The effect and properties of drifts in the heliosphere

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ABSTRACT

We investigate the properties of drifts and their effect on cosmic ray modulation in the heliosphere using a numerical modulation model based on the solution of a set of stochastic differential equations that was derived from the Parker transport equation. The illustrative capabilities of the numerical model are exploited to yield a better understanding of the physical modulation processes involved. Various studies have indicated that drifts need to be scaled down towards solar maximum conditions and the present study looks at how this can be achieved. Drifts are scaled down directly by multiplying the drift coefficient by a factor of less than unity as well as indirectly through the drift–diffusion relation, that is, by modifying the diffusion coefficient so as to cause a change in the drift effects through altered gradients in particle intensity. Contour plots of particle exit positions and exit energies are presented for both of these cases, and it is illustrated that drifts in the model lead to larger energy losses. This is explained with the aid of figures indicating the relative amount of time spent by pseudo-particles in different regions of the heliosphere during the modulation process. These figures also indicate that an increase in diffusion leads to a suppression or reduction of drift effects. Finally, the figures also show that drift effects are reduced as a function of increasing particle energy; even though the drift coefficient increases with particle energy, the total drift effect, taking into account the contribution from the increased diffusion associated with larger energies, causes drift effects to be reduced with an increase in energy.

Key words. Sun:heliosphere – Sun:magnetic fields – Sun:solar wind – Sun:activity – turbulence – scattering

1. Introduction

When Galactic cosmic rays (CRs) enter the heliosphere, their differential intensity and distribution, as specified by the relevant local interstellar spectrum (LIS) at the boundary of the modulation space, are modulated as a function of energy, position, and time. The physical processes responsible for this modulation include convection, particle drifts and diffusion, and adiabatic energy changes; Parker (1958). All of these modulation processes have to be taken into account and understood as comprehensively as possible in order to construct realistic modulation models. See for example Fisk (1999) and Potgieter (2013) for overviews of all the heliospheric transport processes.

In this work, the process of drift is our main concern. During modulation in the heliosphere, CRs will be affected by drifts through the gradient and curvature of the large-scale heliospheric magnetic field (HMF), as well as current sheet drifts at the magnetic equator due to the switch in magnetic polarity across the wavy neutral heliospheric current sheet (HCS); see for example Burger & Potgieter (1989). These drift processes can be influenced by a direct scaling down of the drift coefficient as well as through its relation to the process of diffusion. Furthermore, these drift effects vary as function of CR energy.

Drift effects explain the observed HMF polarity-dependent CR observations (e.g. Jokipii & Kopriva 1979; Potgieter & Moraal 1985; Webber et al. 2005) and also lead to a strong dependence of CR intensity on the solar tilt angle (e.g. Lockwood & Webber 2005). The drift process also has a significant influence on global CR modulation phenomena such as observed latitude gradients (e.g. Heber et al. 1996; Zhang 1997; de Simone et al. 2011) and may also be important for the study of solar energetic particles (e.g. Dalla et al. 2013). Köta (2016) showed that the effect of drifts was relevant even in the heliosheath.

An important property of drifts discussed in this work is that drift effects for CRs of low to intermediate energies have to be suppressed in order to account for observations (e.g. Potgieter et al. 1989). The suppression of drift effects by turbulence has been indicated theoretically and by means of numerical test-particle simulations (e.g. Burger 1990; Jokipii 1993; Fisk & Schwadron 1995; Candia & Roulet 2004; Minnie et al. 2007; Tautz & Shalchi 2012; Engelbrecht & Burger 2015; Engelbrecht et al. 2017). Here, we follow a simpler approach to scaling down drifts by introducing an external scaling function to simulate conditions at extreme solar maximum; the approach we follow, as well as the conditions under which it is best applied, were most recently discussed by Kopp et al. (2021). We note that the results of our study are to be interpreted in this qualitative and comparative context.

The numerical model featured in this work is formulated from a set of stochastic differential equations (SDEs) derived for the relevant Fokker-Planck type equation (e.g. Kopp et al. 2012), that is, the Parker transport equation (Parker 1965), and is based on the model introduced by Strauss et al. (2011a).
2. The modulation model

2.1. The Parker transport equation

An equation that accounts for all of the relevant heliospheric modulation processes was originally derived by Parker (1965) and re-derived by Gleeson & Axford (1967). This equation is known as the Parker transport equation (TPE) and can be expressed as

$$\frac{\partial f}{\partial t} = - (V_{sw} + \langle v_d \rangle) \cdot \nabla f + \nabla \left( \langle K^{(i)} \cdot \nabla f \rangle \right) + \frac{1}{3} \nabla \cdot \left( V_{sw} \right) \frac{\partial f}{\partial \ln p} + Q,$$

in terms of the omni-directional CR distribution function $f(r, p, t)$, where $r$ is the position, $p$ is the particle momentum, and $t$ is time.

The terms, from left to right, respectively represent time-dependent changes, outward convection via the solar wind (SW) flow $V_{sw}$, CR drifts in terms of the pitch-angle averaged guiding centre drift velocity $\langle v_d \rangle$, spatial diffusion through the symmetric diffusion tensor $K^{(i)}$, adiabatic energy changes through the solar wind divergence $\nabla \cdot V_{sw}$, and any possible sources of CRs inside the heliosphere $Q(r, p, t)$ such as Jovian electrons.

