

LETTER TO THE EDITOR

Periodic variations in the first and second moments of broad Balmer emission lines from central accretion disks

XueGuang Zhang*

Guangxi Key Laboratory for Relativistic Astrophysics, School of Physical Science and Technology, GuangXi University, No. 100, Daxue road, Nanning 530004, P.R. China

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ABSTRACT

Broad emission line regions (BLRs) lying in central accretion disks are widely accepted to explain the unique double-peaked broad emission lines in active galactic nuclei (double-peaked BLAGNs). Accepting the accretion disk origin, we theoretically simulated and determined the periodic variations in the central wavelength λ_0 (the first moment) and line width σ (the second moment) of double-peaked broad emission lines. Furthermore, through theoretically simulated periodicities of $T_{\lambda 0}$ and T_{σ} for variations in λ_0 and σ , a periodicity ratio R_{fs} of $T_{\lambda 0}$ to T_{σ} of about 2 was applied to support the hypothesis that the spiral arms are preferred in BLRs in central accretion disks. Then, we determined the periodic variations in λ_0 and σ and showed them in the known double-peaked BLAGN NGC 1097. This led to the parameter $R_{fs} \sim 2$, which can be applied as a clue to support the hypothesis that the spiral arm structure in disk-like BLRs in the central accretion disk is the most compelling interpretation of the variability in the double-peaked broad H α in NGC 1097. The results provide clean criteria for testing the accretion disk origins of double-peaked broad emission lines in AGN.

Key words. galaxies: active – galaxies: nuclei – quasars: emission lines – quasars: supermassive black holes

1. Introduction

Broad emission line active galactic nuclei (BLAGNs) have one special subclass, BLAGNs with double-peaked broad emission lines (double-peaked BLAGNs). The first reports of double-peaked BLAGNs were made in the 1980s by [Chen et al. \(1989\)](#), [Chen & Halpern \(1989\)](#). Since then, large samples of double-peaked BLAGNs have been assembled, such as the 12 double-peaked BLAGNs by [Eracleous & Halpern \(1994\)](#), the 116 double-peaked BLAGNs by [Strateva et al. \(2003\)](#), and the more recent 250 double-peaked BLAGNs by [Ward et al. \(2024\)](#). Two main models have been proposed to explain the double-peaked broad emission lines: the model related to the binary black hole (BBH) system (the BBH model), and the model related to broad emission line regions (BLRs) that lie in the central accretion disk (the accretion disk model).

The BBH model was first proposed to explain the systematic variability of the double-peaked broad Balmer lines in 3C390.3 by [Gaskell \(1996\)](#). However, [Eracleous et al. \(1997\)](#) ruled out the BBH model through spectroscopic monitoring of 3C 390.3 over two decades because they found no long-term systematic changes in the radial velocity, as expected by the BBH model. In addition to the BBH model, the accretion disk model has been accepted for double-peaked broad emission lines, such as the circular accretion disk model by [Chen et al. \(1989\)](#), [Chen & Halpern \(1989\)](#), the elliptical accretion disk model by [Eracleous et al. \(1995\)](#), the accretion disk model considering warped structures by [Hartnoll & Blackman \(2000\)](#), the circular accretion disk plus arms model by [Storchi-Bergmann et al. \(2003\)](#), and the stochastically perturbed accretion disk model by [Flohic & Eracleous \(2008\)](#).

Since the accretion disk models were proposed, more and more studies have shown that the current models are not good enough, but the elliptical accretion disk model and/or the circular accretion disk plus arms model are preferred to explain the emission features of the double-peaked broad lines, especially through the long-term variability in the double-peaked broad lines (see the discussions by [Storchi-Bergmann et al. 2003](#); [Eracleous et al. 2009](#); [Gezari et al. 2007](#); [Lewis et al. 2010](#); [Schimoia et al. 2012](#); [Zhang 2013](#)). The circular accretion disk plus arms model and/or the elliptical accretion disk model were successfully applied to describe the double-peaked broad lines in the literature (as discussed and shown by [Storchi-Bergmann et al. 2017](#); [Hung et al. 2020](#); [Zhang 2021, 2022a, 2024a,b](#); [Ward et al. 2024](#)).

