

Polarized emission of orbiting hot spots near Sagittarius A*

Effects of electromagnetic interaction

Abylaikhan Tlemissov¹, Arman Tursunov¹ and Maciek Wielgus²

¹ Research Centre for Theoretical Physics and Astrophysics, Institute of Physics, Silesian University in Opava,
Bezručovo nám. 13, CZ-74601 Opava, Czech Republic
e-mail: tle0002@slu.cz; arman.tursunov@physics.slu.cz

² Instituto de Astrofísica de Andalucía-CSIC, Glorieta de la Astronomía s/n, E-18008 Granada, Spain
e-mail: maciek.wielgus@gmail.com

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ABSTRACT

We investigated the polarimetric signatures of orbiting hot spots around a Schwarzschild black hole in the presence of an external magnetic field, accounting for the electromagnetic interaction between the charged emitter and the field. Using a general-relativistic model that incorporates synchrotron emission and ray-tracing of light propagation, we analyzed how the electromagnetic interaction parameter modifies the observed polarization patterns, with particular emphasis on the behavior of the electric vector position angle and the time-evolving polarization loops in the Q - U plane. Applying the model to millimeter-wavelength ALMA observations of Sagittarius A*, we explored the parameter space that best reproduces the asymmetry, time ratio, and area ratio of the observed polarization loops. We find that the inclusion of a small positive interaction parameter increases the symmetry of the loops and the time ratio, while a negative magnetic parameter introduces strong asymmetry and fails to reproduce the data. Our results indicate that electromagnetic interaction can lead to ambiguity in the estimation of the system parameters such as orbital inclination or hot-spot velocity.

Key words. Black hole physics — Magnetic fields — Polarization — Galaxies: individual: Sgr A* — Galaxy: center

1. Introduction

Polarimetric observations of black holes provide crucial information on the extreme gravitational and electromagnetic environments in their vicinity. Recent groundbreaking observations by the Event Horizon Telescope (EHT) Collaboration at 230 GHz have opened new avenues for probing the complex interplay between gravity and magnetic fields around supermassive black holes (SMBHs). Specifically, polarization images of the SMBH M87* (Event Horizon Telescope Collaboration et al. 2021) and Sagittarius A* (Sgr A*; Event Horizon Telescope Collaboration et al. 2024a) have unveiled structures suggesting the presence of predominantly poloidal magnetic field components, especially in the latter case (Event Horizon Telescope Collaboration et al. 2024b). Furthermore, observations by the Gravity Collaboration et al. (2018, 2023) in near infrared have detected distinct polarization loops associated with orbiting flaring matter, offering additional constraints on the dynamics and magnetic-field configurations close to an event horizon (GRAVITY Collaboration et al. 2020a,b; Yfantis et al. 2024b).

Complementing these observations, Wielgus et al. (2022b) reported the detection of polarization loops by the Atacama Large Millimeter/submillimeter Array (ALMA), revealing time-dependent polarization signatures associated with orbital motion close to Sgr A*, following a high-energy flare observed by Chandra (Event Horizon Telescope Collaboration et al. 2022a; Wielgus et al. 2022a). These polarization loops, seen as cyclic trajectories in the Stokes Q - U plane of linear polarization, pro-

vide evidence of transient, localized emission components orbiting in the innermost region near the event horizon (see also Vincent et al. 2024). By analyzing these loops, the authors constrained the orbital properties of the emitting regions, such as their orbital radii and periods, revealing details about plasma dynamics and magnetic-field geometry close to Sgr A*. This observational finding significantly enhances our understanding of Sgr A* black hole and its astrophysical environment, offering novel observational constraints on theoretical models (Yfantis et al. 2024a; Levis et al. 2024).

Despite these advances, interpreting polarization signals from near-horizon regions remains challenging due to uncertainties in the plasma motion and magnetic-field structure. One key source of uncertainty arises from the assumption of Keplerian motion for emitting material around black holes, which might not hold true in strong-magnetic-field environments. In reality, the emitting plasma consists of charged particles whose motion can deviate significantly from purely gravitational trajectories due to electromagnetic interactions with vertical magnetic fields (Tursunov et al. 2020). Such interactions can introduce substantial non-Keplerian effects, potentially mimicking observational signatures attributed to black-hole spin or other gravitational phenomena (see, e.g., Appendix C from Wielgus et al. 2022b).

Event Horizon Telescope observations consistently support the presence of dominant vertical magnetic fields, with significantly weaker toroidal or radial components, reinforcing the importance of vertical magnetosphere models in describing the Galactic center environment (Johnson et al. 2015; Jacquemin-Ide et al. 2024). These vertical and large-scale magnetic fields play a crucial role in particle acceleration, jet forma-

tion, and overall energy extraction processes in black-hole systems (Blandford & Znajek 1977; Tursunov & Dadhich 2019).

Motivated by these observations, we propose an extension of the polarized synchrotron emission model by Narayan et al. (2021) and Gelles et al. (2021) from weakly charged hot spots orbiting a Schwarzschild black hole embedded in an vertical magnetic field. In this model, we explicitly account for the electromagnetic interaction between the potentially charged emitting hot spot and the magnetic field, introducing a dimensionless magnetic interaction parameter.

Standard general relativistic magnetohydrodynamic simulations typically assume exact charge neutrality at fluid scales. In reality, a magnetized plasma is only quasi-neutral: a tiny excess of charge carriers can exist without violating global neutrality. The size of this excess is set by electrostatic screening. In a quasi-neutral plasma, the fractional charge imbalance required to sustain a smooth electrostatic potential variation $\Delta\Phi$ across a region of size L can be estimated as (see, e.g., Hazeltine & Waelbroeck 2004; Chen 2016)

$$\frac{n_q}{n} \approx \lambda_D^2 \nabla^2 \frac{e\Phi}{k_B T} \approx \left(\frac{\lambda_D}{L}\right)^2 \frac{e\Delta\Phi}{k_B T}, \quad (1)$$

where $n_q \equiv n_i - n_e$ is the net charge number density, n is a total number density ($n \simeq n_e \simeq n_i$), T is a plasma temperature, and $\lambda_D^2 \equiv k_B T / (4\pi n e^2)$ is the Debye length. Equation (1) is derived by using Poisson's equation $\nabla^2 \Phi = -4\pi e n_q$ with the scale estimate $\nabla^2 \Phi \sim \Delta\Phi / L^2$.

Such charge imbalances can naturally be generated by the relativistic motion of plasma around a black hole in the presence of an ordered magnetic field approximately orthogonal to the orbital plane, which induces charge separation and a nonzero net charge density. In pulsar magnetospheres the same special-relativistic effect produces the well-known Goldreich-Julian charge density (Goldreich & Julian 1969; Muslimov & Tsygan 1992). Neglecting general relativistic corrections for the moment, Maxwell's equations in the lab frame together with Ohm's law imply that the charge density required to reduce the co-moving electric field is

$$\rho_q = \frac{1}{4\pi} \nabla \cdot \mathbf{E} = -\frac{1}{4\pi c} \nabla \cdot (\mathbf{v} \times \mathbf{B}) \approx -\frac{\mathbf{\Omega} \cdot \mathbf{B}}{2\pi c} \quad (2)$$

for rigid rotation, with the speed $\mathbf{v} = \mathbf{\Omega} \times \mathbf{r}$ and a vertical field $\mathbf{B}$. For a hot spot orbiting Sgr A* near the innermost stable circular orbit (ISCO) of a nonrotating black hole and a vertical field of strength $B \approx 10$ G (Event Horizon Telescope Collaboration et al. 2022b, 2024b), the corresponding excess charge number density is of the order of

$$n_q \equiv \frac{|\rho_q|}{e} \approx 10^{-4} \left(\frac{B}{10 \text{ G}}\right) \left(\frac{\Omega}{\Omega_{\text{ISCO}}}\right) \text{ cm}^{-3}. \quad (3)$$

Given the total plasma number density associated with the flare components, $n \sim 10^{7\pm1} \text{ cm}^{-3}$ (Yuan et al. 2003; Eckart et al. 2012; Yfantis et al. 2024a), the ratio of excess to total carriers is $n_q/n \sim 10^{-11\pm1}$. This means that only about one particle in every $10^{10} - 10^{12}$ contributes to the net charge, underscoring that the plasma remains overwhelmingly neutral even when such charges are dynamically relevant.

