

The caustic method applied to The Three Hundred: Prospects for the upcoming CATARSIS and other surveys

B. Callejas-Córdoba^{1,2} , P. Sánchez-Blázquez^{1,2} , A. Gil de Paz^{1,2} , A. Knebe^{3,4,5} , W. Cui^{3,4,6} , C. Catalán-Torrecilla^{1,2} , and R. Dave⁶ 

¹ *Departamento de Física de la Tierra y Astrofísica, Facultad de Ciencias Físicas, Universidad Complutense de Madrid, Plaza de las Ciencias 1, Madrid, Spain*

² *Instituto de Física de Partículas y del Cosmos (IPARCOS-UCM), Facultad de Ciencias Físicas, Universidad Complutense de Madrid, Plaza de las Ciencias 1, 28040 Madrid, Spain*

³ *Departamento de Física Teórica, Facultad de Ciencias, Universidad Autónoma de Madrid, 28049 Madrid, Spain*

⁴ *Centro de Investigación Avanzada en Física Fundamental (CIAFF), Facultad de Ciencias, Universidad Autónoma de Madrid, 28049 Madrid, Spain*

⁵ *International center for Radio Astronomy Research, University of Western Australia, 35 Stirling Highway, Crawley, Western Australia 6009, Australia*

⁶ *Institute for Astronomy, University of Edinburgh, Royal Observatory, Blackford Hill, Edinburgh, EH9 3HJ, UK*

ABSTRACT

We investigate the expected uncertainties involved when recovering galaxy cluster mass profiles from upcoming spectroscopic survey data, based on The Three Hundred Project. Using the caustic technique, which leverages galaxy positions and line-of-sight velocities, we assessed the systematic errors introduced by assumptions regarding velocity anisotropy and demonstrated how an iterative correction method can minimize these errors. We also assessed the impact of survey magnitude limits on cluster mass estimates, highlighting potential biases across different observational strategies. We focused the analysis on our own CATARSIS survey, which is set to obtain redshift measurements for all galaxies with magnitudes of $m_{AB,r} < 22$ within $2 \times R_{200,c}$ for 16 galaxy clusters with redshifts of $0.14 < z < 0.27$. We used the future 8 arcmin² field-of-view TARSIS integral-field spectrograph of the Calar Alto 3.5-m telescope. These data will enable us to mitigate systematic errors in the determination of density profiles. CATARSIS is aimed at enhancing the precision of mass profile estimates by deepening our understanding of the dynamical states and physical characteristics of galaxy clusters.

Key words. Galaxy clusters, caustic technique, cosmological simulations

1. Introduction

The observational determination of the masses of galaxy clusters is of central importance to our understanding of the growth of structure in the Universe and the use of clusters as cosmological probes. Measuring the distribution of mass within galaxy clusters (i.e., the mass profiles) not only enables us to elucidate the nature of dark matter (i.e., warm vs. cold particles; [Blumenthal et al. 1984](#)), but also to distinguish among possible alternatives to the existence of this component, as in the case of modified gravity theories (see, e.g., [Milgrom 2002](#); [McGaugh 2015](#)). The determination of mass profiles in clusters is also an essential reference point for studies of astrophysical processes shaping the properties of the baryons in clusters, within the intracluster medium (ICM) and its member galaxies.

Galaxy cluster masses can be derived using a variety of observational techniques. Each approach has specific assumptions and systematic limitations. Techniques based on dynamical equilibrium, such as X-ray surveys, Sunyaev-Zel'dovich (SZ) determinations ([Sarazin 1988](#)), and Jeans analyses ([The & White 1986](#); [Merritt 1987](#); [Binney & Tremaine 2008](#); [Łokas & Mamon 2003](#); [Łokas et al. 2006](#); [Mamon & Boué 2010](#)), can become unreliable even within $R_{200,c}$ when probing the outskirts of cluster. This is where the assumption of equilibrium starts to break down and clusters continue to accrete galaxies and dark mat-

ter ([Ludlow et al. 2012](#); [Bakels et al. 2021](#)). In contrast, weak gravitational lensing ([Bartelmann 2010](#); [Umetsu et al. 2020](#)) and the caustic technique ([Diaferio & Geller 1997](#); [Diaferio 1999](#); [Serra et al. 2010](#)) provide reliable mass estimates at large radii without requiring equilibrium assumptions. In addition, several techniques based on galaxy tracers have been developed, including mass-richness relations, optical probability distribution functions (OPDF; [Li et al. 2021](#)), intra-cluster light (ICL; [Contreras-Santos et al. 2024](#)), and machine-learning methods ([de Andres et al. 2024](#)), which offer complementary approaches to estimating cluster masses from optical data.

X-ray observations often suffer from biases due to nonthermal pressure sources such as gas accretion, AGN feedback, substructures, turbulence, and cosmic rays, leading to mass underestimations of 10–30% ([Rasia et al. 2012](#); [Nelson et al. 2014](#)). Weak lensing is affected by the presence of interlopers and projection effects, due to the triaxial shape of dark matter halos, which can lead to overestimated masses by up to 40% ([Corless et al. 2009](#); [Becker & Kravtsov 2011](#); [Feroz & Hobson 2012](#)). For completeness, it is worth noting that other mass proxies, such as the integrated SZ signal, can also suffer from limitations, particularly in the cluster outskirts where the signal-to-background ratio becomes low, making SZ-based mass estimates less reliable ([Yoon et al. 2022](#)).

The caustic method provides a dynamical way to estimate cluster masses from the galaxy velocity field by identifying the escape velocity profile in projected phase space. The caustic technique, based on galaxy redshift data, offers an alternative by mapping the positions and velocities of cluster members, revealing a characteristic “trumpet-shaped” profile that traces the mass distribution without assuming equilibrium (Diaferio & Geller 1997; Diaferio 1999; Serra et al. 2010; Gifford et al. 2013). However, the method depends on a calibration factor, $F_\beta(r)$, which encodes the line-of-sight (LoS) projection of the velocity anisotropy profile $\beta(r)$. Adopting a constant or a single average anisotropy for all clusters can introduce systematic biases: simulation studies and empirical tests report overestimates of the enclosed mass in the inner regions of order tens of percent (commonly ~ 30 – 70% at small radii in some tests) and a typical per-cluster scatter of ~ 30 – 50% depending on spectroscopic sampling and projection effects (Diaferio 1999; Serra et al. 2010; Munari et al. 2013; Pizzardo et al. 2023). The bias decreases for well-sampled clusters; namely, with $N_{\text{gal}} \geq 150$ the average bias can be $\lesssim 5\%$, whereas for sparse samples (tens of members), both the bias and scatter rise sharply (Gifford et al. 2013). To mitigate the bias introduced by adopting a constant anisotropy profile, previous works have calibrated an average $F_\beta(r)$ from numerical simulations (Serra et al. 2010; Gifford et al. 2013). Even after this calibration, an additional uncertainty on the order of a few tens of percent must be included to account for residual anisotropy assumptions and projection effects.

Comparative studies have revealed discrepancies between methods. Maughan et al. (2016) found that X-ray masses were 20% higher than caustic masses, potentially due to assumptions about the caustic filling factor (Diaferio 1999; Serra et al. 2010; Gifford et al. 2013). Foëx et al. (2017) studied 10 galaxy clusters and found that Jeans, caustic, and virial mass estimates to be 20%, 30%, and 50% higher than X-ray masses, respectively, with differences diminishing when accounted for substructures. Weak lensing provides direct mass estimates that do not rely on dynamical or hydrostatic equilibrium, making it a critical benchmark for calibrating other techniques. Systematic differences of order ~ 10 – 30% have been reported between weak-lensing and X-ray or dynamical mass estimates, particularly at the cluster outskirts; that is, beyond the virial radius where equilibrium assumptions break down (Hoekstra 2003; Hoekstra et al. 2011; Umetsu et al. 2011; Umetsu 2013; Umetsu et al. 2020). Weak lensing and the caustic technique offer complementary approaches for studying cluster outskirts, with an agreement ranging ~ 20 – 30% (Diaferio et al. 2005; Geller et al. 2013).

Large spectroscopic surveys, such as Cluster Infall Regions in the Sloan Digital Sky Survey (CIRS) and Hectospec Cluster Survey (HeCS), have characterized the mass profiles of ~ 130 galaxy clusters (Rines & Diaferio 2006; Rines et al. 2013). Pizzardo et al. (2021) using the caustic method. Upcoming wide-field surveys with new spectroscopic instruments (Dalton et al. 2012; Tamura et al. 2016; McClintock et al. 2019) and multiwavelength facilities will provide cluster samples selected through scaling relations (e.g., SZ, X-ray luminosity). However, they will require calibrations of systematic effects using refined mass measurements from weak lensing and caustics to achieve robust cosmological constraints.

Cosmological simulations play a crucial role in understanding the formation and evolution of galaxy clusters. Obtaining representative galaxy clusters in hydrodynamical simulations is challenging because within typical cosmological volumes, the number of massive clusters is small. Therefore, dedicated zoom-in simulations are often performed, selecting halos from dark-

matter-only runs and resimulating the cluster regions with higher resolution and baryonic physics (e.g., TNG Cluster, THE THREE HUNDRED project, henceforth, The300; Nelson et al. 2024; Cui et al. 2018). Current simulations enable studies of galaxy clusters under controlled conditions, making it possible to carry out an assessment of biases in mass determination introduced by observational limitations, such as the number of galaxies available, sample selection, or spectroscopic measurement errors (Diaferio 1999; Serra et al. 2010; Gifford et al. 2013; de Andres et al. 2022; Gianfagna et al. 2022; Gianfagna et al. 2023; Ferragamo et al. 2023; Iqbal et al. 2025).

