Spectral break of the density power spectrum in solar wind turbulence

O.W. Roberts\textsuperscript{1,}, Y. Narita\textsuperscript{1,}, R. Nakamura\textsuperscript{1,}, Z. Vörös\textsuperscript{1,\textdagger}

\textsuperscript{1} Space Research Institute, Austrian Academy of Sciences, Schmiedlstrasse 6, 8042, Graz, Austria
\textsuperscript{2} Institute of Earth’s Physics and Space Science, ELRN, Sopron, Hungary

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ABSTRACT

We use density measurements deduced from spacecraft potential to study the power spectral density (PSD) of compressive fluctuations in the solar wind. Typically, plasma measurements do not have a sufficiently high time resolution to resolve density fluctuations down to ion kinetic scales. However, the calibrated spacecraft potential allows for much higher time resolutions to resolve the spectral break between ion inertial and kinetic ranges. We used fast-survey mode data from Magnetospheric MultiScale when the spacecrafts were in the pristine solar wind. The density spectra’s morphology differs from the trace magnetic field fluctuations, with a flattening often occurring between inertial and kinetic ranges. We find that the spectral break of the trace magnetic field fluctuations occurs near the expected frequency for cyclotron resonance or magnetic reconnection. Meanwhile, the spectral break at the start of the ion kinetic range for density fluctuations is often at a higher frequency when compared to the trace magnetic field. We discuss possible interpretations for these observations.

Key words. plasma turbulence, solar wind

1. Introduction

The spectrum of the turbulent magnetic field in the solar wind displays fluctuations over several decades in scale (Kiyani et al. 2015; Verscharen et al. 2019). At large scales, the plasma behaves like a fluid, and the power spectral density (PSD) of magnetic fluctuations has a spectral index close to the Kolmogorov (-5/3) or Iroshnikov-Kraichnan (-3/2) indices (Tu & Marsch 1995; Smith et al. 2006b; Lotz et al. 2023). Some observations show an evolution of the spectra from -5/3 at one astronomical unit (au) to a shallower spectrum closer to -3/2 at smaller heliocentric distances (Chen et al. 2020; Zhao et al. 2022). However, the spread of values is large, and the error bars make differentiating between the two spectral indices difficult (e.g. Lotz et al. 2023). As the turbulent cascade proceeds to smaller scales, near 1 Hz, there is a spectral break (Leamon et al. 1998, 1999, 2000; Smith et al. 2006a; Markovskii et al. 2008; Bourouaine et al. 2012; Smith et al. 2012; Bruno & Trenchi 2014; Chen et al. 2014a; Lotz et al. 2023, and references therein) and the spectrum steepens to a spectral index of around -2.6 (Smith et al. 2006a; Alexandrova et al. 2009, 2012; Roberts et al. 2017, 2022). The location of the spectral break marks the end of the inertial range (where fluid physics is valid). It indicates the beginning of the ion kinetic range where the ion scale lengths are of comparable sizes to the magnetic fluctuations. In this range, energy conversion, dissipation, and dispersion of fluctuations can occur. Improving our understanding of the location of the break and the related physical scale offers insights into the relevant physical process.

Several studies have previously investigated the location of the break in the magnetic fluctuations PSD (Leamon et al. 1998, 1999, 2000; Smith et al. 2006a; Markovskii et al. 2008; Smith et al. 2012; Bruno & Trenchi 2014; Chen et al. 2014a; Borovsky & Podesta 2015; Vech et al. 2018). The interpretation is challenging as solar wind plasma at 1 au because several of the relevant plasma scales (e.g. ion inertial or Larmor radius) are often of similar lengths. Several interpretations have been proposed, such as cyclotron damping (Bruno & Trenchi 2014; Chen et al. 2014a; Lion et al. 2016; Woodham et al. 2018; Duan et al. 2018; Chen et al. 2020; Lotz et al. 2023), Alfvén wave dispersion (Chen et al. 2014a) the ion inertial scale, $d_i$ (which is linked to current sheet thickness in Hall MHD) (Leamon et al. 2000; Vasquez et al. 2007; Chen et al. 2014a; Borovsky & Podesta 2015), and magnetic reconnection (Vech et al. 2018; Terres & Li 2022). However, some results are inconclusive (e.g. Markovskii et al. 2008; Duan et al. 2020). Chen et al. (2014a) studied examples with extreme values of plasma beta (the ratio of thermal to magnetic pressure); for $\beta \gg 1$, the break is consistent with the dispersion of Alfvén waves, and for $\beta \ll 1$, the break is consistent with the ion inertial scale. The spectral break has also been observed to be independent of the angle between the magnetic field and the flow direction (Duan et al. 2018). Further discussion of the relevant plasma scales is included in the Appendix.