One can show that the right-hand side of (1) yields a similar order-of-magnitude estimate. For a hot spot with a characteristic size of $L \sim r_g = GM/c^2 \approx 6 \times 10^{11} \text{ cm}$ and plasma

temperature $T \approx 10^{11} \text{ K}$, moving near the ISCO, the inductive electric field has a magnitude of $E \approx (v/c)B$. This corresponds to a potential difference across the hot spot on the order of $\Delta\Phi \sim EL = (v/c)BL$. Substituting this into (1), we obtain a charge imbalance on the order of $|n_q|/n \sim 10^{-10\pm1}$.

Although this fraction is vanishingly small, the integrated excess charge over a hot spot of size $L \sim r_g$ corresponds to a macroscopic total charge, $Q \approx e n_q L^3 \sim 10^{12} \text{ C}$. Such a charge is large enough for its electrostatic and magnetic coupling to the ambient field, potentially modifying its orbital motion and the polarization signature of its emission. General relativistic extensions of the above estimates (see, e.g., Muslimov & Tsygan 1992; Thorne & MacDonald 1982; Tursunov et al. 2020) yield comparable magnitudes. To estimate the dynamical effect of the excess charge, we define a dimensionless parameter characterizing the strength of the electromagnetic interaction between the charged hot spot and the ambient magnetic field:

$$\mathcal{B} = \frac{QB}{m_{hs}} \left(\frac{GM}{c^4}\right) \equiv \frac{n_q}{n} \times \frac{eB}{m_i} \left(\frac{GM}{c^4}\right), \quad (4)$$

where Q and m_{hs} are the total charge and mass of the hot spot. This parameter represents the ratio of the characteristic Lorentz force to the gravitational force, and governs deviations from purely geodesic motion. For a characteristic ratio of $n_q/n \sim 10^{-11} - 10^{-9}$ we obtain $\mathcal{B} \sim 10^{-5} - 10^{-3}$ for an e^- -p plasma and $\mathcal{B} \sim 10^{-2} - 1$ for an e^- - e^+ plasma. The upper end of the range corresponds to the pair-dominated plasma inertia together with the largest assumed charge imbalance (n_q/n). For smaller charge imbalance or in an electron-proton plasma, substantially lower values of $\mathcal{B}$ are expected. The allowed range of this parameter for Sgr A* has been constrained by Tursunov et al. (2020); see their Eq. (33), which yields $10^{-5} < |\mathcal{B}| < 10$, for 10 G magnetic field near Sgr A*. The sign of the parameter $\mathcal{B}$ depends on the sign of the hot-spot charge and the relative orientation between the magnetic field and the orbital motion, and can be either positive or negative (Frolov & Shoom 2010; Kološ et al. 2015).

Charge separation is also driven by general relativity itself; frame dragging twists magnetic-field lines around a spinning black hole and induces an electric field. Wald (1974) showed that a Kerr black hole in a uniform magnetic field acquires an induced charge, Q_w , set by the spin and the vertical magnetic field. For Sgr A* this constrains the black-hole charge to $|Q_{\text{BH}}| \lesssim 10^{15} \text{ C}$ (Zajaček et al. 2018). Classic analyses of the coupled black hole-magnetosphere system by Ruffini & Wilson (1975) show that, for a wide class of stationary accretion configurations, the global charge of the magnetosphere balances that of a black hole, $Q_{\text{mag}} \approx -Q_{\text{BH}}$, ensuring overall neutrality of the system. These estimates show the significance of electromagnetic interactions; even a tiny deviation from charge neutrality, while still maintaining quasi-neutrality, can sustain dynamically relevant electric fields in the magnetized vicinity of Sgr A*.

In this work we systematically explored how variations in the electromagnetic-interaction parameter can influence polarization signatures discussed by Wielgus et al. (2022b). Further, we compared our theoretical predictions with ALMA observations of polarization loops from Sgr A*, focusing on the effect of the magnetic interaction on the QU -pattern parameters considered by Wielgus et al. (2022b) and Vos et al. (2022), namely the degree of asymmetry of the loop, ratio of the full loop period to inner loop duration, and ratio of the areas of large and small loops on the $Q-U$ plane. The introduced magnetic-field interaction parameter, $\mathcal{B}$, can also be regarded as an effective parameter controlling deviations from the geodesic motion.

The paper is organized as follows. We start from the theoretical description of the dynamics of a charged emitter in a Schwarzschild space-time in the presence of vertical magnetic field given by the exact solution of Wald (1974). We next calculated the polarization vectors and radiative-transfer equations for propagation of the polarized light, and derived the observable parameters in the case of a charged emitter. We then applied our model to the ALMA polarization loops of Sgr A* (Wielgus et al. 2022b) to constrain the magnetic-interaction parameter and assess its impact on orbital dynamics and the interpretation of observational data.

2. Orbiting hot spot with electromagnetic interaction

We start by introducing the line element in a spherically symmetric Schwarzschild space-time,

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (5)$$

where $f(r) = 1 - 2M/r$ is a lapse function and M is the mass of the central object in geometric units. Without loss of generality, we set the black-hole mass to unity, $M = 1$, in the following. The mass will be reinstated later when expressing equations in astrophysically relevant units.

Wald (1974) introduced a method to generate test electromagnetic fields in stationary, axisymmetric space-times with Killing vector symmetries, obtaining a solution of

$$A^\mu = C_t \xi_t^\mu + C_\phi \xi_\phi^\mu, \quad (6)$$

where the constants C_t and C_ϕ are determined by the asymptotic behavior of the field at infinity. In the case of a nonrotating and uncharged black hole, the only relevant Killing vector is the rotational one: $\xi_\phi = \partial/\partial\phi$. Therefore, assuming asymptotic magnetic-field B strength, the resulting vector potential reads

$$A_\phi = \frac{1}{2}Br^2 \sin^2\theta, \quad (7)$$

with the antisymmetric electromagnetic-field tensor components given by

$$F_{r\phi} = Br \sin^2\theta, \quad F_{\theta\phi} = Br^2 \cos\theta \sin\theta. \quad (8)$$

The Hamiltonian governing the motion of a charged hot-spot can be expressed as (Wald 1984; Kološ et al. 2015)

$$H_p = \frac{1}{2}g^{\alpha\beta}(\pi_\alpha - qA_\alpha)(\pi_\beta - qA_\beta) + \frac{1}{2}m^2, \quad (9)$$

where m is the mass of the charged particle and q is its electric charge. The generalized canonical and kinematical four momenta are defined, respectively, as

$$\pi^\mu = p^\mu + qA^\mu, \quad p^\mu = m\dot{x}^\mu. \quad (10)$$

Due to symmetries of the background space-time and the external magnetic field, one can introduce conserved quantities of the energy and the axial component of angular momentum:

$$E = -\pi_t = mf(r)\frac{dt}{d\tau}, \quad (11)$$

$$L = \pi_\phi = mr^2 \sin^2\theta \left(\frac{d\phi}{d\tau} + \frac{qB}{2m} \right). \quad (12)$$

For convenience we used the dimensionless magnetic parameter $\mathcal{B}$, introduced in (4), and introduced the specific energy, $\mathcal{E}$, and specific angular momentum, $\mathcal{L}$, as follows:

$$\mathcal{B} = \frac{qB}{2m}, \quad \mathcal{E} = \frac{E}{m}, \quad \mathcal{L} = \frac{L}{m}. \quad (13)$$

Using the above quantities, one can rewrite the Hamiltonian as

$$H = \frac{1}{2}f(r)p_r^2 + \frac{1}{2r^2}p_\theta^2 + \frac{m^2}{2f(r)^2}(V_{\text{eff}} - \mathcal{E}^2), \quad (14)$$

where the effective potential is taken in the form of

$$V_{\text{eff}} = f(r) \left[1 + \left(\frac{\mathcal{L}}{r \sin\theta} - \mathcal{B}r \sin\theta \right)^2 \right]. \quad (15)$$

The effective potential, representing energetic boundary of the hot-spot, is symmetric under the transformation $(\mathcal{L}, \mathcal{B}) \rightarrow (-\mathcal{L}, -\mathcal{B})$, which implies two equivalent scenarios: either $\mathcal{L} > 0$, $\mathcal{B} < 0$ is equivalent to $\mathcal{L} < 0$, $\mathcal{B} > 0$ or $\mathcal{L} > 0$, $\mathcal{B} > 0$ is equivalent to $\mathcal{L} < 0$, $\mathcal{B} < 0$. We chose the positive angular momentum of the charged hot spot to represent the clockwise motion, which is supported by observations (Gravity Collaboration et al. 2018; Wielgus et al. 2022a)).