2.2. Structure of the heliosphere

The SW profile assumed in this study increases to a constant and supersonic velocity within the first 0.3 AU from the Sun, and is directed radially outwards (Sheeley et al. 1997). During solar minimum activity, the SW profile is dependent on latitude, meaning that there is a slow $V_{sw} \approx 430$ km s$^{-1}$ in the equatorial regions and a fast $V_{sw} \approx 750$ km s$^{-1}$ in the polar regions; this is in accordance with Ulysses observations (e.g. Heber & Potgieter 2006). This latitude-dependent profile is eliminated as solar activity increases with the slow solar wind region expanding over higher latitudes.

The model assumes a spherical termination shock (TS) at $r = r_{TS} = 90$ AU, where $V_{sw}$ drops to subsonic levels. An ad hoc approach is followed for the SW profile when $r_{TS} < r < r_{HP}$, that is, in the inner heliosheath at radial distances falling in between the TS and the heliopause (HP). Inside the inner heliosheath, $V_{sw}$ is assumed to be constant and the divergence $\nabla \cdot V_{sw} > 0$, implying that CRs travelling through these regions lose energy adiabatically. This model contains no acceleration over the TS, because such considerations are characterised by much more involved processes that are beyond the scope of this work; see for example Ferreira & Heber (2006) where the importance of acceleration over the TS is discussed. For more complex approaches than we present in this work, see for example Strauss et al. (2013), Manuel et al. (2015) and Ngobeni & Potgieter (2015). The position of the outer modulation boundary is assumed to be located at the HP, $r_{HP} = 120$ AU.

Regarding the choice of HMF, this study employs the Smith-Bieber modified Parker HMF as introduced by Smith & Bieber (1991). The unmodified Parker HMF (e.g. Parker 1958), is given by

$$B = B_0 \left[ \frac{r_0}{r} \right]^2 \left( e_r - \frac{\Omega(r - r_0)}{V_{sw}} e_\phi \right),$$

where $e_r$ and $e_\phi$ are unit vectors in the radial and azimuthal directions respectively. $B_0$ is a normalisation value which is related to the HMF magnitude at Earth $B_E$ through

$$B_0 = B_E \left[ 1 + \left( \frac{\Omega r_0}{V_{sw}} \right)^2 \right]^{-1/2},$$

and $\Omega = 2.66 \times 10^{-6}$ rad s$^{-1}$ is the average angular rotation speed of the Sun. Equation (2) is valid for $r > r_0$ and is written more compactly as

$$B = B_0 \left[ \frac{r_0}{r} \right]^2 \left( e_r - \tan \psi e_\phi \right),$$

where $\psi$ is the Parker spiral angle, defined as the angle between the radial direction and the direction of the average HMF at any given position; it is expressed as

$$\tan \psi = \frac{\Omega(r - r_0)}{V_{sw}} \sin \theta.$$
2.3. Cosmic-ray diffusion

Cosmic-ray diffusion is included in the TPE through the elements of the symmetric diffusion tensor \( K^{ij} \); each element of this tensor is known as a diffusion coefficient. Following Potgieter et al. (2014), a simplified approach is employed here to describe these coefficients. The parallel diffusion coefficient is expressed as

\[
k_{||} = \bar{k}_{0,||} B_0 \left( \frac{P}{P_0} \right)^{a_2} \left[ 1 + \frac{P}{P_0} \right]^{a_3},
\]

where \( \bar{k}_{0,||} \) is a constant in units of \( 6 \times 10^{20} \) cm\(^2\)s\(^{-1}\) (these units are omitted henceforth). According to Equation (9), the parallel diffusion coefficient is spatially scaled as \( 1/B \). The values of \( P_0 = 1 \) GeV and \( B_0 = 1 \) nT are added on dimensional grounds; \( \beta_c = v/c \) is the ratio of particle speed \( v \) to the speed of light \( c \). Here, \( a_1 \) and \( a_2 \) are dimensionless constants that determine the slope of the rigidity-dependence below and above the rigidity \( P_c \). The quantity \( a_3 \) is another dimensionless constant and determines the smoothness of the transition between the two slopes \( P_c \) and \( P_2 \) at \( P_c \). In this paper, we use \( P_2 = 4.2 \) GV, as in Potgieter et al. (2014). See also Vos & Potgieter (2016) (and references therein) where a clear graphical illustration of these coefficients is presented; these authors also employed these coefficients in a reproduction of observational data.

Diffusion perpendicular to the HMF is accounted for in the modulation model through the perpendicular diffusion coefficient \( k_{\perp} \), which consists of two possibly independent coefficients \( k_{\perp,r} \) and \( k_{\perp,\theta} \) describing the perpendicular diffusion in the radial and polar directions, respectively. In this paper, a simple approach by which \( k_{\perp} \) is scaled as \( k_{||} \) is followed. The ratio of \( k_{\perp,r} / k_{\perp,\theta} \) was found to be between 0.02 and 0.04 by Giacalone & Jokipii (1999). Kóta & Jokipii (1995) proposed the concept of anisotropic perpendicular diffusion with \( k_{\perp,\theta} > k_{\perp,r} \) in the equatorial regions (e.g. Potgieter 1996; Burger et al. 2000) after Ulysses observations revealed that the latitude-dependence of CR protons was significantly less than predicted by classical drift models (Potgieter & Haasbroek 1993). To account for such anisotropic perpendicular diffusion, this work follows e.g. Ngobeni & Potgieter (2011) so that

\[
k_{\perp} = k_{\perp,0} k_{\perp} \quad (\text{10})
\]

and

\[
k_{\perp,\theta} = f(\theta) k_{\perp,0} k_{||} \quad (\text{11})
\]

where \( k_{\perp,0} = k_{\perp,0}^{\perp} = 0.02 \) are dimensionless constants, and

\[
f(\theta) = A^+ + A^- \tanh \left( \frac{1}{\Delta \theta} \left( \theta - \frac{\pi}{2} + \theta_c \right) \right).
\]

Here \( A^+ = (d \pm 1)/2, \Delta \theta = 1/8, \) with \( \theta_c = (\pi - 35^\circ \pi/180^\circ) \) for \( \theta \geq \pi/2 \) and \( \theta < \pi/2 \) respectively; \( d = 3.0 \) is a dimensionless constant defining the spread of the drift coefficient by which \( k_{\perp,0} \) is enhanced from its value in the equatorial plane towards the poles, with respect to \( k_{||} \). See also Ferreira et al. (2000).

