Shell helium-burning hot subdwarf B stars as candidates for blue large-amplitude pulsators

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ABSTRACT

Blue large-amplitude pulsators (BLAPs) are a newly discovered type of variable star. Their typical pulsation periods are on the order of a few tens of minutes, with relatively large amplitudes of 0.2-0.4 mag in optical bands, and their rates of period changes are on the order of $10^{-3}$ yr$^{-1}$ (both positive and negative). They are extremely rare objects and attempts to explain their origins and internal structures have attracted great deal of attention. Previous studies have proposed that BLAPs may be pre-white dwarfs, with masses around 0.3$M_{\odot}$ or core-helium-burning stars in the range of $0.7-1.1M_{\odot}$. In this work, we use a number of MESA models to compute and explore whether BLAPs could be explained as shell-helium-burning subdwarfs type B (SHeB sdBs). The models that best match existing observational constraints have helium core masses in the range of $0.45-0.5M_{\odot}$. Our model predicts that the positive rate of period change may evolve to negative. The formation channels for SHeB sdBs involve binary evolution and although the vast majority of BLAPs do not appear to be binaries (with the exception of HD 133729), the observational constraints are still very poor. Motivated by these findings, we explored the Roche lobe overflow channel. Of the 304 binary evolution models we computed, about half of them are able to produce SHeB sdBs in long-period binaries that evade detection from the limited observations that are currently available.

Key words. (Stars:) binaries: general – (Stars:) oscillations (including pulsations) – (Stars:) peculiar (except chemically peculiar) – (Stars:) subdwarfs

1. Introduction

Pulsating stars are important in astrophysics. For example, through asteroseismology, it is possible to infer the internal structure of pulsating stars and use this information to calibrate some basic parameters of stellar physics (Aerts et al. 2010; Chaplin & Miglio 2013; Bedding et al. 2011; Balona et al. 2011; Huber et al. 2012; Li et al. 2022). Pietrukowicz et al. (2017) discovered a new class of pulsators, with periods of 20 to 40 min and large amplitudes, namely, from 0.2 to 0.4 mags in the optical. By comparison, other pulsators with similar periods, for example, rapidly oscillating Ap stars pulsating in the range of 5 to 24 min have amplitudes from 0.001 to 0.02 mag (e.g., Holdsworth et al. 2018).

Moreover, this new class of pulsators has sawtooth-shaped light curves, similar to those of the fundamental radial mode pulsations of classical Cepheids and RR Lyrae-type stars, and effective temperatures of about 30,000K, similarly to hot subdwarf O/B stars (sdOB), but with much larger luminosities or lower surface gravities that is $\log(g/cm^2) < 5.3$. Long-term observations show that the pulsations are very stable and their rates of period change ($r = \dot{P}/P$) are on the order of $10^{-7}$yr$^{-1}$. Pietrukowicz et al. (2017) named these stars “blue large-amplitude pulsators (BLAPs); see for example Cósico et al. 2019, for a succinct review of their properties.

Such stars are extremely rare: among nearly half a million pulsating stars, the OGLE survey (Pietrukowicz et al. 2017; Sozzi et al. 2013, 2014, 2015ab, 2016; Mróz et al. 2015) found only 14 BLAPs in the Galaxy and none in the Magellanic
and rates of period change ($r \sim 10^{-7}$ yr$^{-1}$) in addition to effective temperatures, surface gravities, and pulsation periods. Based on their study, BLAPs are objects in the middle to late phases of core helium burning stage, with metal-rich models matching $r$ better than metal-poor ones. In their models, however, the H-rich envelope mass is fixed and determined by the total mass. Binary evolution in fact will produce hot subdwarfs with similar total mass but various H-rich envelopes and the evolution properties remarkably depend on the envelope mass (Han et al. 2002, 2003; Heber 2009, 2016; Xiong et al. 2017).

On the other hand, using nonadiabatic analysis with the GYRE stellar oscillation code, Byrne & Jeffery (2020) investigated the pulsation properties of the pre-WD model and the driving mechanism. They showed that, if effects of atomic diffusion and radiative levitation were included, the opacity bump at the iron opacity peak leads to a large instability region, that is, effective temperature from 30,000 K up to 50,000 K at least, log $g$ from 3 to 7, and the periods of unstable fundamental modes from around 100 seconds up to 2-3 hours. This range encompasses both BLAPs and high-gravity BLAPs. Byrne et al. (2021) then further studied formation channels for BLAPs based on the pre-WD model, using a binary population synthesis method, and showed that both common envelope ejection (CEE) and stable Roche lobe overflow (RLOF) can produce these objects. However, CEE generally produces binaries with short orbital periods. The fact that until recently no companions have been detected for BLAPs (Pietrukowicz et al. 2017; Kupfer et al. 2015; Ramsay 2018; McWhirter et al. 2020) seems inconsistent with this channel. However, one object worthy of attention in Table 1 is HD 133729. Pigmulski et al. (2022) reported a BLAP orbiting the main-sequence B-type star HD 133729 on an orbital period of 23.08433 d. The pulsation period of this BLAP is 32.37 min, with an amplitude 0.21 mag and a rate of period change of $-11.5 \times 10^{-7}$ yr$^{-1}$, which is consistent with that of known BLAPs. The BLAP nature of the companion to HD 133729 was previously missed due to the dilution of the observed amplitude by the brighter primary. The companion of HD 133729 is the only BLAP in a binary system that is presently known, but the discovery of this object suggests that (at least some other) BLAPs might be hidden in binaries which might have been misclassified or missed for various reasons.