Depending on the sign of $\mathcal{B}$, one can distinguish two qualitatively distinct cases, differing in the orientation of the effective Lorentz force: for $\mathcal{B} < 0$ the Lorentz force is attractive, acting toward the black hole, while for $\mathcal{B} > 0$ it is repulsive, acting outwards from the black hole. In analogy with the prograde and retrograde motion in Kerr space-time, the two above cases in Schwarzschild space-time can be referred to as Larmor ($\mathcal{B} < 0$) and anti-Larmor ($\mathcal{B} > 0$) motions; these are defined with respect to the magnetic-field orientation.

The extrema points of the effective potential give locations of circular orbits. For a given radius, r , the corresponding specific angular momentum and energy of a hot spot at circular orbit is given by

$$(\mathcal{L}_{\text{c.o.}})_\pm = \frac{-\mathcal{B}r^2 \pm rF}{r-3}, \quad (16)$$

$$(\mathcal{E}_{\text{c.o.}})_\pm = \left(f(r) \left[1 + \left(\frac{(\mathcal{L}_{\text{c.o.}})_\pm}{r} - \mathcal{B}r \right)^2 \right] \right)^{1/2}, \quad (17)$$

where

$$F(r; \mathcal{B}) = \sqrt{\mathcal{B}^2 r^2 (r-2)^2 + r-3}. \quad (18)$$

Thus, using Eq. (12) we find the angular velocity of at the circular orbit,

$$\Omega_\pm = \frac{f(r)}{\mathcal{E}_{\text{c.o.}}} \left(\frac{\mathcal{L}_{\text{c.o.}}}{r^2} - \mathcal{B} \right), \quad (19)$$

where the $\pm$ sign corresponds to the two branches of motion, in analogy with prograde and retrograde orbits in Kerr space-time, here referred to as Larmor and anti-Larmor cases. However, within a narrow radial range of $2 < r < 3$, both branches of the solution reduce to the same orbital orientation, which can be either Larmor-like or anti-Larmor-like depending on the sign of the product, $\mathcal{L}\mathcal{B}$; see details in Frolov & Shoom (2010), Kološ et al. (2015), and Tursunov et al. (2016). With that, the circular motion under $r < 3$ corresponds to the case with the repulsive Lorentz force only. The local radial extrema of $\mathcal{L}_{\text{c.o.}}$

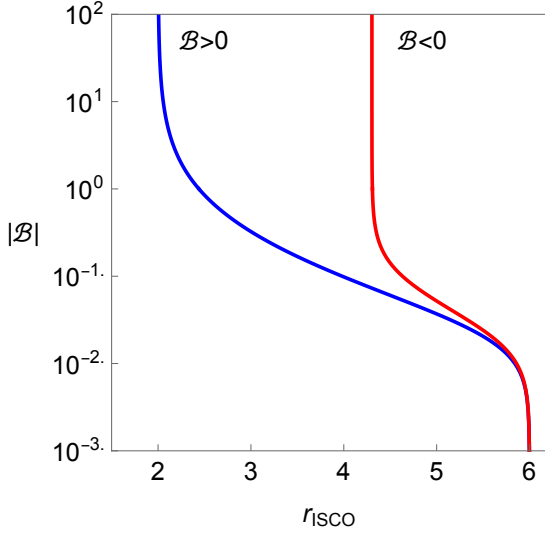


Fig. 1. Dependence of ISCO radius, r_{ISCO} , on the magnetic-field interaction parameter, $\mathcal{B}$, in the presence of a uniform magnetic field. In both cases of positive and negative $\mathcal{B}$, the ISCO radius decreases with increasing $\mathcal{B}$.

correspond to the ISCO, defining their radius, angular momentum, and energy. The ISCO condition reads

$$\mathcal{B}^2 r (2r^3 - 9r^2 + 8r - 12) - \mathcal{B}(r - 6)Y + r/2 - 3 = 0, \quad (20)$$

where

$$Y(r; \mathcal{B}) = \sqrt{\mathcal{B}^2 r^2 (5r^2 - 4r + 4) + 2r}. \quad (21)$$

The behavior of ISCO with respect to the magnetic-field interaction parameter is shown in Fig. 1. In the absence of electromagnetic interaction ($\mathcal{B} = 0$), the ISCO is located at the standard Schwarzschild space-time value of $r = 6r_g$.

3. Polarized signatures of hot spots

3.1. Polarization formalism

In this section we discuss the key aspects of the polarization of light around black hole within a semi-analytic model introduced by Narayan et al. (2021) and Gelles et al. (2021), and modified here by the inclusion of electromagnetic interaction. In this model, radiation is emitted by the fluid from the equatorial plane, $\theta_s = \pi/2$, confined to a point-like emitter located at the dimensionless radius $r_s \equiv rc^2/GM$ in Schwarzschild coordinates. The orbital plane of the hot spot is inclined at an angle, θ_o , relative to a face-on orientation, as seen by a distant observer.

Below, we transform the components of vectors and tensors into the local orthonormal frame at the arbitrary point, P (hereafter, P -frame; the geometry of the different reference frames is described in more detail in Narayan et al. (2021)) in the equatorial plane via

$$V_{(a)} = e_{(a)}^\mu V_\mu, \quad T_{(a)(b)} = e_{(a)}^\mu e_{(b)}^\nu T_{\mu\nu}, \quad (22)$$

where in Schwarzschild space-time

$$e_{(a)}^\mu = \begin{pmatrix} f(r)^{-\frac{1}{2}} & 0 & 0 & 0 \\ 0 & f(r)^{\frac{1}{2}} & 0 & 0 \\ 0 & 0 & r^{-1} & 0 \\ 0 & 0 & 0 & (r \sin \theta)^{-1} \end{pmatrix}. \quad (23)$$

For the chosen magnetic-field configuration given by (8), the components of the magnetic field in the P -frame are

$$F_{\hat{\theta}\hat{\phi}} = -F_{\hat{\phi}\hat{\theta}} = B_{\hat{r}(P)} = \frac{1}{2} B \cos \theta_s, \quad (24)$$

$$F_{\hat{r}\hat{\phi}} = -F_{\hat{\phi}\hat{r}} = B_{\hat{\theta}(P)} = \frac{1}{2} B \sqrt{f(r_s)} \sin^2 \theta_s. \quad (25)$$

In the fluid's rest frame (hereafter, F -frame), the components of the magnetic field take the following form:

$$B_{\hat{r}(F)} = (\cos^2 \chi + \gamma \sin^2 \chi) B_{\hat{r}(P)}, \quad (26)$$

$$B_{\hat{\theta}(F)} = \gamma B_{\hat{\theta}(P)}, \quad (27)$$

where the Lorentz factor is $\gamma = 1/\sqrt{1-\beta^2}$ and the velocity vector is given by

$$\boldsymbol{\beta} = \beta (\cos \chi \hat{r} + \sin \chi \hat{\phi}), \quad (28)$$

with χ being the angle between the velocity vector and the radial direction. In our case, where the emitter lies in the equatorial plane at $\theta_s = \pi/2$, the radial component of the magnetic field, $B_{\hat{r}}$, vanishes. Thus, the only nonzero component of the magnetic field is

$$B_{\hat{\theta}(F)} = \frac{\gamma}{2} B \sqrt{f(r_s)}, \quad (29)$$

which corresponds to the vertical magnetic field in the fluid frame. Now let us consider polarization vector $\mathbf{f}_{(F)}$ in the F -frame as a vector orthogonal to the local magnetic field $\mathbf{B}_{(F)} = (B_{\hat{r}}, B_{\hat{\phi}}, B_{\hat{\theta}})$ and wave vector $\mathbf{k}_{(F)} = (k_{\hat{r}}, k_{\hat{\phi}}, k_{\hat{\theta}})$ direction:

$$\mathbf{f}_{(F)} = \frac{(\mathbf{k}_{(F)} \times \mathbf{B}_{(F)})}{|\mathbf{k}_{(F)}|}, \quad f_{t(F)} = 0. \quad (30)$$