In this paper, we use the numerical simulations of The300 to present a systematic study of the accuracy of galaxy cluster density profiles recovered with the caustic technique. While similar analyses have been performed in the past, our work provides a more detailed assessment of the impact of sample selection, explicitly accounting for the observational constraints of the CATARSIS survey and enabling direct comparisons with other spectroscopic surveys. The galaxy population has been resolved down to stellar mass of $M_\star \gtrsim 10^8 M_\odot$ in the simulations, which is sufficient to study the expected performance of mass estimators; future surveys will probe galaxies of even lower masses. In Sects. 2 and 3, we describe the main characteristics of the CATARSIS survey and THE300 simulations. Sections 4 and 4.2 summarize the caustic method and the modifications introduced in this work, while Sect. 5 applies this method to the expected data that will be collected by CATARSIS. Section 6 summarizes the prospects for other ongoing and future spectroscopic surveys focused on galaxy clusters. Finally, we summarize our findings in Sect. 7. Throughout the paper, we use the Planck cosmological parameters (Planck Collaboration et al. 2016), with $\Omega_{m0} = 0.3089$, $\Omega_{\Lambda 0} = 0.6911$, and $H_0 = 67.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

2. CATARSIS

The Calar Alto Tetra-Armed Super-IFU Spectrograph Survey (CATARSIS; Gil de Paz et al. 2024) new legacy program will be carried out with the 3.5-m telescope at the Calar Alto Observatory (Spain), using the dedicated TARSIS instrument. This integral-field spectrograph (IFS) has a $2.8 \times 2.8 \text{ arcmin}^2$ field of view (FoV) and 2 arcsec spatial resolution, consisting of four spectrographs that collectively cover the wavelength range 320–810 nm. CATARSIS is designed as a blind spectroscopic survey of galaxy clusters in the redshift range $0.14 < z < 0.27$, sampling systems with a broad range of masses and dynamical states. The planned deep exposures will provide an unprecedented number of spectroscopic members, yielding ~ 500 – 1000 redshifts for clusters of mass $(5\text{--}10) \times 10^{14} M_\odot$ at the targeted redshifts. With CATARSIS, we can observe galaxies down to a limiting magnitude of $m_{r,\text{lim}} \sim 21 \text{ mag}$, without any photometric preselection, as the survey is carried out using blind wide-field IFS observations. This approach also enables the recovery of redshifts for dust-reddened cluster members, which are often misclassified as background galaxies in photometrically selected samples. The use of an IFS will therefore significantly reduce the biases introduced by other surveys that rely on the brightest galaxies to confirm cluster membership and are affected by fiber collisions or slit-placement constraints.

CATARSIS aims to deliver major contributions to our understanding of galaxy evolution in dynamically evolving environments. Its scientific goals include probing the nature of dark matter and dark energy, testing cosmological models, and investigating the role of the environment in shaping galaxy properties.

3. The300

THE300 project¹ (Cui et al. 2018) consists of cluster-scale zoom simulations based on a comprehensive sample of the 324 most massive dark matter halos ($M_{\text{vir}} \gtrsim 8 \times 10^{14} h^{-1} M_{\odot}$) selected from the MultiDark simulation (MDPL2; Klypin et al. 2016). MDPL2 assumes cosmological parameters from Planck (Planck Collaboration et al. 2016) and has a periodic cube of comoving length $1 h^{-1} \text{Gpc}$.

For each cluster halo, a cubic region of $15 h^{-1} \text{Mpc}$ comoving size was resimulated at higher resolution using different galaxy formation approaches, including hydrodynamical codes with varying sub-grid physics prescriptions, GADGET-MUSIC (Sembolini et al. 2013), GADGET-X (Rasia et al. 2015; Beck et al. 2016), and (more recently) GIZMO-SIMBA (Cui et al. 2022), as well as semi-analytical models (SAMs). The high-resolution regions were sampled at the mass resolution of the original MDPL2 simulations, using the GINNUNGAGAP code², while the mass of the gas particles is $M_{\text{gas}} = 2.36 \times 10^8 h^{-1} M_{\odot}$. The surrounding regions are represented with progressively lower-resolution particles, allowing the large-scale tidal field to be retained at lower computational cost.

Among its many interesting contributions, THE300 has made significant efforts in obtaining accurate mass estimates and quantifying hydrostatic-equilibrium biases (Ansarifard et al. 2020; Li et al. 2021, 2022; Gianfagna et al. 2022). The collaboration also investigates various aspects of galaxy clusters, including their relationships with connecting filaments (Kuchner et al. 2021; Rost et al. 2021), cluster feeding (Kotecha et al. 2022; Kuchner et al. 2021), backplash galaxies (Haggar et al. 2020; Knebe et al. 2020), and the virial shock radius (Anbajagane et al. 2022; Baxter et al. 2021). Other hydrodynamic simulations further allow detailed studies of cluster properties such as density profiles (Mostoghiu et al. 2019; Li et al. 2020), substructure, baryon content (Arthur et al. 2019; Haggar et al. 2021; Mostoghiu et al. 2021), dynamics, morphologies (Capalbo et al. 2021; De Luca et al. 2021; Zhang et al. 2022; Capalbo et al. 2025), and ICM thermalization (Sayers et al. 2021; Sereno et al. 2021). Additional research addresses the effects of mergers on the properties of brightest cluster galaxies (BCGs; Contreras-Santos et al. 2022) and the effect of environment on galaxy properties (Wang et al. 2018). The project further explores self-interacting dark matter and chameleon gravity, with continuous updates ensuring the robustness and breadth of the simulations.

We based our analysis on the run using the hydrodynamical code GIZMO-SIMBA. Our choice is primarily motivated by two considerations. First, the presence of baryons affects the mass profiles and the mass–concentration relation of halos, which are directly relevant for our analysis. Second, the GIZMO-SIMBA run provides galaxy colors and luminosities that are in better agreement with observations for the calibration adopted in THE300 (Cui et al. 2022). Since an important part of our analysis is focussed on evaluating systematic effects arising from magnitude limits and colour-based selection biases, this run is particularly well suited to the purposes of this work. We emphasize, however, that this choice is tailored to the specific goals of this study. Other runs within THE300 (either hydrodynamical or

semi-analytic) might be more appropriate for different scientific objectives. The run was performed with the GIZMO code (?) and the sub-grid galaxy formation models from the SIMBA simulation (Davé et al. 2019), which include radiative cooling, star formation, and its associated feedback mechanisms, as well as the formation and growth of supermassive black holes. It also incorporates multiple modes of black-hole feedback, which are essential for accurately representing the impact of these massive objects on their host galaxies and the surrounding environment. The parameters of the SIMBA models have been recalibrated for the THE300 runs as described in Cui et al. (2022) due to the lower numerical resolution of the runs compared to the original SIMBA one. Figure 1 shows a 2D projection of two clusters, highlighting the distribution of dark matter and baryonic components at $z = 0.276$.

4. The caustic technique

The caustic technique, as described by Diaferio & Geller (1997), Diaferio (1999), Serra et al. (2010), and Serra & Diaferio (2013), infers the mass profile of a galaxy cluster from the “trumpet-shaped” envelope (i.e., caustics) traced by member galaxies in projected phase space, $r, v_{\text{esc,los}}(r)$. The caustic amplitude, $\mathcal{A}(r)$, approximates the escape velocity at each radius, $v_{\text{esc,los}}^2(r)$, which allows one to infer the enclosed mass under the hypothesis of spherical symmetry,

$$GM(< r) - GM(r_0) = \int_{r_0}^r \mathcal{A}^2(x) \mathcal{F}_{\beta}(x) dx. \quad (1)$$

The function $\mathcal{F}_{\beta}$ is related to the gravitational potential and the anisotropy profile $\beta = 1 - \langle v_{\theta}^2 + v_{\phi}^2 \rangle / 2 \langle v_r^2 \rangle$. (Diaferio & Geller 1997; Serra et al. 2010). This is expressed as

$$\mathcal{F}_{\beta} = -2G\pi g(\beta(r)) \frac{\rho(r)r^2}{\Phi(r)}, \quad (2)$$

$$g(\beta(r)) = \frac{3 - 2\beta(r)}{1 - \beta(r)}. \quad (3)$$

The projected $v_{\text{esc}}(r)$ surface is traced by applying kernel density estimation (KDE) techniques to the distribution of dynamical tracers in the r - v phase space. These tracers are subject to measurement uncertainties in terms of both the position and velocity in observational data. In this study, we considered spectroscopic errors of $\sim 10 \text{ km s}^{-1}$ (equivalent to $0.12 h^{-1} \text{Mpc}$ in normalized coordinates) and astrometric uncertainties of $0.006 h^{-1} \text{Mpc}$, which are the values expected in the CATARSIS survey. Following Gifford et al. (2013), we adopted a fixed multidimensional Gaussian kernel for the KDE implementation, with the bandwidths along r and v directions determined independently from the data sampling (see Silverman 1986). This is expressed as

$$K(r, v) = \left(\frac{4}{3N} \right)^{1/5} \sigma_{r,v}, \quad (4)$$

where N is the number of dynamical tracers in the total phase-space and $\sigma_{r,v}$ is the dispersion along the radial or velocity dimensions. This adaptive kernel ensures that regions with few