Since the origin and structure of BLAPs is still an open question and they have temperatures similar to hot subdwarfs but with inflated envelopes, here we propose that BLAPs are shell-helium-burning (SHeB) hot subdwarfs. In the following, we systematically investigate the observational properties of such models based on a simple asteroseismic analysis and we study the parameter space for producing such objects via binary evolution. Our models are able to aptly replicate existing observations of BLAPs. The paper is organized as follows. First, we describe our construction of certain sdB models, which possess different combinations of H-rich envelope mass and He core mass, and we investigate their period, $P$, and relative pulsation rate of period change, $P/\dot{P}$, along the evolutionary tracks. We then analyze how sdBs can form through a RLOF channel in Section 3, along with estimations of their number in the Galaxy and some of the observational signatures of putative companions. We present our conclusions in Section 4.

## 2. Configuration of BLAPs

Subdwarf B-type stars (hereafter sdBs) are core-helium-burning stars (CHeB) with thin H-rich envelopes, located at the extreme horizontal branch (EHB) of the Hertzsprung Russell diagram
Table 1. List of confirmed and candidate BLAPs and high-gravity BLAPs with some of their parameters from the discovery papers. Symbols √ (+) in the status column denote that their absolute magnitudes and intrinsic colors dereddened by the parallax from Gaia DR2 data are consistent (inconsistent) with the temperatures derived from optical spectra or theoretical predictions (see Ramsay 2018). For BLAPs from Ref (3), the symbols ⬆ denote high-confidence candidates.

<table>
<thead>
<tr>
<th>Variable</th>
<th>P(min)</th>
<th>P/P(10^{-2}yr^{-1})</th>
<th>T_{eff}</th>
<th>log g</th>
<th>log N_{He}/N_{H}</th>
<th>status</th>
<th>Ref.</th>
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<tr>
<td>OGLE-BLAP-001</td>
<td>28.26</td>
<td>2.90±3.70</td>
<td>30800±500</td>
<td>4.61±0.07</td>
<td>-0.55±0.05</td>
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<tr>
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<td>-</td>
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<td>34.87</td>
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<td>33.62</td>
<td>4.82±0.39</td>
<td>30900±2100</td>
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<td>-</td>
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<tr>
<td>HD 133729</td>
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<td>-11.5</td>
<td>29000</td>
<td>4.5</td>
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<td>(4)</td>
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</table>

(1) Pietrukowicz et al. (2017), (2) Kupfer et al. (2019), (3) McWhirter & Lam (2022), (4) Pigulski et al. (2022), (5) Lin et al. (2022). The P/P of OGLE-BLAP-002,-004,-011 are from Wu & Li (2018) and it of ZGP-BLAP-01/TMTS-BLAP-1 is from Lin et al. (2022).

(e.g., Heber 2009, 2016; Geier 2015). The shell helium-burning (SHeB) follows the CHB phase and we show in this section that some SHeB sdBs may aptly reproduce the properties of BLAPs.

We constructed a certain number of sdB models and study the pulsation periods and their changes along evolutionary tracks. The sdB models are computed with the stellar evolution code named MESA (Modules for Experiments in Stellar Astrophysics, version 15140, see Paxton et al. 2011, 2013, 2015, 2018, 2019). Our study is for Population I stars (i.e., with a metallicity Z = 0.02, as in Pietrukowicz et al. 2017). We adopted the nuclear network pp_cno Extras_ne22.net, which includes all relevant reactions for H and He burning, and the opacity table OPAL type II, which allows the abundances of C and O to vary with time. The mixing length parameter, α_{MLT}, is set to 2. For simplicity, no stellar wind or other mass loss mechanism is included in our calculations. The input physics used here is similar to that of previous studies of Population I sdB stars by (Xiong et al. 2017, see also Schindler et al. 2015).

2.1. The sdB model

The main characteristics of sdBs are determined by the helium core mass, M_c, the H-rich envelope mass, M_e, and the element abundances in the envelope. We start by constructing some zero-
and log

Fig. 1. Evolutionary tracks for constructed sdBs on the \((T_{\text{eff}} - \log g)\) diagram (left). In each panel the core mass \(M_c\) is reported, along with different values of H-rich envelope mass, \(M_e\), indicated with different linestyles. Four BLAPs (red) and four high-gravity BLAPs (green) with known \(T_{\text{eff}}\) and \(\log g\) are overplotted with error bars for comparison (see Table 1). In the top panel, the loops on the tracks with \(M_e = 0.025M_\odot\) and \(0.031M_\odot\) are caused by helium core breathing pulses (Li 2012, 2017; Li et al. 2018). To the right, we show the rate of period change \(\Delta \log(T_{\text{eff}})\) to \(\Delta \log g\) (shown on the left panel). Purple dots (open circles highlighting stars with spectroscopic parameters), green ticks (shown as green squares on the left panel), and grey ticks (not shown on the left panel). Purple \(\times\) indicates HD 133729 on all panels. There are some markers on both panels to indicate special evolutionary points, and the same markers on the left and right panels are for the same point. \(\Delta\) indicates the starts of stable SHeB, and \(\square\) indicates where are the maximum radius of SHeB phases. So SHeB sdBs evolve from \(\Delta\) to \(\square\). When the core masses are small \((0.45\) or \(0.5M_\odot))\), the SHeB sdBs expand to \(\square\) and then shrink to WD \((\Delta \rightarrow \square \rightarrow \text{outside})\); when the core masses are large \((0.75\) or \(1.0M_\odot))\), the SHeB sdBs directly expand to giant branch \((\Delta \rightarrow \text{outside})\). The arrows on right panels also indicate the evolutionary direction. We note that the \(r\)-axis on Part B applies symlog (symmetrical log) scale, which allows positive and negative values by setting a range around zero within the plot to be linear instead of logarithmic.

age sdBs, that is at the onset of helium burning quietly in the center. We adopt four core masses that is \(M_c=0.45, 0.5, 0.75,\) and \(1.0\) \(M_\odot\), respectively\(^1\). For each core mass, we explore a series of envelope masses starting from an initial value of \(M_e = 0.001M_\odot\) and increasing it in steps of \(0.002M_\odot\), until the evolutionary tracks are beyond the region in the \(T_{\text{eff}} - \log g\) space where BLAPs are found. In total, we computed more than 80 models, some of which are displayed in Figure 1. In the core, the adopted helium abundance is \(Y=0.98\), while the number ratio of helium to hydrogen, \(\log N_{He}/N_{H}\), in the envelope is set to \(-0.55\) for simplicity. The latter value corresponds to the best atmosphere model fit for OGLE-BLAP-001, which is the prototype object of BLAPs (see Table 1 and Pietrukowicz et al. 2017).