When the line profile variability in the double-peaked broad emission lines was studied, the following parameters were determined: the positions and intensities of the two peaks, the line widths (full width of the profile at half maximum and/or quarter-maximum) of the double-peaked broad lines, and so on. Quasi-periodic variations in the line parameters are basically expected according to the accretion disk model, such as the simple results shown in Fig. 31–36 in [Lewis et al. \(2010\)](#). However, the expected quasi-periodic variations disagree with results from multi-epoch spectroscopic results, probably for two main reasons. On the one hand, the peak positions are not apparent in the observational line spectra, which leads to large uncertainties in the peak positions and/or peak intensities. On the other hand, the peak of the double-peaked broad lines mixed with narrow emission lines probably leads to apparent effects of the determined line widths.

We do not determine the information of the peaks of the double-peaked broad emission lines, but determine the first

* Corresponding author: xgzhang@gxu.edu.cn

moment (central wavelength, λ_0) and second moment (line width, σ) of the double-peaked broad emission lines based on the accretion disk model. We then compare them with the observational results. The peak positions of the line profiles slightly affect the estimates of λ_0 and σ , which probably leads to more apparent periodic variations in λ_0 and σ . This is our main objective. Section 2 presents our main results and the necessary discussion of the variability in λ_0 and σ of the double-peaked broad lines. Section 3 gives our main conclusions. We adopted the cosmological parameters $H_0 = 70 \text{ km} \cdot \text{s}^{-1} \text{ Mpc}^{-1}$, $\Omega_\Lambda = 0.7$, and $\Omega_m = 0.3$.

2. Main results and necessary discussions

As noted in the Introduction, we mainly considered the elliptical accretion disk model and the circular accretion disk plus arms model. The elliptical accretion disk model by Eracleous et al. (1995) has seven free model parameters: the inner radius r_0 (in units of the Schwarzschild radius R_G) and the outer radius r_1 (in units of R_G) of the emission region, the inclination angle i of the emission region, the line emissivity power-law index $f_r \propto r^{-q}$, the local turbulence broadening parameter σ_L (in units of km/s), the eccentricity e of the emission region, and the orientation angle ω . The circular accretion disk plus arms model by Storchi-Bergmann et al. (2003) has four model parameters in addition to those listed for the elliptical accretion disk model and the eccentricity is zero: the contrast ratio A for the arms relative to the rest of the disk, the width w and the pitch angle p for the arms, and the starting radius r_s (in units of R_G) of the arms.

With the theoretical models, we determined the variability in the central wavelength (the first moment, λ_0) and in the line width (the second moment, σ) with the following five steps. First, except for the model parameter of ω , the other model parameters were randomly selected within the accepted limited ranges listed in Table 1. In Table 1 the range from 0.3 to 0.9 for the $\sin(i)$ means an inclination angle larger than 18 degrees, but smaller than 64 degrees. These are common values for BLAGNs, as discussed in Zhuang et al. (2018). Second, 30 values of ω in a cycle were randomly collected from 0 to 2π . Third, combining the 30 values of ω with the other model parameters, we created 30 line profiles in one cycle with different ω and with the same other model parameters within wavelengths from 6100 Å to 7200 Å for the expected double-peaked broad H α (6564.61 Å as the theoretical central wavelength in the rest frame). Fourth, based on the definitions of the first moment λ_0 and second moment σ by Peterson et al. (2004), we calculated λ_0 and σ of the 30 simulated line profiles within one cycle. Fifth, to repeat the four steps above 1000 times, there were 1000 cases with determined λ_0 and σ in 30 different phases in one rotation cycle of the disk-like BLRs.

