Excitation of Langmuir waves at shocks and solar type II radio bursts

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ABSTRACT

Context. In the solar corona, shocks can be generated due to the pressure pulse of a flare and/or driven by a rising coronal mass ejection (CME). Coronal shock waves can be observed as solar type II radio bursts in the Sun’s radio radiation. In dynamic radio spectra, they appear as stripes of an enhanced radio emission slowly drifting from high to low frequencies. The radio emission is thought to be plasma emission, that is to say the emission happens near the electron plasma frequency and/or its harmonics. Plasma emission means that energetic electrons excite Langmuir waves, which convert into radio waves via non-linear plasma processes. Thus, energetic electrons are necessary for plasma emission. In the case of type II radio bursts, the energetic electrons are considered to be shock accelerated.

Aims. Shock drift acceleration (SDA) is regarded as the mechanism for producing energetic electrons in the foreshock region. SDA delivers a shifted loss-cone velocity distribution function (VDF) for the energetic electrons. The aim of the paper is to study in which way and under which conditions a shifted loss-cone VDF of electrons excites Langmuir waves in an efficient way in the corona.

Methods. By means of the results of SDA, the shape of the resulted VDF was derived. It is a shifted loss-cone VDF showing both a loss-cone and a beam-like component. The growth rates for exciting Langmuir waves were calculated in the framework of Maxwell-Vlasov equations. The results are discussed by employing plasma and shock parameters usually found in the corona at the 25 MHz level.

Results. We have found that moderate coronal shocks with an Alfvén-Mach number in the range $1.59 < M_A < 2.53$ are able to accelerate electrons up to energies sufficient enough to excite Langmuir waves, which convert into radio waves seen as solar type II radio bursts.

Conclusions.

Key words. Sun: flares – Sun: radio radiation – Sun: particle emission – Sun: shock waves

1. Introduction


An example of a solar type II radio burst is presented in Figure 1 (adapted from Magdalenic et al. (2020)). It was observed with the novel radio telescope LOFAR (LOw Frequency ARray) (van Haarlem et al. 2013) in the frequency range 10-90 MHz. This example shows all typical features of solar type II radio bursts (Smerd et al. 1962, Wild and Smerd 1972, see as reviews e.g. Nelson and Melrose (1985) and Mann (1995)). In dynamic radio spectra, type II bursts appear as stripes of enhanced radio emission. These stripes drift slowly from high to low frequencies. As in the presented example, type II bursts show a fundamental harmonic structure, that is at 15:15:00 UT the fundamental and harmonic band start at \( \approx 30 \) and 60 MHz, respectively. The fundamental band drifts from 40 MHz to 14 MHz in the period 15:09-15:21 UT, leading to a drift rate of the fundamental band of \(-0.036 \) MHz s\(^{-1}\). It is a typical value in this frequency range (Mann 1995, Dorovsky et al. 2015). Both the fundamental band and the harmonic one are divided into sub-bands. This is called band-splitting, which is a usual phenomenon of solar type II radio bursts (Smerd et al. 1962, Vršnak et al. 2001, 2002, Magdalenic et al. 2020). Furthermore, rapidly drifting stripes of enhanced radio emission are seen as fine structures in both the fundamental and harmonic band. They have typical drift rates of about \( \pm 1 \) MHz s\(^{-1}\) (Mann & Klassen 2005). They are called ‘herringbones’ (Nelson & Melrose 1985, Cairns & Robinson 1987, Mann et al. 2018) and are regarded as radio signatures of electron beams accelerated at the shock wave associated with the type II burst.

In LOFAR’s frequency range, that is 10–240 MHz, the Sun’s non-thermal radio emission is thought to be plasma emission as originally proposed by Ginzburg & Zheleznyakov (1958). In this case, energetic electrons excite electrostatic Langmuir waves which convert into escaping radio waves due to non-linear plasma processes (Ginzburg & Zheleznyakov (1958) and Melrose (1985) as a textbook). Thus, the radio waves are emitted near the local electron plasma frequency and/or its harmonics. As a result, the appearance of energetic electrons is a necessary requirement for exciting Langmuir waves and non-thermal radio emission.