In this scenario, the intensity of synchrotron radiation along the wave vector is proportional to

$$f^\mu f_\mu = \sin^2 \zeta |\mathbf{B}_{(F)}|^2, \quad (31)$$

with the pitch angle

$$\sin \zeta = \frac{|\mathbf{k}_{(F)} \times \mathbf{B}_{(F)}|}{|\mathbf{k}_{(F)}| |\mathbf{B}_{(F)}|}. \quad (32)$$

One can use an inverse Lorentz boost to transform the polarization vector, which yields

$$f_{(P)}^{\hat{t}} = \gamma f_{(F)}^{\hat{t}} + \gamma \beta \cos \chi f_{(F)}^{\hat{r}} + \gamma \beta \sin \chi f_{(F)}^{\hat{\phi}} \quad (33)$$

$$f_{(P)}^{\hat{r}} = \gamma \beta \cos \chi f_{(F)}^{\hat{t}} + (1 + (\gamma - 1) \cos^2 \chi) f_{(F)}^{\hat{r}} + (\gamma - 1) \cos \chi \sin \chi f_{(F)}^{\hat{\phi}} \quad (34)$$

$$f_{(P)}^{\hat{\phi}} = \gamma \beta \sin \chi f_{(F)}^{\hat{t}} + (\gamma - 1) \cos \chi \sin \chi f_{(F)}^{\hat{r}} + (1 + (\gamma - 1) \sin^2 \chi) f_{(F)}^{\hat{\phi}} \quad (35)$$

$$f_{(P)}^{\hat{\theta}} = f_{(F)}^{\hat{\theta}}. \quad (36)$$

In further calculations, it is important to know the polarization vector expressed in global coordinates, which is obtained by applying the inverse transformation from the local P -frame:

$$f^t = \frac{f_{(P)}^{\hat{t}}}{\sqrt{f(r_s)}}, \quad f^r = \sqrt{f(r_s)} f_{(P)}^{\hat{r}}, \quad (37)$$

$$f^\phi = \frac{f_{(P)}^{\hat{\phi}}}{r_s}, \quad f^\theta = \frac{f_{(P)}^{\hat{\theta}}}{r_s}.$$

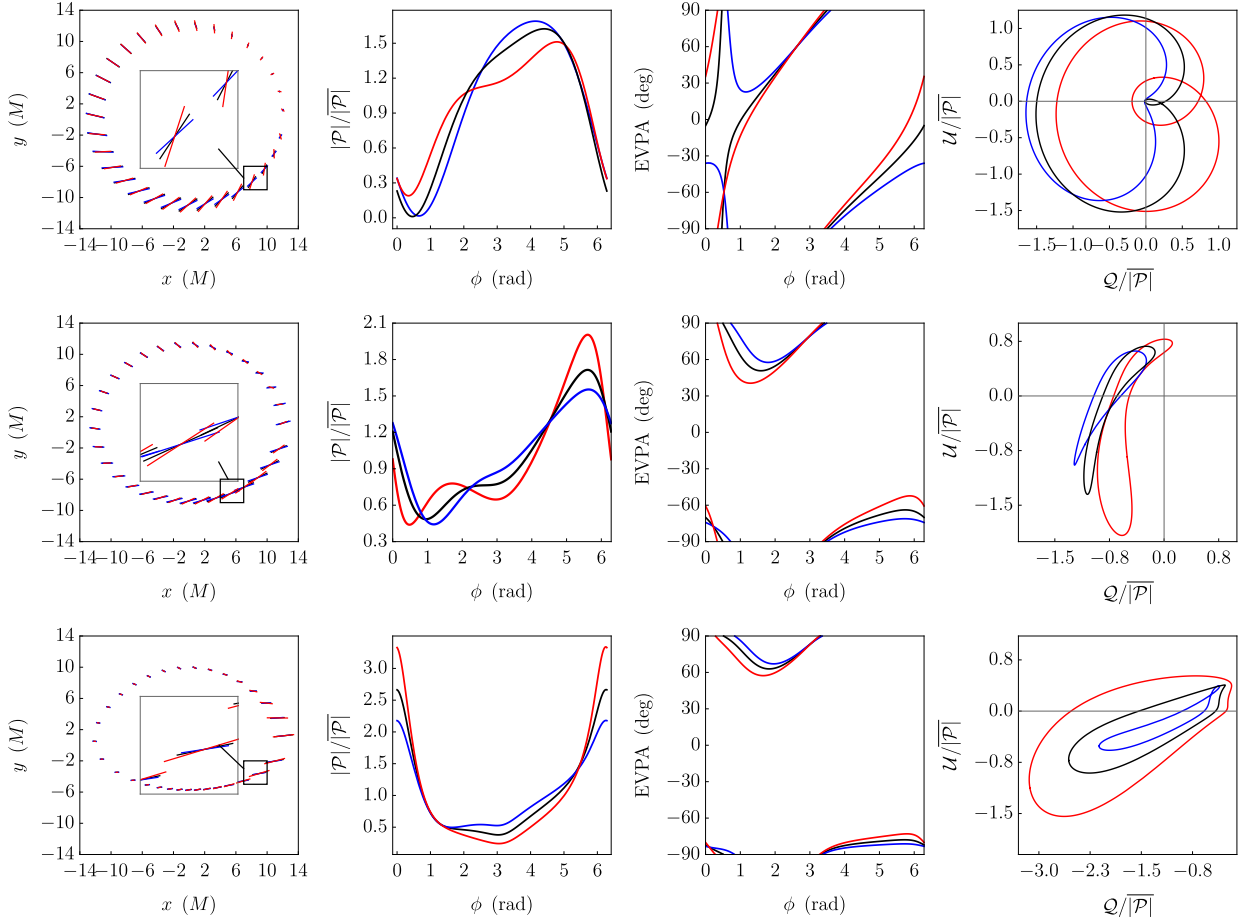


Fig. 2. From left to right: Polarization patterns, polarization intensity values, EVPA, and Q - U fluxes shown in the case of $r_s = 11$, $\chi = -\pi/2$, $\beta = \beta_R$ and for different inclination angles: 20°, 40°, 60° (from top to bottom), with curves corresponding to different magnetic-field parameters: $\mathcal{B} = 0.01$ (blue), $\mathcal{B} = 0$ (black), and $\mathcal{B} = -0.01$ (red). In each polarization panel, the polarization segment lengths are normalized relatively to the maximum intensity. The intensity and Q - U fluxes are normalized by the average polarization magnitude. This average can be defined as an angular average, $|\overline{\mathcal{P}}| = \frac{1}{2\pi} \int_0^{2\pi} |\mathcal{P}(\phi)| d\phi$. Since the angular frequency, ω , is constant in time, the same quantity can equivalently be expressed as a time average over one period, $T = 2\pi/\omega$, namely $|\overline{\mathcal{P}}| = \frac{1}{T} \int_0^T |\mathcal{P}(\omega t)| dt$.

3.2. Propagation of linearly polarized light

Light travelling from source to observer undergoes redshift, which leads to the following expression for photon momentum in the geodesic local Cartesian P -frame,

$$\begin{aligned} k_{(P)}^{\hat{t}} &= \frac{1}{\sqrt{f(r_s)}}, & k_{(P)}^{\hat{\phi}} &= -\frac{\sin \xi \sin \alpha}{\sqrt{f(r_s)}}, \\ k_{(P)}^{\hat{r}} &= \frac{\cos \alpha}{\sqrt{f(r_s)}}, & k_{(P)}^{\hat{\theta}} &= \frac{\cos \xi \sin \alpha}{\sqrt{f(r_s)}}. \end{aligned} \quad (38)$$

Here, α is the emission angle and ξ is defined by

$$\cos \xi = \frac{\cos \theta_0}{\sin \psi}, \quad \sin \xi = \frac{\sin \theta_0 \cos \phi}{\sin \psi}, \quad (39)$$

where ψ is the angle between $\hat{r}$ and the unit vector, $\hat{n}$, toward the observer.