2.4. Cosmic-ray drifts

In this work, we refer to a ‘global drift pattern’, which describes the drift motion of particles over the polar and equatorial regions during different polarity cycles, that is, during an \( A > 0 \) cycle, positively (negatively) charged particles will drift inward (outward) over the polar regions and outward (inward) along the HCS; the pattern is reversed during an \( A < 0 \) cycle.

The expressions for gradient and curvature drifts are presented in detail by Raath et al. (2015), while those for current sheet drifts are treated extensively by Raath et al. (2016). In this work, it is therefore sufficient to only present the expression for the drift coefficient \( k_A \) from Burger et al. (2000) and Burger et al. (2008), that is,

\[
k_A = k_{A,0} P_1 \left( \frac{P}{P_0} \right)^2 \left( 1 + \left( \frac{P}{P_0} \right)^2 \right)^{-1/2},
\]

where \( P_0 = 0.16 \) GV, following Potgieter et al. (2014); \( k_{A,0} \in [0, 1] \) is used to scale drift from zero to one hundred percent (Potgieter et al. 1989; Webber et al. 1990). Furthermore, Ferreira & Potgieter (2004) showed that \( k_{A,0} \in [0.0, 0.1] \) was needed to produce solar maximum observations; see also Ndiitwani et al. (2005), Manuel et al. (2011) and Síllyszký et al. (2011). It is important to note that the drift coefficient, like the diffusion coefficient, is scaled as \( 1/B \) through \( r_1 = mv/|q|B = P/Bc \), where \( m \) is particle mass, \( q \) the charge, and \( P \) the particle rigidity.

In this study, a simple function is introduced with the aim of reducing drifts towards solar maximum so that drifts are scaled down to zero at \( \alpha = 75^\circ \). The function is defined in terms of the tilt angle \( \alpha \) by

\[
f_\alpha(\alpha) = \left[ \cos \left( \frac{\pi}{150^\circ} \alpha \right) \right] c_1,
\]

where \( c_1 = 100 \) is a constant measured in degrees; the tilt angle is defined in its simplest form, that is, the angle between the solar rotational and magnetic axes, and is not to be confused with the concept of an ‘effective’ tilt angle as discussed by both Raath et al. (2016) and Moloto et al. (2018). The function \( f_\alpha(\alpha) \) does not change the global features of the intensity versus tilt-angle profiles for small \( \alpha \), but closes the loop in the \( A > 0 \) and \( A < 0 \) intensity versus \( \alpha \) solutions at \( \alpha = 75^\circ \). The scaling down function is implemented by multiplying the factor \( f_\alpha(\alpha) \) with the drift scaling coefficient \( k_{A,0} \) in Equation (13) so that \( k_{A,0} \in [0, f_\alpha(\alpha)] \).

For a more thorough investigation of drift reduction at high solar activity, we refer to Kopp et al. (2021).

2.5. The proton local interstellar spectrum

The local interstellar spectrum (LIS) for protons, assumed to be located at the position of the HP (e.g. Guo & Florinski 2014; Kóta & Jokipii 2014; Zhang et al. 2015) is as given by Vos & Potgieter (2015) and Potgieter & Vos (2017). For work done on the modulation of Galactic CRs in the outer heliosheath, that is, beyond the HP, see e.g. Scherer et al. (2011), Herbst et al. (2012), Florinski et al. (2013), Strauss et al. (2013) and Luo et al. (2015).

In this work, the LIS serves as the primary input spectrum or boundary condition in the numerical modulation model. See for example Bisschoff & Potgieter (2016) and Bisschoff et al. (2019) for studies of the LIS.
The numerical model featured in this work is formulated from a set of SDEs derived for the TPE in Equation (1). The model is time-stationary and spatially three-dimensional, including energy as a fourth dimension. It is based on the model introduced by Strauss et al. (2011a). This model has been benchmarked and implemented extensively (e.g. Strauss et al. 2011a,c, 2012; Raath et al. 2015, 2016; Raath & Potgieter 2017).

The set of SDEs corresponding to the TPE when written in spherical coordinates is given by

\[d r = \left[ \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \kappa_{rr} \right) + \frac{1}{r \sin \theta} \frac{\partial \kappa_{r\theta}}{\partial \theta} - V_{\text{sw}} - \langle \nu_r \rangle \right] ds + \sqrt{2 \kappa_{rr}} \frac{dW_r}{\sqrt{\kappa_{r\theta}}} \]

\[d \theta = \left[ \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \kappa_{\theta\theta} \right) - \frac{\langle \nu_{\theta} \rangle}{r} \right] ds + \frac{\sqrt{2 \kappa_{r\theta}}}{r \sin \theta} dW_{\theta} \]

\[d \phi = \left[ \frac{1}{r^2 \sin^2 \theta} \frac{\partial \kappa_{\phi\phi}}{\partial \phi} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial r} \left( r \kappa_{rr} \right) - \frac{\langle \nu_{\phi} \rangle}{r \sin \theta} \right] ds + \frac{\sqrt{2 \kappa_{\phi\phi}}}{r \sin \theta} dW_{\phi} \]

\[d E = \left[ \frac{1}{3r^2} \frac{\partial}{\partial r} \left( r^2 V_{\text{sw}} \right) \Gamma \right] ds, \]  

\text{(15)}

for each of the coordinates \(r, \theta, \) and \(\phi,\) as well as for the particle kinetic energy \(E; dW = [dW_r, dW_{\theta}, dW_{\phi}]\) is the multidimensional Wiener process with each of its elements containing a Gaussian distributed number. The quantity \(\Gamma\) is defined by

\[\Gamma = \frac{E + 2E_0}{E + E_0}, \]  

\text{(16)}

where \(E_0\) is the particle rest energy. The infinitesimal backward time increment \(ds = 0.004\) days, a value found to be small enough to ensure a sufficiently small truncation error (e.g. Strauss et al. 2011b; Strauss & Effenberger 2017).