¹ <https://www.nottingham.ac.uk/astronomy/The300/index.php>

² GINNUNGAGAP generates initial conditions for zoom-in simulations by identifying the Lagrangian region of a halo and sampling it at the desired mass resolution while embedding it in the larger-scale volume. More information is available at: <https://ginnungagapgroup.github.io/ginnungagap/>

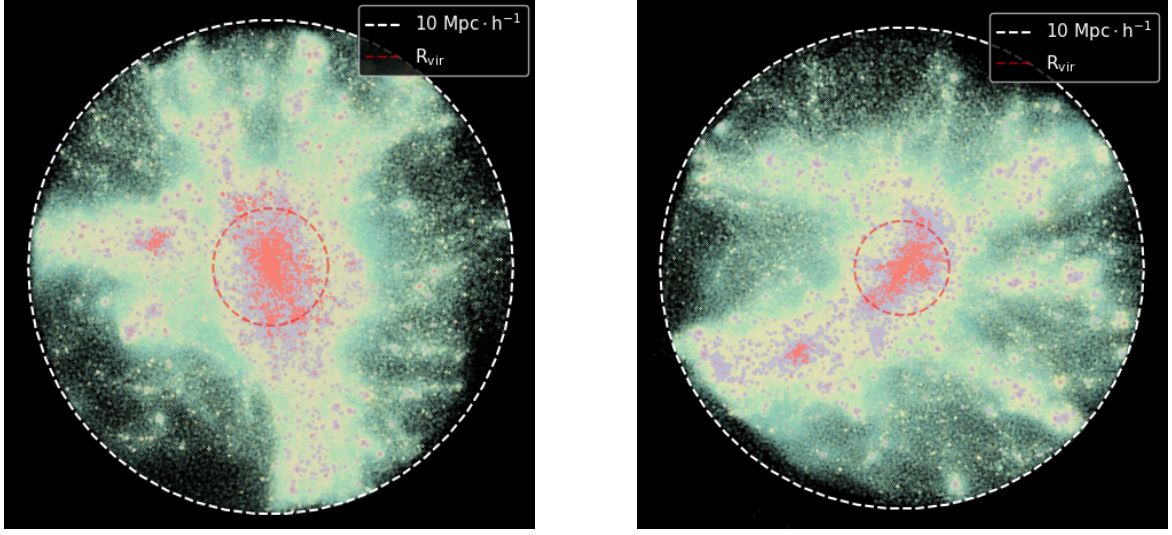


Fig. 1. 2D projection of the NewMDCLUSTER_0002 and NewMDCLUSTER_0006 clusters onto the XY plane at snapshot 117 ($z = 0.276$). The figure shows the spatial distribution of dark matter and baryonic particles within each cluster. The stellar component is shown in pink, the gas component in purple, and the dark matter particles in green. The simulations were performed using the GIZMO-Simba code, a hybrid hydrodynamic and N-body solver designed for cosmological simulations. The clusters are centered in the simulation box for reference. The red dashed line indicates the virial radius (R_{vir}) of each cluster.

tracers are smoothed more heavily, while densely sampled regions are smoothed less, maintaining a balanced representation of the phase-space density.

The ratio between the sizes of the smoothing windows in position and velocity, q , balances the relative contribution of these parameters in the identification of the caustic (Diaferio & Geller 1997; Geller et al. 1999) and needs to be adjusted to the observational errors. We used the expected errors in velocity and position for the CATARSIS survey and adopted $q = 21$. Previous studies (see, e.g. Geller et al. 1999; Rines et al. 2000, 2002) have shown that the derived cluster mass profile is relatively insensitive to the exact choice of q , as long as it remains within a reasonable range. In Sect. 4.1, we discuss the impact of these parameters in the determination of the caustic. Next, a physically motivated constraint was applied to the escape velocity profile, $v_{\text{esc}}(r)$. According to Diaferio (1999), any realistic galaxy cluster model must satisfy

$$\frac{d \ln v_{\text{esc}}}{d \ln r} \leq \zeta, \quad (5)$$

where ζ limits how steeply the escape velocity can change with radius. The rationale is that very steep gradients in $v_{\text{esc}}(r)$ would be unphysical, corresponding to unrealistic mass distributions. If an iso-density contour violates this constraint along its surface, the corresponding $v_{\text{esc}}(r)$ value is replaced with a corrected one.

In practice, following Serra et al. (2010), we adopted a looser limit of $\zeta = 2$, rather than the more conservative $\zeta = 1/4$ suggested by Diaferio (1999). This choice allows for greater flexibility in identifying the caustics in realistic cluster data, accounting for observational noise and sampling effects, while still ensuring that the inferred escape velocity profiles remain physically plausible.

To derive cluster mass profiles with the caustic method, an accurate determination of the cluster center is required. In numerical simulations, the center of a halo is often defined as the minimum of the gravitational potential. However, alternative definitions are also used. For instance, some studies adopt the

position of the most bound particle, which might be slightly offset from the potential minimum, while halo finders such as AHF³ (Knollmann & Knebe 2009) identify the center as the peak of an adaptively smoothed density field (see, e.g., Cui et al. 2016). However, as our aim here is to assess biases relevant to forthcoming observations, we estimated the center using a KDE approach, which infers the projected surface density from the discrete positions of observed galaxies. The cluster center is defined as the location of the global maximum (mode) of the resulting density field. When using only the coordinates, it is assumed that the number of halos (and not their mass) can trace the density field of the cluster. To check the influence of this assumption in the determination of the center, we applied a method using a weighting with the luminosity of galaxies as a proxy for the mass. Figure 2 shows an example of the position of the center determined with the different methods.

The figure also displays the projected density map of a cluster obtained with both KDE implementations. In the luminosity-weighted case, brighter galaxies have a stronger influence on the density estimate, leading to a more concentrated distribution. However, the inferred cluster centers in both cases differ by less than 2% for the whole sample. The centers obtained with this method deviate only slightly from the position of the most bound particle. Across the full cluster sample, the median offset is 7% for the standard KDE and 5% for the luminosity-weighted KDE, with corresponding 84th-percentile values of 17% and 14%, respectively. This error in the center position has a negligible effect on the determination of the caustics (see Serra et al. 2010).

We note that the results might depend on the choice of the LoS due to the nonspherical nature of galaxy clusters. In this work, we adopt a single LoS (i.e., along the simulation box Z-axis) to provide an assessment of the performance of the caustic method in The300 simulations from a representative set of observational data. A more detailed analysis, including projection effects along different sightlines, substructure, rotation, has been

³ <http://popia.ft.uam.es/AHF>

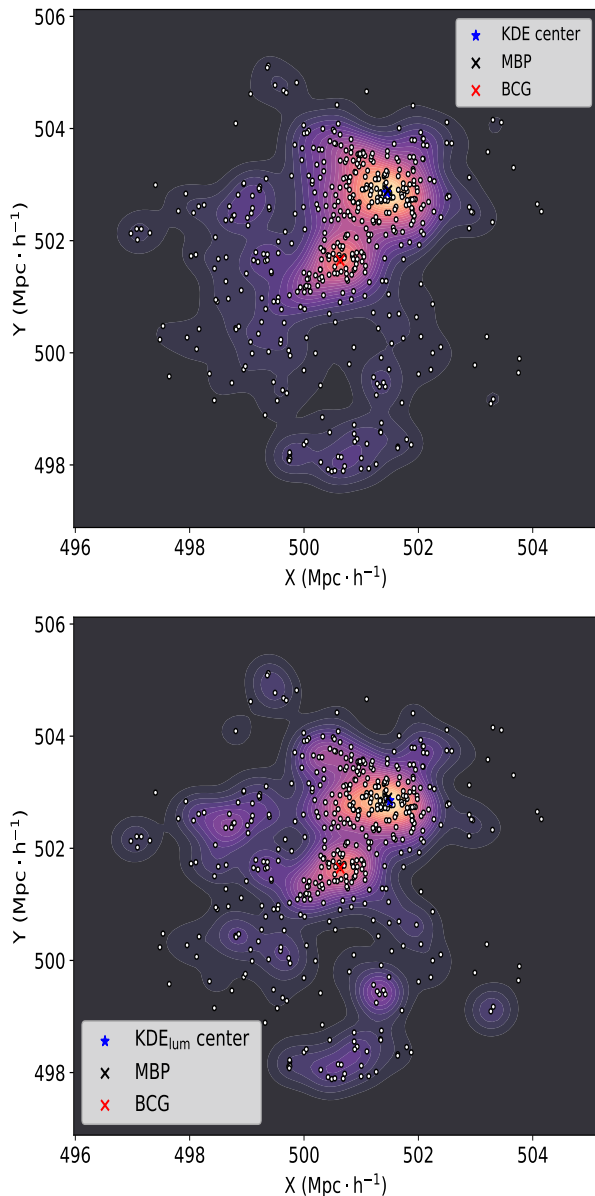


Fig. 2. Projected maps obtained with the KDE method for the cluster NewMDCLUSTER_0013 at redshift $z=0.276$. In the top panel only the positions of the galaxies have been used, while the bottom panel shows the results of weighting the positions with galaxy luminosities. The contours represent isodensity levels, with higher density regions indicated by the innermost lines. The star marks the center obtained from the KDE method, while the black cross indicates the bound more particle (BMP) center, identifying the most gravitationally bound region of the cluster, and the red cross shows the BCG center. The white dots correspond to galaxies in the cluster.

left for future work, where we will aim to improve the caustic method beyond spherical assumptions.