While there has been considerable effort in studying magnetic field spectra, density spectra are not as well studied. This is partially because magnetic field measurements are operationally simpler to perform at ion kinetic scales than density measurements. Oftentimes, density measurements from plasma instruments do not have the necessary time resolution or signal-to-noise ratio (S/N) and calibrated spacecraft potential needs to be used. The morphology of the density spectrum is more variable than its magnetic counterparts. Sometimes, a flattening exists between the ion inertial and ion kinetic ranges (e.g. Celnikier et al. 1983, 1987; Chen et al. 2012a,c; Safrankova et al. 2013; Roberts et al. 2020a,b,c; Montagud-Camps et al. 2021). However, spectra which resemble the trace magnetic field more closely have also...
been observed. This has been modeled as a combination of MHD scale slow waves that are passively cascaded by the Alfvénic component and an active cascade of kinetic Alfvén waves (the extension of the Alfvén wave to large $k_z$, which becomes compressive at ion kinetic scales) at small scales (Howes et al. 2008; Chandran et al. 2009). The crossover between these two processes yields a flattening. Some alternative possibilities are as follows: (1) charge separation begins at these scales (Treumann et al. 2019); (2) the Hall electric field becomes important, affecting the density fluctuations (Narita et al. 2019); (3) a separate fast or magnetosonic cascade (Gary 2015; Narita et al. 2022); or (4) nonlinear waves (Narita & Hada 2018).

This letter presents a study of the break scales, with both the trace magnetic and the density spectra measured by the Magnetospheric MultiScale Mission. In the following section, the data will be presented followed by the results and concludes with a summary.

2. Data

This study uses fast-survey mode data from the Magnetospheric MultiScale mission (Burch et al. 2016). Magnetic field data are obtained from the fluxgate magnetometer (Russell et al. 2016) with a sampling rate of 16 Hz. We used the magnetic field data to calculate the trace magnetic power. Furthermore, we estimate the compressive magnetic power by using the magnetic field magnitude. This is a good approximation for the compressive fluctuations when the amplitudes are low (e.g. Perrone et al. 2016, 2017). For the density data, we used the calibrated spacecraft potential (Roberts et al. 2020a), where the original spacecraft potential data were obtained from the SDP instrument at a sampling rate of 32 Hz (Lindqvist et al. 2016). Although the fast-survey mode data have significantly lower time resolutions than burst-mode data, they are more than sufficient to resolve the ion spectral break, which typically occurs near 1 Hz (Leamon et al. 1998, 1999, 2000; Smith et al. 2006a; Markovskii et al. 2008; Bourouaine et al. 2012; Smith et al. 2012; Bruno & Trenchi 2014; Chen et al. 2014a) at one au. For calculation of the electron inertial length, we use the plasma measurements from the fast plasma investigations dual electron spectrometers (FPI-DES), which have a time resolution of 0.2 Hz (Pollock et al. 2016). For the ion parameters, the Fast Plasma Investigation’s Dual Ion Spectrometer (FPI-DIS) is unsuitable for estimating the temperature of the solar wind (e.g. Bandyopadhyay et al. 2018; Roberts et al. 2021; Wilson et al. 2022). Therefore, we used OMNI data (King & Papitashvili 2005) for the bulk velocity, and we use the time shift from the OMNI data to find the corresponding ion perpendicular and parallel temperatures from Wind (as only total temperature measurements are available in the OMNI data set).

