and gray-shaded areas correspond to $\varepsilon_{\text{turb}} = 1$ and 0.1, respectively. The orange-shaded region corresponds to the one found in Roper Pol et al. (2022a), where the limiting case was analysed, in which no sound waves are formed and all the kinetic energy from the plasma motion is in the form of MHD turbulence. It is clear that including the sound-wave contribution to the SGWB spectrum enlarges the range of magnetic field parameters compatible with a detectable GW signal.

Examples of possible evolutionary paths for helical and non-helical fields are also shown in Fig. 4 by inclined lines starting at the boundaries of the initial condition regions, with arrows pointing at the parameter values expected to be reached at recombination. The evolutionary paths depend on the magnetic field helicity. Helical magnetic fields with maximal helicity $h = 1$ are expected to follow the evolutionary path with $\tilde{B} \propto \tilde{\lambda}_B^{-1/2}$ in the parameter space $(\tilde{B}, \tilde{\lambda}_B)$, due to helicity conservation (see dash-dotted purple lines in Fig. 4; Banerjee & Jedamzik 2004; Durrer & Neronov 2013). The decay laws of non-helical fields are not entirely understood. The hypothesis of selective decay of short-range modes would lead to the decay law $\tilde{B} \propto \tilde{\lambda}_B^{-5/2}$, while the hypothesis of conservation of the Hosking invariant (Hosking & Schekochihin 2023; Zhou et al. 2022) suggests the