The projected positions were used to build the surface density maps, while the LoS velocity component, including the Hubble flow, was adopted to construct the redshift-space phase-space diagram. This setup provides a consistent and observationally motivated representation of each simulated cluster, serving as the basis for the application of the caustic technique. In this

work, we use the galaxy catalogs provided by CAESAR⁴, which include all galaxies identified within each cluster halo, without applying an additional membership selection. We note that in contrast to the simulations, the identification of true members in observations is considerably more uncertain. The impact of observational selection effects and FoV limitations is discussed in Sects. 5 and 6.

4.1. Impact of smoothing kernel size

Although some studies have reported that the choice of smoothing kernel only has a slight impact on the determination of density profiles (see, e.g., Rines et al. 2002), we find that this effect varies from cluster to cluster and, in a significant fraction of our sample, it exerts a non-negligible influence on the caustic determination. The smoothing is typically parameterized by the scale of the position kernel s and by the ratio between the velocity and position kernels q . Although the typical positional uncertainties are $\sim 0.02 h^{-1} \text{ Mpc}$, s can be converted to km s^{-1} using the Hubble constant, so that it can be directly compared with the velocity kernel when computing $q = h_v/h_r$. Here, we investigate whether an optimal combination of these parameters exists that minimizes the errors in mass determination and whether suitable values can be inferred for individual clusters based on their observed properties.

We first explored the distribution of derived masses for different combinations of s and q , with s ranging from 0.01 to 0.1 in increments of 0.01 and q from 5 to 50 in increments of 5. Figure 3 shows, for one representative cluster, a two-parameter map of the masses within r_{200} and r_{500} . As illustrated, several parameter combinations yield local minima, corresponding to better mass estimated, but these do not converge toward a well-defined region in parameter space. Moreover, the optimal combinations are not always consistent between the two mass estimated.

To quantify the impact of these parameters, we compared the mean mass values obtained over the full grid of (q, s) combinations with those derived from the optimal pair (i.e., the one yielding the smallest difference with respect to the true mass). Figure 4 illustrates these comparisons. The deviation between the mean and optimal values provides an empirical measure of the robustness of the mass determination with respect to the kernel parameters: a small deviation indicates that the result is stable across the explored parameter space, while larger deviations reveal a stronger dependence on the particular parameter choice. The 20% threshold shown as the shaded region in Fig. 4 was adopted as a practical criterion to distinguish between robust and parameter-sensitive cases. This value roughly represents the typical variation observed across the cluster sample and serves as a convenient reference for identifying outliers whose mass estimates are particularly sensitive to the adopted parameters.

Figure 5 summarizes this analysis for all clusters in the sample. For each cluster, we identified the combination of (q, s) that minimizes the difference between the recovered and true mass, and the corresponding best-fitting values. Although regions of parameter space around this minimum can yield similarly good fits, we report the single best-fitting pair for clarity and consistency across the sample. In general, smaller angular-distance kernel sizes yield better results when combined with a redshift kernel that is either five or fifty times larger, depending on the cluster. However, the behavior varies significantly from cluster to cluster and no clear trend could be established.

⁴ <https://caesar.readthedocs.io/en/latest/>

Finally, we examined the possible relationships between the best (q, s) combinations and other cluster properties, such as mass and richness. Figures 6a and 6b show that the optimal smoothing kernels do not correlate with these parameters. Although the KDE distributions show a global minimum at lower values of q and s , several local minima are also present, corresponding to different parameter combinations. The optimal values therefore vary across clusters. We were able to further quantify this by computing the correlation coefficients between the best-fit parameters (q, s) and cluster properties (mass and richness), finding no significant correlations ($\rho \leq 0.3$). Consequently, the current analysis does not allow us to identify a unique pair of (q, s) values that performs optimally for all galaxy clusters.

4.2. Impact of the anisotropy profile: An iterative method to improve the caustic determination

The velocity anisotropy profile, $\beta(r)$, plays a crucial role in the determination of the caustic amplitude (see Eq. 3), as it governs the relation between the observed LoS velocity distribution and the underlying escape velocity profile. Previous studies have often adopted simplified prescriptions, either assuming a constant anisotropy (e.g., Diaferio 1999; Gifford et al. 2013) or imposing a fixed radial dependence motivated by theoretical models or by averaging $\beta(r)$ over halo samples from numerical simulations (e.g., Mamon & Lokas 2005).

All these approaches present important limitations. A constant anisotropy neglects the expected radial variation of galaxy orbits, while adopting a fixed functional form may introduce systematic biases if it does not accurately represent the dynamical structure of the specific cluster under study. To address these limitations, we adopted an iterative approach to estimate cluster masses. We began by assuming a constant anisotropy and applying the caustic method to derive the mass and density profiles. The concentration parameter was then obtained by fitting a Navarro-Frenk-White (NFW) profile to the inferred density distribution.

Using these initial results, we recomputed the caustic profile adopting two radially varying anisotropy models: (i) a data-driven profile derived directly from the definition of $\beta(r)$ and (ii) the analytical prescription proposed by Mamon & Lokas (2005). The empirical $\beta(r)$ profile was constructed by dividing the galaxies into logarithmically spaced radial bins from the cluster center, considering all bound galaxies within $R_{\max}$, and computing the ratio between the tangential and radial velocity dispersions in each bin (see Fig. 7).

We find that both anisotropy profiles provide a good description up to R_{200} . Beyond this radius, the theoretical profile shows a decrease that coincides with the transition between the virialized region and the surrounding infall zone, where galaxy orbits become increasingly radial. In this outer region, the number of tracer galaxies per radial bin is smaller, which increases the statistical uncertainty of β but does not systematically bias its value. At larger radii, the profile rises again as the number of sources increases and the infall motion becomes more dominant. Figures 8 and 9 show the resulting $R_{200,c}$ and $M_{200,c}$ values obtained with the iterative caustic method.

As can be seen, the results obtained when applying the iterative method better reproduce the true values. For the full sample of 324 clusters, the noniterative approach systematically overestimates the recovered masses. Using a constant $F_\beta = 0.65$, we obtain a median ratio $M_{200,c}/M_{200,true} = 1.16$, corresponding to a bias of +16% with a scatter of 0.23. When adopting the fit-

ted F_β profile ($c = 5$), the median ratio decreases to 1.12, with a bias of +12% and a scatter of 0.22. In contrast, the iterative method significantly reduces both the bias and the scatter. Using the fitted F_β profile, we obtain $M_{200,c}/M_{200,true} = 0.94$, with a bias of -6% and a scatter of 0.18. Using the Mamon & Lokas anisotropy model, the recovered ratio is 0.93, with a bias of -7% and a scatter of 0.2. This improvement is particularly relevant for the least massive systems, where the noniterative method can overestimate the masses by factors of ~ 1.5 –2. The following results are based on the iterative method employed together with our anisotropy profile.

5. Bias in the mass determination due to sample selection

In observational studies of galaxy clusters, the selection of galaxies used to trace the gravitational potential can introduce biases in the mass determination. These biases arise from various factors, including the preferential use of red sequence (RS) galaxies, magnitude-limited samples, and line-flux cuts, which can systematically over- or under-estimate cluster masses. In this section we investigate how different selection criteria affect the mass estimates obtained with the caustic method. Specifically, we sought to examine: (i) the impact of restricting the analysis to RS galaxies and (ii) the effects of observational limits in apparent magnitude and $H\alpha$ flux, which influence the detectability of galaxies with different star formation rates. Our goal is to quantify these biases and to assess how the planned CATARSIS survey, with its deep, unbiased spectroscopic observations, will mitigate them.

5.1. Red sequence

In spectroscopic surveys of galaxy clusters, it is common practice to prioritize galaxies lying on the RS. Selecting this population, clearly identifiable as a tight locus in color-magnitude space, helps to reduce contamination from foreground and background objects, improves the efficiency of spectroscopic target selection. However, focusing solely in these galaxies, this method can introduce a bias that typically leads to an underestimation of the true cluster mass as these objects are abundant near the dense cluster central regions⁵. To quantify this effect, we isolated RS galaxies as those located within a $\pm 5\%$ interval of our best-fitting RS in the $m_g - m_r$ versus m_r color-magnitude diagram. Besides objects in the blue cloud and the green valley, this strategy also eliminates galaxies that appear above the RS because of dust-reddened star formation. Once the RS has been defined, we can calculate the cluster mass using the caustic method. As shown in Fig. 10, the masses derived from this approach are consistently lower than the actual values, and we achieve more accurate results when the method is applied without preselecting RS galaxies.

5.2. $H\alpha$ flux and magnitude limits

This subsection investigates the impact of sample selection; specifically, the magnitude and $H\alpha$ flux limits on the determination of cluster masses using the caustic method. The $H\alpha$ fluxes were derived from the star formation rates (SFRs) using the standard conversion of Kennicutt (1998).

⁵ The higher relative abundance of luminous RS galaxies in the center of the clusters is a direct consequence of the well-known morphology-density relation first found by Dressler (1980).