In the next step, one must rewrite the emission angle α in terms of ψ . To do this, we calculated the angle ψ by assuming the observer is located at infinity as

$$\psi = \int_{r_s}^{\infty} \frac{dr}{r^2 \sqrt{\frac{1}{b^2} - \frac{f(r)}{r^2}}}, \quad (40)$$

together with

$$\sin \alpha = \frac{b}{r_s} \sqrt{f(r_s)}, \quad (41)$$

which helped us to find the relationship between the ψ and α emission angle. The relation (41) was found using the fact that $\tan \alpha = \sqrt{u^\psi u_\psi} / \sqrt{u^r u_r}$ (for more details, see Beloborodov 2002). Finally, we obtained the four momentum of the photon in the fluid frame by using the Lorentz transformations as

$$k_{(F)}^{\hat{t}} = \gamma k_{(P)}^{\hat{t}} - \gamma \beta \cos \chi k_{(P)}^{\hat{r}} - \gamma \beta \sin \chi k_{(P)}^{\hat{\phi}}, \quad (42)$$

$$k_{(F)}^{\hat{r}} = -\gamma \beta \cos \chi k_{(P)}^{\hat{t}} + (1 + (\gamma - 1) \cos^2 \chi) k_{(P)}^{\hat{r}} + (\gamma - 1) \cos \chi \sin \chi k_{(P)}^{\hat{\phi}}, \quad (43)$$

$$k_{(F)}^{\hat{\phi}} = -\gamma \beta \sin \chi k_{(P)}^{\hat{t}} + (\gamma - 1) \cos \chi \sin \chi k_{(P)}^{\hat{r}} + (1 + (\gamma - 1) \cos^2 \chi) k_{(P)}^{\hat{\phi}}, \quad (44)$$

$$k_{(F)}^{\hat{\theta}} = k_{(P)}^{\hat{\theta}}. \quad (45)$$

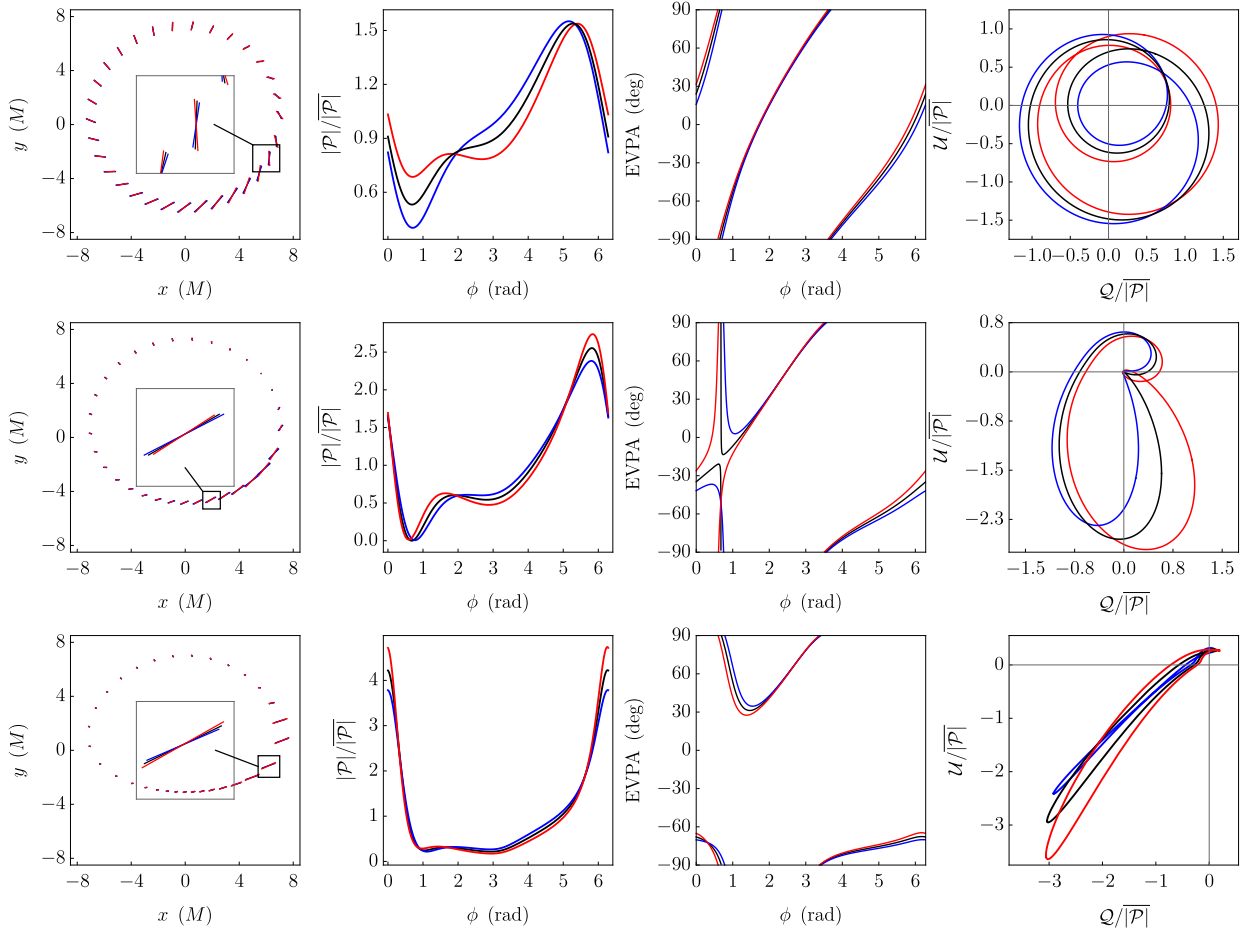


Fig. 3. Same as Fig. 2 but for $r_s = 6$. Strong gravity effects are more prominent.

3.3. Stokes parameters

At this point, we were ready to introduce the complex Penrose–Walker constant (Walker & Penrose 1970):

$$\kappa = \kappa_1 + i\kappa_2, \quad (46)$$

$$\kappa_1 = \psi_2^{-1/3} (k^t f^r - k^r f^t), \quad (47)$$

$$\kappa_2 = \psi_2^{-1/3} r_s^2 (k^\phi f^\theta - k^\theta f^\phi), \quad (48)$$

where the photon momentum and polarization components are expressed in Boyer–Lindquist coordinates. These components are obtained via an inverse transformation from the local frame:

$$\begin{aligned} k^t &= \frac{k_{(P)}^t}{\sqrt{f(r_s)}}, & k^r &= \sqrt{f(r_s)} k_{(P)}^r, \\ k^\phi &= \frac{k_{(P)}^\phi}{r_s}, & k^\theta &= \frac{k_{(P)}^\theta}{r_s}, \end{aligned} \quad (49)$$

together with the Weyl scalar,

$$\psi_2 = \frac{M}{r_s^3}. \quad (50)$$

The celestial coordinates (x, y) for a photon moving toward the observer at infinity are described as follows:

$$x = -\frac{r_s k_{(P)}^\phi}{\sin \theta}, \quad (51)$$

$$y = r_s \sqrt{(k_{(P)}^\phi)^2 - (k_{(P)}^r)^2} \cot^2 \theta \operatorname{sgn}(\sin \phi). \quad (52)$$

The expression for the normalized electric field, $\mathbf{E}$, associated with synchrotron radiation, as it propagates along a geodesic in the observer’s sky coordinates, is given as follows:

$$E_{x,\text{norm}} = \frac{y\kappa_2 + x\kappa_1}{\sqrt{(\kappa_1^2 + \kappa_2^2)(x^2 + y^2)}}, \quad (53)$$

$$E_{y,\text{norm}} = \frac{y\kappa_1 - x\kappa_2}{\sqrt{(\kappa_1^2 + \kappa_2^2)(x^2 + y^2)}}, \quad (54)$$

$$E_{x,\text{norm}}^2 + E_{y,\text{norm}}^2 = 1. \quad (55)$$

The components of the observed electric field and the intensity of synchrotron radiation, as received by an observer at infinity, are dependent on multiple factors and expressed as

$$E_{x,\text{obs}} = \delta^{\frac{3+\alpha_\gamma}{2}} l_p^{1/2} |\mathbf{B}|^{\frac{1+\alpha_\gamma}{2}} \sin^{\frac{1+\alpha_\gamma}{2}} \zeta E_{x,\text{norm}}, \quad (56)$$

$$E_{y,\text{obs}} = \delta^{\frac{3+\alpha_\gamma}{2}} l_p^{1/2} |\mathbf{B}|^{\frac{1+\alpha_\gamma}{2}} \sin^{\frac{1+\alpha_\gamma}{2}} \zeta E_{y,\text{norm}}, \quad (57)$$

$$E_{x,\text{obs}}^2 + E_{y,\text{obs}}^2 = \delta^{3+\alpha_\gamma} l_p |\mathbf{B}|^{1+\alpha_\gamma} \sin^{1+\alpha_\gamma} \zeta, \quad (58)$$

