LETTER TO THE EDITOR

Estimating the energy flux of transverse waves associated with Kelvin-Helmholtz instability in solar coronal loops

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ABSTRACT

Context. The energy flux of kink waves in coronal loops has been estimated in previous studies. Recent numerical simulations have revealed that kink oscillations can induce a Kelvin-Helmholtz Instability (KHI) in magnetic flux tubes. This non-linear process breaks the assumptions that have typically been included in previous eigenmode analyses. Therefore, the current analytical expressions of energy flux need to be re-examined.

Aims. In the present work, we aim to compare our numerical energy flux with previous analytical formulae and establish modifications to the estimation of the energy flux of kink waves in coronal loops.

Methods. Working within the framework of ideal magnetohydrodynamics (MHD), we conducted three-dimensional (3D) simulations of kink oscillations in coronal cylinders. Forward models were also employed to translate our numerical results into observables using the FoMo code.

Results. We find that the previous estimation of the energy flux of kink waves is reasonable up to the point before the KHI is fully developed. However, as small vortices develop, the energy flux derived from the analytical formula becomes smaller than the total Poynting flux calculated from our numerical results. Furthermore, when degrading the original numerical resolution to match a realistic instrumental resolution, for instance, the Extreme Ultraviolet Imager (EUI) on board the Solar Orbiter (SO), the energy flux becomes much smaller than the numerical value.

Conclusions. The energy flux calculated from the analytical formula should be modified by multiplying it by a factor of about 2.

Key words. Magnetohydrodynamics (MHD) – Sun: corona – Sun: oscillations

1. Introduction

Kink waves and oscillations have been frequently reported in solar coronal loops (see e.g., Nakariakov et al. [2021], for a recent review). The energy carried by these waves is of great importance, since it may be associated with the coronal heating problem (e.g., Van Doorsselaere et al. [2020]). In transversely structured loops, it is generally believed that the damping of kink oscillations and waves depends on mechanisms such as resonant absorption (e.g., Goossens et al. [2011]) and mode coupling (e.g., Pascoe et al. [2010]), respectively. Through these processes, azimuthal local Alfvén waves can be induced and, following a phase mixing process (e.g., Heyvaerts & Priest [1983], Browning & Priest [1984], Guo et al. [2019b]), the Kelvin-Helmholtz instability (KHI) can be enhanced. The development of transverse wave-induced KH vortices has been confirmed by recent numerical progress (e.g., Terradas et al. [2008], Antolin et al. [2014], Magyar & Van Doorsselaere [2016], Howson et al. [2017a,b]). Thus, wave energy dissipation can occur at these small scales (e.g., Karpimpelas et al. [2017], Guo et al. [2019b], Shi et al. [2021], Guo et al. [2023]).

An essential issue is to calculate the energy content of kink waves in the corona. Goossens et al. [2009, 2012] pointed out that kink waves have an Alfvénic property in a thin tube (TT) limit, which is a good approximation for coronal loops that usually exhibit a large aspect ratio. This means that we need to be careful when estimating the energy flux in kink waves since they are different from bulk Alfvén waves [Van Doorsselaere et al. [2008], which are usually considered when computing wave energy in previous observations (e.g., Tomczyk et al. [2007], McIntosh et al. [2011]). A detailed energy description of kink Alfvénic waves has been given by Goossens et al. [2013]. The spatial variation in the energy of kink Alfvénic waves leads to a significant overestimate of the energy flux compared with previous consideration of bulk Alfvén waves. Van Doorsselaere et al. [2014] further extended this work by considering a volumetric filling factor in a bundle of magnetic flux tubes. They obtained the following expression for the average energy flux of kink waves:

\[ F_k = \frac{1}{2} f(p_i + \rho_o) \left( \frac{2\pi}{P_{\text{obs}}} \right)^2 \xi_{\text{obs}}^2 v_{gr}, \]