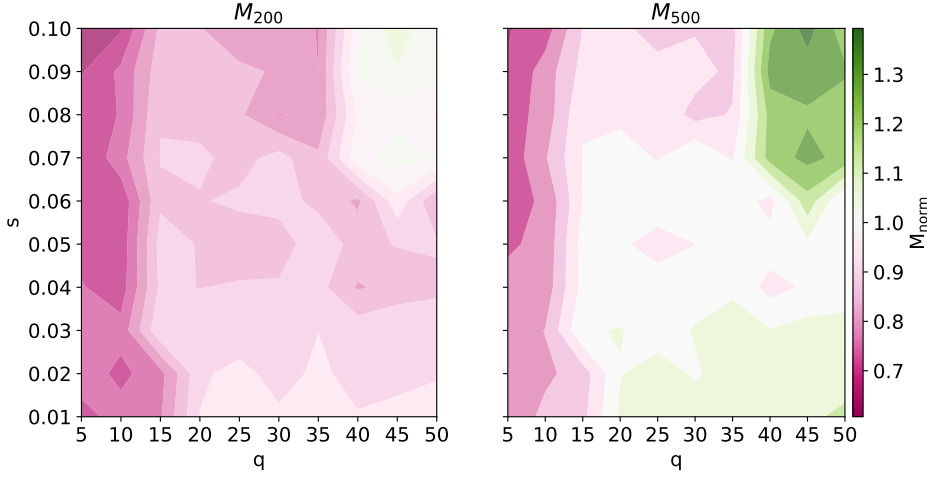


Fig. 3. Biparametric map of s and q for the cluster NewMDCLUSTER_0007 at $z = 0.276$. The color bars represent the ratio between the estimated and true values $M_r/M_{r,\text{true}}$ for masses between R_{200} (M_{200}) and R_{500} (M_{500}).

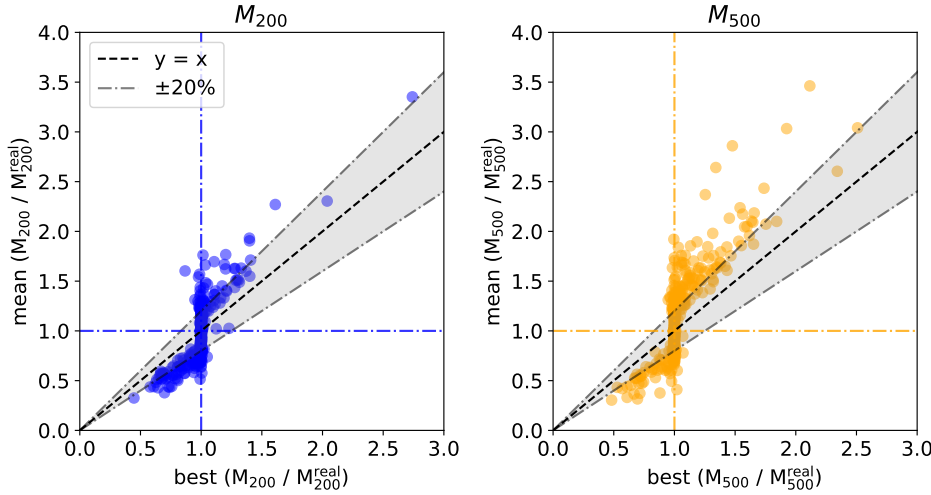


Fig. 4. Comparison between the best estimated values of M_{200} and M_{500} (x-axis) and the average values computed using all combinations of q and s explored in this work (y-axis). Each point corresponds to a cluster from the THE300 sample. The shaded region indicates deviations smaller than $\pm 20\%$ from the identity relation ($y = x$).

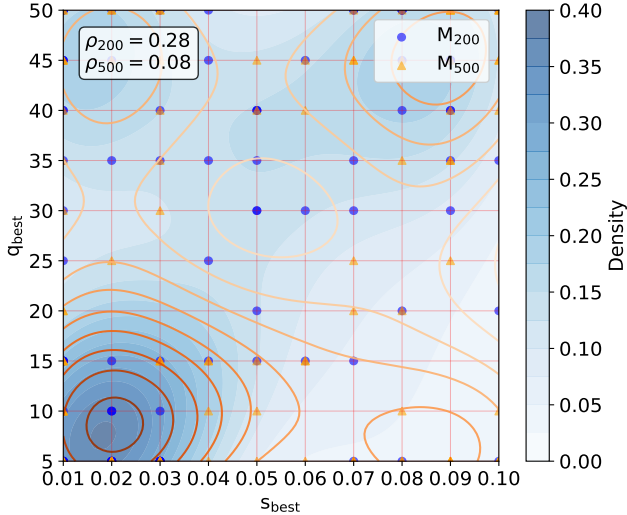


Fig. 5. Distribution of the best-fit parameters in the q - s plane for the cluster sample. Blue circles and orange triangles represent the values obtained using the M_{200} and M_{500} criteria, respectively. The shaded background and overlaid contours show the two-dimensional KDE of the parameter distribution, with the color scale indicating the local density across the parameter space. The Pearson correlation coefficients between q and s for each mass definition are showed in the inset.

Figure 11 illustrates, for a representative cluster, the relation between $H\alpha$ flux and (uncorrected) apparent magnitude in the SDSS r band. The shaded regions indicate the adopted selection cuts in magnitude and $H\alpha$ flux, defined according to the specifications of the TARSIS integral field spectrograph and the current observational strategy of the CATARSIS survey (Gil de Paz et al. 2024).

Galaxies were further classified according to their SFR into three regimes: $\text{SFR} > 0.1, M_{\odot}, \text{yr}^{-1}$, $0.01 < \text{SFR} \leq 0.1, M_{\odot}, \text{yr}^{-1}$, and $\text{SFR} \leq 0.01, M_{\odot}, \text{yr}^{-1}$. As expected, galaxies with lower SFRs tend to exhibit weaker $H\alpha$ emission and fainter apparent magnitudes, making them more challenging to detect within the survey limits. Consequently, these systems are preferentially excluded by the adopted cuts. In contrast, galaxies with higher SFRs are more easily detected and remain well represented in the selected sample, although some incompleteness persists due to the imposed observational thresholds.

Regarding our observations cuts in line-flux and apparent (continuum) magnitude, we chose $\log(f_{H\alpha, \text{lim}} [\text{erg s}^{-1} \text{cm}^{-2}])$ between -16.5 and -15.5 and $m_{r, \text{lim}}$ from 22 to 20.5 mag. Figure 12 shows the results of M_{200} that we have obtained for the whole set of THE300 clusters. As we cut in apparent magnitude, the average value shifts to higher values, resulting in an overestimation of the actual cluster masses. As for the cuts in flux, as we exclude galaxies with progressively fainter $H\alpha$ flux, we obtain greater dispersion compared to the real (true) mass values.

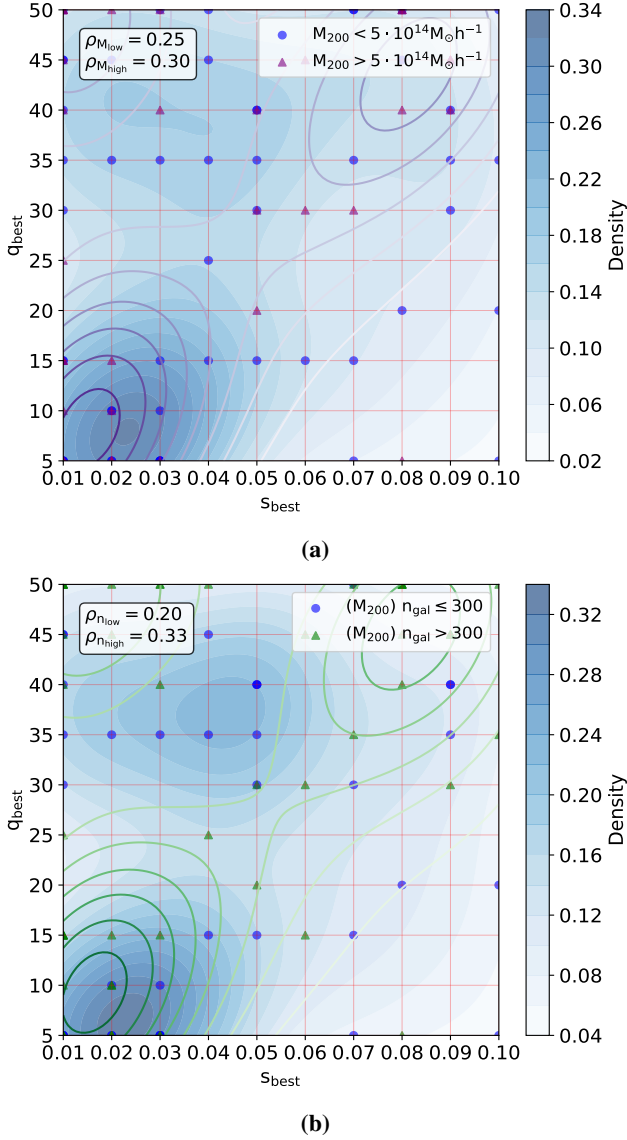


Fig. 6. Same as Fig. 5 but dividing the clusters into: clusters with M_{200} larger (triangles) and lower (circles) than $10^{14} M_{\odot} h^{-1}$ (panel a); and clusters with more (green) and fewer (blue) than 300 members (panel b). This division allows us to investigate how the distributions of the best-fit values vary as a function of the total mass and the cluster richness.