The data intervals were selected from the already calibrated and spin-removed data presented in Roberts et al. (2020a, 2021). Initially, this data set consists of 96 intervals that are (on average) 3 hours long. However, several intervals were removed because there are bumps in the magnetic spectra indicative of connection to the foreshock (this eliminates 13 intervals from our analysis) or the error on the break location is large (eliminating 28 intervals). In some cases, only one power law is clearly visible (five cases). One spectrum is also removed as it is noisy even after spin removal. After removing such cases, we are left with 49 different intervals to calculate the density power spectra and the trace and compressive magnetic field power spectra.

The break locations are obtained by investigating each spectrum initially by eye, then fitting either three or two power laws to the spectra. In our data set, we have 15 spectra fitted with two power laws and 34 spectra fitted with three power laws. The power laws are fitted on either side of the spectral break and the break is determined by the intersection of the two power laws (Bruno & Trenchi 2014). This is done by performing linear regression on the logarithm of the PSD and the logarithm of the frequency. Standard errors are calculated for the linear regressions based on the residuals and by propagating those errors, an error can be obtained for the spectral break. If this error is large, it may be that the type of fit is unsuitable (i.e. a two-power law fit is performed to a three-power law spectrum). After fitting, we check the error on the breaks and see if our fit is appropriate. The results will now be presented.

3. Results

Three different morphology types are observed for the density spectrum, and different examples are presented in Fig 1. The spectra are organised with the density spectra on the left (a,d,g) and the corresponding trace magnetic field in the middle (b,e,h) and the magnetic field magnitude on the right (c,f,i). The three types are as follows: (a) a single power law, (c) a double power law, and (e) a triple power law. The different characteristic scales are also indicated and so, are the fits. The fits are performed with the dots indicating the range of the fits. Despite the different morphologies of these three types of density spectra, the trace magnetic field spectra for the three cases are similar.

In a case of a single power law density spectra, the corresponding spectral index of the magnetic field in the kinetic range (b) is very moderate (−2.20) compared to the others (−2.60 and −3.03). However, in panel (c) the magnetic field magnitude spectra more closely resembles the trace magnetic spectra rather than the density spectra. In panel (d), a double power law fit is shown. In this case, the density spectra and the magnetic field magnitude spectra are similar to the typical magnetic field spectrum, namely, a near Kolmogorov-like inertial range and a steeper kinetic range separated by a clear break. However, the break in the trace magnetic and the density spectra are at different locations. The three power-law spectra shown in (g,i) have a significant flattening between the inertial and the kinetic ranges (e.g. Celnikier et al. 1983; Chandran et al. 2009; Chen et al. 2012b; Safronova et al. 2013; Narita et al. 2019; Treumann et al. 2019; Roberts et al. 2020a; Montagud-Camps et al. 2021). We now concentrate on the location of the spectral break and split our spectra into three power law and two power law fits.

The results for the break locations are summarised in Figure 2. Different colours represent different characteristic scales, and the black line denotes $f_{break} = f_{scale}$ with the $\chi^2$ values indicated where the model is assumed to be the black line. In other words, if we expect the characteristic scale to be related to the break, then $f_{scale} = f_{break}$ would be expected. In total, there are eight panels where (a) denotes the density break where there are only two power laws, (b) presents the results of the location of the first break where a flattening is observed, and (c) presents the results of the second break after the flattening. In the next row, the breaks of the magnetic field magnitude spectra are shown. These are separated according to the morphology of the magnetic field magnitude spectra. The density and magnetic field magnitude morphologies do not always correspond well to one another, but in most cases they are in agreement. The magnetic field breaks are shown in panels (g) and (i), where the intervals have been separated so that panel (g) corresponds to panel (a) and panel (i) corresponds to panels (b), and (c). Error bars on the x-axis are estimated from the errors on the fits, and error bars on
The  frequency of the first break is much lower than the second break and tends to occur nearer the cyclotron frequency at around 0.1 Hz. Although the best agreement is seen with the cyclotron frequency, this is only likely to be a coincidence. This is because ion cyclotron waves are not strongly compressive and should not greatly influence the density spectrum. A possible explanation for this is that kinetic slow waves become damped here. There is ample evidence (Yao et al. 2011; Howes et al. 2012; Verscharen et al. 2017; Roberts et al. 2018, 2020c) that the compressive fluctuations in the inertial range in the solar wind exhibit strong anti-correlations between magnetic field magnitude and density.
Fig. 2. Figures showing the measured spectral breaks and their relations to the corresponding measured physical scales. The points are separated into eight plots. The top row (a,b,c) corresponds to the density measurements. (a) shows the break when there are only two power laws, while (b) and (c) denote the results for three power laws for the first and second breaks, respectively. The middle row shows the breaks for the magnitude spectra, these are split according to their morphologies with (d) showing two power laws, (e) showing the first break when there are three power laws fit, and (f) showing the second break when there are three power laws. Although most magnetic field magnitude spectra have similar morphologies to the density spectra, this is not the case for all the spectra. The corresponding magnetic field spectral breaks are presented for completeness in panels (g) and (h), where the intervals have been split according to the density spectra’s morphology. The different colours indicate the different physical scales, and the black line denotes $f_{\text{scale}} = f_{\text{break}}$, $\chi^2$ values are calculated concerning this line.

