

Comprehensive analysis of constructing hybrid stars with a renormalization group-consistent Nambu-Jona-Lasino model

Jan-Erik Christian^{1,*}, Ishfaq Ahmad Rather^{2,*}, Hosein Gholami^{3,*}, and Marco Hofmann³

¹ Hamburger Sternwarte, University of Hamburg, Gojenbergsweg 112, 21029 Hamburg, Germany

² Institut für Theoretische Physik, Goethe Universität, Max-von-Laue-Str. 1, D-60438 Frankfurt am Main, Germany

³ Technische Universität Darmstadt, Fachbereich Physik, Institut für Kernphysik, Theoriezentrum, Schlossgartenstr. 2, D-64289 Darmstadt, Germany

Received 2 April 2025 / Accepted 22 July 2025

ABSTRACT

We investigate the properties of hadronic and quark matter that would allow for a first-order phase transition within neutron stars. To this end, we use a parameterizable relativistic mean-field description for the hadronic phase and a renormalization group-consistent Nambu-Jona-Lasino model for the quark phase. This also enables us to consider sequential phase transitions involving a two-flavor color-superconducting and a color-flavor-locked phase. We find large ranges for all parameters that permit a phase transition, even when constrained by current astrophysical data. We further attempt to filter out parameter sets with a high chance of detectability by mass-radius measurement, i.e., stars with an identical mass but different radii, so-called twin stars. However, we find that such configurations lie outside the constrained parameter spaces. Instead, most of the mass-radius relations that feature a phase transition appear to be indistinguishable from a purely hadronic description.

Key words. dense matter – stars: neutron – pulsars: general

1. Introduction

Neutron stars are among the densest objects in the Universe. As such, they can provide unique insights into the behavior and composition of bulk matter under extreme conditions. The equation of state (EoS), which describes such matter by connecting energy density and pressure, is strongly linked to the mass, radius, and tidal deformability of neutron stars and can therefore be probed by measuring those quantities. As such, astrophysical observations play a significant role in constraining the EoS of neutron star matter (Annala et al. (2018, 2022), Landry et al. (2020), Dietrich et al. (2020), Chatziioannou (2020)). The first observation that ruled out a significant number of possible EoSs was the detection of several neutron stars with masses of around $2 M_{\odot}$ (Demorest et al. (2010), Antoniadis et al. (2013), Fonseca et al. (2016), Cromartie et al. (2019), Romani et al. (2022)). This high mass necessitates that the pressure rises steeply as a function of energy density. In other words, the EoS is required to be stiff. While neutron star masses can be accurately measured, determining their radii remains more challenging. However, recent observations from the NICER collaboration have provided valuable constraints on neutron star radii (Miller et al. (2019, 2021), Riley et al. (2019, 2021), Raaijmakers et al. (2019, 2021), Salmi et al. (2024), Dittmann et al. (2024), Choudhury et al. (2024)), and can be used to further constrain the EoS. However, the most influential data point comes from the detection of GW170817, the first gravitational wave signal from a binary neutron star merger with an electromagnetic counterpart (Abbott et al. 2017, 2018, 2019, 2020). This event puts stringent

limits on the tidal deformability of a $1.4 M_{\odot}$ neutron star, thereby constraining the EoS. Notably, the tidal deformability points to a softer EoS, in contrast to the $2 M_{\odot}$ constraint.

While a significant number of EoSs in the literature contain only hadronic matter, it has been shown that a phase transition to quark matter can help ease this apparent tension between the maximum mass and tidal deformability constraint (Paschalidis et al. (2018), Christian et al. (2019), Montana et al. (2019), Sieniawska et al. (2019), Christian & Schaffner-Bielich (2020)). Such an EoS could lead to a neutron star with a quark matter core and a hadronic envelope, a so-called hybrid star (Ivanenko & Kurdgelaidze (1965), Itoh (1970), Alford et al. (2005), Coelho et al. (2010), Chen et al. (2011), Masuda et al. (2013), Yasutake et al. (2014), Zacchi et al. (2016), Xie & Li (2021)). Hybrid stars cannot inherently be distinguished from purely hadronic neutron stars (Alford et al. (2005)); however, if the phase transition is suitably strong, certain indicators would be detectable in mass, radius, and tidal deformability. One of the more extreme versions of such an indicator would be a so-called twin star configuration (Kämpfer (1981), Glendenning & Kettner (2000), Schertler et al. (2000), Schaffner-Bielich et al. (2002), Zdunik & Haensel (2013), Blaschke & Alvarez-Castillo (2016), Zacchi et al. (2017), Blaschke et al. (2020), Tan et al. (2020)). Twin stars are defined as two stars with the same mass but different radii. They require the existence of two stable branches in the mass-radius diagram, separated by an unstable region. This configuration cannot be achieved without a phase transition. Apart from the strength of the phase transition, the choice of quark matter model plays a significant role in the construction of hybrid and twin stars. Specifically, a stiff quark matter model is more likely to exhibit a disconnected second branch than a soft one (Alford et al. 2013). Therefore, modeling the quark phase with