6. Comparison with other surveys

In this section, we evaluate the expected biases in the determination of the caustic masses with other ongoing and upcoming spectroscopic surveys of galaxy clusters. The analysis highlights the differences in redshift range, spectroscopic depth, sampling strategy, and spatial coverage, which ultimately determine the feasibility of dynamical mass measurements based on galaxy redshifts.

6.1. WEAVE cosmological clusters survey

The WEAVE cluster survey (Jin et al. 2024) will soon target approximately 70 SZ-selected clusters at $z < 0.5$, using both its LIFU (for $z > 0.25$) and MOS (for $z < 0.25$) configurations with the low-resolution setup ($R \approx 5000$). The MOS mode covers a

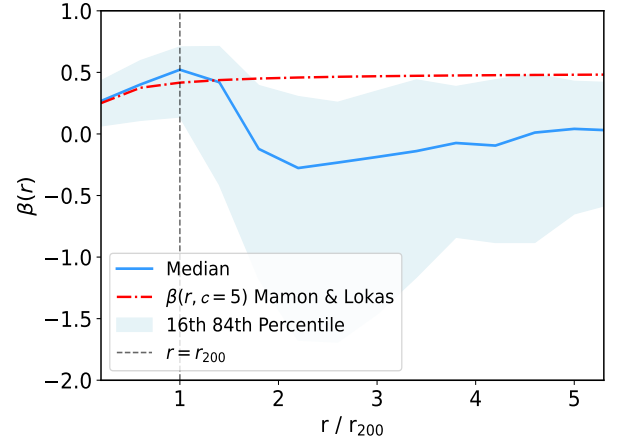


Fig. 7. Anisotropy profile derived from the theoretical definition of the anisotropy parameter $\beta(r)$ for the whole sample (blue line). The thick line shows the median profile across all clusters, while the shaded areas represent the 16th and 84th percentiles, respectively. The red line corresponds to the anisotropy profile from (Mamon & Lokas 2005) with a concentration parameter fixed to $c = 5$. In future work, we will explore the differences arising when clusters exhibit rotational effects, which can cause the two definitions of $\beta(r)$ to diverge. In this work, we compute $\beta(r)$ using the radial and tangential velocity dispersions (σ_r and σ_t) in each radial bin, instead of the ideal definition based on the mean squared velocities. This approach is appropriate for clusters without significant rotational support, where both definitions coincide.

wide 2° diameter FoV that is ideal for low redshift (corresponding to ~ 3.9 kpc/arcsec at $z = 0.25$), while the LIFU mode provides a significantly smaller coverage of $90'' \times 78''$ (~ 6.2 kpc/arcsec at $z = 0.5$). The limiting magnitude is not explicitly provided, but values of $r \approx 20.5$ – 21 at $z = 0.25$ are a reasonable assumption. The large FoV in physical units at $z < 0.25$ ensures adequate sampling for dynamical analyses, while at higher redshift the limited LIFU coverage restricts observations to the cluster core, preventing the application of the caustic method.

6.2. Hectospec Cluster Survey (HeCS) and HeCS-SZ

The Hectospec Cluster Survey (Rines et al. 2016) includes 58 X-ray selected clusters at $0.1 < z < 0.3$, while its SZ extension (HeCS-SZ) targets 21 systems at $0.05 < z < 0.20$. The survey employs a 1° diameter FoV, corresponding to radial scales from ~ 1 Mpc at $z = 0.05$ to ~ 4.5 Mpc at $z = 0.3$. The magnitude limit is $r = 20.8$, with typical sampling of ~ 300 galaxies per cluster and velocity uncertainties around 50 km s^{-1} .

The combination of depth, wide FoV, and high spectroscopic completeness of HeCS makes it one of the most suitable surveys for the caustic method. This was demonstrated by Rines et al. (2013), who obtained mass profiles out to large radii for 58 X-ray selected clusters; HeCS masses have since been widely used as a reference for cluster mass calibrations and studies of mass accretion (e.g., Pizzardo et al. (2021); Maughan et al. (2016); Andreon (2016)).

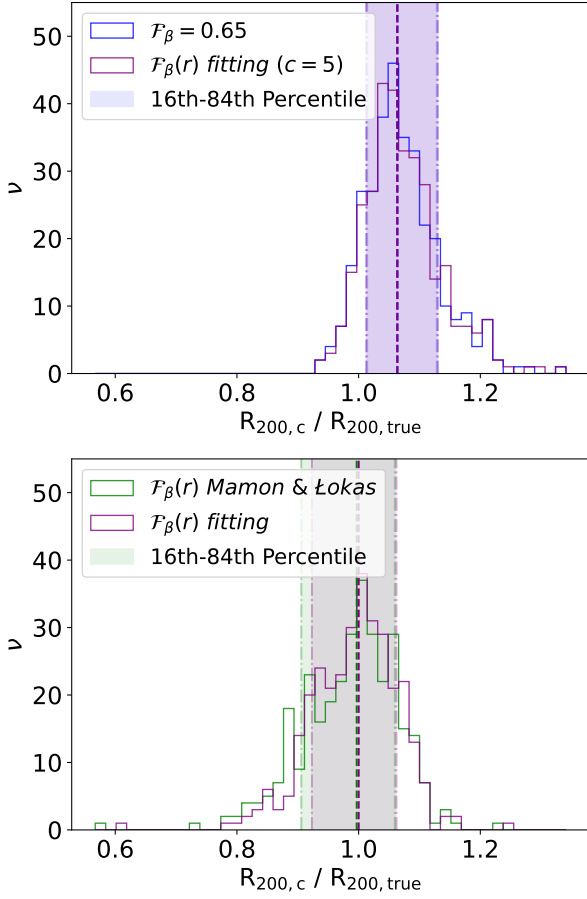


Fig. 8. Ratio between the derived and true R_{200} values. Top panels: Noniterative approach, comparing a constant F_β profile (blue) and a Mamon & Łokas anisotropy model with fixed concentration $c = 5$ (purple). Bottom panels: Results obtained using the iterative method, comparing a Mamon & Łokas profile (green) and a fitted F_β model (purple). In all panels, bold lines indicate the median values, while shaded regions correspond to the 16th and 84th percentiles. For the noniterative approach, we obtain a median ratio of $1.06^{+0.07}_{-0.05}$, indicating a systematic overestimation. In contrast, the iterative method yields a median ratio of $1.00^{+0.06}_{-0.09}$.

6.3. 4MOST galaxy cluster surveys: CHANCES and eROSITA follow-up

The CHANCES Evolution program (Sifón et al. 2025) targets ~ 50 massive clusters in the range $0.07 < z < 0.45$, selected above $M_{200} > 7 \times 10^{14} M_\odot$. The spectroscopic coverage extends out to $5 \times r_{200}$, ensuring radial completeness well into the cluster outskirts. The limiting magnitude is $r < 20.5$, providing sufficient sampling depth for member identification. The combination of large radial coverage, depth, and focus on massive systems makes CHANCES highly suitable for caustic-based analyses, with the potential to complement and extend results from HeCS to higher redshift.

The eROSITA Galaxy Cluster Survey (Finoguenov et al. 2019) will provide redshifts for systems up to $z < 0.7$, targeting galaxies with $K < 18$ (roughly $r \approx 19$). Current estimates indicate magnitude limits of $z_{\text{DECam}} \leq 21.5$ for cluster members and $r_{\text{DECam}} \leq 18.5$ for the filament survey (with no color cut). Given the low number of spectroscopic members per cluster, the

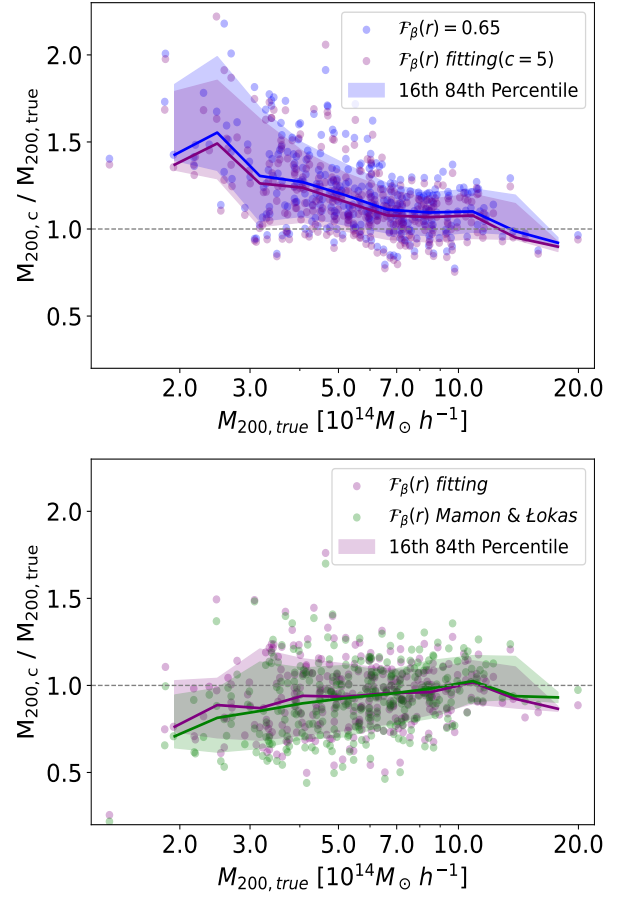


Fig. 9. Comparison of the ratio between the caustic mass estimate and the profile mass, $M_{200,c}/M_{200,true}$, as a function of M_{200} , for two different approaches. Top panel: $F_\beta(r)$ fitting with fixed parameters $F_\beta = 0.65$ (blue) and concentration $c = 5$ (purple). Bottom panel: $F_\beta(r)$ fitting with free parameters (purple) compared to the Mamon & Łokas method (green). In both panels, points show individual measurements, solid lines represent the median ratio in logarithmic mass bins, and shaded regions indicate the 16th to 84th percentile range. The gray dashed horizontal line marks the unity ratio, $M_{200,c} = M_{200,true}$.

full caustic methodology will only be applicable to a subset of clusters, requiring stacking of multiple systems to obtain reliable dynamical measurements.