which are a signature of slow waves (Howes et al. 2012; Verscharen et al. 2017; Roberts et al. 2018, 2020c). The damping of slow waves would be expected at lower frequencies than for kinetic Alfvén waves (e.g. Narita & Marsch 2015). We note that the Taylor microscale may occur near this break for the density; however, higher time resolution data and linear formations of tetrahedra are necessary to estimate this characteristic scale.

The breaks according to the magnetic field magnitude are very similar to the density spectra and generally show the same tendencies as the density spectra. However, some density and magnetic field magnitude spectra do not correspond well in terms of morphology. We interpret this difference because the magnetic field magnitude only approximates the compressive magnetic fluctuations when fluctuations are small $\delta B/B_0 \ll 1$. Should there be large amplitudes in the fluctuations, this approximation will not hold. Therefore the density spectrum can be considered a ‘cleaner’ measurement of compressible turbulence. Furthermore, the noise level and the sampling rate of the density measurement allow a measurement of the PSD to higher frequencies, when compared to the magnetic field magnitude.

If we compare panels Figs. 2(a) and 2(c) (as well as Figs. 2(d) and 2(f)) as to compare the start of the kinetic range for both the two and three power-law cases. In terms of $\chi^2$, the ion inertial, sound, Larmor radius, and dispersion scale are all reasonable candidates. However, unlike the trace magnetic field, the cyclotron and the disruption scales do not explain the data well.

The difference in the break location of magnetic and density spectra is investigated in Appendix A. It is clear from Fig 3 that the spectral break for the compressive fluctuations is almost always at a higher frequency when compared to the trace magnetic field break.

The difference in the break location between compressive and incompressive spectra be explained in the framework proposed by Chandran et al. (2009), the transition from Alfvén wave to kinetic Alfvén wave occurs near in $k \rho_i = 1$ (e.g. Hasegawa & Chen 1975; Howes et al. 2008) for a high $\beta$ plasma, however, the maximum compressibility may be at scales slightly larger than $k \rho_i = 1$, for instance, see the kinetic theory results in Fig. 3 of Chandran et al. (2009). Therefore the KAW could result in slightly different breaks for the trace magnetic and the
density spectra. In this hypothesis, the kinetic scale fluctuations of the magnetic and density fluctuations arise from the kinetic Alfvén wave and the large-scale compressive fluctuations from slow waves. Alternatively, the magnetic component may be a superposition of kinetic Alfvén and ion cyclotron waves. The ion cyclotron waves are damped at the combined scale and the kinetic Alfvén waves continue to smaller scales. In this interpretation, the compressive component at large scales is still provided by slow waves.

Another possibility is that the magnetic and density spectra at kinetic scales result from different waves, namely, that magnetoacoustic waves (see Gary (2015); Narita et al. (2022)) could contribute in this range. However, the anti-correlation of compressive magnetic and density fluctuations has been observed to persist to the ion kinetic scale Roberts et al. (2018, 2020b). Magnetoacoustic waves may be present at smaller scales; however, measurements at electron scales have not been able to be performed yet. Finally, some other possibilities are the Hall effect Treumann et al. (2019). Regarding the Hall MHD, current sheets form near the inertial range scale and the density break is often located near the inertial scale.