The diffusion tensor has here been converted to spherical coordinates. Its elements are related to the coefficients in HMF aligned coordinates through

\[\kappa_{\theta\theta} = \kappa_{\theta\theta} = \kappa_{\phi\phi} = \kappa_{\phi\phi} = 0 \]

\[\kappa_{rr} = k_3 \cos^2 \psi + k_{s3} \sin^2 \psi \]

\[\kappa_{\theta\theta} = \kappa_{\phi\phi} = (k_{s2} - k_0) \cos \psi \sin \psi \]

\[\kappa_{r\theta} = k_{s1} \sin \psi \cos \psi \]

\text{(17)}

We note that the full propagation tensor would include the drift coefficient \(k_A;\) however, the numerical scheme employed here rather uses the drift velocity so that \(k_A\) is not included in the tensor. Furthermore, the tensor transformation of Eq. (17) is only valid for Parker-like HMF profiles, that is, for an HMF that does not contain a \(\theta\)-component.

Solving the TPE in a time backward fashion implies that a number of pseudo-particles, \(N,\) are traced from Earth outward to the boundary of the heliosphere, stepping in \((r, \theta, \phi)\) and calculating energy losses according to Eq. (15). At the HP, where \(r_{\text{HP}} = 120\) AU (i.e. the assumed outer modulation boundary), particle relative intensities are convolved with the LIS and an energy spectrum at Earth is obtained; this convolution implies that the exit energy is weighted with the LIS, that is, high-energy particles are much more rare than low-energy particles. As it is assumed that no modulation occurs beyond the HP, the spectrum at the HP is synonymous to the LIS (Potgieter et al. 2014). A reflective inner modulation boundary is used at \(r_m = 0.1\) AU, so that any pseudo-particle for which \(r < r_m\) is reflected back into the modulation volume, i.e. \(r_m < r < r_{\text{HP}}.\)

### 3. Results

#### 3.1. The direct scaling down of drift effects

To illustrate the effect of the direct reduction of drift effects, the top panel of Figure 1 shows intensity versus tilt-angle profiles for 500 MeV protons at Earth and using \(k_{s0} = 0.75 f_{0}(\alpha)\) and \(k_{s0} = 0.50 f_{0}(\alpha)\) in addition to the case for \(k_{s0} = 1.00 f_{0}(\alpha);\) the zero-drift solution is also shown, and \(f_{0}(\alpha)\) is as given by Equation (14). We note that the actual value of the diffusion coefficient is unimportant, because we are only comparing relative intensities. In each case, the upper solution represents the \(A > 0\) polarity cycle, while the lower solution represents the \(A < 0\) polarity cycle. The loop formed between the \(A > 0\) and \(A < 0\) solutions covers the largest area in the case of \(k_{s0} = 1.00 f_{0}(\alpha),\) decreasing successively with \(k_{s0};\) this is indicative of the drift effects that become smaller when the drift coefficient is reduced. The bottom panel of Figure 1 shows the \(A > 0\) to \(A < 0\) intensity ratios, which is a measure of the extent of drift effects, and it clear that this ratio decreases as the drift coefficient is scaled down.

The direct scaling down of drifts can also be illustrated by constructing contour plots of the exit positions and exit energies of pseudo-particles. Because the modulation model works in a time backward fashion, these exit positions and energies should rather, from a physical point of view, be interpreted as entry positions and energies. In the remainder of this paper, whenever exit energies or positions are referred to, these are to be understood from a modelling perspective; when entry energies or positions are referred to, these are to be understood from a physical point of view. It is important to point out that the value of the exit energy compared to the energy at Earth will then give an indication of particle energy losses suffered during the modulation from the HP to the Earth.

The first contour plot constructed is shown in the top-left panel of Figure 2, and is for 500 MeV protons during a zero-drift scenario using \(1 \times 10^4\) pseudo-particles; drift has been directly scaled down to zero by setting \(k_{s0} = 0\) in Equation (13); the values for \(k_{ij}\) need not be the same as in the case illustrated in Figure 1 because we only compare the panels of individual cases. The diffusion was set to a small value \(\kappa_{ij} = 5\) so that, for illustrative purposes, drift effects were not overly suppressed (see Section 3.2 for more on this effect). The contour plot was constructed using bins of 2.5° and 333 MeV so that each bin contained the number of particles exiting at the latitudes and energies ascribed to the particular bin. This number of particles is influenced by the statistical processes of the stochastic model so that any contour plot subsequently constructed will not be smooth. Such contours are presented primarily for their qualitative significance and, as such, no colour bar is presented. Exit energies are shown on the vertical axis, while the exit latitudes \((\theta - 90°)\) are shown on the horizontal axis. The middle-left and
bottom-left panels of Figure 2 show the exit positions and exit energies for the $A > 0$ and $A < 0$ polarity cycles respectively, assuming a tilt angle of $\alpha = 15^\circ$ and $k_{\alpha,0} = 1.00 f_{\beta}(\alpha)$. All contour plots are normalised individually so that the colour scale ascends from the lowest (dark blue) to highest intensities (red).