As examples in Fig. 1, panels (a) and (d) show the simulated lines in different phases in one cycle according to the elliptical accretion disk model and the circular accretion disk plus arms model. The apparent variability in the line profiles is confirmed. Panels (b) and (e) show the corresponding variability in λ_0 in different phases for the simulated lines shown in panels (a) and (d). The periodic variations in λ_0 in a rotation cycle are clear. Panels (c) and (f) show the corresponding variability in σ in different phases for the simulated lines shown in panels (a) and (d). The periodic variations in σ in a rotation cycle are again clear. Instead of a pure sine function, considering ω as the function argument, we applied a sine function plus a linear trend to describe λ_0 and σ on ω related to accretion disk models, as shown in Fig. 1.

Table 1. Limited ranges for the model parameters.

Par	Units	Arms	Elli
r_0	R_G	[100, 600]	[100, 600]
r_1	R_G	$r_0 + [300, 1800]$	$r_0 + [300, 1800]$
$\sin(i)$		[0.3, 0.9]	[0.3, 0.9]
q		[-7, 7]	[-7, 7]
e		0	[0, 0.95]
σ_L	km/s	[400, 1500]	[400, 1500]
A		[1, 3]	
W	degree	[10, 60]	
p	degree	[-45, 45]	
r_s	R_G	$[r_0, 0.8r_1]$	

Notes. The first column shows the applied model parameter. The second column shows the units for the model parameters. The third column shows the limited range ([lower boundary, upper boundary]) of each model parameter in the circular accretion disk plus arms model. The fourth column shows the limited range of each model parameter in the elliptical accretion disk model.

To determine the periodic variability in λ_0 and σ of the simulated double-peaked broad lines, we applied the widely accepted Lomb-Scargle periodogram technique (Lomb 1976; Scargle 1982; Zechmeister & Kurster 2009; VanderPlas 2018) to the light curves $\lambda_0(\omega)$ and $\sigma(\omega)$ in the 1000 cases according to the theoretical accretion disk models. No real time information is available for the model-created curves of λ_0 and σ on ω . We therefore applied ω to trace the time information. By applying the Lomb-Scargle technique, we set the periodicity range we searched (the range for the x-axis in the left panels of Fig. 2) to be larger than 0.1 (in units of rad) and smaller than 20 (in units of rad), in order to show the clear peaks around 2π (rad). In Fig. 2, panels (a) and (b) show the Lomb-Scargle power of the $\lambda_0(\omega)$ and $\sigma(\omega)$ shown in panels (b), (c), and (e), (f) in Fig. 1. Interestingly, clear peaks with a confidence level higher than 99% (the corresponding false-alarm probability is 0.01) are found in the Lomb-Scargle power. This shows that 860 and 894 of the 1000 cases according to the elliptical accretion disk model and according to the circular accretion disk plus arms model, respectively, show reliable peaks with a confidence level higher than 99% in the Lomb-Scargle power. The corresponding peak distributions are shown in panels (c) and (d) in Fig. 2 to support the clear periodic variations in λ_0 and σ for the double-peaked broad lines from BLRs that lie in the central accretion disks. Furthermore, we applied the two-sided Kolmogorov-Smirnov test to all the listed model parameters in Table 1 to confirm each model parameter for the 860 and 894 cases with periodic results and all the 1000 cases with the same distributions with a probability higher than 99%. The higher probability of more than 86% (860 out of 1000, and 894 out of 1000) for the periodic results of the simulated results and the same model parameter distributions for the 1000 cases and the 860 and 894 cases strongly indicate that the coverage of the model parameter space affects the detected periodic results very little.