where \( f \) is the filling factor and \( p_i (\rho_o) \) represents the internal (external) loop density. Also, \( P_{\text{obs}} (\xi_{\text{obs}}) \) is the observational wave period (displacement) and \( v_{gr} \) represents the group speed. This formula has been used, for example, in Guo et al. [2022] to calculate the total energy of decayless kink oscillations recently observed by the Atmospheric Imaging Assembly (AIA, Lemen et al. [2012]). It has also been used in Petrova et al. [2023], Shivastav et al. [2023], and Li & Long [2023] to calculate the energy...
The analytical expressions of energy flux obtained by Goossens et al. (2013) and Van Doorsselaere et al. (2014) need to be modified when considering kink oscillations associated with turbulent structures induced by the KHI. As demonstrated in, for instance, Guo et al. (2020), resonant absorption and phase mixing are ideal linear processes. The eigenmode analyses conducted in Goossens et al. (2013) and Van Doorsselaere et al. (2014) still remain in this linear regime with some assumptions, such as pressureless MHD and a piecewise constant density distribution in the transverse direction. In the nonlinear regime, however, the development of the transverse wave-induced small scales extends the resonant layer from a loop boundary to almost the whole cross-section of the loop (e.g., Karampelas et al. 2017; Magyar & Van Doorsselaere 2016) also found that the dynamics of transverse oscillating coronal loops deviate from linear damping theory due to the development of the KHI. In addition, the consideration of non-zero plasma $\beta$ in coronal loops enables the analysis of internal energy, which makes a significant contribution to the total energy. The energy carried by kink waves has been calculated in non-zero plasma $\beta$ studies by, for instance, Geeraerts (2022); Yuan et al. (2023).

Given the importance of energy flux calculation in future studies, especially in observational studies, it is essential to modify the current formula (1). In this paper, we conduct a numerical simulation to mimic the decayless kink oscillations reported in coronal loops and compare the numerical Poynting flux with the energy flux deduced from Equation (1). The current formula is modified to make it more reasonable for computing the total energy flux in the nonlinear regime. Section 2 describes our numerical model, including the equilibrium and numerical setup. In Section 3 we present the numerical results and forward modeling results, along with the modification of formula in Equation (1). Section 4 summarizes our findings, ending with some concluding remarks.

2. Model description

We considered a similar three-dimensional model as in Guo et al. (2019b). The magnetic flux tube is density enhanced and embedded in a uniform background corona. The magnetic field is directed along the $z$-direction. The density profile is given by:

$$\rho(x, y) = \rho_0 + (\rho_i - \rho_0) \zeta(x, y),$$

with

$$\zeta(x, y) = \frac{1}{2} \left[ 1 - \tanh \left( b \left( \sqrt{x^2 + y^2 / R^2} - 1 \right) \right) \right].$$

The internal density is $\rho_i = 2.5 \times 10^{-15}$ g cm$^{-3}$ and the density ratio is $\rho_i/\rho_0 = 3$. The parameter $b = 8$ gives the width of the inhomogeneous layer $l \approx 0.4 R$, with $R = 1$ Mm being the radius of the tube. The loop length, $L$, is set to be 150 Mm. The temperature is 1 MK throughout the entire computational domain. To maintain the magnetostatic balance, the magnetic field has a slight variation from internal $B_z = 50$ G to external $B_z = 50.07$ G.

We employed the PLUTO code (Mignone et al. 2007) to solve the time-dependent MHD equations. A parabolic spatial scheme is used for reconstruction, and the numerical fluxes are computed by a Roe Riemann solver. A Runge-Kutta method was used for time marching. The whole computational domain is $[-8, 8]$ Mm $\times$ $[-8, 8]$ Mm $\times$ [0, 150] Mm. A uniform grid of 100 points was adopted in the $z$-direction. In the $x$- and $y$-directions, 256 stretched grid cells were adopted, respectively. The highest resolution is 20 km in $[x, y] \ll 2$ Mm.