6.4. Comparison of cluster surveys for caustic mass reconstruction

From this comparison, we conclude that HeCS/HeCS-SZ and CHANCES offer the most favorable conditions for applying the caustic method given their wide spatial coverage and high spectroscopic completeness. The results obtained from the CHANCES and HeCS surveys are consistent with the findings reported in Sect. 5.2 (see Fig. 12). WEAVE provides sufficient sampling only for low-redshift systems ($z < 0.25$ in MOS mode), while its higher-redshift LIFU configuration is limited to the cluster core, making caustic mass reconstruction unfeasible. In contrast, the eROSITA Galaxy Cluster Redshift Survey, despite its cosmological reach, does not provide the necessary sampling per cluster and therefore does not constitute a practical dataset for caustic analyses. Finally, CATARSIS, given its ability to

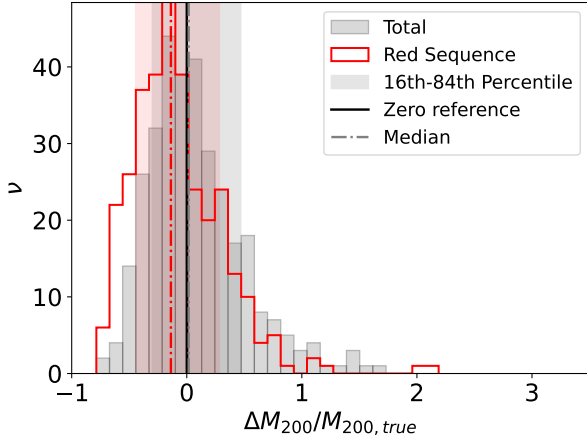


Fig. 10. Relative error in the determination of M_{200} using the caustic method for the full galaxy sample (grey) and for the RS sample (red). Shaded areas indicate the 16th to 84th percentiles of the distributions; dashed lines show the median values, and the black line marks the reference value of zero. For the full sample, we obtain $0.02^{+0.45}_{-0.32}$. For the RS sample, we obtain $-0.14^{+0.42}_{-0.30}$.

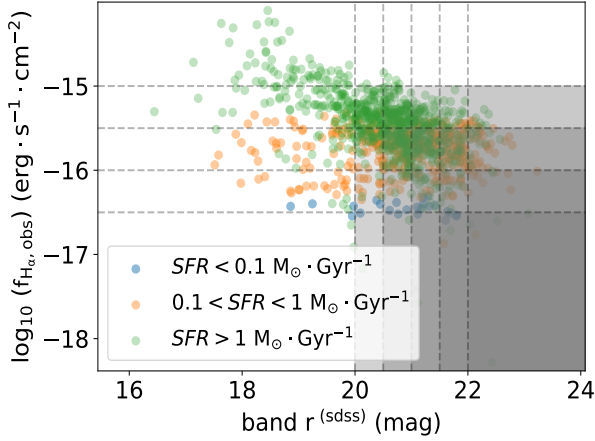


Fig. 11. Magnitude and $H\alpha$ flux distributions for the cluster NewMDCLUSTER_0001 at snapshot 117 ($z = 0.276$). Galaxies are colored according to three star formation rate (SFR) bins: low (blue), medium (orange), and high (green), illustrating how star formation activity correlates with both r -band magnitude and $H\alpha$ flux. The shaded regions indicate the magnitude and $H\alpha$ flux cuts adopted for the analysis, with the magnitude selection performed in the r -band.

reach galaxies down to $m_r \approx 21$, will enable the study of fainter cluster members, offering a depth comparable to other surveys based on preimaging, while adding galaxies selected by line emission down to $\log(f_{H\alpha,lim}[\text{erg s}^{-1} \text{cm}^{-2}]) = -16.5$. Therefore, it has been established as a promising option for applying the caustic method with minimal mass bias and cluster-by-cluster dispersion.

7. Conclusions

In this work, we assess the reliability of the caustic technique for recovering galaxy cluster mass profiles, using THE300 simulations as a preparation for the forthcoming CATARSIS survey. Our study demonstrates that the standard method suffers from

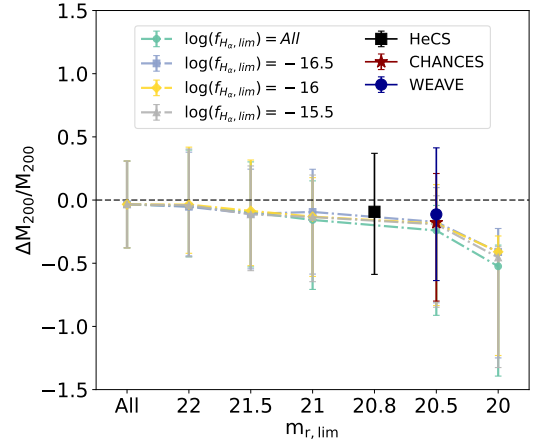


Fig. 12. Relative M_{200} values obtained for different cuts in r -band magnitude and $H\alpha$ flux. The y-axis shows the values normalized to the true cluster mass, while the x-axis represents the magnitude cuts. Colors indicate the $H\alpha$ flux cuts in logarithmic scale. A value of zero corresponds to perfect agreement between the calculated and true cluster mass. Error bars indicate the 16th and 84th percentiles of the distribution across THE300 clusters.

systematic biases, but these can be significantly reduced through a refined implementation.

We introduce an iterative approach that combines anisotropy profile fitting with the caustic mass reconstruction, improving the agreement with true cluster masses. We find that the method exhibits a systematic trend with halo mass, with biases in the low-mass regime reaching values of order 40%, depending on the modelling assumptions. This behaviour is consistent with previous studies of the caustic technique in both numerical and observational contexts, which have shown that mass estimates are generally unbiased, but often subject to significant scatter driven by projection effects, limited phase-space sampling, and uncertainties in the velocity anisotropy profile (Serra et al. 2010; Gifford et al. 2013; Armitage et al. 2019; Pizzardo et al. 2023). In particular, the iterative scheme reduces the systematic deviation, leading to an overall improvement in the mass calibration of order 20% when compared to the noniterative case. However, we note that the inferred masses remain sensitive to the assumed velocity anisotropy profile, consistent with the well-known mass–anisotropy degeneracy in caustic reconstructions.

The analysis further highlights the critical role of galaxy sampling. Restricting measurements to RS galaxies consistently underestimates cluster masses, while the blind integral-field spectroscopy of CATARSIS ensures a more complete representation of the cluster population and eliminates this bias. The influence of magnitude and $H\alpha$ flux limits was also investigated: bright magnitude cuts tend to overestimate masses, whereas relatively bright flux thresholds increase the scatter. The planned depth of CATARSIS strikes a balance between these effects, providing both precision and statistical robustness.

When compared with other spectroscopic surveys, CATARSIS emerges as uniquely powerful thanks to its combination of depth and wide spatial coverage. It will deliver dynamical mass profiles of comparable quality to those from benchmark surveys such as HeCS, while also offering an essential complement to weak-lensing studies. Overall, CATARSIS will provide a decisive step forward in the precise calibration of galaxy cluster masses, thereby enhancing the cosmological constraints that can be derived from these structures.

Acknowledgements. This work acknowledges support from the following grants funded by MCIN/AEI/10.13039/501100011033/FEDER, EU: PID2022-138621NB-I00 (TARSIS) and PID2022-138855NB-C31 (CATARSIS). This work has also been made possible by the 'The Three Hundred' collaboration.⁶ A. Knebe acknowledges support from the Ministerio de Ciencia e Innovación (MICINN) under research grant PID2021-122603NB-C21 as well as by project PID2024-156100NB-C21 financed by MICIU /AEI/10.13039/501100011033 / FEDER, UE. He further thanks The Cranberries for 'zombie'.