\(^1\) Since we are not interested in the ignition process, nor do we follow the evolution of a star before reaching the sdB phase, in this work we start our analysis from helium-burning quietly in the core.
We then evolved these sdBs to reach the SHeB phase and continue far beyond the location of BLAPs on the Kiel diagram \( (T_{\text{eff}} > 50,000 \, \text{K or log g < 4.0}) \). The evolutionary tracks on the \( T_{\text{eff}} - \log g \) plane are presented on the left panels of Fig. 1, where the legend indicates the core and envelope mass, and different line-styles mark models with the same envelope mass on the left and right panels. For clarity, not all models are shown in this figure. For comparison, four BLAPs (in red) and four high-gravity BLAPs (in green) with known \( T_{\text{eff}} \) and \( \log g \) are plotted with their error bars (see Table 1), while the purple \( \times \) indicates HD 133729. For \( M_c = 0.45M_\odot \) (upper panel), we see several loops on the evolutionary tracks when \( M_e = 0.025M_\odot \) and 0.031\( M_\odot \), which are caused by helium core breathing pulses after the exhaustion of the central He (Caloi 1989; Li 2012, 2017; Li et al. 2018; Ostrowski et al. 2021).

2.2. Pulsations and rate of period change, \( P/P \)

The large amplitude, short periods, and small rates of period change observed in BLAPs still pose a challenge to pulsation theory (Pietrukowicz et al. 2017). A few pioneering asteroseismic investigations have been carried out (for example Romero et al. 2018; Córsico et al. 2018; Wu & Li 2018; Byrne & Jeffery 2020), but none of them can fully explain the existence and the pulsation properties of BLAPs.

In this paper, rather than focusing on a detailed asteroseismic analysis, we explore a new evolutionary pathway to BLAPs. Our goal is thus to compute a broad range of evolutionary models and use these to inform on possible formation channels for BLAPs via binary evolution. While we defer a detailed asteroseismic analysis to a future paper (Wu et al., in prep.), we use the fact that the the lightcurves of BLAPs are similar to those of classical pulsators like Cepheid and RR Lyrae-type stars that exhibit just the radial fundamental mode (see for example Córsico et al. 2019, for a discussion of pulsation modes). Thanks to this similarity, we can use simple scaling relations to estimate the pulsation period, \( P \), and rate of period change, \( r = \dot{P}/P \), for our models. We describe our method further below.

Confidence in this approach comes from the fact that \( P \) and \( r \) inferred from scaling relations agree to within 10 percent (5 percent for most BLAPs models) with those computed with detailed asteroseismic analysis via GYRE (Wu et al., in prep.). This suffices for the qualitative investigation of Fig. 1. Also, despite our here on SHeB sdB models, we also constructed some ~ 0.3\( M_\odot \) pre-WD models; the values of \( r \) we derive when these models are located in the BLAPs region of the HR diagram are on the order of \( 10^{-5} \), similar to those obtained by Wu & Li (2018) through detailed asteroseismic analysis.

For \( p \)-mode oscillations, we used Eq. (1) to estimate the frequency, \( \nu \), with radial order, \( n \), and spherical harmonic degree, \( l \) (Tassoul 1980), which is not dependent on the driving mechanism

\[
\nu(n, l) \approx (n + l/2 + \epsilon) \Delta \nu,
\]

where \( \epsilon \) is the phase constant (= 2.625 according to the best-fitting model in Wu & Li 2018) and \( \Delta \nu \) is the large frequency separation, which is defined as the frequency spacing between adjacent radial order mode with the same spherical harmonic degree. The value of \( \Delta \nu \) strongly depends on mean density and is estimated by Tassoul (1980)

\[
\Delta \nu \approx \Delta \nu_0 \sqrt{-\mu/P} = \Delta \nu_0 \sqrt{M/M_\odot} \left(\frac{R}{R_\odot}\right)^{3/2},
\]

where \( \mu \), \( M \), and \( R \) are the mean density, mass, and radius of our sdBs models, respectively. \( \nu(0, 0) \approx \Delta \nu \) is the frequency of fundamental mode, and \( \Delta \nu_0 = 135.1 \mu \text{Hz} \) (Huber et al. 2011). The period is the reciprocal of the frequency, \( P = 1/\nu(0, 0) \). Then, following the definition of Pietrukowicz et al. (2017) the rate of period change can be estimated as

\[
r = \frac{\Delta P}{\Delta t} = \frac{1}{P^2} \frac{\partial P}{\partial t},
\]

which we calculate in this work as:

\[
r_n = \frac{P_{n+1} - P_n}{t_{n+1} - t_n} \frac{1}{P_n},
\]

where \( n \) refers to each point along a track computed through MESA. In the right panels of Fig. 1, the rates of period change for our SHeB sdB models are compared to the relative period changes measured by Pietrukowicz et al. (2017) using data from OGLE-III and OGLE-IV: \( P/P = \frac{\nu}{\nu_0} \approx \frac{P_{n+1} - P_n}{P_n} \).
Figure 1 shows that SHeB models pass through the $T_{\text{eff}} - \log(g)$ region inhabited by BLAPs when the H-rich envelope has the following masses: $M_e = 0.013M_\odot - 0.031M_\odot$ for $M_e = 0.45M_\odot$, $M_e = 0.007M_\odot - 0.011M_\odot$ for $M_e = 0.5M_\odot$, $M_e = 0.005M_\odot - 0.037M_\odot$ for $M_e = 0.75M_\odot$, and $M_e = 0.009M_\odot - 0.081M_\odot$ for $M_e = 1.0M_\odot$. For the high-gravity BLAPs, we obtain $M_e = 0.001M_\odot - 0.013M_\odot$ for $M_e = 0.45M_\odot$, and $M_e = 0.001M_\odot - 0.005M_\odot$ for $M_e = 0.5M_\odot$.