Assuming plasma emission for type II radio bursts, the individual frequencies are emitted at different height levels because the electron plasma frequency depends on the square root of the electron number density and the density is gravitationally stratified in the corona. Hence, the up- and downward movement of a radio source manifests in a negative and positive drift in the dynamic radio spectrum, respectively. The density model by Mann et al. (1999) results from a special solution of Parker’s (1958) wind equation and describes the radial behaviour of the electron number density well from the corona up to 1 AU in the interplanetary space. It gives a relationship between the emission frequency and its radial localization in the corona.

In the special example presented in Figure 1, the fundamental band drifts from 40 MHz to 14 MHz within 720 s. According to the density model of Mann et al. (1999), the 40 MHz and 14 MHz level correspond to a radial distance \( 2.25 R_\odot = 1.57 \times 10^6 \) km \((R_\odot, \text{Sun’s radius})\) and \(1.68 R_\odot = 1.17 \times 10^6 \) km, respectively. Thus, the shock travels a radial distance of \(4.00 \times 10^6 \) km within 720 s resulting in a radial velocity of \(556 \) km s\(^{-1}\). We note that this is the radial shock velocity. The real one can be much larger in the case of the shock travelling horizontally along the solar surface. The herringbones shooting away from both the fundamental and harmonic band towards higher and lower frequencies have drift rates of \(\pm 1.0 \) MHz/s in the example presented in Figure 1, which is typical for these features (Mann & Klassen 2002, Dorovsky et al. 2015). Such a drift rate results in a radial velocity of 25,000 km s\(^{-1}\) which should be regarded as a typical velocity of shock-accelerated electron beams in the corona (Mann & Klassen 2005). However, there are also cases in which the electrons associated with the herringbones have velocities up to 100,000 km s\(^{-1}\) as reported by Dorovsky et al. (2015).

The type II burst presented in Figure 1 appears around 25 MHz in the fundamental band. The 25 MHz level corresponding to an electron number density \( N_e = 7.75 \times 10^{10} \) cm\(^{-3}\) is typically located at a distance of \( \approx 2 R_\odot \) from the centre of the Sun (Mann et al. 1999). At this height, a magnetic field \( B \) of 0.5 G (Dulk & McLean 1978) is obtained. The model by Dulk & McLean (1978) approximately describes the radial behaviour of the magnetic field strength across active regions, where solar type II radio bursts occur. With these parameters, one gets an electron plasma frequency \( \omega_{pe} = (4\pi e^2 N_e/m_e)^{1/2} = 1.57 \times 10^8 \) s\(^{-1}\) (e, elementary charge; m\(_e\), electron mass), electron cyclotron frequency \( \omega_{ce} = eB/m_ec = 8.79 \times 10^7 \) s\(^{-1}\) (c, velocity of light), a ratio between the electron plasma frequency to the electron cyclotron frequency \( \omega_{pe}/\omega_{ce} \approx 18 \), and an Alfvén speed \( v_A = 365 \) km s\(^{-1}\). Assuming \( T = 1.4 \times 10^6 \) K as a typical temperature in the corona (Koutchmy 1994), a thermal electron velocity \( v_{th,e} = (k_B T/m_e)^{1/2} = 4600 \) km s\(^{-1}\) \((k_B, \text{Boltzmann’s constant})\) and sound speed \( c_s = (\gamma k_B T/\mu m_p)^{1/2} = 180 \) km s\(^{-1}\) \((m_p, \text{proton mass}; \gamma = 5/3, \text{ratio of the specific heats}; \mu = 0.6, \text{mean molecular weight})\) are obtained. Then, the ratio \( \beta \) between the thermal and the magnetic pressure, that is \( \beta = (8\pi B^2)/N_k q_T = 2c_s^2/(\gamma v_A^2) \), is found to be 0.3. The Debye length \( \lambda_{De} = v_{th,e}/\omega_{pe} \) is found to be 3 cm. These parameters are regarded as typical plasma parameters of type II radio burst sources in the following study.

The aim of the paper is to study in which way and under which conditions energetic electrons accelerated at coronal shock waves are able to excite Langmuir waves, which are necessary for non-thermal radio radiation as observed in terms of solar type II radio bursts. The energetic electrons are considered to be produced by shock drift acceleration (SDA). Therefore, a brief description of SDA is given in Section 2. In Section 3, the excitation of Langmuir waves is studied by means of the Maxwell-Vlasov equations (Baumjohann & Treumann 1997, Treumann & Baumjohann 1997). The numerical results delivered in Section 3 are discussed in Section 4.

\[1 \text{For the same event, Magdalenic et al. (2020) obtained a radial velocity of the type II burst source of 800 km s}^{-1} \text{ by employing a 3.5-fold Saito (1970) density model, whereas the density model by Mann et al. (1999) is adopted in this paper leading to the difference in the radial type II burst velocities.}\]
The paper is closed with a summary of the obtained results (Sect. 5).