4. Summary

We summarise the findings of our study of the density and the magnetic field spectral breaks as follows:

1. The trace magnetic field spectra show a spectral break consistent with the cyclotron or disruption scales. This is consistent with many previous studies (Vech et al. 2018; Lotz et al. 2023, and references therein). Unfortunately, we cannot distinguish these two scales from the available data. The compressive spectra can have various morphologies with one, two, or three power laws. (3) The spectral break before the sub-ion scale of the compressive spectrum is typically at a higher frequency when compared to the magnetic field. We offer several possibilities in the text to explain this. (4) In cases with three power laws, the second break is most consistent with the ion Larmor radius, ion sound gyroradius, or the dispersion scale. (5) In cases where there are two power laws, the break is consistent with the scale where Alfvén waves become dispersive, or the inertial Larmor radius. (6) The magnetic field magnitude spectrum gives a useful proxy for the compressive fluctuations and mostly does look similar to the density.

Unfortunately, with the magnetic field and the density spectrum, several scales are close to one another, which makes distinguishing the exact scale particularly difficult. Moreover, different processes may be present in different intervals (e.g. Chen et al. 2014a). Therefore, each interval ought to be studied in terms of its parameters that would then be compared with different candidate models. Comparisons to numerical simulations will also help improve our understanding the spectra of density and magnetic field and their associated spectral breaks.

Acknowledgements. The data sets analyzed for this study can be found in the MMS science data archive: https://lasp.colorado.edu/mms/sdc/public/. This includes the original spacecraft potential, which is calibrated to the electron density. The calibrated electron density data, as well as several useful codes to analyze spacecraft potential data are available at https://www.lswf.oeaw.ac.at/en/research/researchgroups/space-plasma-physics/sc-plasma-interaction/mmsaspc-data-analysis/. Analysis of the spacecraft potential data at IWF is supported by Austrian FFG projects ASAP15/873685.

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5. Appendix A—Comparison of the break locations

In this appendix, we present a supplementary analysis of the comparison of the break locations. Figure 3 (a) shows the comparison between the trace magnetic and the compressive magnetic and density fluctuations spectral break. Panel b shows the comparison of the compressive magnetic and density spectral breaks.

Here, the data points from Fig. 2 (a and c) are combined, along with those from Fig. 2 (d and f), and compared to the corresponding trace magnetic field breaks. The comparison is presented in Fig. 3 (a), where the compressive spectral breaks $n_c$ (blue) and $|\Omega|$ (red) are on the $x$-axis and the trace break is on the $y$-axis. Linear fits to the data (red and blue dashed lines) and the line $y = x$ (black) are also shown. Where the two compressive breaks are compared in Fig. 3 (b) we see some scatter, but in general, the compressive breaks agree much better with one another.

6. Appendix—Relevant physical scales

Many scales may be relevant for the ion kinetic spectral break. Some can be compared with neutral fluid scales, for instance, the Taylor microscale, while others have no counterpart. We briefly summarise them below.

Firstly, we discuss the ion cyclotron frequency (in linear frequency units, Hz), which is given by Eq. 1 as follows:

$$f_c = \frac{qB}{2\pi m_i},$$

where $q$ is the fundamental unit of charge, $B$ is the magnetic field magnitude, and $m_i$ is the ion (proton) mass. This is the angular frequency units (rad s$^{-1}$) as

$$\Omega_c = \frac{qB}{m_i}.$$

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but occurs before dissipative effects truly take hold. The Taylor microscale can be measured from the curvature of the autocorrelation function near the origin. However, the estimation of the Taylor microscale requires high time-resolution data (so that the correlation function can be calculated close to the origin). The fast survey mode data do not allow an accurate measurement of this quantity. In Bandyopadhyay et al. (2020), a multispacecraft measurement of the Taylor microscale estimated a value of \( \sim 7000 \text{km} \), which for that interval corresponded to a frequency of \( \sim 0.05 \text{ Hz} \). We do not calculate this scale here due to the data limitations, but it may be a scale that should be considered should the data allow it.

The ion Larmor radius is defined as:

\[
\rho_i = \frac{v_{\perp \text{th}}}{c_i},
\]  
(4)

where, \( v_{\perp \text{th}} = \sqrt{2 k_B T_{\perp i}} \) is the ion perpendicular thermal speed, \( k_B \) is the Boltzmann constant and \( T_{\perp i} \) is the perpendicular ion temperature. This is the radius that a particle makes when it gyrates about the magnetic field.