Moreover, as the results in panels (c) and (d) in Fig. 2 show, the distributions of R_{fs} (ratio of the periodicity in units of rad in $\lambda_0(\omega)$ to that in $\sigma(\omega)$) shown in panels (e) and (f) in Fig. 2 indicate that circular accretion disk plus arms model can lead to $R_{fs} \sim 1.98 \pm 0.08$ (the mean value plus/minus the standard deviation), but the elliptical accretion disk model should lead to $R_{fs} \sim 1.22 \pm 0.30$ (the mean value plus/minus the standard deviation). The very different R_{fs} can be applied to determine

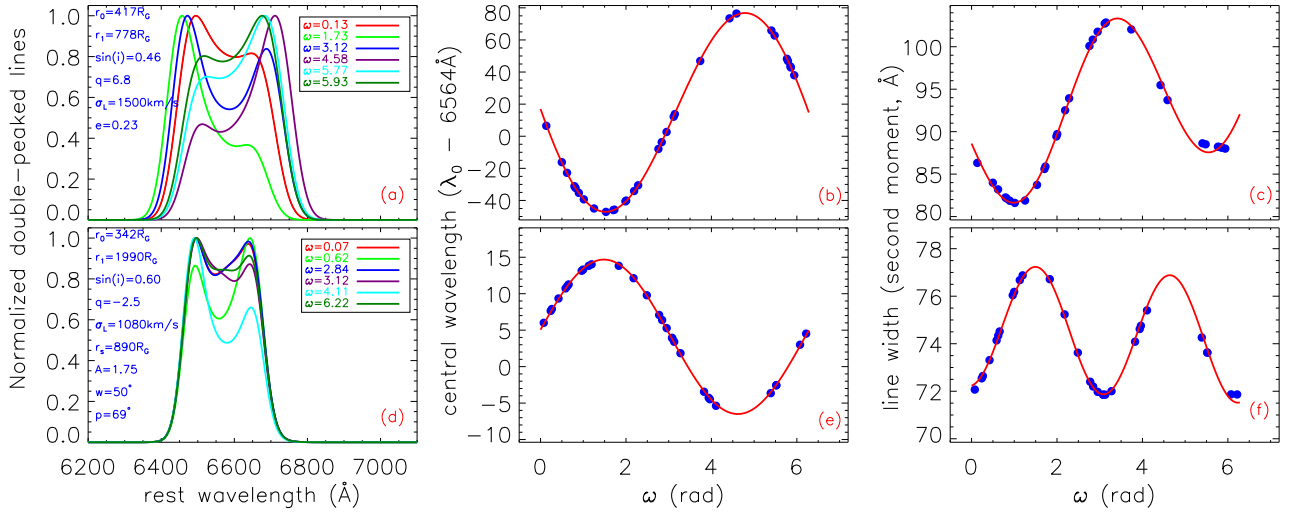


Fig. 1. Panel (a): Examples of the simulated double-peaked broad emission lines in different phases according to the elliptical accretion disk model. The solid lines in different colors show the line profiles in different phases with different orientation angles ω , as shown in the legend in the top right corner. Panels (b) and (c): Dependence of the first moment λ_0 and the second moment σ on the orientation angle ω for the case shown in panel (a). The solid blue circles show the λ_0 and σ calculated through simulated results with 30 randomly collected values of ω , and the solid red line shows the best descriptions of $\lambda_0(\omega)$ and $\sigma(\omega)$ based on applications of a sine function plus a linear trend. Panels (d), (e), and (f): Corresponding results according to the circular accretion disk plus arms model.