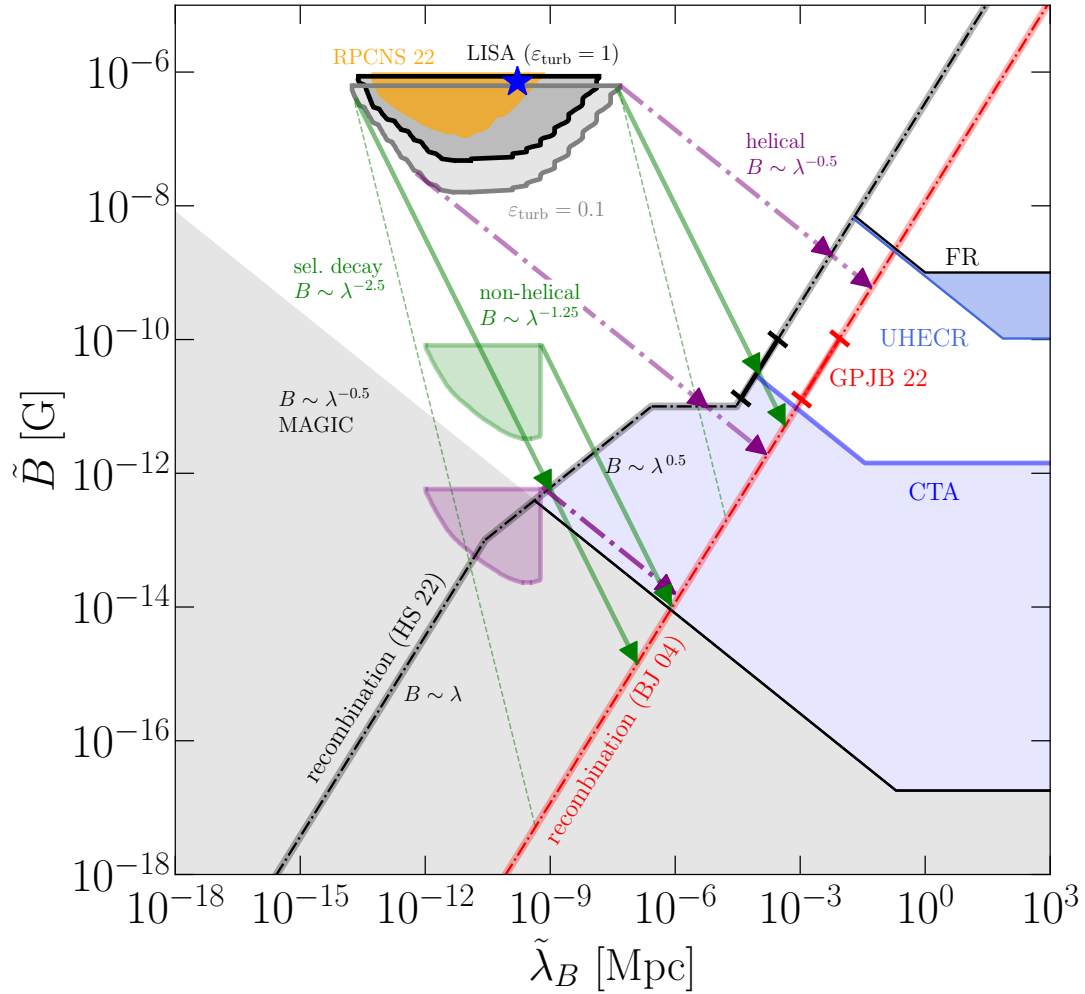


Fig. 4. Expected initial and final comoving strength and correlation length of a cosmological magnetic field generated at a first-order phase transition with parameters leading to an SGWB detectable by LISA. The black and grey contours in the upper left corner of the figure show the parameter space of initial conditions if one assumes that, respectively, the totality ($\epsilon_{\text{turb}} = 1$) and 10% ($\epsilon_{\text{turb}} = 0.1$) of the sound-wave kinetic energy is eventually transferred to MHD turbulence (i.e., magnetic and vortical kinetic energy) at the time of nonlinearities development. The orange-shaded region is the one found in Roper Pol et al. (2022a, RPCNS 22), assuming that the SGWB is sourced exclusively by MHD turbulence. The blue star corresponds to the phase transition parameters for which the SGWB spectrum is shown in Fig. 1, considered for illustrative purposes. The inclined arrows show the envelopes of the evolutionary tracks. The dash-dotted purple arrows apply to helical fields, $B \sim \lambda^{-0.5}$, as given in Banerjee & Jedamzik (2004); the solid green arrows apply to non-helical fields, $B \sim \lambda^{-1.25}$, as given in Hosking & Schekochihin (2023); the dashed green arrows correspond to the hypothesis of selective decay of short-range modes for non-helical fields, $B \sim \lambda^{-2.5}$ (Banerjee & Jedamzik 2004). The dash-dotted black and red lines show the possible endpoints of the magnetic field evolution, corresponding to the IGMF present in the voids today, assuming respectively that the reconnection timescale dominates the magnetic field dynamics (Hosking & Schekochihin 2023) (HS 22) and that the Alfvénic timescale dominates the magnetic field dynamics (Banerjee & Jedamzik 2004) (BJ 04). The gray-shaded region at the bottom left of the plot is excluded by the lower bound on the IGMF established by the MAGIC γ -ray observatory (Acciari et al. 2023). The thin dark blue and black lines show the upper limit on the IGMF from, respectively, ultra-high-energy cosmic rays (Neronov et al. 2023) and Faraday rotation measures (Pshirkov et al. 2016). The blue-shaded area shows the range of IGMF parameters that will be probed by the γ -ray observatory CTA (Korochkin et al. 2021). The red and black ticks over the BJ 04 and HS 22 recombination lines correspond to the range of magnetic field strengths obtained in Galli et al. (2022), which would induce enough baryon clumping to help alleviate the Hubble tension, as proposed in Jedamzik & Pogosian (2020). The green and purple areas denote, respectively, the range of non-helical and helical magnetic field parameters that would arise from a first-order phase transition occurring at $T_* \sim 100$ GeV and sourcing a SGWB detectable by LISA, fixing the smallest possible value ϵ_{turb} while still satisfying MAGIC’s lower bound. These values of ϵ_{turb} are $O(10^{-9})$ (non-helical) and $O(10^{-13})$ (helical).

decay law $\tilde{B} \propto \tilde{\lambda}_B^{-5/4}$ (respectively, dashed and solid green lines in Fig. 4).