* Corresponding authors: jan-erik.christian@uni-hamburg.de, rather@astro.uni-frankfurt.de, mohammadhossein.gholami@tu-darmstadt.de

a constant speed of sound (CSS) approach with high values for the sound speed is a popular method (Alford et al. (2013, 2015), Alford & Sedrakian (2017), Christian et al. (2018, 2024), Christian & Schaffner-Bielich (2020), Li et al. (2024)). Here we use a quark matter model with a stronger basis in microphysics.

We employ a relativistic mean-field (RMF) EoS (Johnson & Teller (1955), Duerr (1956), Walecka (1974), Boguta & Bodmer (1977), Serot & Walecka (1986), Mueller & Serot (1996), Typel et al. (2010), Todd-Rutel & Piekarewicz (2005), Chen & Piekarewicz (2014), Fattoyev et al. (2018), Hornick et al. (2018)) to describe hadronic matter. This is motivated by its high degree of parameterizability combined with its microphysically well-motivated approach. We combine it with a color-superconducting (CSC) quark matter description. This type of quark matter is characterized by Cooper pairing between the quarks, leading to a nonzero energy gap in the quasiparticle excitation spectrum (Barrois (1977), Bailin & Love (1979, 1984), Alford et al. (1998, 1999a), Rapp et al. (1998), Son (1999), Schäfer & Wilczek (1999)). In the two-flavor color-superconducting (2SC) phase, up-and-down quark flavors of two colors (e.g., red and green) participate in the pairing, while one color (blue) and the strange quarks (if abundant) remain unpaired (Alford et al. 1999b). In the color-flavor locked (CFL) phase, quarks of all colors and flavors participate in the pairing (Alford et al. 1999a), Schäfer (2000), Shovkovy et al. (2005)). The Nambu-Jona-Lasinio (NJL) model is a microscopic model for quark matter that allows for a self-consistent calculation of chiral symmetry breaking and sequential CSC phases, including the 2SC and CFL phases. It has been widely used and extended to describe quark matter in compact star physics (e.g., Baldo et al. 2003; Buballa et al. 2004; Klähn et al. 2007; Pagliara & Schaffner-Bielich 2008; Bonanno & Sedrakian 2012; Klähn et al. 2013; Baym et al. 2018; Ranea-Sandoval et al. 2017; Alaverdyan 2021, 2022; Gao et al. 2024; Yuan et al. 2025). For the present work, it is important that quark matter at the core densities of the most massive star configurations (n_B up to ~ 10 times nuclear saturation density) can be reliably calculated. To ensure this, we use the three-flavor renormalization group-consistent NJL (RG-NJL) model (Gholami et al. 2025a,b) to ensure that our calculations at high densities are not spoiled by cutoff artifacts. We assume the transition between hadronic and quark matter to be of the first order and model it with a Maxwell construction.

The role of CSC in hybrid stars has become an active field of research in recent years, with various quark matter descriptions applied, such as nonlocal NJL models (Shahrbaf et al. (2023), Blaschke et al. (2023), Ivanytskyi (2025)), confining relativistic density functional approaches (Ivanytskyi & Blaschke (2022a,b), Gärtlein et al. (2023, 2025)), and parameterized models incorporating strong perturbative quantum chromodynamic corrections (Zhang & Mann 2021).