Since our models are able to explain many properties of the BLAPs, in the following discussion, we explore their possible formation channels. An important aspect that will be addressed in future works is to analyze the pulsation stability properties of (some of) our BLAP models. We remark that the study of the excitation and stability of pulsations strongly relies on both microscopic and macroscopic physical processes of elements, such as element diffusion, radiative levitation, and opacity tables (for example Byrne & Jeffery 2018). We plan to analyze pulsation stability across the stellar parameters identified with our models in a future investigation by using non-adiabatic theory and radial stellar pulsation theory to try reproducing light curves and radial velocities observed in BLAPs.

The computation of non-adiabatic pulsations can provide powerful insights into the stability of pulsation modes. For example, in their work on pre-WD models to explain BLAPs, Romero et al. (2018) found the need of super-solar metallicity to drive pulsations via the $\kappa$ mechanism. The effect of metallicity on the stability of pulsation is particularly interesting, given the lack of BLAPs detected thus far in the metal-poor regime (Pietrukowicz 2018). Since the fraction of close binaries increases at low metallicities (Moe et al. 2019), if our models are not pulsationally unstable at low metallicity, this could have implication for the viability of evolutionary scenarios relying on close period binaries (see the next section).

3. Formation channel(s)

Given the success of our SHeB sdB models in reproducing a number of BLAPs’ properties, in this section we investigate their formation channel(s). Han et al. (2002, 2003) developed a binary model for the formation of sdB stars. This model explained almost all properties of sdB stars, including single sdBs, short- and long-period sdBs, and thus has been widely accepted in the literature. Our study for BLAPs is based on this model.

Except for HD 133729 (Pigulski et al. 2022), no evidence for companion stars has been found in BLAPs so far (Pietrukowicz et al. 2017; Kupfer et al. 2015; Ramsay 2018; McWhirter et al. 2020). This however does not necessarily imply that BLAPs are single stars. For example, they could be in long-period binaries which have escaped detection or, otherwise, in short-period binaries, where very bright companions dominate the light budget and dilute the pulsation amplitude (similar to HD 133729, see Pigulski et al. 2022). It might also be possible that BLAPs themselves have very faint companions. Radial velocity monitoring might be used to detect binaries, but so far only few BLAPs have spectroscopic observations.

In Han’s model, hot subdwarfs can be produced through three channels, namely, CEE, stable RLOF, or a merger of two He WDs (Han et al. 2002, 2003). For the CEE channel, the donor loses most of the H-rich envelope mass close to the tip of RGB during common envelope ejection and the remnant evolves to short-period sdBs. This channel is inconsistent with the -albeit limited- observations that no BLAP shows sign of being in a short-period binary, except for the case of the companion of HD 133729. Since the CEE process is complicated and is not yet well understood (see Ivanova et al. 2013), it is hard to know the envelope mass of sdBs in this way from binary calculation. However, the locations of observed short-period sdBs in the $T_{\text{eff}} - \log(g)$ diagram suggest that the H-rich envelopes of sdBs from the CEE channel are massive enough to account for BLAPs (Xiong et al. 2017). If we only consider the envelope mass of sdBs, we cannot exclude the possibility that the CEE produces BLAPs\footnote{If sdBs from the CEE channel cannot evolve into the BLAPs, the abundances in the envelope could be the cause since element abundances are crucial for driving the pulsations. There is evidence showing that short-orbital period sdBs resulted from the CEE channel have surface abundances that are significantly different from that of the RLOF (Geier et al. 2022).}. Therefore, in Section 3.3, we give the expected number of BLAPs also from this channel.

As we have already pointed out, HD 133729 is the only object for which a BLAP has been identified in a binary system and given its parameters it was likely produced via stable RLOF rather than CEE. The (present-day) primary in HD 133729, has a mass of $2.85 \pm 0.25 M_\odot$ which means that the donor (which we speculate to have evolved into the BLAP) probably had a mass around $\sim 3.0 M_\odot$ (McWhirter & Lam 2022) and non-degenerate core before the RLOF took place. Such a star may become an sdB star (and then a BLAP) if mass transfer starts during the Hertzsprung gap. Based on the work of Han et al. (2003), sdB binaries produced by non-degenerate progenitors via RLOF during a Hertzsprung gap have orbital periods in the range between 10 hours and dozens of days, consistent with the observations of HD 133729.

For the merger channel, two He WDs could merge due to gravitational wave radiation and produce a single hot subdwarf star, with its mass in the range of $0.3 - 0.8 M_\odot$ (Han et al. 2002, 2003; Zhang et al. 2009, 2017). During the merger of double He WDs, the less massive He WD is disrupted and accreted onto the more massive one. Helium is ignited through He flashes, and the merged object becomes an sdB star when He burns stably in the center (Zhang & Jeffery 2012). In this case, however, little hydrogen survives the series of He flashes that is the maximum hydrogen mass is only $0.002 M_\odot$ in the study of Hall & Jeffery (2016, see also Zhang & Jeffery 2012). This value agrees with the $M_e$ determined for high-gravity BLAPs but it is far less than that required for the other BLAPs ($>0.005 M_\odot$ as discussed in Section 2). Hence, if BLAPs are single sdBs, it is unlikely that they form through the merger of two He WDs\footnote{Although it is typically quoted that non-degenerate cores form beyond $2 M_\odot$, with the models used in this work this happens for $>1.99 M_\odot$.}.