The ion inertial length is defined as:

\[
d_i = \frac{v_A}{\rho_i}. 
\]  
(5)

This is the scale where the ions decouple from the magnetic field. In other words, the ions can no longer respond to changes in the magnetic field with smaller scale lengths than this.

The combined (or the cyclotron resonant) scale. This scale combines the aforementioned inertial length and the Psuedo Larmor radius (e.g. Woodham et al. 2018) as \( d_i + \sigma_i \) and this is the scale where cyclotron resonance would be expected to occur. The definition of the Psuedo-Larmor radius is:

\[
\sigma_i = \frac{v_{\parallel \text{th}}}{c_i}. 
\]  
(6)

In the literature, \( d_i + \rho_i \) is often used for the combined scale (e.g. Leamon et al. 1998; Bruno & Trenchi 2014; Chen et al. 2014b). This approximation is valid when \( v_{\perp \text{th}} \approx v_{\parallel \text{th}} \).

If we consider the cyclotron resonance condition in Eq. 7,

\[
\omega_r + k_{||} v_{\parallel \text{th}} = \Omega_{ci},
\]  
(7)

and insert the dispersion relation for a parallel propagating cyclotron wave \( \omega = k_{||} v_A \), we obtain the resonant scale as,

\[
\lambda_{\text{Resonance}} = (\rho_i + \sigma_i). 
\]  
(8)

This is the scale at which cyclotron resonance is expected for a parallel propagating Alfvén wave.

If we were to replace \( v_{\perp \text{th}} \) in Eq 4 with the sound speed, \( v_s = \sqrt{\frac{k_B T_e}{m_i} + \frac{3 k_B T_i}{m_i}} \), (Gary 1993) then we would obtain the ion sound gyroradius:

\[
\rho_s = \frac{v_s}{\omega_{ci}} 
\]  
(9)

We note that oftentimes the definition \( v_s = \sqrt{\frac{k_B T_e}{m_i}} \) is used, which assumes the electron temperature is larger than the ion temperature (Chen et al. 2014b; Vech et al. 2018; Terres & Li 2022). This is typically true in the slow solar wind (Bruno & Carbone 2013; Salem et al. 2023).

If we consider the dispersion relations for the MHD scale Alfvén wave are given as:

\[
\omega^2 = k_{||}^2 v_A^2, 
\]  
(10)

and the kinetic Alfvén wave as:

\[
\omega^2 = k_{||}^2 v_A^2 \left( \frac{1 + v_s^2/v_A^2}{1 + v_{\parallel \text{th}}^2/v_A^2} \right), 
\]  
(11)

as was done in Chen et al. (2014a) then for a low \( \beta \) plasma the transition between the two regimes occurs at:

\[
\lambda_{\text{Dispersion}} = \left( d_i^{-2} + \rho_s^{-2} \right)^{1/2}. 
\]  
(12)

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**Fig. 3.** Comparison between the spectral break locations for different spectra. a: Comparison between the trace magnetic field and density and magnetic field magnitude spectral break. The blue line denotes the \( y=x \) and the red line denotes the fitting to the data points. b: Comparison of the density break and the magnetic field magnitude spectral break.

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For a high $\beta$ plasma, the transition between the Alfvén and kinetic Alfvén wave will occur at $\rho_i$.

Finally, in the model of Mallet et al. (2017); Loureiro & Boldyrev (2017), current sheets are thought to break up through magnetic reconnection at the disruption scale. The disruption scale is given below:

$$\lambda_{\text{Disruption}} = C_D L_\perp^{1/9} (d_e \rho_s)^{4/9},$$

(13)

where $L_\perp$ is the turbulence outer scale (the size of the largest eddy), which has been estimated as $7.4 \times 10^5$ km (Vech et al. 2018), and $C_D$ is a constant on the order of 1. We use the value of 4.7 estimated by Vech et al. (2018). Also, $d_e$ is the electron inertial length and $\rho_s$ is the sound gyroradius defined in Eq 9. A study of the spectral break by Vech et al. (2018) found that both the cyclotron resonant scale and the disruption scale occur near the observed break frequency.