The boundary conditions were specified as follows. All components of the velocity at $z = L$ are set to zero, meaning that this loop end is fixed. The remaining variables there have zero gradients. Regarding the $z = 0$ plane, the $z$-component of the velocity is set to be antisymmetric, while $v_x, v_y$ are described by a continuous, dipole-like driver (e.g., Pascoe et al. 2010; Karampelas et al. 2017). This driver is used to excite decayless kink oscillations in the loop. In the internal loop region ($r < R$), the time-dependent velocity is

$$v_t = v_0 \cos \left( \frac{2\pi}{P_k} t \right),$$

where $v_0 = 4$ km s$^{-1}$ is the amplitude of the driver. The period $P_k = 87$ s, corresponding to the eigenperiod of the fundamental kink mode (Edwin & Roberts 1983). In the outside loop region, the driver is spatially dependent. It takes the form:

$$v_e = v_0 R^2 \cos \left( \frac{2\pi}{P_k} \right) \left( \frac{x-x'}{P_k} \right)^2 \left( \frac{y-y'}{P_k} \right)^2.$$

Here, $x'$ is the displacement of the driver to follow the motion of the footpoint. Similar setups can be found in, for instance, Karampelas et al. (2017); Guo et al. (2019b); Pelouze et al. (2023).

3. Results

The detailed dynamical evolution of the footpoint-driven loop has been extensively discussed in previous works from the literature (e.g., Karampelas et al. 2017; Guo et al. 2019b). Figure 1 illustrates a snapshot of the density structure after the KHI is fully developed. The loop apex cut shows that the small-scale structures extend almost over the entire loop cross-section. The increasing velocity around the vortices indicates the onset of resonant azimuthal Alfvén waves in the boundary layer. Compared with previous linear models (e.g., Guo et al. 2020), the current resonance layer is distorted and distributed over a larger region of the loop. Therefore, the wave energy that is enhanced in a resonant layer is now spread over the loop cross-section. A previous work conducted by Goossens et al. (2013) concluded that most of the wave energy is confined to the boundary layer, regardless of whether it is a thick boundary layer or a thin boundary layer (TB) limit. Here, we extend this conclusion by stating that most of the wave energy is confined from the boundary towards the loop centre due to the extension of the KHI eddies across the magnetic field. To clearly show this cross-field effect, the predominantly $z$-directed magnetic field is presented in the left panel of Figure 1.

Now, we take this study a further step to examine the energy flux in the current loop model. Since the energy flux given by Van Doorsselaere et al. (2014) is the spatially averaged total energy flux in a loop system, we can thus calculate the input energy...
flux in the current numerical model for a direct comparison. Before proceeding, it should be noted that the input energy equals the total energy changes in a numerical model. As previously demonstrated in [Guo et al. (2019b), Karampelas et al. (2019a)], the simulation maintains an energy balance. Therefore, the input energy flux is equivalent to the total energy flux changes in the loop. We compute the input energy flux by considering the Poynting flux at the driven footpoint. The spatially averaged Poynting flux is given by

\[ S(t) = \frac{1}{A} \int_A S \cdot dA, \]  

where \( A \) represents the surface area of \( |x| \leq 2.83R, |y| \leq 2.83R \).

\[ S = E \times B/\mu_0 \]  

is the Poynting flux, and \( dA \) represents the normal surface vector. Figure 2 shows this input energy flux as solid black curves. We note that this energy flux curve starts from zero, although it may not be clearly visible due to the rapid excitation of the kink oscillation in the loop. We can see that the energy flux increases over time – before about 600 s, and then saturation is achieved after around 750 s, indicating that the KHI is fully developed. This scenario has been discussed in, for instance, [Karampelas et al. (2019a) and Guo et al. (2019b)].