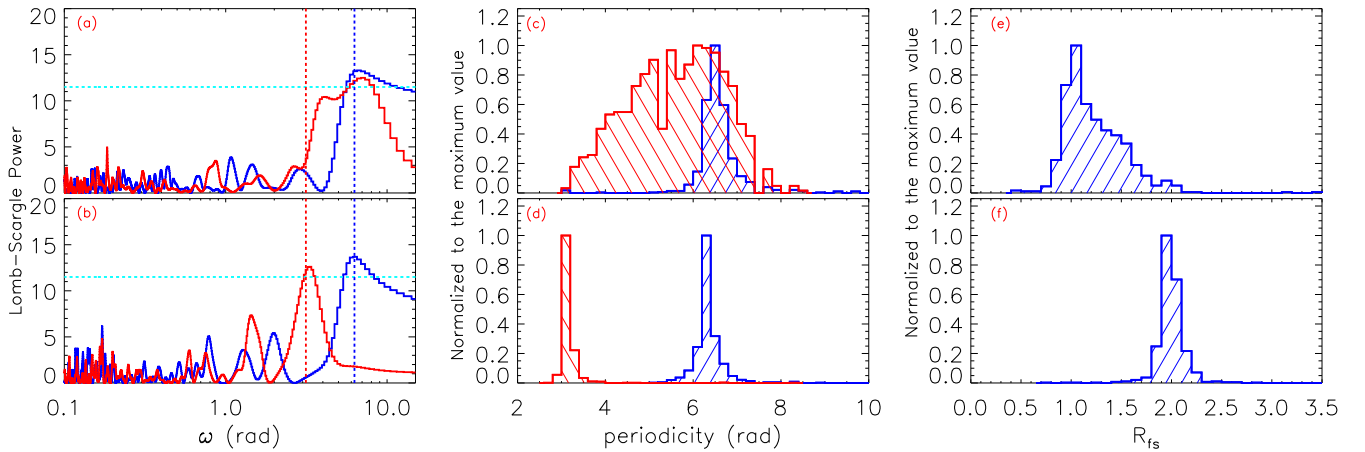


Fig. 2. Panels (a) and (b): Lomb-Scargle power of the $\lambda_0(\omega)$ (solid blue line) and $\sigma(\omega)$ (solid red line) shown in panels (b), (c), and (e), (f) of Fig. 1. The horizontal dashed cyan lines mark the confidence level of 99% (corresponding to a false-alarm probability of 0.01) for the periodicities. Panels (c) and (d): Periodicity (in units of rad) distributions for the 864 and 894 cases according to the elliptical accretion disk model and the circular accretion disk plus arms model, respectively. The blue and red histograms show the results for the periodicity distributions in $\lambda_0(\omega)$ and $\sigma(\omega)$, respectively. Panels (e) and (f): Distributions of R_{fs} according to the elliptical accretion disk model and the circular accretion disk plus arms model, respectively.

which model, elliptical accretion disk model or circular accretion disk plus arms model, is preferred to explain the double-peaked broad emission lines.

In addition to the results for the theoretical accretion disk models, it is interesting to determine whether periodic variations can be detected in a real double-peaked BLGN. We chose the known double-peaked BLGN NGC 1097 to do this. Through the reported 11 high-quality double-peaked broad H α in Storchi-Bergmann et al. (2003, host galaxy contributions have been removed) and the one public spectrum collected from the Hubble Space Telescope mission (ID:8684, PI: Dr. Eracleous) that was described in Zhang (2022b), we measured the first and second moments after masking the narrow emission lines in the observer frame. The time evolution of the first moment $\lambda_0(t)$ and the second moment $\sigma(t)$ is shown in the left panels of Fig. 3 for the 12 high-quality double-peaked broad

H α from November 2, 1991 (MJD = 48 563) to July 24, 2004 (MJD = 53 211).

By considering a model function including a sine function plus a linear trend to describe $\lambda_0(t)$ and $\sigma(t)$, with the real time t as the function argument and based on the maximum likelihood method combined with a Markov chain Monte Carlo (MCMC) technique (Foreman-Mackey et al. 2013), we determined the best descriptions and the corresponding 1RMS scatter. They are shown in the left panels of Fig. 3. The determined robust periodicities are $T_{\lambda_0} = 2630_{-55}^{+168}$ days (T_{λ_0} about $k_{10} \sim 47.8$ times longer than its negative uncertainty margin of 55 days and about $k_{11} \sim 15.6$ times longer than its positive uncertainty margin of 168 days) in $\lambda_0(t)$ and $T_{\sigma} = 1322_{-64}^{+72}$ days (T_{σ} about $k_{20} \sim 20.6$ times longer than its negative uncertainty margin of 64 days and $k_{21} \sim 18.4$ times longer than its positive uncertainty margin of 72 days) in $\sigma(t)$. The MCMC