2. Shock drift acceleration

As already mentioned in Section 1, solar type II radio bursts are the radio signature of shock waves travelling through the corona. A fast magnetosonic shock is accompanied by compressions of both the density and the magnetic field (Priest 1982). Hence, such a shock represents a moving magnetic mirror at which charged particles can be reflected and accelerated. This process is usually called shock drift acceleration (SDA). The acceleration happens due to the electric field, which is induced in the shock transition region. A much more detailed description of SDA is given in the papers by Holman & Pesses (1983), Ball & Melrose (2001), Mann & Klassen (2005), and Mann et al. (2018). Here, SDA is treated in a classical (i.e. non-relativistic) manner, whereas Mann et al. (2009) described the SDA in a fully relativistic manner.

The SDA is usually described in the de Hoffmann-Teller frame (see Sect. 4 in Mann & Klassen (2005)), where the shock wave is at rest and the motional electric field is removed. Hence, the reflection process can be treated by conservation of the energy

\[
\frac{m_e}{2} \left( V_{i,\parallel,HT}^2 + V_{i,\perp,HT}^2 \right) = \frac{m_e}{2} \left( V_{r,\parallel,HT}^2 + V_{r,\perp,HT}^2 \right) - e\phi_{HT}
\]

and the magnetic moment

\[
\frac{V_{i,\perp,HT}^2}{B_1} = \frac{V_{r,\perp,HT}^2}{B_2}
\]

where \( V_{i,\parallel} \) and \( V_{i,\perp} \) denote the particle velocity parallel to the ambient magnetic field before and after the reflection process, respectively. Furthermore, \( B_1 \) and \( B_2 \) are the magnetic field vectors in the upstream and downstream region, respectively. The index \( HT \) denotes the quantity in the de Hoffmann-Teller frame. The cross-shock potential \( \phi_{HT} \) must be taken in the de Hoffmann-Teller frame since it is frame dependent. The velocity components in the de Hoffmann-Teller frame are related to those in the laboratory frame by

\[
V_{i,HT} = V_i - v_s \sec \theta
\]

\[
V_{r,HT} = V_r
\]

(see Eqs. (13) and (14) in Mann & Klassen (2005)), where \( \theta \) is the angle between the upstream magnetic field \( B_1 \) and the shock normal \( n_s \). At SDA, the velocity component parallel to the magnetic field changes its sign. As a result of SDA, the velocity gain is found to be

\[
V_{r,\parallel} = 2v_s \sec \theta - V_{i,\parallel}
\]

with \( v_s \) as the shock speed in the laboratory frame (Holman & Pesses 1983, Ball & Melrose 2001). The velocity perpendicular to the magnetic field stays unchanged during the reflection.
The Alfvén-Mach number $M_A$ on $X$ according to Eq. (9) for $\beta = 0.29$ and $\gamma = 5/3$.

Due to the different inertia of electrons and protons, an electric field is established across the shock (see e.g. Goddrich & Scudder (1984) and Kunic et al. (2001)). It is directed from the downstream into the upstream region. In the de Hoffmann-Teller frame, the (cross-shock) potential related to this electric field is given by

$$e\phi_{HT} = \frac{\gamma}{(\gamma - 1)} \cdot k_B T_1 \cdot \left[ \frac{T_2}{T_1} - 1 \right]$$  \hfill (6)

(see Eq. (10) in Mann & Klassen (2005)), where $\phi_{HT} = 0$ was chosen in the far upstream region. Furthermore, $T_1$ and $T_2$ denote the temperatures in the upstream and downstream region, respectively.