Considering the top-left panel (zero-drift), there is a preference for protons to enter at intermediate to high latitudes between $-45^\circ$ and $-75^\circ$ (red and yellow regions). Significant numbers of particles (light blue regions) enter over all latitudes, but the preference for higher latitudes is clear; the reason for this can only be the fact that parallel diffusion $k_{\parallel}$ is so much larger than perpendicular diffusion $k_{\perp}$, which means that particles will more easily diffuse inwards via the progressively more open magnetic field lines at higher latitudes rather than across closed field lines at lower latitudes. The maximum number of particles do not enter at the highest latitudes closest to the poles, but rather about $15^\circ$ to $45^\circ$ away from the poles. This is the effect of the Smith-Bieber modification to the magnetic field. It is therefore evident that the Smith-Bieber modification influences modulation in the polar regions, even without the presence of drifts.

The effects of drifts are illustrated in the middle-left and bottom-left panels of Figure 2 where the exit positions and exit energies for the $A > 0$ and $A < 0$ polarity cycles are considered respectively. Compared to the zero-drift case, the $A > 0$ case indicates that turning on drifts in the model has caused exit positions to shift to higher latitudes. There are still some pseudo-particles that exit at lower latitudes, but the vast majority (red and yellow regions) now leave modulation space in the higher latitudinal regions between $-50^\circ$ and $-90^\circ$ from the equator. From a physical point of view, this is to be expected because $A > 0$ drift will cause positively charged particles to drift inward over the polar regions and toward the HCS at lower latitudes; therefore more particles will enter in the polar regions. Far fewer particles now enter at latitudes below $15^\circ$, because the HCS will drift protons outwards in these regions. Considering the $A < 0$ polarity cycle, significant numbers of particles enter in the region occupied by the HCS, that is, at latitudes below $15^\circ$. Because the extent of the HCS is relatively small with the selected value $\alpha = 15^\circ$, there is still a majority of particles entering at intermediate latitudes, as explained for the zero-drift scenario.

The effects of drifts are also evident in the exit energies of the pseudo-particles; keep in mind that the energy loss and resulting flux are determined by the time spent weighted by the rate of cooling. In the zero-drift case shown in the top-left panel of Figure 2, the exit energy is centred around $\sim 2.30$ GeV and the most significant fraction of particles (colour scale down to light blue) are distributed very roughly over $\sim 1.10$ GeV above and below this value. There is a slight asymmetry in the distribution above and below the central value, in the sense that particles are more spread out above than below this value; this should be due to the energy-dependence of the energy loss term in Equation (15). In all cases that follow, the indicated spread can be viewed as a ‘minimum spread’ in the sense that it is chosen so as to include at least the energies spread out below the central value. The above-mentioned values correspond to an energy loss of $\sim 1.80 \pm 1.10$ GeV where $\pm$ is used simply to indicate the spread in energy, which is not to be confused with a margin of error.

Comparing the middle-left panel to the zero-drift case, where $A > 0$ drift was now turned on, the exit energy was centred around $\sim 5.25$ GeV and spread out over $\sim 3.75$ GeV above and below this value so that the energy loss was $\sim 4.75 \pm 3.75$ GeV; it was therefore spread out wider around the central value than in the zero-drift case. Also, the spread becomes wider with increasing latitude. The bottom panel, where $A < 0$ drift was turned on, shows that the exit energy was centred around $\sim 5.00 \pm 3.00$ GeV, corresponding to an energy loss of $\sim 4.5 \pm 3.00$ GeV. It is therefore clear that the effect of including drifts is to increase the energy losses and also to spread out the distribution over energy. These results are consistent with those of Kopp et al. (2017).

The three panels on the right-hand side of Figure 2 illustrate why turning on drifts has the effect of increasing exit energies, consequently increasing the relative energy losses. These panels were compiled from the trajectories of 50 pseudo-particles (due to limited resources) in each of the cases for zero-drift (top panel), $A > 0$ drift (middle panel), and $A < 0$ drift (bottom panel); the figure shows where pseudo-particles spend most of their time before exiting modulation space at the modulation boundary. We note that using 50 pseudo-particles indeed provides us with a very large number of positions in modulation space and the results are therefore statistically significant even though the number of pseudo-particles is small. It is important to point out that particles will spend most of their time in regions where they experience the most ‘difficulty’ in propagating; these regions show up as high-activity regions (red) in Figure 2. Regions where particles spend the least amount of time are the regions where propagation is the most effortless and these regions do not show up in Figure 2 or in any similar figures that follow. It is also important to point out that figures such as Figure 2 are by no means an indication of the exit positions - there is no obvious or direct correlation. All these contour plots have been normalised in a similar fashion to the panels on the left-hand side of Fig. 2; in this case, the darkest colour on the scale (red) corresponds to the largest fraction of total modulation time.

For the zero-drift case, the majority of the modulation time is spent at radial distances of less than $\sim 20$ AU and latitudes of less than $\sim 25^\circ$, but a significant amount of activity is also spread out over larger radial distances and latitudes; light-blue filled regions are found up to almost 110 AU and 70° latitude. The reason for the greater amount of time spent at smaller radial distances is the fact that the HMF magnitude becomes very large at such small radial distances, according to Equation (6); both the processes of diffusion and drift will then be small because both coefficients scale as $1/r$ according to Equations (13) and (9 to 12).