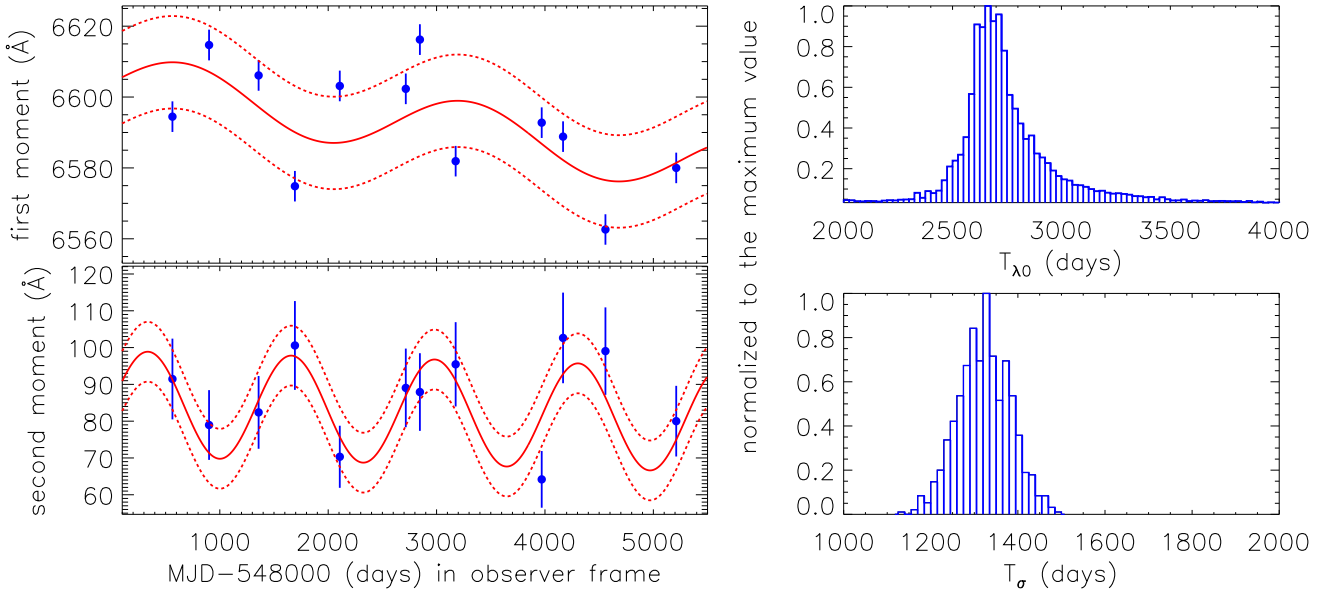


Fig. 3. Left: Variability in the first moment $\lambda_0(t)$ and second moment $\sigma(t)$ of the double-peaked broad H α in NGC 1097. The solid blue circles plus error bars show the measured results for the 12 high-quality spectra after the host galaxies were removed and the narrow emission lines were masked out. The solid and dashed red lines show the best descriptions and the corresponding 1RMS scatter for $\lambda_0(t)$ and $\sigma(t)$ determined by the maximum likelihood method combined with the MCMC technique. Right: Periodicity distributions of $T_{\lambda 0}$ in $\lambda_0(t)$ and T_{σ} in $\sigma(t)$.