We move on to examine the analytical expression in Equation (1) for the energy flux. As mentioned in [Van Doorsselaere et al. (2014)], this formula is valid when the filling factor, \( f \), is less than 0.1. Therefore, we chose a computational region of \([-2.83R, 2.83R] \times [-2.83R, 2.83R] \times [0, 150R] \), which gives a filling factor of \( f = 0.1 \). This corresponds to the second configuration of a loop system shown in Figure 2 in [Van Doorsselaere et al. (2014)]. The period, \( P_{\text{obs}} \), is the period of our driver, while the displacement, \( \xi_{\text{obs}} \), is the observed transverse displacement of the loop. In the following numerical data analysis, we use the displacement of the centre of mass of the loop. The group speed, \( v_{\text{gr}} \), is chosen to be the kink speed under the assumption of a long wavelength approximation for a typical coronal loop. The kink speed, \( c_k \), is a density-weighted average value [Edwin & Roberts 1983] given by

\[ c_k = \sqrt{\frac{\rho_i v_{\text{Ai}}^2 + \rho_e v_{\text{Ae}}^2}{\rho_i + \rho_e}}, \]  

where \( v_{\text{Ai}} (v_{\text{Ae}}) \) represents the internal (external) Alfvén speed.

Then, the analytical energy flux computed from Equation (1) is illustrated in Figure 2, shown as red lines.

We then compared the numerical energy flux with the analytical results in Figure 2. When the kink oscillation is not formed, the displacement of the centre of mass is minimal. Therefore, the two curves show different behavior for the initial period. The two curves match well before the KHI is fully developed, indicating that the analytical expression in Equation (1) is a good description of the energy flux in the linear regime. The deviation is probably caused by the simple transverse density profile considered in the eigenmode analysis. However, when the system enters the nonlinear regime, the formula fails to accurately describe the total energy flux. In this case, a factor of about 1.77 needs to be added to Equation (1) in order to accurately describe the total energy flux in the nonlinear regime. Therefore, we modify Equation (1) to obtain:

\[ F_k(t) = \frac{1}{2} \alpha f (\rho_i + \rho_e) \left( \frac{2\pi}{P_{\text{obs}}} \right)^2 \xi_{\text{obs}}^2 v_{\text{gr}}, t > t_c, \]  

where \( \alpha = 1.77 \pm 0.01 \) and \( t_c \sim 750 \text{ s} \), which indicates the onset time of the KHI. The factor \( \alpha \) is the slope of the linear fit of the scatter plots of the numerical energy flux versus the analytical results at each instant when \( t > t_c \), as shown in Figure 2. The Pearson correlation coefficients are 0.9931 and 0.9903 for the two different fits, respectively.

The parameter \( \alpha \) may vary for different numerical setups. However, it is impossible to exhaust all the related parameters. Here, we conducted a parametric survey by changing the amplitude of the footpoint driver to \( v_0 = 6 \text{ km s}^{-1} \). Then, following the above-described procedure, we find \( \alpha = 2.19 \pm 0.02 \). The factor...
\[ F_k = \frac{1}{2} \alpha eui \left( \rho_i + \rho_e \right) \left( \frac{2\pi}{\rho_{obs}} \right)^2 \xi_{obs}^2 v_{gr}, \quad t > t_c, \] (11)

where \( \alpha = 1.77 \pm 0.01 \) (\( \alpha = 2.19 \pm 0.02 \)) for a driver of \( v_0 = 4 \text{ km s}^{-1} \) (\( v_0 = 6 \text{ km s}^{-1} \)), \( t_c \) is the onset time of the KHI, \( f \) represents the filling factor of a loop, \( \rho_i + \rho_e \) represents the internal (external) loop density, \( P_{obs} (\xi_{obs}) \) is the observational wave period (displacement), and \( v_{gr} \) represents the group speed.

3. For the SO/EUI observations, the energy flux of kink waves becomes much smaller when employing the previous energy flux computing formula. Therefore, it should also be modified when using the SO/EUI observational data to calculate the energy flux \( F_k \). The updated formula is given by

\[ F_k = \frac{1}{2} \alpha eui \left( \rho_i + \rho_e \right) \left( \frac{2\pi}{\rho_{obs}} \right)^2 \xi_{obs}^2 v_{gr}, \quad t > t_c, \] (12)

where \( \alpha = 3.46 \pm 0.03 \) (\( \alpha = 4.08 \pm 0.196 \)) for a driver of \( v_0 = 4 \text{ km s}^{-1} \) (\( v_0 = 6 \text{ km s}^{-1} \)).