The endpoint of the evolution is also uncertain, depending on whether the evolution timescale is determined by the Alfvénic (Banerjee & Jedamzik 2004) or the reconnection speed (Hosking & Schekochihin 2023). A straightforward estimate of the largest processed eddies at recombination suggests that the modes with $\tilde{\lambda} \lesssim v_A t_{\text{rec}}$ are processed by turbulence (Banerjee & Jedamzik 2004), with v_A being the Alfvén velocity,

defined in Eq. (11), and t_{rec} the Hubble time at recombination. In this case, setting $\tilde{\lambda}_B \equiv v_A t_{\text{rec}}$, the final strength and the correlation length satisfy the relation (Durrer & Neronov 2013)

$$\tilde{B} \simeq 10^{-8} \left[\frac{\tilde{\lambda}_B}{1 \text{ Mpc}} \right] \text{G}, \quad (21)$$

shown by the thick dash-dotted red line in Fig. 4. On the other hand, the evolution model put forward in

Hosking & Schekochihin (2023) proposes that the relevant timescale for the turbulent evolution is the one determined by reconnection, and that the decay law for non-helical fields is governed by the conservation of the Hosking integral, related to local fluctuations of helicity. Within this model, the correlation length at recombination can be smaller than the estimate in Eq. (21) by a factor of 20 for magnetic fields with final strength higher than 10^{-11} G, and even smaller for weaker fields—as much as by a factor of $10^{5.5}$ when the field strength is below 10^{-13} G (Hosking & Schekochihin 2023). The possible endpoints of the reconnection-driven evolution are represented by the thick dash-dotted black line in Fig. 4.

Figure 4 shows that the strength and correlation length of the IGMF resulting from the initial conditions leading to an SGWB detectable by LISA vary over a wide range: 10^{-9} Mpc $< \tilde{\lambda}_B < 10^{-1}$ Mpc and 10^{-17} G $< B < 10^{-9}$ G, depending on the fractional helicity h , on the uncertainty about the evolutionary path of non-helical fields, and on the uncertainty about the locus of the evolutionary endpoints.

It is also interesting to note that the phase transition parameters $(\alpha, \beta, v_w, T_*, \varepsilon_{\text{turb}})$ providing a detectable SGWB correspond to a relic IGMF that could be probed with γ -ray observations or through its imprint on recombination. Given the uncertainty of the magnetic field helicity $0 < h < 1$, none of the $(\alpha, \beta, v_w, T_*, \varepsilon_{\text{turb}})$ combinations detectable by LISA are currently ruled out by known upper bounds on the magnetic field inferred from the CMB (Ade et al. 2016), Faraday rotation (FR) measures (Pshirkov et al. 2016), or ultra-high-energy cosmic rays (UHECRs; Neronov et al. 2023). It has been proposed that primordial magnetic fields would also affect the CMB anisotropies through the production of small-scale baryonic density fluctuations (Jedamzik & Saveliev 2019). This leads to a stringent upper limit on the magnetic field amplitude, shown by the upper ticks on the solid red and black lines in Fig. 4 (Jedamzik & Saveliev 2019; Galli et al. 2022). According to this upper limit, a small range of the parameters detectable by LISA (i.e., large B_* and λ_* or, equivalently, large α , small β/H_* , and small T_*) would be ruled out if the magnetic field is helical.

Remarkably, within the evolution model of Hosking & Schekochihin (2023), all initial conditions corresponding to a detectable SGWB would also result in an IGMF consistent with the lower bounds of the MAGIC γ -ray observations (Neronov & Vovk 2010; Acciari et al. 2023). Moreover, if the magnetic field is non-helical, the resulting field strength and correlation length would be within the sensitivity reach of the next-generation γ -ray observatory CTA (Korochkin et al. 2021), again for all initial conditions compatible with a detectable SGWB. Only a present-day IGMF with strength $\tilde{B} \gtrsim 10^{-11}$ G may not be accessible to CTA: this would correspond to most of the initial condition parameter space if the magnetic field is helical. However, in this case, the field should have left an observable imprint at recombination, detectable in the CMB (Jedamzik & Saveliev 2019; Galli et al. 2022). In any case, within the evolution model of Hosking & Schekochihin (2023), first-order phase transitions providing a detectable SGWB would have a complementary multi-messenger signature, either in the γ -ray or in the CMB data, even if the fraction of kinetic energy transferred to MHD turbulence is minimal.

This does not occur, however, within the evolution model of Banerjee & Jedamzik (2004). This model proposes that the Alfvén speed determines the relevant timescale for the turbulent evolution, and that non-helical fields decay following the hypothesis of selective decay of short-range modes. Following this evolution, a non-helical magnetic field could have a

present-day strength that is below the γ -ray lower bound. In this case, γ -ray observations would not rule out the corresponding $(\alpha, \beta, v_w, T_*, \varepsilon_{\text{turb}})$ combination; they just imply that the observed IGMF must have been produced by a different process and that the field generated by the phase transition is not detectable on top of the dominant IGMF.