This work aims to provide a comprehensive analysis of the possible parameter space for vector and diquark couplings within an RG-consistent framework. We constrain the parameter space of the RMF-NJL hybrid model with measurements from NICER, the tidal deformability measurement of GW170817, and by requiring that the EoS allows for neutron stars with a maximum mass of at least $2 M_\odot$. We find that a phase transition to quark matter prefers a stiff hadronic EoS. This means that the tidal deformability constraint imposes significant limitations on possible parameter variations for both phases. Nevertheless, a large number of hybrid EoSs are within the astrophysical constraints. We also consider possible twin star configurations, showing that the $2 M_\odot$ constraint in combination with the tidal

deformability constraint from GW170817 reduces the parameter space for twin stars to such an extent that they can effectively be ruled out, within our approach.

Our work is organized as follows. In Section 2 we provide a detailed description of the EoS for the hadron and quark phase, as well as the hadron-quark transition. In Section 3 we present the results from an extensive analysis of $\sim 700\,000$ parameter combinations, incorporating the mass constraint, as well as NICER and GW170817 observations. Finally, in Section 4 we discuss our findings.

2. Equation of state

2.1. Hadronic phase

For the crust EoS, we employed the well-known Baym-Pethick-Sutherland EoS (Baym et al. (1971), Negele & Vautherin (1973)). The hadronic part of the core is described using a RMF parametrization (Johnson & Teller (1955), Duerr (1956), Walecka (1974), Boguta & Bodmer (1977), Serot & Walecka (1986), Mueller & Serot (1996), Typel et al. (2010), Todd-Rutel & Piekarewicz (2005), Chen & Piekarewicz (2014)) within the limits explored in Hornick et al. (2018). We refer to this as the Frankfurt-Barcelona (FB) EoS. In this setup, the symmetry energy, J , the slope parameter, L , and the effective mass, m^*/m , were fixed at saturation density n_0 in a way that is compatible with chiral effective field theory calculations. Furthermore, we varied these parameters within the ranges allowed by experiments (Lattimer 2023), allowing us to provide a full picture of the possible parameter space for phase transitions to our quark model. Small effective masses correspond to stiff EoSs (Boguta & Stöcker 1983), where stiff refers to the pressure as a function of energy density rising quickly when compared with a soft EoS. Stiff EoSs exhibit higher masses, radii, and tidal deformabilities than soft EoSs. As a result, our hadronic model requires a value of $m^*/m > 0.65$ to be consistent with the tidal deformability of GW170817 (Abbott et al. 2019). A value of $m^*/m = 0.65$ is only within the limits if the other parameters, fixed at saturation density, are adjusted such that they contribute to a soft EoS (Hornick et al. (2018), Ghosh et al. (2022)). However, following the detection of GW170817, it has been shown that its constraining power on the hadronic EoS can be mitigated with a suitable phase transition to quark matter (Paschalidis et al. (2018), Christian et al. (2019), Montana et al. (2019), Sieniawska et al. (2019), Christian & Schaffner-Bielich (2020)). This is the case because hybrid stars are more compact than purely hadronic neutron stars, allowing previously ruled-out hadronic phases to regain compatibility. With this in mind, we varied the effective mass within the entire range, $m^*/m = 0.55\text{--}0.75$, laid out by Hornick et al. (2018) instead of limiting ourselves to the range constrained by GW170817. The slope parameter was varied between $L = 40\text{--}60$ MeV, the symmetry energy between $J = 30\text{--}32$ MeV, and finally we varied the value of the nuclear saturation density, $n_0 = 0.15\text{--}0.16$ fm $^{-3}$. The incompressibility coefficient at saturation density was set to $K(n_0) = 240$ MeV, in accordance with Hornick et al. (2018). The influence of J , L , and n_0 on mass, radius, and tidal deformability is minimal but not negligible. In Section 3, we show that the ability to combine a given quark EoS with the hadronic envelope is affected by these parameters as well.

It could be argued that the parameter range of $L = 40\text{--}60$ MeV is too small, given that there is some tension between different measurements (Zhao et al. 2024). Generally, the slope parameter is expected to be between $L = 50 \pm 10$ MeV

(Adhikari et al. (2022), Zhang & Chen (2023), Lattimer (2023)) and $L = 121 \pm 47$ MeV (Adhikari et al. (2021), Zhang & Chen (2023), Lattimer (2023)). Our main analysis only considers the former, since this value is in better agreement with chiral effective field theory (Essick et al. (2021), Lim & Schwenk (2024)). However, in order to solidify our findings, we compared our results to parameter sets up to $L = 160$ MeV, which did not change our conclusions, the reasons for which are outlined in Section 3.