The modified expressions for both numerical and forward modelling results reveal that the formula given by Van Doorsselaere et al. (2014) underestimates the total energy flux in a transversely oscillating loop with the appearance of the KHI. For previous EUV observations, Petrova et al. (2023), for instance, the energy flux of kink oscillations should be 6.89 kW m\(^{-2}\) (22.59 kW m\(^{-2}\)) for the shorter (longer) frequency oscillations if Equation (10) is considered. This means that the energy flux of the high-frequency kink oscillations reported before can be comparable to the total energy losses (> 10 kW m\(^{-2}\)), Withbroe & Noyes (1977) in the active region corona.

In addition, we also degraded the original forward modelling results to roughly match the resolution of SDO/AIA at 17.1 nm, which has a spatial resolution of 1.2 arcsec and a time cadence of 12 s. A similar procedure can be found in (Guo et al. 2019b). Then we can obtain \( \alpha_{aia} = 3.65 \pm 0.09 \). Note that the value of \( \alpha_{aia} \) is slightly larger than \( \alpha_{aui} \) due to the lower resolution of AIA. In previous numerical simulations by Karampelas et al. (2019b), there is no clear correlation between the amplitude of kink oscillation observed by AIA and the input energy from a footpoint driver. This probably indicates that the oscillating amplitudes captured by imaging instruments hide a possibly large input energy flux. In the current work, from a different quantitative perspective for each individual simulation, we stress that the energy flux should be larger than analytical expectations. Given the results of Karampelas et al. (2019b) and the current work, we can conclude that the energy flux of kink oscillations in coronal loops is underestimated by the imaging observations.

Apart from varying the amplitude of the velocity driver, we also conducted a higher resolution run. In this case, we increased the highest resolution to 14.8 km in the domain of \([x, y] \leq 2 \text{ Mm}\). We obtained a slightly increased value of \( \alpha = 1.94 \pm 0.009 \), compared with the original value of \( \alpha \). It is known that changes in energy flux are sensitive to the energy dissipation within the system (e.g., Klimesch 2015 Prokopyzyn et al. 2019 Howson 2022). However, before the full development of the energy cascading process, the growth of KHI is still closely related to...
Fig. 2. Energy flux changes in the numerical model and forward model. (a) Energy flux obtained from the numerical results given by Equation (1) (black line), analytical results given by Equation (1) (red line), and the energy flux calculated using Equation (1) with the synthetic displacement in Figure 3b (blue line). (b) Scatter plots of the energy flux when \( r > t \), The horizontal axis represents the analytical (red) and synthetic EU (blue) energy flux, while the vertical axis shows the numerical energy flux. Dashed lines illustrate the linear fits of the data, and their slopes correspond to the factors \( \alpha \) and \( \sigma_{\text{EUI}} \).

Fig. 3. Forward-models for the numerical simulation. Upper panel (a) shows the time–distance map of the normalized intensities with the full numerical resolution at the loop apex with a LoS angle along the y-direction. Lower panel (b) shows a degraded resolution result comparable to SO/EUI. The dashed line represents the oscillation profile of the intensities, obtained by calculating the centre of gravity of the intensity in panel (b).

The resonant component of the broadband driver can be selected by the loop, thus facilitating efficient energy injection. Nevertheless, regardless of the form of the driver employed, the qualitative results of the current study are not expected to change.

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We note that the resonant driver used in the current simulation may seem artificial. To achieve an efficient energy flux, a driver with an eigenfrequency is a common choice. However, from a more realistic perspective, the use of a broadband driver has been discussed, for instance, by Afanasyev et al. (2019), Pagano et al. (2020), Howson & De Moortel (2023). In this case, the resonant component of the broadband driver can be selected by the loop, thus facilitating efficient energy injection. Nevertheless, regardless of the form of the driver employed, the qualitative results of the current study are not expected to change.

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