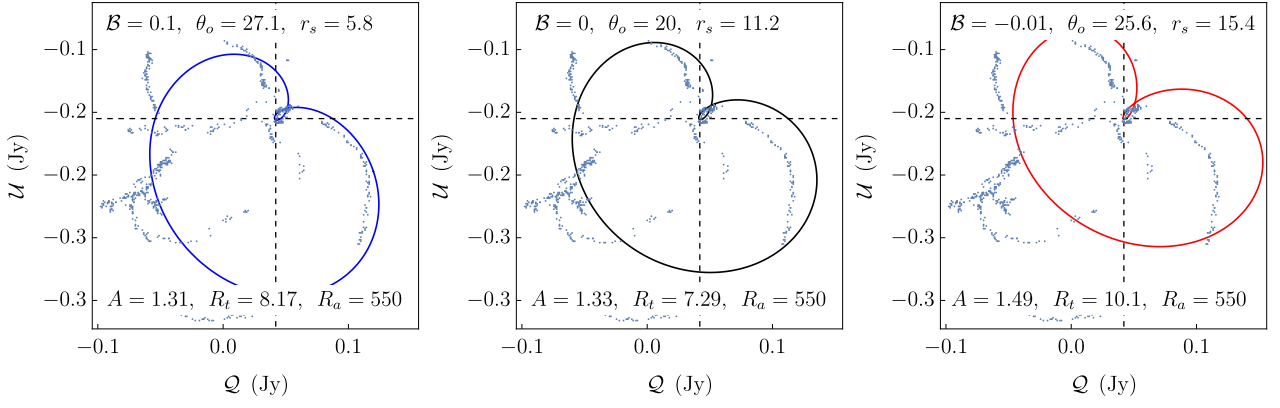


Fig. 4. Comparison between ALMA $Q-U$ loop data with semi-analytical ring model prediction for $\mathcal{B} = 0.1$ (left), $\mathcal{B} = 0$ (middle), and $\mathcal{B} = -0.01$ (right).

where $\delta = 1/k_{(F)}^{\hat{t}}$ is the Doppler shift factor and α_ν is the spectral index. In the following, we assume the emitted spectrum follows a power-law distribution of $I_\nu \propto \nu^{-\alpha_\nu}$. As a consequence, the angular dependence of the emission varies as $\sin^{1+\alpha_\nu} \zeta$. Therefore, for the spectral index, α_ν , the observed flux density is related to the hot spot's emitted flux density by

$$\frac{I_{\nu, \text{obs}}}{I_{\nu, \text{HS}}} = \delta^{3+\alpha_\nu}. \quad (59)$$

The other quantity, l_p , describes photon path length, which is usually written in the form

$$l_p = \frac{k_{(F)}^{\hat{t}}}{k_{(F)}^{\hat{\theta}}} H, \quad (60)$$

where H is the height of the disk. In the following, we set $\alpha_\nu = 1$ as a representative value at 230 GHz frequency (Goddi et al. 2021). As a result, the intensity (I) and electric vector position angle (EVPA) of polarized light becomes

$$I = |P| = \delta^4 l_p |\mathcal{B}|^2 \sin^2 \zeta, \quad (61)$$

$$\text{EVPA} = 0.5 \text{Arg}(P) = 0.5 \text{Arg}(Q + iU), \quad (62)$$

where the Stokes parameters Q and U are defined as

$$Q = E_{y, \text{obs}}^2 - E_{x, \text{obs}}^2, \quad U = -2E_{x, \text{obs}}E_{y, \text{obs}}. \quad (63)$$

In our model, circular polarization is neglected. Moreover, the linear polarization fraction is equal to a constant p and total intensity of radiation satisfies $\sqrt{Q^2 + U^2}/I = p$. We took $p = 1$ for the sake of simplicity; hence, $|P| = \sqrt{Q^2 + U^2} = I$. The Stokes parameters Q and U were calculated using Eq. (63).

4. Results for an orbiting point-like hot spot

For hot-spot modeling, we assumed a localized isotropic emitting region of characteristic size, $L \sim r_g$, confined to the equatorial plane. The primary radiative-transfer and polarization calculations are performed in the point-source approximation, in which only the trajectory of the centroid of the emitting region is followed. This approximation is justified provided that the source size remains small compared to the characteristic spatial scales over which relevant quantities vary along the orbit, including the magnetic-field structure, gravitational lensing, and relativistic polarization transport across neighboring geodesics.

Effects of extended sizes are incorporated in the depolarization calculation by integrating over the spatial extent of the equatorial emitting patch. These effects become significant when relevant quantities vary appreciably across the emitting region. We address this in Sect. 5.

4.1. Effect of electromagnetic interaction on polarization

We were then ready to construct the polarization images of the hot-spot-orbiting black hole in the equatorial plane at a fixed radius of $r = r_s$. We chose the motion to be clockwise, corresponding to an angle of $\chi = -\pi/2$. The associated relativistic velocity measured in the locally nonrotating frame is given by

$$\beta_R = \Omega \frac{r_s}{\sqrt{f(r_s)}}, \quad (64)$$

where the angular velocity, Ω , is defined by formula (19) in the previous section. The orbital period of the hot spot around the black hole can be determined using the relation

$$T = 2\pi t_g \sqrt{r^3 (2\mathcal{B}^2 r^2 (r-2) + 2\mathcal{B} r F + 1)}, \quad (65)$$

where $t_g = GM/c^3$ is the gravitational timescale and $\mathcal{B}$ is a magnetic-field interaction parameter defined in (13). In Figs. 2 and 3 we illustrate how the magnetic interaction parameter $\mathcal{B}$ affects every major polarization observable for an orbiting hot spot, and how those effects vary with inclination angle. From left to right we show the instantaneous polarization-vector maps on the sky (the ticks are normalized to the same maximum length in each panel), the total polarization intensity, the EVPA, and the $Q-U$ trajectory over one full 2π of azimuthal phase, ϕ . Rows correspond to the inclinations of 20° , 40° , 60° (from top to bottom), while colors show $\mathcal{B} = +0.01$ (blue), 0 (black), and -0.01 (red); that is, the interaction parameter is relatively weak. Figure 2 shows the hot spot at radius $r = 11 r_g$, while Fig. 3 repeats the same analysis at $r = 6 r_g$, bringing the hot spot deeper into the strong-gravity zone.