2.2. Quark phase

For the quark phase, we employed the three-flavor RG-NJL model with a diquark interaction, allowing for the formation of homogeneous CSC condensates. The NJL model is non-renormalizable, implying that the model's regularization dependence cannot be absorbed into renormalized quantities. In the literature, the NJL model is typically regularized with a sharp three-momentum cutoff, Λ' , which is fit to vacuum properties. The RG-consistent description of the NJL model (Gholami et al. 2025b) removes artifacts of the conventional regularization and allows for a consistent investigation of the phase structure at high chemical potentials (Braun et al. 2019).

As is discussed in Gholami et al. (2025b), using a conventional cutoff regularization can introduce unphysical artifacts. Even evaluating the model at chemical potentials as small as 50% of the cutoff, Λ' , can lead to this effect. To prevent this issue, a renormalization-group consistent regularization scheme was proposed in Braun et al. (2019) and adapted for a neutral three-flavor NJL model with color superconductivity in Gholami et al. (2025b). The concept of RG consistency can be straightforwardly formulated in the context of the functional renormalization group, in which the full quantum effective action is calculated by solving a flow equation for a scale-dependent average effective action, which is initialized at some initial ultraviolet scale, Λ (Wetterich 1993). RG consistency then requires the initial scale, Λ , for the flow equation to be set sufficiently high for the full quantum effective action to become independent of Λ in the infrared limit (Braun et al. 2019). In the context of the mean-field approximation, this large initial scale effectively acts as a high-momentum cutoff. For further details on the importance of RG consistency in mean-field models, see Braun et al. (2019) and Gholami et al. (2025b). In short, the RG-consistent regularization employs a large cutoff scale, Λ , while the divergent vacuum contributions are regularized with the traditional cutoff, Λ' . RG consistency is obtained for $\Lambda \gg \Lambda'$. Additionally, when diquark condensates are present, new medium-dependent divergences emerge, which must be subtracted by appropriate counterterms to ensure that the physical results remain independent of the large cutoff scale. These counterterms can be motivated by different renormalization schemes. In this work, we employ the so-called massless scheme, which was also used for calculating a phase diagram and the EoS in Gholami et al. (2025a,b).

The different contributions to the Lagrangian density along with the effective potential in the RG-consistent regularization for the model are outlined in detail in Gholami et al. (2025a). The grand potential per volume, $\Omega(\mu_B, T)$, is a function of the baryon chemical potential, μ_B , and temperature, T . The pressure, p , is obtained as

$$p = -(\Omega - \Omega_0) - B, \quad (1)$$

with $\Omega_0 = \Omega(\mu_B = 0, T = 0)$. The parameter B acts as an additional bag constant, which fixes the pressure in vacuum to

$p(\mu_B = 0, T = 0) = -B$. For thermodynamic consistency, the energy density, ϵ , is increased for nonzero B as $\epsilon \rightarrow \epsilon + B$.

Overall, the quark matter model includes three free parameters, which were varied to determine the EoS. We then assessed its compatibility with the astrophysical data, ultimately constraining them. These are the diquark coupling, η_D , the vector coupling, η_V , and the additional bag constant, B . The stiffness and the composition of the quark phase at a fixed density can be varied by adjusting the couplings, η_D and η_V . In an earlier study, we investigated the stability of hybrid stars with a 2SC-CFL phase transition (Gholami et al. 2025a). In this study, we employed parameter ranges of $\eta_V = 0.00$ – 1.50 and $\eta_D = 0.90$ – 1.80 . Nearly all of the maximum-mass configurations of the parameter sets surpassing $2 M_\odot$ contained a CFL quark phase at their center. Note, however, that these calculations were done exclusively with a vanishing bag constant, $B = 0$.

The NJL model lacks confinement and is only used for the high-density quark matter description of our hybrid model. Consequently, the vacuum pressure calculated within the NJL model does not necessarily have to match the vacuum pressure derived from the RMF model. In this work, we use the freedom to choose a nonzero B to effectively change the density of the hadron-quark phase transition. This is a standard procedure in the literature (Rather et al. (2021, 2020), Burgio et al. (2002), Steiner et al. (2000)). Increasing B lowers the pressure of quark matter at every point and moves the hadron-quark phase transition to higher densities.