For the RLOF channel, if the donors have low initial masses ($\leq 2 M_\odot$) and degenerate He cores after central H burning, the produced hot subdwarfs have masses around $0.5 M_\odot$ (Han et al. 2002, 2003) and orbital periods around 1400 days (Chen et al. 2013). If the donors have masses with non-degenerate He cores, the resulting hot subdwarfs have a wide mass range, namely, from 0.3 – 0.8$M_\odot$, and the orbital periods are relatively short, namely, from several days to more than one hundred days, depending on the initial mass ratio, initial orbital period, and the assumptions for mass and angular momentum loss (Han et al. 2002, 2003).
2000; Chen & Han 2002, 2003). In the following, we focus on the RLOF channel with donors ranging from evolved to degenerate He cores and we create long orbital period binaries.

### 3.1. Binary evolution calculations

Using MESA, we investigate the binary evolution for several binaries consisting of a giant star (the progenitor of sdBs) and an MS or WD companion. The study is based on Population I stars and the basic physics inputs are the same as introduced in Sect. 2. We only evolve donors and consider companions as point sources. In our calculation, the mass transfer rate is calculated by the scheme of Kolb & Ritter (1990), and the mass transfer process is completely non-conservative, that is, all mass lost from the donor onto the MS star is not actually accreted by the MS star, but is lost from the MS and, hence, from the system. The angular momentum lost from the system is then given by the mass loss times the specific angular momentum (i.e., angular moment per unit mass) of the MS star. No wind loss has been included in the whole evolutionary process. We discuss this point further in Sect. 3.4.

We go on to explore eight values for the initial mass of the giant (the donor: \(M_{1i} = 0.79, 0.89, 1.0, 1.12, 1.26, 1.4, 1.58, \) and 1.78 \(M_\odot\)) and three values for initial giant and MS mass-ratio \(q_i = 1.1, 1.25 \) and 1.5. The upper limit of \(q_i = 1.5\) here is consistent with the critical mass ratio for dynamically stable mass transfer when the donor is a giant (Han et al. 2002; Chen & Han 2008). For each \((M_{1i}, q_i)\), we increase the initial orbital separation, \(A_0\), in steps of equal \(A_0\), from the minimum separation to produce sdB stars, to the point where the donor cannot fill its Roche lobe on the red giant branch.

### 3.2. Outcomes from binary evolution

From our binary evolution calculations, we obtain in total 24 sets of long-period SHeB sdBs. For each \((M_{1i}, q_i)\), we obtain some SHeB sdBs passing through the position of BLAPs and that of high-gravity BLAPs. We show an example in Fig. 2, with initial parameters \((M_{1i}, q_i) = (0.79M_\odot, 1.5)\): the upper panel displays evolutionary tracks in the \(T_{\text{eff}} - \log g\) plane, while the lower one shows those in the \(P - r\) diagram. A total of 11 evolutionary tracks (including 5 tracks overlapping in the thick gray line) are shown in Fig. 2, with initial orbital periods (~ orbital periods before RLOF) equally spaced in terms of \(\log P\). When the initial orbital periods are short, the produced sdBs nearly have no H-rich envelope due to delayed He flashes (Xiong et al. 2017) and the evolutionary tracks are overlapped, as shown with thick grey line in the upper panel.

As shown in the upper of Fig. 2, several SHeB sdBs with relatively high envelope mass match the location of BLAPs, while some with relatively low envelope mass match the location of high-gravity BLAPs, similarly to what has already been shown in Fig. 1. The pulsation period and its rate of change match the observations in terms of the right order of magnitude. All models have orbital period of ~ 1400 days and the companion is a 0.53\(M_\odot\) main sequence star due to no accretion in the RLOF process (mass transfer is completely nonconservative).

The results for the other explored sets of \((M_{1i}, q_i)\) are qualitatively similar to those shown here, (presented in Figs. A1-A23 in the appendix). All sdBs we produce have very similar evolutionary tracks in both the \(T_{\text{eff}} - \log g\) and \(P - r\) diagrams. They also have orbital periods greater than 1000 days, with companions in the mass range of 0.53 – 1.62\(M_\odot\).

![Fig. 2. Evolution of sdBs produced via binary evolution for \((M_{1i}, q_i) = (0.79M_\odot, 1.5)\). The thick gray line identify overlapping evolutionary tracks (there are five tracks overlapping in this figure, see text for details). Panel (a): Evolutionary tracks in the \(T_{\text{eff}} - \log g\) plane. Each linestyle corresponds to a different set of \((M_\odot, M_\ast, P_{\text{orb}})\) at the start of stable CHeB (see legend). Panel (b): Rate of period change, \(r\), versus pulsation period, \(P\), for the models in panel (a). The final orbital periods for these models are ~ 1400 days. Observations are the same as in Fig. 1.](image)

We remark that it is somewhat difficult for our models to explain the relatively high positive \(r\) values observed in some BLAPs; however, we can clearly see in Fig. 1 that some models may reproduce it. This happens more easily for high He-core masses \(M_\ast = 0.75M_\odot\) and 1.00\(M_\odot\), and even for \(M_\ast = 0.5M_\odot\) when fine-tuning envelope masses. This seems to suggest that RLOFs with non-degenerate donors might also be a viable channel for BLAP formation and they are worthy of more investigations in a future work.