In the left panels of Figs. 2 and 3 illustrating the polarization patterns, systematic offsets (zoomed-in view) are visible: a positive $\mathcal{B}$ (blue) twists the polarization direction forward in the direction of motion, while negative $\mathcal{B}$ (red) twists it backward. Changing the viewing angle shows that at low inclinations the local magnetic interaction torque more directly alters the projected field geometry, whereas at high inclinations the lensing effect dominates. The total-intensity, I , plots exhibit peaks when the

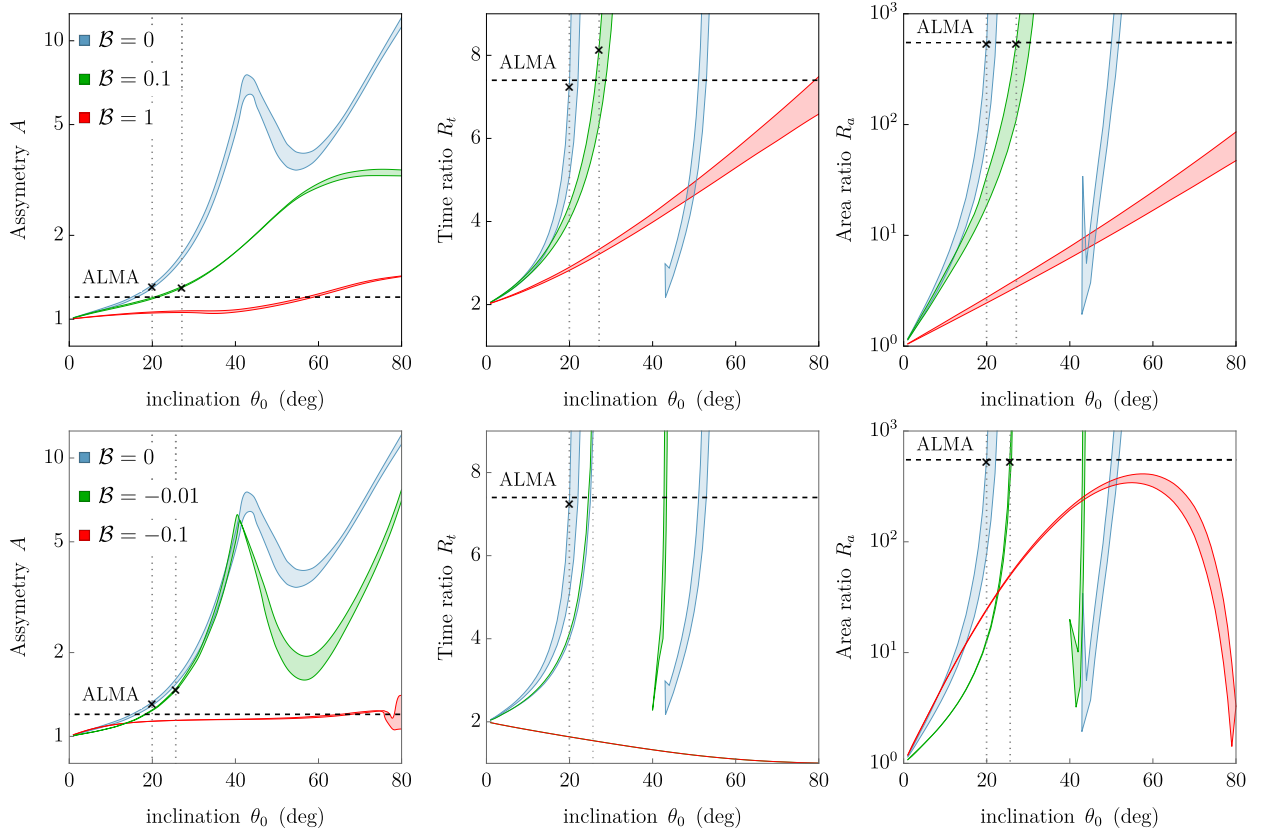


Fig. 5. Asymmetry (A), time (R_t), and area (R_a) ratio in the case of negative magnetic parameters (*top*) and positive ones (*bottom*) with respect to inclination angle. The lowest inclination angle at which $R_a = 550$ is indicated by a cross.

emitting hot spot passes behind the black hole due to maximal Doppler boosting. As with the other observables, the absolute difference between positive and negative B grows as the system is viewed more face-on. This shows that low-inclination polarized flares provide a better sensitivity to the magnetic-interaction parameter, B .

The third columns of Figs. 2 and 3 show the EVPA versus orbital phase. Interestingly, even small $B = \pm 0.01$ shifts the entire EVPA curve by ~ 5 – 10° and introduces a slight asymmetry between the incoming and outgoing rotations. Since the EVPA can in principle be measured with sub-degree precision in bright flares, even a small persistent offset can potentially provide a sensitive probe of B . A limiting factor at 230 GHz frequency is the variable internal Faraday screen of Sgr A* (Wielgus et al. 2024), which we neglect in this analysis. Finally, the rightmost panels of Figs. 2 and 3 illustrate the Q – U trajectories over one orbit. The magnetic-interaction parameter does not change the sense of the loop rotation, but its shape and enclosed area clearly depend on the sign and magnitude of B . We quantified this in the next section when applied to the observational data.

4.2. Polarization loops from Sgr A* with ALMA

Wielgus et al. (2022b) reported an exceptional radio flare from Sgr A* detected by the ALMA, whose EVPA traces clear, quasi-closed loops in the Stokes Q – U plane. By fitting a synchrotron hot spot orbiting a black hole, they extracted three key parameters of each loop: asymmetry, A – the largest ratio of perpendicularly projected diameters of the loops; time ratio, R_t – the ratio of the full loop period to the time; and area ratio, R_a – the ratio of area of the large loop to that of the small loop. Based on the

ALMA measurements Wielgus et al. (2022b) constrained these parameters to

$$A = 1.3 \pm 0.1, \quad R_t = 6.5 \pm 0.5, \quad R_a = 550 \pm 50, \quad (66)$$

and inferred an observer inclination of $\theta_0 \approx 20^\circ$. They also derived that the magnetic field near the black hole is dominated by the vertical component.

We extended their analysis by incorporating electromagnetic interaction between the synchrotron-emitting hot spot and those vertical magnetic fields parametrized by B . If one considers $B = 0$, corresponding to a purely Keplerian hot spot, the emitter is located at approximately $r \approx 10.6 \pm 0.6 r_g$, corresponding to a range from $r_{\min} = 10 r_g$ to $r_{\max} = 11.2 r_g$. However, for nonzero B , the situation changes considerably, as we demonstrate in Figs. 4 and 5.

Figure 4 shows a comparison between the Q – U fluxes predicted by the simple ring model and the ALMA observations acquired on April 11, 2017 for a different B . In Fig. 5 we show the dependence of A , R_t , R_a for different values of the inclination angle. The overall result is consistent with that obtained by Wielgus et al. (2022b), and it predicts a small tilt angle of approximately $\theta_o \approx 20^\circ$ for $B = 0$, $\theta_o \approx 26^\circ$ for $B = -0.01$, and $\theta_o \approx 27^\circ$ for $B = 0.1$, respectively. Interestingly, including either the positive or negative magnetic parameter, both lead to larger inclination angles. In our comparison, it is evident that large negative values of the magnetic parameter around $B \approx -0.1$ fail to accurately reproduce the time ratio observed in the ALMA data (see the bottom middle panel). Likewise, large positive values of B also struggle to reproduce the area ratio (see the top right panel). Therefore, one can infer that the magnetic parameter likely lies within the range of $-0.1 < B < 1$.

In the figures, the area ratio of the loops, $R_a = 550$, remains unchanged. In this scenario, in order to better reproduce the observations, we set $r_s = r_{\max}$ in each figure, corresponding to the smallest tilt angle (as illustrated in Fig. 5, smaller θ_o produce more symmetric loops). As shown in the figures, introducing a positive magnetic parameter makes the loop more symmetrical, although it also increases the R_l ratio. In the case of a negative magnetic parameter, the situation changes dramatically. Including a small negative $\mathcal{B}$ increases not only the time ratio, it also makes the loops more asymmetric. Therefore, it can be inferred that if a magnetic parameter is present, it likely has a positive effective sign, corresponding to either $q > 0$, $B > 0$ or $q < 0$, $B < 0$.

5. Depolarization of a finite-sized hot spot

We quantified the depolarization arising from the nonzero spatial extent of a synchrotron-emitting hot spot in the equatorial plane of a spherical symmetric space-time by comparing its net linear polarization to that of an equivalent point-like emitter. We considered a symmetric, optically thin hot spot centered at radius r_c and azimuth $\phi_c = -\Omega_c t$ at emission time t , with emissivity-profile scaling modeled as

$$J(r, \phi) \sim \exp \left[-4 \log 2 \left(\frac{\Delta x^2 + \Delta y^2}{w^2} \right) \right] = \exp \left[-4 \log 2 \left(\frac{\Delta r^2 + 4r_c(r_c + \Delta r) \sin^2 \frac{\Delta \phi}{2}}{w^2} \right) \right], \quad (67)$$

where the minus sign in $\phi_c = -\Omega_c t$ denotes clockwise motion, $\Delta \phi = \arg e^{i(\phi - \phi_c)}$ is the wrapped phase difference, and $\Delta r = r - r_c$ is the radial displacement from the spot center. The parameter w is the full width at half maximum (FWHM) of the Gaussian profile. In the following, we identify the hot-spot scale as $L \sim w$.