In particular, the discussion in Mann et al. (2018) (as well as the inspection of Eq. (5)) shows that just quasi-perpendicular shocks (i.e. $\theta \rightarrow 90^\circ$) are able to accelerate electrons very efficiently. In the case of nearly perpendicular shocks, that is $\theta \rightarrow 90^\circ$, the Rankine-Hugoniot relationships (Priest 1982) in magnetohydrodynamics (MHD) describe the jump in temperature and magnetic field across the shock

$$\frac{T_2}{T_1} = 1 + \frac{(\gamma - 1)}{2} \cdot \frac{v_A^2}{c_s^2} \cdot (M_A)^2 \cdot \left( 1 - \frac{1}{X^2} \right)$$  \hfill (7)

and

$$\frac{B_2}{B_1} = X$$  \hfill (8)

(see Eqs. (A8) and (A9) in Mann et al. (2018)), respectively. Also, $X = N_2/N_1$ denotes the ratio between the particle number densities in the upstream ($N_1$) and downstream ($N_2$) region. The Alfvén-Mach number $M_A = v_e/v_{A1}$ is related to $X$ by

$$M_A = \sqrt{X \cdot \left[ \frac{\gamma(\beta + 1) + X(2 - \gamma)}{[\gamma + 1] - X(\gamma - 1)} \right]}$$  \hfill (9)

(see Eq. (10) in Mann et al. (2018)). For instance, Figure 2 presents the dependence of $M_A$ on $X$ for $\beta = 0.29$ and $\gamma = 5/3$. With the expression for the jump of the temperature across the shock (see Eq. (7)), the cross-shock potential (see Eq. (6)) and the related velocity $V_e = (2e\phi_{HT}/m_e)^{1/2}$ is found to be

$$e\phi_{HT} = k_B T_1 \cdot \frac{\gamma}{2} \cdot \frac{v_A^2}{c_s^2} \cdot (M_A)^2 \cdot \left( 1 - \frac{1}{X^2} \right)$$  \hfill (10)

and

$$V_e = v_{\text{th,e}} \cdot \sqrt{\frac{\gamma}{2} \cdot \frac{v_A^2}{c_s^2} \cdot (M_A)^2 \cdot \left( 1 - \frac{1}{X^2} \right)}$$  \hfill (11)

respectively.

The conditions under which the reflection takes place can be derived by using Eqs. (2) and (3) to be

$$V_{\perp,HT} > \tan \alpha_K \cdot \sqrt{V_{\perp,HT}^2 + V_{\parallel,e}^2}.$$  \hfill (12)

The so-called loss-cone angle $\alpha_K$ is defined by $\alpha_K = \arcsin[(B_1/B_2)^{1/2}] = \arcsin[(1/X)^{1/2}]$ because of Eq. (8).

After the reflection, the velocity distribution function (VDF) $f_e$ of the electrons has the shape of a shifted loss-cone distribution (Leroy & Mangeney 1984, Wu 1984) (see also Eq. 15 in Mann & Klassen (2005)) as illustrated in Figures 3 and 4 in Mann et al. (2018). The VDF $f_e$ has both a beam-like and loss-cone-like shape. Hence, there are special regions with both $\partial f_e/\partial V_{\parallel} > 0$ and $\partial f_e/\partial V_{\perp} > 0$ giving rise to the excitation of plasma waves. Even regions with $\partial f_e/\partial V_{\parallel} > 0$ lead to the excitation of electrostatic Langmuir waves (Baumjohann & Treumann 1997).

In Section 3, the excitation of Langmuir waves is discussed for the case of an initial Maxwellian VDF

$$f_{\text{mf}}(V_\parallel, V_\perp) = \frac{1}{(2\pi v_{\text{th,e}}^2)^{3/2}} \cdot e^{-[(V_\parallel - V_{\parallel,e})^2/2v_{\text{th,e}}^2]}$$  \hfill (13)

in the upstream region. It is normalized to unity. After the reflection via SDA, the reduced VDF

$$F_{\text{acc}} = \Theta(V_\parallel - V_{\parallel,e}) \cdot F_b \cdot \exp \left[ - \frac{(V_\parallel - V_{\parallel,e})^2}{2v_{\text{th,e}}^2 \cos^2 \alpha_K} \right]$$  \hfill (14)

is found with $V_b = V_{\parallel,e}(1 + \cos^2 \alpha_K)$ and

$$F_b = \frac{1}{(2\pi v_{\text{th,e}}^2)^{1/2}} \cdot e^{-[(v_A^2 \sin^2 \alpha_K + v_s^2 \sin^2 \alpha_K)/2v_{\text{th,e}}^2]}$$  \hfill (15)

for the reflected electrons (see Eqs. (17), (18) and (19) in Mann et al. (2018)). Here, the $\Theta(x)$ is the well-known step function, that is $\Theta(x) = 1$ and $\Theta(x) = 0$ for $x > 0$ and $x < 0$, respectively. The reduced VDF is obtained by integrating the usual VDF with respect to the velocity component perpendicular to the magnetic field. We note that $F_{\text{acc}}$ represents a distribution function of an electron beam with the velocity $V_b$ and the width $2^{-1/2}v_{\text{th,e}} \cos \alpha_K$.