Apart from η_V , η_D , and B , the remaining coupling constants of the model and the bare quark masses are fit to the pseudoscalar meson octet in vacuum (Rehberg et al. 1996). They take the same values as in our previous work (Gholami et al. 2025b). Since the present model is RG-consistent, it can be evaluated at arbitrary high scales with the condition that the large cutoff scale, Λ , is chosen sufficiently large. For this work, $\Lambda = 10\Lambda' = 6023$ MeV is found to be sufficiently high to study the densities reached in neutron stars.

2.3. Phase transition

The hadron-quark phase transition in this work was modeled using a Maxwell construction. In this setup, the baryon chemical potential and pressure are kept constant at the point of transition, p_{trans} , but the (energy) density is expected to jump. A prerequisite for this construction is a “strong” phase transition, which is more likely to lead to noticeable changes in the mass-radius relation than the more involved Gibbs construction, where the charge chemical potential is conserved across the mixed phase as well (Hempel et al. 2009). As the Maxwell construction requires a point where $\mu_B(p)_{\text{HM}} = \mu_B(p)_{\text{QM}}$, a match between a given hadronic EoS and a quark EoS is not always possible. Additionally, we imposed that a phase transition can only occur at densities larger than n_0 and smaller than $10 n_0$. The upper-density limit is not physically motivated but was chosen to keep our phase transitions close to neutron star densities. For our description of hadronic and quark matter, the parameter choices affect a possible match to different degrees. The nuclear parameters, m^*/m and L , have the largest impact on the allowed values of η_V and η_D . We discuss this in detail in Section 3.

The presence of a phase transition can lead to a significant impact on the mass-radius relation and therefore on astrophysical observables. This usually takes the form of a kink in the mass-radius relation or even a scenario in which the sequence becomes unstable when $p_{\text{central}} = p_{\text{trans}}$ and regains stability at higher central pressures, forming a stable second branch

containing hybrid stars. If such a second branch forms, two neutron stars with the same mass but different radii and tidal deformability can occur, i.e., twin stars. The significance of these twin stars lies solely in them being indicators of a phase transition that can be checked with mass and radius (and tidal deformability, Λ) measurements. In the following, we explore what conditions allow for a phase transition and which of these phase transitions lead to twin star configurations.

3. Results

3.1. Matching hadronic and quark phase

In this work, we investigated over 700 000 parameter combinations, of which about 125 000 feature a possible transition point and 22 000 comply with astrophysical constraints. The ranges and steps in which our parameters were varied are listed in Table 1. We found that the last stable star for every parameter set either has quark matter in the CFL phase at its center or a purely hadronic core; there are no cases in which the densest star features a 2SC center. However, for some EoSs there are less dense stars that can feature a pure 2SC quark core.

The effects of the nuclear parameters n_0 , J , L , and m^*/m on the $\eta_V - \eta_D$ parameter space are shown in Figs. 1 and 4. In each of these figures, we denote areas in which a particular nuclear parameter is kept constant with a designated color. For each point in such an area, at least one matching parameter set exists for fixed η_V and η_D as well as the fixed nuclear parameter. For example, at least one hadronic parameter set with an effective mass of $m^*/m = 0.65$ can be combined with a quark phase that is characterized by $\eta_V = 0.80$ and $\eta_D = 1.30$. However, we cannot assume that all hadronic sets with $m^*/m = 0.65$ can be combined with such a quark phase.

Before we begin with the parameter variations in Fig. 1, we note that we found the choice of symmetry energy, J , to be inconsequential for the possibility of matching a hadronic EoS with a quark matter parameter set. The value of saturation density, n_0 , is only slightly more relevant. By increasing $n_0 = 0.15 \text{ fm}^{-3}$ to $n_0 = 0.16 \text{ fm}^{-3}$, we noticed a decrease in the allowed η_V range of 0.1 at most, while the range of η_D is unaffected.

The parameters m^*/m and L have a much larger impact on the possibility of matching hadronic and quark matter than n_0 and J . This becomes clear in Fig. 1, in which we show the $\eta_V - \eta_D$ space that has at least one parameter set with a given effective mass, m^*/m (left), or a given slope parameter, L (right), available to them. In addition, Fig. 1 has three rows, where B is increased in each row by 100 MeV, starting at $B = 0 \text{ MeV}$ in the first row. This allows us to visualize not only the effect of m^*/m and L , but B as well.