The middle panel on the right-hand side of Fig. 2 shows the effect when $A > 0$ drift is turned on. The distribution is now much less spread out with both radial distance and latitude, showing that the majority of modulation time is spent at very small radial distances of less than $\sim 5$ AU. This explains the increased energy loss in the $A > 0$ case, because the energy loss term in Equation (15) is particularly large at small values of $r$. With such a large amount of time spent at very small radial distances, the energy losses are larger than in the zero-drift case. From a physical perspective, when $A > 0$ drift is turned on, this difference between the zero-drift and $A > 0$ cases can be explained as follows: the $A > 0$ drift cycle will drift positive particles inward over the poles so that these particles now easily travel towards Earth with the aid of both diffusion and drift, only encountering difficulties at small radial distances when the HMF magnitude is very large so that both the diffusion and drift coefficients, scaled as $1/r$, are small. It is noticeable that this high-activity region is also limited to latitudes $\theta < \alpha = 15^\circ$, that is, it occurs on the HCS; the HCS drift now contributes to the difficulty of inward propagation towards Earth by drifting particles outward, in opposition to the inward component of diffusion.
Considering the $A < 0$ case, most time is spent at small radial distances of less than $\sim 20$ AU and away from the HCS. This is understood in the sense that, close to the HCS, particles will now both diffuse and drift inward so that they reach Earth quickly and therefore these regions do not light up on the plot. However, when the particles have travelled inwards down to $\sim 20$ AU, and then happen to leave the HCS, they encounter outward drift and only have diffusion to advance inward propagation; hence, most of the modulation time is spent at radial distances of less than $\sim 20$ AU and below the HCS. Therefore, in this case, the energy losses are also much higher than in the zero-drift case.

However, the amount of modulation is not only determined by how much time particles spend in modulation space, but also by the location in modulation space where they spend this time, as is apparent from the discussion of the right-hand column of Figure 2. The column on the right side of Figure 3 is analogous to that of Figure 2, but for increased diffusion $k_{\perp,0} = 20$. If the top panels of the columns on the right of Figures 2 and 3 are compared, that is, the zero-drift cases, it is evident that the activity is much more spread out in the radial dimension for the high diffusion case so that a less disproportionate amount of time is spent at the smallest radial distances where most energy is lost. In all the cases that follow, the word ‘disproportionate’ is used to indicate the contrast between time spent at larger radial distances and time spent at smaller radial distances.

This is even more clear from comparing the middle panels on the right-hand side of Figures 2 and 3, that is, the $A > 0$ polarity cycles. When there is relatively little diffusion $k_{\perp,0} = 5$, the high-activity regions are limited to radial distances of smaller than about 5 AU and also below the HCS extent at $\theta \sim 15^\circ$. For larger diffusion, $k_{\perp,0} = 20$, the distribution is significantly more spread out in both the radial and latitudinal directions with the majority of time now being spent (considering the red to yellow/green filled regions) at radial distances as far as $\sim 60$ AU or even 80 AU and at latitudes of up to $\sim 20^\circ$ above the extent of the HCS; however, a clear majority of time is still spent on the HCS. Overall, a less disproportionate amount of time is spent at small radial distances where energy losses are the greatest; the result is that less energy is lost in the higher diffusion case. The middle panel on the right of Figure 3 shows a narrow band of intensity around $\sim 1.00 \pm 0.50$, similar to the zero-drift case. This again corresponds to an energy loss of $0.50 \pm 0.50$ GeV, which is less than in the low-diffusion case.

Very much the same conclusions are drawn from comparing the bottom panels on the right side of Figures 2 and 3, that is, the $A < 0$ polarity cycle. When there is relatively little diffusion $k_{\perp,0} = 5$, the activity is limited to smaller radial distances below $\sim 25$ AU and latitudes above the HCS. In the high diffusion case $k_{\perp,0} = 20$, the high-activity regions are significantly more spread out over both the radial and latitudinal dimensions, meaning that less time is spent in the regions where most energy is lost. The result of this more spread out distribution is that less energy is lost and the bottom panel on the right-hand side of Figure 3 shows an energy loss of $\sim 0.50 \pm 0.50$, similar to the zero-drift and $A > 0$ cases. Furthermore, for the high diffusion case, the energy losses do not increase when drifts are turned on; this can only be interpreted as further indication of the reduced effect of drifts when diffusion is increased.

When comparing the panels of Figures 2 and 3, it must be kept in mind that each of these panels was normalised so that the colour-filled regions give an indication of the amount of time spent relative to the total modulation time in that particular case. The consequence of this is that the panels of Figure 3, that is, the high-diffusion case, show a greatly increased number of red filled regions, even though the total amount of time spent by pseudo-particles in modulation space is less than in the low-diffusion case of Figure 2. All contours showing these high-activity regions must be interpreted in this way.

### 3.2. The drift–diffusion relation

Drift effects can also be influenced by altering the amount of diffusion during the modulation process. When diffusion is increased (decreased), the intensity gradients of cosmic ray particles will decrease (increase) and this will — through the TPE given in Equation (1) — decrease (increase) the extent of drift effects experienced by these particles. This indirect scaling of drift effects can also be illustrated with the help of contour plots similar to those shown in Figure 2. Figure 3 is similar to Figure 2, but with both $k_1$ and $k_2$ increased by a factor of four so that $k_{\perp,0} = 20$. The first panel on the left-hand side shows the zero-drift result, while the middle and bottom panels show the $A > 0$ and $A < 0$ results, respectively, i.e., with $k_{\perp,0} = 1.00 f_{\perp}(A)$. We highlight the difference when compared to Figure 2: the vertical axis is zoomed in so as to only go up to 3.0 GeV. This enables a more effective comparison with Figure 2, i.e. the case for $k_{\perp,0} = 5$.