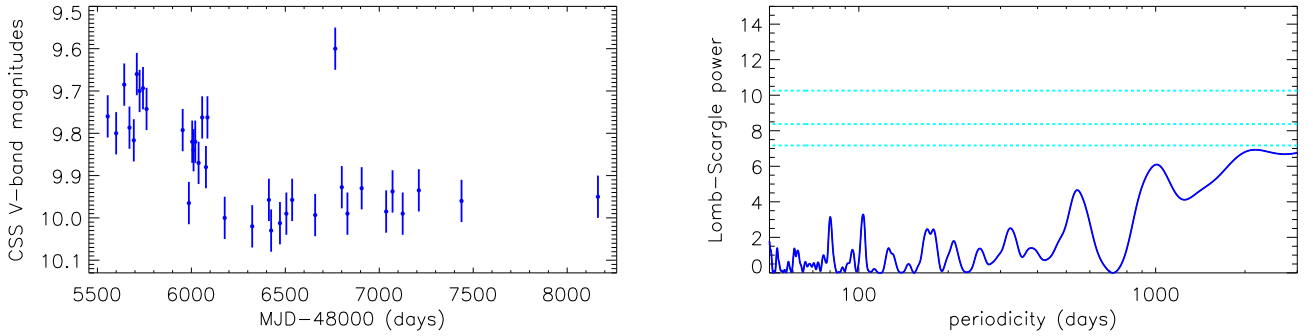


Fig. 4. Left: CSDR2 V-band light curve of NGC 1097. Right: Corresponding Lomb-Scargle power. The horizontal dashed cyan lines from top to bottom show the 90%, 50%, and 10% confidence levels (the corresponding false-alarm probability is 0.1, 0.5, and 0.9), respectively.

technique determined posterior distributions in the right panels of Fig. 3, which mean that the parameter $R_{fs} = 1.99^{+0.23}_{-0.14}$ in NGC 1097 is well consistent with the theoretically simulated results shown in panel (f) in Fig. 2. The results support the hypothesis that the circular accretion disk plus arms model is preferred for the double-peaked broad H α in the known double-peaked BLAGN NGC 1097. Moreover, R_{fs} in panel (e) in Fig. 2 shows that for the elliptical accretion disk model, R_{fs} is larger than 1.85 and smaller than 2.22 for only 33 of the 860 cases (the lower and upper values for NGC 1097). In other words, the probability is only 3.8% (33/860) that the elliptical accretion disk model can explain the double-peaked broad H α in NGC 1097.

Only 12 data points are available for $\lambda_0(t)$ and $\sigma(t)$, and the application of the sine function plus a linear trend lead to the well-accepted descriptions. We therefore do not discuss applications of other methods or techniques further (e.g., the Lomb-Scargle technique) to determine periodic results in NGC 1097. Furthermore, based on the same time information of $\lambda_0(t)$ and $\sigma(t)$ shown in Fig. 3, 100 000 artificial pairs $[[\lambda_0(t, a)], [\sigma(t, a)]]$ were created by the corresponding 12 values of $\lambda_0(t, a)$ and $\sigma(t, a)$ that were randomly collected within the range from min-

imum $\lambda_0(t)$ to maximum $\lambda_0(t)$ for each $\lambda_0(t, a)$ and from minimum $\sigma(t)$ to maximum $\sigma(t)$ for each $\sigma(t, a)$. We also applied the sine function plus a linear trend to determine the probable periodicity $T_{\lambda, a}$ and $T_{\sigma, a}$ in the 100 000 artificial $[[\lambda_0(t, a)], [\sigma(t, a)]]$. Then, when we assume that the determined periodicities in $\lambda_0(t)$ and $\sigma(t)$ are not random, but are intrinsically true in NGC 1097, it is necessary to determine how many artificial cases have similar periodic results as those in NGC 1097. Based on the criteria that $T_{\lambda, a}$ and $T_{\sigma, a}$ are longer than 13 (shorter than k_{10} and k_{11}) and 16 (shorter than k_{20} and k_{21}) times their corresponding uncertainties and that $T_{\lambda, a}$ lies within the range of 2630–55 days to 2630 + 168 days and $T_{\sigma, a}$ lies within the range of 1322 – 64 days to 1322 + 72 days, 156 of the 100 000 artificial results lead to a periodicity of about 2630 days and 1322 days in $\lambda_0(t, a)$ and $\sigma(t, a)$. This indicates the probability is lower than 0.156% (156 out of 100 000) that the detected periodic results are random in NGC 1097. We did not apply a broad range of periodicities in the artificial cases. If we had done so, the collected artificial cases would have very different ratios of $T_{\lambda, a}$ to $T_{\sigma, a}$ from the $R_{fs} \sim 1.99$ in NGC 1097. Therefore, the periodic results in NGC 1097 are not random but robust enough, with confidence levels at least higher than 99.84% (1–0.156%).