References

- Anbajagane, D., Chang, C., Jain, B., et al. 2022, *MNRAS*, 514, 1645
- Andreon, S. 2016, *A&A*, 587, A158
- Ansarifard, S., Rasia, E., Biffi, V., et al. 2020, *A&A*, 634, A113
- Armitage, T. J., Kay, S. T., Barnes, D. J., et al. 2019, *MNRAS*, 482, 3308
- Arthur, J., Pearce, F. R., Gray, M. E., et al. 2019, *MNRAS*, 484, 3968
- Bakels, L., Ludlow, A. D., & Power, C. 2021, *MNRAS*, 501, 5948
- Bartelmann, M. 2010, *Classical and Quantum Gravity*, 27, 233001
- Baxter, E. J., Adhikari, S., Vega-Ferrero, J., et al. 2021, *MNRAS*, 508, 1777
- Beck, A. M., Murante, G., Arth, A., et al. 2016, *MNRAS*, 455, 2110
- Becker, M. R. & Kravtsov, A. V. 2011, *ApJ*, 740, 25
- Binney, J. & Tremaine, S. 2008, *Galactic Dynamics* (Princeton University Press)
- Blumenthal, G. R., Faber, S. M., Primack, J. R., et al. 1984, *Nature*, 311, 517
- Capalbo, V., De Petris, M., Ferragamo, A., et al. 2025, *A&A*, 698, A201
- Capalbo, V., De Petris, M., De Luca, F., et al. 2021, *MNRAS*, 503, 6155
- Contreras-Santos, A., Knebe, A., Cui, W., et al. 2024, *A&A*, 683, A59
- Contreras-Santos, A., Knebe, A., Pearce, F., et al. 2022, *MNRAS*, 511, 2897
- Corless, V. L., King, L. J., & Clowe, D. 2009, *MNRAS*, 393, 1235
- Cui, W., Dave, R., Knebe, A., et al. 2022, *MNRAS*, 514, 977
- Cui, W., Knebe, A., Yepes, G., et al. 2018, *MNRAS*, 480, 2898
- Cui, W., Knebe, A., Yepes, G., et al. 2016, *MNRAS*, 456, 2566
- Dalton, G., Trager, S., Abrams, D., et al. 2012, in *SPIE Conference Series*
- Davé, R., Anglés-Alcázar, D., Narayanan, D., et al. 2019, *MNRAS*, 486, 2827
- de Andres, D., Cui, W., Ruppin, F., et al. 2022, *Nature Astronomy*, 6, 1325
- de Andres, D., Cui, W., Yepes, G., et al. 2024, *MNRAS*, 528, 1517
- De Luca, F., De Petris, M., Yepes, G., et al. 2021, *MNRAS*, 504, 5383
- Diaferio, A. 1999, *MNRAS*
- Diaferio, A. & Geller, M. J. 1997, *ApJ*, 481, 633
- Diaferio, A., Geller, M. J., & Rines, K. J. 2005, *ApJ*, 628, L97
- Dressler, A. 1980, *ApJ*, 236, 351
- Feroz, F. & Hobson, M. P. 2012, *MNRAS*, 420, 596
- Ferragamo, A., de Andres, D., Sbrigliio, A., et al. 2023, *MNRAS*, 520, 4000
- Finoguenov, A., Merloni, A., Comparat, J., et al. 2019, *The Messenger*, 175, 39
- Föex, G., Böhringer, H., & Chon, G. 2017, *A&A*, 606, A122
- Geller, M. J., Diaferio, A., & Kurtz, M. J. 1999, *ApJL*, 517, L23
- Geller, M. J., Diaferio, A., Rines, K. J., et al. 2013, *ApJ*, 764, 58
- Gianfagna, G., Rasia, E., Cui, W., et al. 2022, *EPJ Conf*, 257, 00020
- Gianfagna, G., Rasia, E., Cui, W., et al. 2023, *MNRAS*, 518, 4238
- Gifford, D., Miller, C., & Kern, N. 2013, *ApJ*, 773, 116
- Gil de Paz, A., Iglesias-Páramo, J., Carrasco, E., et al. 2024, in *Society of Photo-Optical Instrumentation Engineers (SPIE) Conference Series*, Vol. 13096, *Ground-based and Airborne Instrumentation for Astronomy X*, ed. J. J. Bryant, K. Motohara, & J. R. D. Vernet, 1309620
- Haggar, R., Gray, M. E., Pearce, F. R., et al. 2020, *MNRAS*, 492, 6074
- Haggar, R., Pearce, F. R., Gray, M. E., et al. 2021, *MNRAS*, 502, 1191
- Hoekstra, H. 2003, *MNRAS*, 339, 1155
- Hoekstra, H., Hartlap, J., Hilbert, S., et al. 2011, *MNRAS*, 412, 2095
- Iqbal, A., Majumdar, S., Rasia, E., et al. 2025, *A&A*, 704, A334
- Jin, S., Trager, S. C., Dalton, G. B., et al. 2024, *MNRAS*, 530, 2688
- Kennicutt, R. C. 1998, *ARA&A*, 36, 189
- Klypin, A., Yepes, G., Gottlöber, S., et al. 2016, *MNRAS*, 457, 4340
- Knebe, A., Gámez-Marín, M., Pearce, F. R., et al. 2020, *MNRAS*, 495, 3002
- Knollmann, S. R. & Knebe, A. 2009, *ApJ Supplement Series*, 182, 608
- Kotecha, S., Welker, C., Zhou, Z., et al. 2022, *MNRAS*, 512, 926
- Kuchner, U., Aragón-Salamanca, A., Rost, A., et al. 2021, *MNRAS*, 503, 2065
- Li, Q., Cui, W., Yang, X., et al. 2020, *MNRAS*, 495, 2930
- Li, Q., Han, J., Wang, W., et al. 2022, *MNRAS*, 514, 5890
- Li, Q., Han, J., Wang, W., et al. 2021, *MNRAS*, 505, 3907
- Li, Q., Han, J., Wang, W., et al. 2021, *MNRAS*, 505, 3907
- Łokas, E. L., Wojtak, R., Gottlöber, S., et al. 2006, *MNRAS*, 367, 1463
- Ludlow, A. D., Navarro, J. F., Li, M., et al. 2012, *MNRAS*, 427, 1322
- Mamon, G. A. & Boué, G. 2010, *MNRAS*, 401, 2433
- Mamon, G. A. & Łokas, E. L. 2005, *MNRAS*, 363, 705
- Maughan, B. J., Giles, P. A., Rines, K. J., et al. 2016, *MNRAS*, 461, 4182
- McClintock, T., Varga, T., Gruen, D., et al. 2019, *MNRAS*, 482, 1352
- McGaugh, S. S. 2015, *Can. J. Phys.*, 93, 250
- Merritt, D. 1987, *ApJ*, 313, 121
- Milgrom, M. 2002, *New Astron. Rev.*, 46, 741
- Mostoghiu, R., Arthur, J., Pearce, F. R., et al. 2021, *MNRAS*, 501, 5029
- Mostoghiu, R., Knebe, A., Cui, W., et al. 2019, *MNRAS*, 483, 3390
- Munari, E., Biviano, A., Borgani, S., et al. 2013, *MNRAS*, 430, 2638
- Nelson, D., Pillepich, A., Ayromlou, M., et al. 2024, *A&A*, 670, A1
- Nelson, K., Lau, E. T., Nagai, D., et al. 2014, *ApJ*, 782, 107
- Pizzardo, M., Di Gioia, S., Diaferio, A., et al. 2021, *AA*, 646, A105
- Pizzardo, M., Geller, M. J., Kenyon, S. J., et al. 2023, *A&A*, 675, A56
- Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2016, *A&A*, 594, A13
- Rasia, E., Borgani, S., Murante, G., et al. 2015, *ApJL*, 813, L17
- Rasia, E., Meneghetti, M., Martino, R., et al. 2012, *New J Phys*, 14, 055018
- Rines, K. & Diaferio, A. 2006, *AJ*, 132, 1275

⁶ <https://www.the300-project.org>

- Rines, K., Geller, M. J., Diaferio, A., et al. 2013, *ApJ*, 767, 15
- Rines, K., Geller, M. J., Diaferio, A., et al. 2002, *AJ*, 124, 1266
- Rines, K., Geller, M. J., Diaferio, A., et al. 2000, *AJ*, 120, 2338
- Rines, K. J., Geller, M. J., Diaferio, A., et al. 2016, *ApJ*, 819, 63
- Rost, A. & et al. 2021, *MNRAS*, 502, 714
- Sarazin, C. L. 1988, *X-ray Emission from Clusters of Galaxies* (Cambridge University Press)
- Sayers, J., Sereno, M., Ettori, S., et al. 2021, *MNRAS*, 505, 4338
- Sembolini, F., Yepes, G., De Petris, M., et al. 2013, *MNRAS*, 429, 323
- Sereno, M., Lovisari, L., Cui, W., et al. 2021, *MNRAS*, 507, 5214
- Serra, A. L. & Diaferio, A. 2013, *ApJ*, 768, 116
- Serra, A. L., Diaferio, A., Murante, G., et al. 2010, *MNRAS*, no
- Sifón, C., Finoguenov, A., Haines, C. P., et al. 2025, *A&A*, 697, A92
- Silverman, A. N. 1986, *American Journal of Physics*, 54, 1092
- Tamura, N., Takato, N., Shimono, A., et al. 2016, in *Proc. SPIE*, Vol. 9908, *Ground-based and Airborne Instrumentation for Astronomy VI*, ed. C. J. Evans, L. Simard, & H. Takami, 99081M
- The, L. S. & White, S. D. M. 1986, *AJ*, 92, 1248
- Umetsu, K. 2013, *ApJ*, 769, 13
- Umetsu, K., Broadhurst, T., Zitrin, A., et al. 2011, *ApJ*, 729, 127
- Umetsu, K., Sereno, M., Lieu, M., et al. 2020, *ApJ*, 890, 148
- Wang, Y., Pearce, F., Knebe, A., et al. 2018, *ApJ*, 868, 130
- Yoon, J. H., Rasia, E., Biffi, V., et al. 2022, *MNRAS*, 516, 4084
- Zhang, B., Cui, W., Wang, Y., et al. 2022, *MNRAS*, 516, 26
- Lokas, E. L. & Mamon, G. A. 2003, *MNRAS*, 343, 401