The upper left panel of Figure 3 indicates that the exit positions are more spread out over latitude in the $k_{\perp,0} = 20$ case and also that a significant number of particles now enters the modulation space all the way down to the equatorial regions. However, there is still a preference for the intermediate to higher latitudinal regions due to $k_1 > k_2$, as explained above. The highest latitude exit positions are still not found directly at the poles because of the Smith-Bieber modification.

The middle-left panel shows the case for the $A > 0$ polarity cycle, and there is a significant preference for the higher latitudinal regions when compared to the zero-drift case. This is due to the $A > 0$ drift bringing in particles over the polar regions, as explained in the case of Figure 2. However, the preference is less pronounced than in the case for $k_{\perp,0} = 5$, as when considering that significant numbers of pseudo-particles now also leave the modulation space at the lower latitudes and especially below the extent of the HCS $\theta = 15^\circ$. This clearly indicates the weaker relative effect of drifts in the high diffusion case. Looking at the $A < 0$ case, where protons now drift inwards along the HCS, there is not much difference in the distribution with latitude between the $k_{\perp,0} = 20$ and $k_{\perp,0} = 5$ cases; this is most probably due to the small extent of the HCS.

The top panel on the left hand side of Figure 3 shows a narrow band of intensity around $\sim 1.00 \pm 0.50$ GeV in the zero-drift case, including the light blue regions. This corresponds to an energy loss of $0.50 \pm 0.50$ GeV, which is now notably less than in the low-diffusion case considered above. This result is as expected, because greater diffusion will more effectively transport protons inwards to the Earth so that they spend less time in modulation space, meaning that they are modulated to a lesser extent.

We note that, when these results are plotted on the same scale as Figure 2 (not shown), it also becomes clear that the distribution of these exit energies is less spread out than in the lower diffusion case.

### 3.3. Drift effects as function of energy

The energy-dependence can also be illustrated using contour plots. The columns on the left hand sides of Figures 4 and 5 show exit positions and exit energies of pseudo-particles where 100 MeV and 5 GeV protons are used, respectively; diffusion
was set to $k_{\text{drift}} = 5$ for the same reasons as pointed out earlier. The top panel on the left in each of these figures shows the zero-drift case. Comparing the cases for 100 MeV and 5 GeV protons, there is not much difference in the distribution with latitude: both show a preference for intermediate to high latitudes that can be ascribed to $k_{\text{drift}} \gg k_{\rho}$, and the effect of the Smith-Bieber modification over the polar regions.

Though the distribution with latitude does not appear to differ significantly, the distribution of the exit energies is more spread out in the case for 100 MeV protons. The exit energies in the 100 MeV case are centred around $\sim 2.25 \pm 1.50$ GeV, implying an energy loss of $\sim 50\%$. This is a fractional energy loss of between $\sim 0.87$ and $\sim 0.97$, i.e., protons that arrive at Earth with an energy of 100 MeV have lost between $\sim 87\%$ and $\sim 97\%$ of the energy they had when they entered the heliosphere at the modulation boundary. The exit energies in the 5 GeV case are centred around $\sim 0.60 \pm 0.50$ GeV, implying a lower energy loss of $\sim 0.10 \pm 0.50$ GeV; this is a fractional energy loss of only between $\sim 0.09$ and $\sim 0.23$, meaning that protons that arrive at Earth with an energy of 5 GeV have lost only about $\sim 9\%$ to $\sim 23\%$ of the energy they had when they originally entered modulation space. We must bear in mind that these numbers indicate the minimum spread, as explained above. In this instance, it is important to point out that the spread above the central value may actually reach up to $\sim 7.50$ GeV. This would imply an energy loss of up to $\sim 33\%$, which is still significantly less than in the 100 MeV case. This result is as expected, because particles of higher energy must experience less modulation because of the increased effect of inward diffusion.

Consider now the effect of turning on $A > 0$ drifts in each of the 100 MeV and 5 GeV cases, i.e., the middle-left panels of Figures 4 and 5, respectively. The exit positions in both the cases for 100 MeV and 5 GeV now shift towards higher latitudes. However, the exit positions in the case of 5 GeV protons are slightly more spread out over latitude than in the case for 100 MeV; this can be seen for example by comparing the red to yellow regions and indicates that the shift of exit positions towards higher latitudes was slightly weaker in the 5 GeV case, which shows that the effect of the globally imposed drift pattern is weaker. Much the same conclusions can be reached from comparing the $A < 0$ drift cases (see the bottom-left panels of Figures 4 and 5), but the effect is less clear due to the small latitudinal extent of the HCS.

The difference in the drift effect between the 100 MeV and 5 GeV cases is best illustrated by looking at the respective exit energies. This is illustrated above in the case for 500 MeV protons, where, when drift effects are turned on, the energy losses increase. In the case for 100 MeV, the energy losses are increased from $\sim 2.15 \pm 1.50$ GeV in the zero-drift case to $\sim 4.90 \pm 3.50$ GeV for the $A > 0$ polarity cycle; this is an increase in energy losses of $\sim 128\%$, using only the values around which these energy losses are centred. In the case of 5 GeV, the energy losses are increased from $\sim 1.00 \pm 0.50$ GeV in the zero-drift case to $\sim 1.50 \pm 1.50$ GeV for the $A > 0$ polarity cycle; this is an increase in energy losses of $\sim 50\%$. The relative increase in the energy loss from the zero-drift case to the $A > 0$ case is therefore significantly smaller in the 5 GeV than in the 100 MeV case, clearly illustrating the weaker drift effects for the higher energy scenario. The same argument can be constructed for the $A < 0$ polarity cycle.