### 3. Excitation of Langmuir waves

The resonant interaction of the beam electrons with the surrounding plasma waves leads to the excitation of Langmuir waves. The resonance condition is given by

$$0 = \omega_L - V_{\parallel e}k,$$  \hfill (16)
where $\omega_L$ is the frequency of the Langmuir wave and $k$ is the wave number of the Langmuir wave (Baumjohann & Treumann 1997). In addition, $V_{res}$ is the velocity of the electrons resonantly interacting with the Langmuir waves, and $\omega_L$ is related to $k$ by the dispersion relation

$$\omega_L = \sqrt{\omega_{pe}^2 + 3k^2v_{th,e}^2}$$

(Baumjohann & Treumann 1997). Introducing dimensionless quantities according to $\nu = \omega_L/\omega_{pe}$ and $q = kA_{De}$ with the Debye length $A_{De}$, the dispersion relation (see Eq. (17)) and the resonance condition (see Eq. (16)) can be written as

$$\nu = \sqrt{1 + 3q^2}$$

and

$$U_{res} = \frac{\nu}{q} = \sqrt{\frac{1 + 3q^2}{q^2}} = \sqrt{\frac{3\nu^2}{\nu^2 - 1}}$$

with $U_{res} = V_{res}/v_{th,e}$ leading to

$$q = \sqrt{\frac{1}{U_{res}^2 - 3}}$$

and

$$\nu = \frac{U_{res}}{U_{res}^2 - 3},$$

respectively. The growth rate $\gamma$ of Langmuir waves is given by Eq. (4.3) in Treumann & Baumjohann (1997), which can be expressed by

$$\frac{\gamma}{\omega_{pe}} = \frac{v}{2q^2} \frac{dF}{dU}$$

taken at $U = U_{res} = \nu/q$. Here, $F(U)$ is the reduced VDF, where $U$ denotes the electron velocities normalized to the thermal one. As is well known, wave growth and damping occurs for $dF/dU > 0$ and $dF/dU < 0$, respectively.

The full reduced VDF consists of the VDF of the background plasma $F_M$, which is a Maxwellian one (see Eq. (13))

$$F_M = \frac{1}{(2\pi)^{1/2}} e^{\nu^2/2}$$

and of the VDF $F_{acc}$ of the accelerated electrons (see Eqs. (15) and (16))

$$F_{acc} = \Theta(U - U_s) \cdot \frac{F_b}{(2\pi)^{1/2}} \cdot e^{-(U - U_s^2)/2\cos^2 \alpha_u}$$

with

$$F_b = e^{-(U_s^2 \sin^2 \alpha_u + U_s^2 \tan^2 \alpha_u)/2},$$

where $U = U/\nu_v$, $U_s = V_s/\nu_v$, and $U_b = V_b/\nu_v$. The complete VDF $F$ is given by $F = F_{acc} + F_M$ with Eqs. (23) and (24). For illustration purposes, it is presented in Figure 3 for a temperature of 1.4 MK and a shock with the parameters $X = 1.9$, $M_s = 1.94$, and $\theta = 88.14^\circ$ as well as $\beta = 0.29$ and $\gamma = 5/3$. Employing the plasma parameters at the 25 MHz level in the corona as given in Section 1, this shock has a velocity of 708 km s$^{-1}$ and of 21,813 km s$^{-1}$ in the laboratory and de Hoffmann-Teller frame, respectively. It provides an electron beam with the velocity of 32,145 km s$^{-1}$ corresponding to a kinetic energy of 3 keV. We note that such a shock is discussed in more detail in Sect. 4. The component of the thermal and accelerated electrons is evidently seen. The VDF shows a local minimum and maximum at $U = 6.10$ and $U = 7.00$, respectively. Hence, $F$ has a positive slope in the region $6.10 < U < 7.00$, where an instability and, consequently, wave growing occurs.