The range of IGMF parameters that correspond to a field strength in recombination that could help alleviate the Hubble tension via baryon clumping (Jedamzik & Pogosian 2020; Galli et al. 2022), is particularly interesting. It is indicated by the GPJB 22 label and the red interval in Fig. 4, if one assumes that the Alfvénic speed determines the final values at recombination (Banerjee & Jedamzik 2004); and by the black interval if one considers the relevant timescale to be that of reconnection (Hosking & Schekochihin 2023). We find that for almost any beyond-the-Standard-Model scenario resulting in a first-order phase transition that produces an SGWB detectable by LISA, there exists some fractional helicity h , for which the magnetic field generated at the phase transition evolves into a sufficient strength at recombination to induce enough baryon clumping and to help alleviate the Hubble tension. In this case, by combining a possible estimation of the magnetic field at recombination inferred from CMB data, with the other possible estimations inferred from the detection of a SGWB with LISA, one could fix the start and end points of the evolutionary path of the magnetic field and, in this way, provide a measurement of the initial helicity of the field.

Finally, note that the transfer from the sound-wave kinetic energy to MHD turbulence does not need to be very efficient to guarantee the multi-messenger signature. Even if almost all of the energy in sound waves dissipated before leading to the development of nonlinearities and MHD turbulence, i.e., $\varepsilon_{\text{turb}} \ll 1$, then the SGWB would simply be dominated by the sound-wave contribution, and it would still be detectable for appreciable ranges of the phase transition parameters $(\alpha, \beta, v_w, T_*)$. Correspondingly, even if $\varepsilon_{\text{turb}} \ll 1$, there may still exist regions in the magnetic field parameter space that are compatible with the lower bounds of γ -ray observations. Fixing $T_* = 100$ GeV, we find that this happens provided that $\varepsilon_{\text{turb}} \gtrsim \mathcal{O}(10^{-13})$ for non-helical magnetic fields, and $\varepsilon_{\text{turb}} \gtrsim \mathcal{O}(10^{-9})$ for helical fields. These initial conditions are shown, respectively, by the green and purple regions in Fig. 4.

5. Conclusions

We analysed the possibility of obtaining a multi-messenger signature of a cosmological first-order phase transition if it leads to both a stochastic gravitational wave background detectable by LISA, and the generation of a primordial magnetic field that: i) leaves an imprint at recombination observable in the CMB data, and ii) populates the voids of the LSS at present time with a strong enough intergalactic magnetic field to satisfy the lower bounds from γ -ray observations. This multi-messenger signature could be present even if a tiny fraction of the sound-wave kinetic energy is transferred to MHD turbulence.

Adopting a phenomenological description of the phase transition in terms of five parameters $(\alpha, \beta, v_w, T_*, \varepsilon_{\text{turb}})$, we found the ranges of these parameters for which the gravitational wave background is within LISA's sensitivity reach (Fig. 2). For each of such parameter combinations, we estimated the initial strength and correlation length of the primordial magnetic field that could be generated at the phase transition (Fig. 3). From these initial conditions, we traced the field evolution in time until the present-day Universe, depending on the initial fractional helicity

of the field (Fig. 4). To do so, we adopted two possible evolution scenarios, proposed in Banerjee & Jedamzik (2004) and in Hosking & Schekochihin (2023). We find that in the latter case, all first-order phase transition parameter combinations that provide a gravitational wave background detectable by LISA result in a magnetic field that would be detectable either via γ -ray observations, for example, with the CTA telescope, or through its imprint on the CMB. If the cosmological evolution of the magnetic field follows the model of Banerjee & Jedamzik (2004), it may happen that the resultant magnetic field from the phase transition is too weak to explain the intergalactic magnetic field.

Data availability

The templates of the different contributions (i.e., sound waves and turbulence) to the SGWB, the analysis tools and results of this work (accompanied by a tutorial), and the different constraints on the IGMF, are available as part of the open-source, public Python package COSMOGW, stored on GitHub (Roper Pol 2024). The tutorial is available [here](#).

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