The observed Stokes parameters were computed by incorporating all relevant general relativistic radiative-transfer effects. The observed intensity on the image plane is expressed as

$$I(\rho, \varphi) = J(\rho, \varphi) I_{v, \text{obs}}, \quad (68)$$

where (ρ, φ) denote the polar coordinates on the observer's screen, defined as

$$\rho = \sqrt{x^2 + y^2}, \quad \varphi = \arg(x + iy). \quad (69)$$

In evaluating Eqs. (61)–(63), we included the redshift correction through the observed specific intensity $I_{v, \text{obs}}$ defined by Eq. (59).

The net observed polarization from the extended hot spot was obtained by integrating the Stokes parameters over the image of the emitting region,

$$Q_{\text{tot}} = \iint Q(\rho, \varphi) dA, \quad U_{\text{tot}} = \iint U(\rho, \varphi) dA, \quad (70)$$

$$I_{\text{tot}} = \iint I(\rho, \varphi) dA, \quad (71)$$

leading to a polarization fraction

$$p_{\text{finite}} = \frac{|Q_{\text{tot}} + i U_{\text{tot}}|}{I_{\text{tot}}} = \frac{\sqrt{Q_{\text{tot}}^2 + U_{\text{tot}}^2}}{I_{\text{tot}}}. \quad (72)$$

For comparison, the polarization fraction of a point-like emitter located at (r_c, ϕ_c) is $p_{\text{point}} = 1$. Therefore, we define the depolarization factor as

$$D = \frac{p_{\text{finite}}}{p_{\text{point}}}. \quad (73)$$

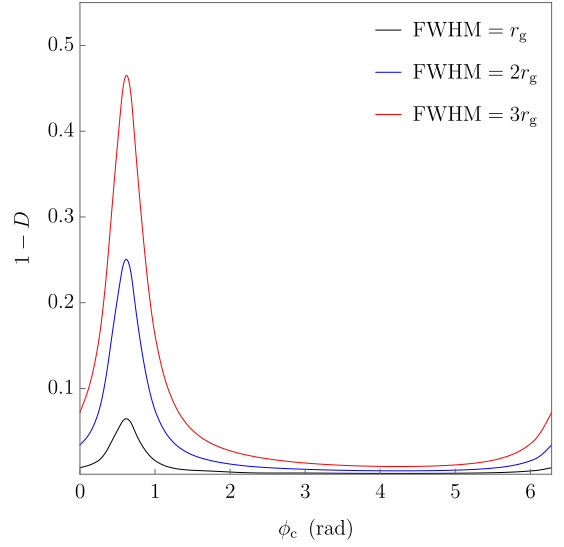


Fig. 6. Depolarization of emitted radiation as a function of the orbital phase, ϕ_c , for an orbiting hot spot with different sizes measured with the FWHM. The observer inclination angle is fixed at $\theta_o = 20^\circ$, while the hot-spot center is located at $r_c = 10 r_g$. The figure demonstrates how the extent of the emitting region influences the phase-dependent depolarization behavior.

The net observed Stokes parameters were obtained by numerically integrating over the observer's image plane. In practice, we summed the local flux densities $I(x, y)$, $Q(x, y)$, and $U(x, y)$ over all pixels in an image with a $30 r_g$ field of view and resolution 512×512 . Depolarization arises from spatial variations of the EVPA across the image of the emitting region, leading to partial cancellation in the vector sum of local polarized fluxes. For $w = r_g$ and $r_c = 10 r_g$, the variation of the EVPA across the hot spot is small, resulting in weak depolarization. The net depolarization fraction is given by

$$(1 - \langle D \rangle) \times 100\% \approx 0.72\%, \quad (74)$$

where $\langle D \rangle$ is the time-averaged depolarization factor over one orbit, which remains close to unity.

In Fig. 6, the weakest finite-size depolarization occurs near $\phi_c \sim 7\pi/5$, corresponding to the hot-spot position in front of the black hole. In contrast, the depolarization becomes strongest around $\phi_c \sim \pi/5$, when the hot spot is located behind the black hole. In this configuration, photons emitted from different regions of the source propagate along substantially different geodesics due to strong gravitational lensing, enhancing variations in the polarization properties across the emitting region and leading to a stronger net depolarization effect. For comparison, in the case of a static hot spot the depolarization extrema occur at exactly $\phi_c = \pi/2$ (maximum) and $\phi_c = 3\pi/2$ (minimum). The slight shift of these extrema in the orbiting case is caused by the nonzero orbital velocity of the emitting plasma. We note, however, that this effect becomes significant for emission originating close to the black hole, where strong-field lensing amplifies variations in the transported polarization vectors. Our results indicate that strong depolarization occurs when the characteristic source size becomes comparable to the distance of the source center from the black hole, namely $w/r_c \sim 1$. In this regime, differential transport across the emitting region becomes substantial, producing large variations in the transported polarization directions and consequently leading to a significant reduction of the integrated polarization degree.

6. Conclusions

Our results demonstrate that even within a quasi-neutral plasma, small charge imbalances can lead to electromagnetic interactions with significant dynamical and observational consequences near Sgr A*. In particular, the new magnetic interaction parameter, $\mathcal{B}$, produces characteristic, inclination-dependent imprints on all polarization observables of an orbiting hot spot. In practice, the strongest effects can be measurable from modeling the EVPA offset and the Q – U loop area.

In this study, we adopted two distinct simplifications: (1) the black hole was assumed to be nonrotating; and (2) only the primary (direct) image was considered, while higher order images were neglected. However, black-hole spin can contribute noticeably, and secondary images may influence the detailed structure of the Q – U loop (Gelles et al. 2021; Yfantis et al. 2024a). Including them would help capture spin-dependent effects more accurately, but treating higher order images is challenging within this framework, so they are not included in this analysis. Moreover, in a rotating Kerr black hole, the frame dragging of space-time twists the magnetic-field lines, inducing an electric field. This effect is encoded in Wald’s solution as a nonzero temporal component of the vector potential, which gives rise to a radial and polar electric field near the black hole (Komissarov 2022).

Our results indicate that the depolarization associated with the finite extent of the emitting region is weak for compact hot spots, with the orbit-averaged value remaining below 1% and maximum along the orbit of around 5% for $w \sim r_g$ at an orbital radius of $r_c = 10 r_g$. However, a systematic increase is observed for larger hot-spot sizes ($w > r_g$), leading to more pronounced depolarization, reaching orbit-averaged value of $\approx 6.4\%$ and a maximum along the orbit of over 40% for $w = 3 r_g$.

The inclusion of the $\mathcal{B}$ -term in our emission model represents a direct coupling between the hot-spot electrons and an external, large-scale magnetic field around the black hole. A nonzero $\mathcal{B}$ applies a small torque on the emitting electrons, tilting their pitch-angle distribution and hence the instantaneous polarization vector. As the hot-spot orbits, this torque accumulates a phase shift that appears as a coherent EVPA offset and altered Q – U morphology. Simultaneously, the model allows the emission to occur closer to the black hole.

Even a small charge imbalance in the hot spot (e.g., due to charge separation originating from turbulence) renders it sensitive to a Lorentz force that can subtly alter its trajectory, and hence its EVPA features. We conclude that only weak electromagnetic coupling, $|\mathcal{B}| \lesssim 0.1$, remains consistent with ALMA’s polarization loops. These limits on $\mathcal{B}$ translate into constraints on the product of the hot-spot charge-to-mass ratio and the ambient field strength at radii of $r \sim 5$ – $10 r_g$. In the absence of electromagnetic interaction, stable circular orbits around a Schwarzschild black hole are only possible at radii greater than $r = 6 r_g$. Therefore, any emission originating from $r < 6 r_g$ may result from electromagnetic effects, which shift the location of stable orbits closer to the event horizon. Future high-precision astrometric monitoring will directly localize the hot spot’s orbital phase and radius, breaking the current degeneracy with inclination and $\mathcal{B}$ and thereby tightening constraints on both the coupling parameter and the magnetic-field strength.

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