With $F = F_{acc} + F_M$ and Eqs. (23) and (24), one gets

$$\frac{dF}{dU} = \frac{1}{(2\pi)^{1/2}} \times \frac{(U_b - U)}{\cos^2 \alpha_{lc}} \cdot F_b \cdot e^{-(U - U_s)^2/2\cos^2 \alpha_u} - (U_b - U) \cdot e^{(U - U_s)/2}$$

in the region $U > U_s$. Wave growing requires $dF/dU > 0$ and hence, $U < U_b$. The first term on the right-hand side of Eq. (26) has a local maximum at $U_{max} = U_b - \cos \alpha_{lc}$, that is to say wave growing is mostly efficient for electrons with velocities

$$U_{res} = U_{max} = U_b - \cos \alpha_{lc} = U_s(1 + \cos^2 \alpha_{lc} - \cos \alpha_{lc})$$

(Eq. (27) means that a definite resonance velocity $U_{res}$ is related to a given shock speed $U_s$. In other words, each individual shock has its own resonance velocity. Inserting $U = U_{res}$ into Eq. (26) and the thusly obtained expression for $dF/dU$ taken at $U = U_{res}$ into Eq. (22), one gets for the maximum growth rate

$$\gamma_{max}/\omega_{pe} = \sqrt{\frac{\pi}{8}} \frac{\gamma_{res}}{U_{res}^2 - 3} \left[ F_{b,\max} \cdot e^{-1/2} - U_{res} \cdot e^{U_{res}^2/2} \right]$$

with

$$F_{b,\max} = \exp \left( -\frac{1}{2} \left( \frac{U_{res}^2 + \cos \alpha_{lc}}{1 + \cos^2 \alpha_{lc}} \right) \cdot \sin^2 \alpha_{lc} + U_s^2 \cdot \tan^2 \alpha_{lc} \right)$$

Here, Eqs. (20) and (21) have been used. For illustration purposes, the dependence of the growth rate $\gamma_{max}/\omega_{pe}$ on the resonant electron velocity $U_{res}$ is drawn in Figure 4. It shows...
that thermal wave damping occurs until $U_{res} = 5.97$. There, wave damping changes into wave growing. Beyond this value, the growth rate rapidly increases up to $\gamma_{max}/\omega_{pe} = 1.17 \times 10^{-6}$ at $U_{res} = 6.3$. Subsequently, it falls down exponentially. Concerning Figure 4, we note once again that the resonance velocity $U_{res}$ is related to the shock speed $U_s$ according to Eq. (27), in other words, a shock with the speed $U_s$ belongs to a fixed resonance velocity $U_{res}$.

Figure 4 shows that the growth rate $\gamma_{max}/\omega_{pe}$ has a clear maximum. In Table 1, the dependence of the maximum $\gamma_{max}/\omega_{pe}$ of the growth rate on the shock strength expressed by the density jump $X$ across the shock is calculated and presented in Figure 5. Inspecting Table 1, the velocity of electrons resonantly interacting with the electrons for growing Langmuir waves decreases from $U_{res} = 7.3$ down to $U_{res} = 6.3$ with increasing $X$ in the range $X < 1.9$ and, subsequently, it increases with increasing $X$. A shock with $X = 1.9$ and $M_A = 1.94$ produces, in the most efficient way, energetic electrons which generate Langmuir waves with a maximum growth rate of $\gamma_{max}/\omega_{pe} = 1.17 \times 10^{-6}$. The electrons resonantly interact with the Langmuir waves which have a velocity $U_{res} = 6.3$ in this case.

### Table 1. Dependence of the maximum growth rate $\gamma_{max}/\omega_{pe}$ on the density jump $X$ across the shock and the velocity $U_{res}$ of the resonant electrons (see also Fig. 5) in the case of $\gamma = 5/3$ and $\beta = 0.29$.

<table>
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<tr>
<th>$X$</th>
<th>$U_{res}$</th>
<th>$\gamma_{max}/\omega_{pe}$</th>
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</table>