### 3.3. Number of BLAPs in the Galaxy

According to our calculation, about 25 to 50 percent (marked with \(r_1\)) of sdBs would evolve to become BLAPs during their lives \(^7\). Considering that the typical lifetime of sdBs is \(t_{\text{sdB}} \sim 80\) and \(t_{\text{sdB}} \sim 160\) pass through the location of BLAPs. This range is due to the

---

\(^7\) We have computed a total of 304 evolution tracks and between ~ 80 and ~ 160 pass through the location of BLAPs. This range is due to the
10^6 yrs and the lifetime of SHEB sdBs is \( t_{\text{shell}} \approx 5 \times 10^7 \) yrs, we can simply estimate the number of SHEB sdBs produced in this way as \( N_{\text{shell}} = r_1 \times N_{\text{sdB}} \times t_{\text{shell}} / t_{\text{sdB}} \), where \( N_{\text{sdB}} \) is the number of sdBs produced from the RLOF channel in the Galaxy.

From the study of Han et al. (2003), \( N_{\text{sdB}} = 4.36 \times 10^6 \). We therefore have \( N_{\text{shell}} \approx 0.5 - 1 \times 10^6 \). The number of BLAPs depends on the parameter space defined for BLAPs. A typical SHEB sdB stays \( \sim 2 \times 10^6 \) yrs in the BLAP region defined by Pietrukowicz et al. (2017), only about 1/20 (marked with \( r_2 \)) of the SHEB sdB time. Also, not all stars in the instability region show pulsations. For example, percent of subdwarfs in the rapid variability region (sdBVr, Østensen et al. 2010) and 75 percent in the slow variability region (sdBVs, Green et al. 2003) show pulsations. We hence assume the fraction of stars showing pulsation to be in the range \( r_3 \approx 0.1 - 1 \). The number of BLAPs produced through the RLOF channel therefore can be estimated as \( N_{\text{BLAP}} = r_2 \times r_3 \times N_{\text{shell}} \), ranging from \( \sim 2500 \) to \( \sim 50000 \).

Observationally, we can estimate the number of BLAPs from existing data. For example, McWhirter & Lam (2022) discovered 22 BLAP candidates (six of which are high-gravity ones) by cross-matching Gaia DR2 and ZTF DR3 using a sample of over 162 million sources satisfying a number of photometric and astrometric cuts. Excluding for simplicity the impact of these cuts on sample selection, a plain scaling to the number of stars in the Milky Way (which is in the range 100-400 billion stars), gives an estimated number of BLAPs candidates between \( 10^4 \) and \( 10^5 \). If half of them are BLAPs, their number should be of a few tens of thousands, which finely matches our estimates obtained from SHEB sdBs formed through RLOF channel, although uncertainties are still admittedly large. Our estimates are also consistent with the range 280 to 280000 derived by Meng et al. (2020) from the OGLE survey under the two extreme assumptions that all BLAPs had been discovered, or that 99 percent of them had been missed, respectively.

If we consider that the CEE channel may also contribute to the formation of BLAPs, and assume sdBs from CEE have an envelope mass as similar to that from RLOF, from the study of Han et al. (2003), we have \( N_{\text{CEE}} = 6.18 \times 10^6 \) for both the CEE+RLOF channels. Keeping the other assumptions unchanged, the estimated \( N_{\text{BLAP}} \) ranges from \( \sim 3500 \) to \( \sim 70000 \), still consistent with the observations. In further work, a detailed stellar population synthesis (like Byrne et al. 2021, and Alexey et al. in prepared) may give a more precise number and the distribution of properties of BLAPs.

### 3.4. Putative BLAPs companions

In our calculations, all SHEB sdBs likely to be observed as BLAPs have relatively long orbital periods, \( P \sim 1400 \) days. The companions are main-sequence stars with lower mass limit, since the mass transfer is set to be completely non-conservative and the companion will not accrete any material during the RLOF. In this section, we explore and quantify the impact of such companions on photometry and radial velocities.

In Fig. 3, we show the spectral energy distribution (SED) of a star with BLAP-like stellar parameters, in comparison to typical A- to M-type main sequence stars. We use synthetic fluxes at solar metallicity from the grid of Castelli & Kurucz (2003), rescaled by the square of the adopted stellar radius and interpolated at the appropriate \( T_{\text{eff}} \) and \( \log g \) (see Table 2 for the adopted uncertainty on the location of BLAPs in the \( T_{\text{eff}} - \log g \) diagram. For example, the first 14 BLAPs discovered by Pietrukowicz et al. (2017) cover a much smaller region than that reported in the most recent works.

### Table 2. Physical parameters adopted for the SED of main sequence stars orbiting a BLAP. For the latter, we adopted the values from one of our binary models. For main sequence companions, we used literature parameters for an archetypal spectral types (Zombeck 2007).

<table>
<thead>
<tr>
<th>Spec. type</th>
<th>( T_{\text{eff}} ) (K)</th>
<th>( \log g )</th>
<th>( R(R_\odot) )</th>
<th>( M(M_\odot) )</th>
<th>archetypal</th>
</tr>
</thead>
<tbody>
<tr>
<td>BLAP</td>
<td>27718</td>
<td>4.51</td>
<td>0.62</td>
<td>0.469</td>
<td></td>
</tr>
<tr>
<td>A0</td>
<td>10800</td>
<td>4.15</td>
<td>2.50</td>
<td>3.20</td>
<td>CrB A</td>
</tr>
<tr>
<td>F0</td>
<td>7240</td>
<td>4.44</td>
<td>1.30</td>
<td>1.70</td>
<td>Vir</td>
</tr>
<tr>
<td>G0</td>
<td>5920</td>
<td>4.44</td>
<td>1.05</td>
<td>1.10</td>
<td>Com</td>
</tr>
<tr>
<td>K0</td>
<td>5240</td>
<td>4.47</td>
<td>0.85</td>
<td>0.78</td>
<td>Oph A</td>
</tr>
<tr>
<td>M0</td>
<td>3800</td>
<td>4.80</td>
<td>0.51</td>
<td>0.60</td>
<td>Lacaille 8760</td>
</tr>
</tbody>
</table>

Values). All stars are assumed to be at the same distance. For comparison, we also plot a number of filter transmission curves from the ultraviolet to the mid-infrared. From panel (a1) we can clearly see that for most spectral types the flux of the BLAP is dominant over the main-sequence companion. Sufficiently hot main sequence stars, however, can become dominant in the optical (A-type) or in the infrared (F-type), depending on the adopted radii. Panel (a2) shows similar comparison, but this time for the monochromatic magnitude difference between the flux of the main-sequence companion and that of the BLAP.