Some insights can also be obtained from the contour plots constructed from particle trajectories; these are shown on the right hand sides of Figures 4 and 5, respectively, for 100 MeV and 5 GeV. First, comparing the top panels, that is, the zero-drift scenarios, as the energy is increased, the contrast between time spent at larger radial distances and the time spent at smaller radial distances becomes less pronounced: a less disproportionate amount of time is spent at smaller radial distances. In the case for 100 MeV, the majority of activity (red to yellow/green filled regions) is restricted to below radial distances of $\sim 15$ AU, with little to no significant activity (light-blue filled regions) occurring above about $\sim 100$ AU. In the case for 500 MeV protons, the majority of activity is found up to radial distances of $\sim 120$ AU. In the 5 GeV case, the highest activity regions are found up to radial distances of $\sim 110$ AU, while significant activity now occurs over all values of $r$. The consequence, as indicated earlier, is successively smaller energy losses (relative to the energy at the modulation boundary) because of the $1/r$ dependence of the energy loss term in Equation (15).

The same conclusions are drawn from comparing the middle right-hand-side panels of Figures 4 and 5, that is, the $A > 0$ solutions. For the 100 MeV case, the majority of activity (red to yellow/green regions) is restricted to radial distances below $\sim 10$ AU, while, in the case for 5 GeV, this majority of activity was found to reach up to $\sim 35$ AU or even $\sim 45$ AU. In the case for 100 MeV, significant activity (light blue regions) did not reach higher than about $\sim 15$ AU, while in the case for 5 GeV it reached all the way up to about $\sim 60$ AU. This would again account for the larger energy losses of 100 MeV protons, because those particles spend more time at the smallest radial distances. Once again, the same conclusions can be drawn when considering the $A < 0$ solutions: the activity regions are much more spread out over higher radial distances in the case for 5 GeV protons, leading to smaller energy losses.

The reason why the activity regions spread out over $r$ with increasing energy can be understood as follows. When the energy is low, at 100 MeV, particles have more difficulty propagating inward via diffusion. This causes particles to spend a disproportionately large amount of time at the smallest values of $r$ where the HMF is strongest; it can be stated that particles ‘get caught’ or ‘held up’ at these smaller radial distances due to the strong HMF. When the energy is increased to 500 MeV in Figure 3, diffusion is larger and particles can now more easily diffuse inward, including at the smallest values of $r$, and the amount of modulation time spent in these regions is now less disproportionate; hence, the distribution spreads out with $r$. When the energy is increased to 5 GeV in Figure 5 — following the same reasoning — the distribution over $r$ spreads out even more, meaning that the relative energy loss decreases even further.

4. Summary and Conclusions

Making use of the illustrative capabilities of the SDE-based modulation model, this work provides useful insights into the workings of the drift modulation process. Drifts were altered directly by changing the value of the drift coefficient, and indirectly through the drift–diffusion relation, by modifying the value of diffusion.

We illustrate the effect of the direct scaling down of drifts by comparing contour plots showing exit positions and exit energies of pseudo-particles. We find that turning on drifts in the model leads to greater energy losses. This is explained with the help of contour plots showing the regions in the heliosphere where pseudo-particles spend the most time during modulation; these regions are referred to as high-activity regions. We explain that turning on drifts leads to different distributions of these high-
activity regions over both radial distance and latitude and that this is a crucial factor when determining energy losses.

We also explain how the extent of drift effects can be influenced indirectly through changing the amount of diffusion that occurs during the modulation process. This is again illustrated with the help of contour plots, this time using an increased value for diffusion to be compared with the low-diffusion case. Contour plots showing the distribution of the high-activity regions over modulation space once again provide an explanation of the respective energy losses.

Finally, drift effects are considered as a function of energy. The energy-dependence is also illustrated with the help of contour plots such as those described above, comparing the cases for 100 MeV and 5 GeV, respectively. Comparisons show that drift effects are reduced as the energy is increased. Reasoning in terms of the high-activity regions is applied again to explain this fact.

The results of this work provide an example of how the illustrative faculties of SDE-based models can be used effectively to obtain valuable insights into modulation processes in the heliosphere, such as drifts.

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Fig. 1. Intensity versus tilt-angle profiles for 500 MeV protons at Earth. The top panel shows these profiles where the value of $\kappa_{A,0}$ is changed from $\kappa_{A,0} = 1.00f_d(\alpha)$ (red lines) to $\kappa_{A,0} = 0.75f_d(\alpha)$ (blue lines) and $\kappa_{A,0} = 0.50f_d(\alpha)$ (green lines) respectively; for each of these cases, the upper line presents the $A > 0$ solution, while the lower line represents the $A < 0$ solution. The bottom panel shows the corresponding $A > 0$ to $A < 0$ ratios.
Fig. 2. Exit latitudes and energies for $1 \times 10^4$ pseudo-particles in the case of 500 MeV protons shown in the left hand column. The top, middle, and bottom panels show these positions and energies in the case of zero-drift, $A > 0$, and $A < 0$, respectively. Diffusion is set to $\kappa_{ij} = 5$. The top, middle, and bottom panels in the right hand column show the high-activity regions, as defined in the text, for the zero-drift situation, $A > 0$, and $A < 0$ polarity cycles, respectively. The colour scale ascends from blue to red.
Fig. 3. Similar to Figure 2, but with $\kappa_{i,0} = 20$. 
Fig. 4. Exit latitudes and energies (left column) as well as high-activity regions (right column) of $1 \times 10^4$ pseudo-particles in the case of 100 MeV protons, using $\kappa_{\|0} = 5$. 
Fig. 5. Same as Figure 4, but for 5 GeV.