4. Discussion

In Section 3, it was demonstrated that the electrons accelerated by SDA are able to excite Langmuir waves under plasma conditions at the 25 MHz-level in the corona. These Langmuir waves convert non-linearly into radio waves (Melrose 1985) observed as type II radio bursts. Which growth rate of Langmuir waves is needed for type II radio bursts? Looking to Figure 1, where a typical example of a solar type II radio burst is presented, the herringbones are the elementary features as mentioned in Section 1. They appear for $\approx 1$ s at a fixed frequency. Hence, an enhanced Langmuir wave level should be produced within 0.1 s. The enhancement of the electric field $E$ related to the Langmuir wave by a factor of 10, 100, and 1000 above the thermal one $E_{therm}$ according to

$$\frac{E}{E_{therm}} = \exp(\gamma_{max}t)$$

require a growth rate $\gamma_{max}/\omega_{pe} = 1.47 \times 10^{-7}$, $2.93 \times 10^{-7}$, and $4.40 \times 10^{-7}$, respectively. The energy density $w_{L,therm}$ of the thermal level of Langmuir waves is given by

$$w_{L,therm} = \left(\frac{E^2}{8\pi} \right)_{therm} = \frac{k\beta T}{\Lambda_{De}}$$
(Krarl & Trivelpiece 1973). Adopting the plasma parameters of the 25 MHz level, \( w_{\text{therm}} = 7.0 \times 10^{-13} \) W s m\(^{-3}\) is found with \( \lambda_{D_e} = 3 \) cm and \( k_B T = 120 \) eV. Hence, the Langmuir waves are associated with electric fields of 0.4 V m\(^{-1}\) on the thermal level. For instance, an enhancement of the electric field of 100 with respect to the thermal one is considered for discussion. Then, the sources of solar type II bursts (see seventh paragraph in Section 1) will be employed for doing that. Because of Eq. (8), the jump in the magnetic field is \( B_y/B_x = X = 1.9 \), leading to a loss-cone angle \( \alpha_{L} = 46.51^\circ \). The shock speed \( v_s = M_A \lambda_A \) is found to be 710 km s\(^{-1}\) by using \( M_A = 1.94 \) and an Alfvén speed \( s_A = 365 \) km s\(^{-1}\). In this special case, the Langmuir waves responsible for the radio waves are generated by electrons with a velocity of 29,000 km s\(^{-1}\) because of \( U_{\text{res}} = 6.3 \) (see Table 1) and \( v_{\text{the}} = 4600 \) km s\(^{-1}\) as the thermal electron velocity. Because of \( U_{\text{res}} = U_B - \cos \alpha_{L} = U_B (1 + \cos \alpha_{L}) - \cos \alpha_{L} \) (see Eq. (27)) \( U_B = 7.00 \) is derived and leads to a beam velocity of 32,000 km s\(^{-1}\) corresponding to a kinetic energy of 2.9 keV. Furthermore, the shock speed \( U_s = U_b/(1 + \cos \alpha_{L}) = 4.75 \) (or 22,000 km s\(^{-1}\)) in the de Hoffmann-Teller frame is obtained by using a thermal electron velocity of \( v_{\text{the}} = 4600 \) km s\(^{-1}\). We note that the shock speed \( v_s \) in the de Hoffmann-Teller frame is defined by \( v_{\text{S}} = v_{\text{H}} \cdot \sec \theta \). Then, \( \theta = 88.14^\circ \) is found for the angle between the upstream magnetic field and the shock normal, that is to say it is a nearly perpendicular shock geometry. Hence, the Alfvén-Mach number \( M_{\text{A}} = v_{\text{S}} / \sec \theta / v_{\text{A}} = M_{\text{A}, \text{sec} \theta} \) in the de Hoffmann-Teller frame is found to be 60. For the special event on 25 August 2014 (see Figure 1), a radial velocity \( v_{r, x} = 556 \) km s\(^{-1}\) (see sixth paragraph of Section 1) was derived for the shock associated with the solar type II radio burst. Since a typical shock velocity \( v_s = 710 \) km s\(^{-1}\) is expected at the 25 MHz level (see seventh paragraph of Section 1) the shock normal takes an angle \( \alpha_s = \arccos(v_{x, r} / v_s) = 38.45^\circ \) with respect to the radial direction. Here, there are two different angles. That is the angle \( \theta \) of the shock normal and the upstream magnetic field and the angle \( \alpha_s \) between the shock normal and the radial direction. We note that the magnetic field does not necessarily need to be radially directed in the source region of solar type II radio bursts.

The growth rate \( \gamma_{\text{max}} / \omega_{\text{pe}} = 1.17 \times 10^6 \) leads to \( \gamma_{\text{max}} = 183 \) s\(^{-1}\) and \( \omega_{\text{max}} = 5.4 \) ms, that is the energy density of Langmuir waves is rapidly enhanced above the thermal level. According to Eq. (21), the excited Langmuir waves have a frequency \( \omega_{\text{L}} = 1.04 \omega_{\text{pe}} \).