Observationally, however, this hypothetical binary systems must have been unresolved so far. This means that only the total flux can be observed. To account for this, in panel (b1) we compare the flux of a single BLAP (red) against that of a BLAP with a companion. The slope of the SED in the ultraviolet is largely determined by the \( T_{\text{eff}} \) of the BLAP, but it is altered by the flux of a companion at increasingly longer wavelengths. This excess amounts to \( \sim 0.1 \) mag for an M companion in the mid-infrared and it is considerably larger for earlier spectral types. However, it must be kept in mind that the effective temperature and the radius of a BLAP might vary quite considerably while pulsating, thus complicating the detectability of potential companions.

Now let us consider the possibility of looking for binarity through radial velocity variations. From our models, a BLAP with mass \( 0.458 M_\odot \) orbiting a companion of \( 1.62 M_\odot \) on a period of 1366 days will have an orbital speed of 19 km/s, assuming circular orbit. A model with a significantly less massive primary on similar period (0.465, 0.72, 1372) has an orbital speed of 11 km/s. These values will decrease with the inclination angle at which the system is observed. If BLAPs are instead composed of binaries with short periods, the radial velocity signal will be significantly higher. It is thus reasonable to aim for a radial velocity precision on the order of one km/s to test whether BLAPs have companions.

To explore this possibility in more detail, we simulated the radial velocity precision that can be achieved with spectra of various resolution (\( R \)) and signal-to-noise ratio (\( S/N \)). For the sake of simplicity, we fixed the parameters of the BLAP at \( T_{\text{eff}} = 30000 \) K, \( \log g = 4.5 \) dex, and solar metallicity using a TLUSTY synthetic spectrum (Lanz & Hubeny 2003). We generated 100 spectra in the wavelength range 3800 7400 Å for several values of resolution and signal-to-noise, and then estimated the radial velocity precision by computing the standard deviations of the radial velocities that are measured via cross-correlation function (Tonry & Davis 1979) As shown in Fig. 4, radial velocity errors decrease with increasing spectral resolution and signal-to-noise, achieving a km/s precision when \( R > 4000 \) and \( S/N > 25 \). It should, however, be pointed out that our results are idealized as they do not include uncertainties arising from instrumental effects, nor the contribution from the pulsations of
3.5. Discussion

Our study assumes that the mass transfer processes in this part of parameter space are completely nonconservative, namely, that all the mass lost from the giant is lost from the system. If the mass transfer is somehow conservative, the results of RLOF will change. Also, different assumptions for mass and angular momentum loss will result in different parameter spaces for producing sdB stars that is, the range of \( A_i \) for a given \( M_{1i} \) and \( q_i \).

However, as donors in our study have degenerate cores, the results satisfy a unique sdB mass - orbital period relation (Chen et al. 2013). This means that the orbital period will not change if the sdB mass stays constant, and only the mass of the companion changes. Therefore, the sdB binaries formed through RLOF channel have the same characteristics of our results (i.e., core mass, envelope mass, and orbital period), although the companion would become more massive than that given in this study and may become a blue straggler (see Chen & Han 2008). And a more massive companion would be easier to be detected via SED and the BLAP would have a higher radial velocity (as described in Sect. 3.4).

We have not considered the case where the companion is a WD in our model. We explain this as follows. To ensure the mass transfer process is dynamically stable, the pre-mass-transfer mass of the giant is very restricted, that is, the mass ratio of the giant to the WD is expected to be below a value of \( \sim 1.1 - 1.3 \). Presently we cannot rule out the possibility that the donor is on the giant branch, indicating that the WD mass could be lower than believed for stable mass transfer. We estimated the impact of the critical mass ratios and obtained a few sdB+WD binaries from the RLOF channel indeed based on the criterion of the Ge et al. (2020). But the number is very small and the WD companions to BLAPs should consequently be very rare. For the case of neutron star (NS) or black hole companions, the mass transfer is more likely to be stable due to massive WDs. In fact, Han et al. (2003) have not obtained any sdB stars with a WD companion in their model. However, based on an adiabatic mass loss model, Ge et al. (2013, 2015) obtained significantly larger critical mass ratios for the stability of mass transfer when the donor is on the giant branch, indicating that the WD mass could be lower than believed for stable mass transfer. We estimated the impact of the critical mass ratios and obtained a few sdB+WD binaries from the RLOF channel indeed based on the criterion of the Ge et al. (2020). But the number is very small and the WD companions to BLAPs should consequently be very rare. For the case of neutron star (NS) or black hole companions, the mass transfer is more likely to be stable due to massive WDs. According to the study of Wu et al. (2019), one percent of sdBs have NS companions with long periods (~ 1000 days), we thus may expect ~ 20 – 500 BLAPs with NS companions – in a similar way to what is explained in Section 3.3.