In addition to the accretion disk origin for double-peaked broad emission lines, we discuss the BBH model (Shen & Loeb 2010; Eracleous et al. 2012; Doan et al. 2020; D’Orazio & Charisi 2023; Songshen & Wang 2023) for NGC 1097 as follows. For double-peaked broad lines related to assumed BBH systems, optical quasi-periodic oscillations (QPOs; Graham et al. 2015a,b; Zheng et al. 2016; Zhang 2024c) could be detected in optical light curves. Unfortunately, the 7.1-year-long optical V-band light curve from the Catalina Survey Data Release 2 (CSDR2; Drake et al. 2009, 2019) with MJD from 53 554 (July 2, 2005) to 56 163 (August 23, 2012; shown in the left panel of Fig. 4) shows no signs of probable QPOs, especially according to the Lomb-Scargle power shown in the right panel of Fig. 4. The time duration is about 7 years of the CSDR2 light curve of NGC 1097, which is comparable to the periodicity in λ_0 in NGC 1097. Therefore, the BBH model is not preferred in NGC 1097 because no optical QPOs are detected.

Furthermore, for common accretion disk models, the disk precession period (Storchi-Bergmann et al. 2003) is about $T_p \sim 1040 M_8 R_3^{2.5}$ years, with M_8 the BH mass in units of $10^8 M_\odot$, and R_3 the distance in units of $10^3 R_G$ between the BLRs and the central BH. Considering that the commonly used values (Ward et al. 2024) of R_3 are not lower than 0.2, we expect periodic variations in the first and second moment in optical spectra for 10-year-long multi-epoch observations in double-peaked BLAGNs with central BH masses lower than $5.4 \times 10^7 M_\odot$. In the near future, we will test the periodic variations in the first and second moments of λ_0 and σ in double-peaked BLAGNs with lower-mass BHs.

Before we conclude this section, we note two points. First, we tried to confirm the spectroscopic results for other double-peaked BLAGNs in the literature, such as 3C390.3 (Zhang 2011) and Arp 102B (Shapovalova et al. 2013; Popovic et al. 2014). However, the other double-peaked BLAGNs have shorter time spans and/or only several spectra that do not lead to an apparent periodic variability in at least one cycle. It is therefore hard to confirm the periodic results in $\lambda_0(t)$ and in $\sigma(t)$ in other double-peaked BLAGNs. Second, as discussed by Bon et al. (2009), Hung et al. (2020), the observed broad emission lines could include additional nondisk components that are not related to the BLRs in the central accretion disks, which has a strong effect on the expected periodic results. However, for specific cases, if the nondisk component were from common virialized BLRs, the variability in the nondisk component is expected to be far lower than the variability in the disk component, and the effects of the nondisk component might be weak, which would probably lead to similar periodic results as we proposed here. It is currently unfortunately not possible to quantify the effects without information about the variability intensity ratio of the nondisk component to the disk component from the central disk.

3. Conclusions

The results based on theoretical accretion disk models and also through real observational results in the known double-peaked BLAGN NGC 1097 strongly indicate that the periodic variations in the first and second moment of the broad emission lines can be accepted as indications that the broad emission lines come from BLRs that lie in the central accretion disks in BLAGN. Moreover, the periodicity ratio of about 2 of the periodic variations in the first moment to the periodic variations in the second moment

are indications that support the structure of spiral arms in the BLRs in the double-peaked BLAGNs. The results further provide an independent method for testing the accretion disk origin of the double-peaked broad emission lines through the moments of the profiles of broad emission lines alone, and the physical properties of theoretical model-determined parameters do need to be considered.

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