The number density of accelerated electrons can be calculated by integrating the reduced distribution function \( F_{\text{acc}}(U) \) with respect to \( U \), providing

\[
N_{\text{acc}} / N_i = \exp(-[U_s^2 \sin^2 \alpha_{L} + U_r^2 \cos^2 \alpha_{L}]/2) \\
x \cos \alpha_{L} / 2 [1 + 2 \Phi_0(U \cos \alpha_{L})]
\]

(32)

with

\[
\Phi_0(\xi) = \frac{1}{\sqrt{2\pi}} \int_0^\xi d\xi e^{-\xi^2}/2.
\]

(33)

Employing the above given parameters, one finds \( N_{\text{acc}} / N_i = 5.0 \times 10^{-8} (N_{\text{acc}} = 0.39 \text{ cm}^{-3}) \), that is only a very tiny part of the upstream electrons are finally accelerated up to an energy of a few keV. These electrons carry an energy density \( w_b \approx N_{\text{acc}} m_e V_{\text{e}}^2 / 2 = 1.8 \times 10^{-9} \text{ erg cm}^{-3} \).

Solar type II radio bursts have typical fluxes of 1000 sfu (see e.g. Nelson & Melrose 1985) at 1 AU (1 AU = 1.496 × 10^8 km, 1 astronomical unit) (1 s.f.u. = 10^{-22} W m^{-2} Hz^{-1} cm^{-2}) with typical frequencies of 10^10 Hz. The radio flux in the frequency interwell \( \Delta f \) at 1 AU is related to the energy density \( w_{\text{radio}} \) of the radio waves at the emission site in the corona by

\[
S_{\text{1AU}} = \frac{R_3^2}{R_{\text{IAU}}^2} \frac{w_{\text{radio}} c}{\Delta f},
\]

(34)

where \( c \) is the velocity of light. Then, \( w_{\text{radio}} \approx 1.5 \times 10^{-16} \text{ erg cm}^{-2} \) is obtained with \( \Delta f = 1 \) MHz. Thus, the energy density of the electron beam is much greater than the energy density of the emitted radio waves, that is to say the electrons accelerated by SDA at a coronal shock have sufficiently enough energy to generate the radio waves which are observed as type II radio bursts.

5. Summary

Solar type II radio bursts are regarded as the radio signature of shock waves travelling through the corona. Such shocks can be generated either by blast waves due to the pressure pulse of a flare and/or driven ahead of CMEs. If these shocks penetrate into interplanetary space, they can be observed as interplanetary type II radio bursts. Coronal shocks are able to accelerate electrons by means of SDA. SDA produces a shifted loss-cone VDF for the energetic electrons. This VDF has both a beam-like and a loss-cone component. The beam-like component is able to excite Langmuir wave, which can be converted into escaping radio waves seen as type II radio emission.

Under coronal circumstances, Langmuir waves are efficiently excited by a shock wave with moderate strength, that is the density jump \( X \) across the shock and the Alfvén-Mach number are in the ranges \( 1.55 < X < 2.37 \) and \( 1.59 < M_A < 2.53 \), respectively. Thus, solar type II radio bursts are associated with shocks of moderate Alfvén-Mach numbers. This result explains why type II radio burst observations provide only moderate Alfvén-Mach numbers of the associated shocks (see e.g. Vršnak et al. (2002), Warmuth & Mann (2005) and Magdalenic et al. (2020)). For plasma conditions at the 25 MHz level in the corona, a shock wave with \( X = 1.9 \) and \( M_A = 1.94 \) produces Langmuir waves in the most efficient way. In this case, the maximum growing rate is \( \gamma_{\text{max}} = 184 \) s\(^{-1}\), which rapidly provides an
enhanced Langmuir wave level above the thermal one within 5.4 ms. The conversion of Langmuir waves into escaping radio waves has been treated by Schmidt & Cairns (2012a,b) and Schmidt et al. (2014), for instance.

Coronal shock waves are able to produce electrons up to energies of a few keV by SDA. However, only a small fraction of the order of $10^{-8}$ are finally accelerated. These electrons carry an energy density of $1.8 \times 10^{-9}$ erg cm$^{-3}$, but that is sufficient enough to generate radio waves with an energy density of $1.5 \times 10^{-16}$ erg cm$^{-3}$ at the emission site in the corona in order to provide the observed type II radio fluxes at 1 AU.

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