The CEE channel will generate sdBs in short period binaries. However, it is still an open question how much H-rich envelope will be left after the CEE (Xiong et al. 2017), due to the huge uncertainties in common envelope evolution (see Ivanova et al. 2013). Presently we cannot rule out the possibility that some sdBs from CEE have relatively thick H-rich envelopes and...
show characteristics similar to BLAPs. Their companions could be discovered from radial-velocity variations (see footnote 8). HD 133729 proves that BLAPs in short-period binary do exist, implying that the CEE channel might be a viable formation scenario. Because the primary is a B-type main sequence with $\sim 2.5M_\odot$, the progenitor of this BLAP would be an intermediate mass star, which is outside of our grid. According to Han et al. (2003), HD 133729 also could be produced by the RLOF channel, in which the RLOF occurs on the Hertzsprung gap. We will investigate this possibility in future work.

4. Conclusions

In this paper, we explore whether SHeB sdBs can reproduce the properties of BLAPs and we discuss their formation channels, based on binary evolution. Our study has shown that some SHeB sdBs aptly reproduce the properties of BLAPs, namely, effective temperature, surface gravity (or luminosity), and the rate of period change. Given that the purpose of this paper is to explore the formation of BLAPs through the RLOF channel. We will investigate the formation of BLAPs through radial velocity monitoring. Since the fraction of companions would require dedicated radial velocity monitoring.

We performed a series of binary evolution calculations to investigate the formation of BLAPs through the RLOF channel. From our grid of 304 models, we have found that 142 SHeB sdBs could be good BLAP candidates. All these BLAP candidates have long orbital periods, that is, $\geq 1400$ days. Because of these periods and the faintness of known BLAPs, the detection of companions would require dedicated radial velocity monitoring.

We also briefly discuss how CEE and the double He WDs merger scenarios might produce sdBs with very thin hydrogen envelopes – far less than required for high-gravity BLAPs. We believe that this is the first time that such a formation channel has been suggested. The merger of double He WDs leads instead to the formation of single sdBs with very thin hydrogen envelopes – far less than required for high-gravity BLAPs. We believe that this is the first time that such a formation channel has been suggested. The merger of double He WDs leads instead to the formation of single sdBs with very thin hydrogen envelopes – far less than required for high-gravity BLAPs. We believe that this is the first time that such a formation channel has been suggested. The merger of double He WDs leads instead to the formation of single sdBs with very thin hydrogen envelopes – far less than required for high-gravity BLAPs. We believe that this is the first time that such a formation channel has been suggested. 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\(M_{1i} = 0.79M_\odot, q_i = 1.1\)

\(M_{1i} = 0.79M_\odot, q_i = 1.25\)

Fig. A.1. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for \((M_{1i}, q_i) = (0.79M_\odot, 1.1)\).

Fig. A.2. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for \((M_{1i}, q_i) = (0.79M_\odot, 1.25)\).

Appendix A: \(T_{\text{eff}} - \log g\) and \(P - r\) diagram of SHeB SdBs produced by RLOF channel

The evolutionary tracks on \(T_{\text{eff}} - \log g\) and \(P - r\) diagrams from binary evolution (see Sect. 3.2). In Sect. 3.2, we only display the results for one \((M_{1i}, q_i)\), here, we show other results.
Fig. A.3. The evolution of sdBs produced by binaries, which is same as in Fig. 1, but for \((M_1, q_1) = (0.89M_\odot, 1.1)\).

Fig. A.4. The evolution of sdBs produced by binaries, which is same as in Fig. 1, but for \((M_1, q_1) = (0.89M_\odot, 1.25)\).
Fig. A.5. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for \((M_{i1}, q_i) = (0.89 M_\odot, 1.5)\).

Fig. A.6. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for \((M_{i1}, q_i) = (1.0 M_\odot, 1.1)\).
Fig. A.7. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for \((M_{1i}, q_i)=(1.0 \, M_\odot, 1.25)\).

Fig. A.8. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for \((M_{1i}, q_i)=(1.0 \, M_\odot, 1.5)\).
$M_1 = 1.12 M_\odot$, $q_1 = 1.1$

$M_1 = 1.12 M_\odot$, $q_1 = 1.25$

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**Fig. A.9.** Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for $(M_1, q_i) = (1.12 M_\odot, 1.1)$.

**Fig. A.10.** Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for $(M_1, q_i) = (1.12 M_\odot, 1.25)$. 
$M_1 = 1.12 M_\odot$, $q_i = 1.5$

$M_1 = 1.26 M_\odot$, $q_i = 1.1$

Fig. A.11. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for $(M_1, q_i)=(1.12 M_\odot, 1.5)$.

Fig. A.12. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for $(M_1, q_i)=(1.26 M_\odot, 1.1)$.
Fig. A.13. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for $(M_{1i}, q_i) = (1.26 M_\odot, 1.25)$.

Fig. A.14. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for $(M_{1i}, q_i) = (1.26 M_\odot, 1.5)$. 
Fig. A.15. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for ($M_{1i}, q_i$) = (1.41$M_\odot$, 1.1).

Fig. A.16. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for ($M_{1i}, q_i$) = (1.41$M_\odot$, 1.25).
Fig. A.17. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for \((M_1, q_i) = (1.41M_\odot, 1.5)\).

Fig. A.18. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for \((M_1, q_i) = (1.58M_\odot, 1.1)\).
Fig. A.19. Evolution of sdBs produced by binaries, which is same to Fig. 1, but for \((M_{1i}, q_i) = (1.58M_\odot, 1.25)\).

Fig. A.20. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for \((M_{1i}, q_i) = (1.58M_\odot, 1.5)\).
**Fig. A.21.** Evolution of sdBs produced by binaries, which is same to Fig. 1, but for \((M_1, q_i) = (1.78 M_\odot, 1.1)\).

**Fig. A.22.** Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for \((M_1, q_i) = (1.78 M_\odot, 1.25)\).
Fig. A.23. Evolution of sdBs produced by binaries, which is same as in Fig. 1, but for \((M_{1i}, q_i) = (1.78 \, M_\odot, 1.19)\).