We find that the effective mass and slope parameters affect the $\eta_V - \eta_D$ space in opposite ways. An increase in m^*/m reduces the parameter space, while an increase in L enlarges it. What is common to these variations is a stiffening of the hadronic EoS. This is also the case for decreasing n_0 . It is therefore clear that stiff EoSs are more easily combined with a quark phase than soft ones, as one would expect. We further note that the η_V range is strongly affected by softening the hadronic EoS, whereby much smaller values are required to make a match possible. The hadronic EoS influence on the allowed range for η_D is minimal. Instead, η_D seems to be limited by the bag constant, B , which in turn does not influence the allowed range for η_V . This is because increasing B increases the transition density from the hadronic to the quark phase, which introduces parameter sets

Table 1. Parameter ranges and step sizes.

Parameter	Lower bound	Upper bound	Step size
m^*/m	0.55	0.75	0.05
L [MeV]	40	60	5
J [MeV]	30	32	1
n_0 [fm^{-3}]	0.15	0.16	0.01
η_V	0.0	1.5	Var
η_D	0.9	2.0	Var
B [MeV/ fm^3]	0	200	10–50

Notes. The effective mass, m^*/m , vector coupling, η_V , and diquark coupling, η_D , are dimensionless. The symmetry energy, J , and slope parameter, L , are in mega-electronvolts. The saturation density, n_0 , and bag constant, B , are in units of particle per cubic femtometer and mega-electronvolts per cubic femtometer, respectively. Step sizes for η_V and η_D vary for efficiency. B uses steps of 10 MeV/ fm^3 up to 50 MeV/ fm^3 , then steps of 50 MeV/ fm^3 .

with the density at the transition, n_{trans} , previously outside the investigated range.

In Fig. 2 we show the mass-radius, and the dimensionless tidal deformability–mass relations for a small number of representative EoSs. Since it is not possible to perfectly represent the entire parameter space we investigate, we chose to include examples to highlight the different possible types of phase transition, as well as EoSs that feature potentially detectable peculiarities, such as twin stars. This also includes parameter sets (PSs) that are not compatible with astrophysical observations, such as PS2 and PS3. The hadronic and quark phase parameters used to generate these mass-radius relations are listed in Table 2. The solid lines in Fig. 2 represent the purely hadronic stars. Stars on the dashed line feature a 2SC core and stars on the dash-dotted line have quark matter in the CFL phase at their center. The transition point from the hadronic phase to the quark phase is marked by a solid dot. A square symbol marks the transition from the 2SC phase to the CFL one. The cross symbol denotes a direct transition from hadronic matter to the CFL phase and the star symbol indicates the last stable star of the mass-radius relation. It is followed by the dotted line, which represents the unstable region. On the left side of the figure, the mass-radius relation is shown in conjunction with the NICER constraints for PSR J0030+0451 (Miller et al. (2019), Riley et al. (2019), Raaijmakers et al. (2019)), PSR J0740+6620 (Miller et al. (2021), Riley et al. (2021), Raaijmakers et al. (2021)), and PSR J0437–4715 (Choudhury et al. (2024), Salmi et al. (2024), Dittmann et al. (2024)) as well as the HESS J1731–347 data point (Doroshenko et al. 2022). PS1 has a very soft hadronic phase and an early onset of the quark phase at about $1.3 n_0$. It is therefore a very extreme example. In addition, the mass-radius relation does not exhibit any clear signs of a phase transition taking place; it would be indistinguishable from a purely hadronic EoS. The only hint of its more exotic makeup is its placement at lower radii. Conversely, PS2 and PS3 lead to twin stars. In the case of PS2, the transition from the hadronic phase to the CFL phase is accompanied by a sizable jump in energy density (latent heat) of $\Delta\epsilon = 370 \text{ MeV}/\text{fm}^3$. This leads to the location of the hybrid star branch being moved to smaller masses and radii compared to PS3, which has a much smaller latent heat. The smaller latent heat of PS3 places its second branch at nearly the same mass as the hadronic maximum. Nevertheless, twin stars are present, which means that an observer unfamiliar with the EoS could infer the presence of a

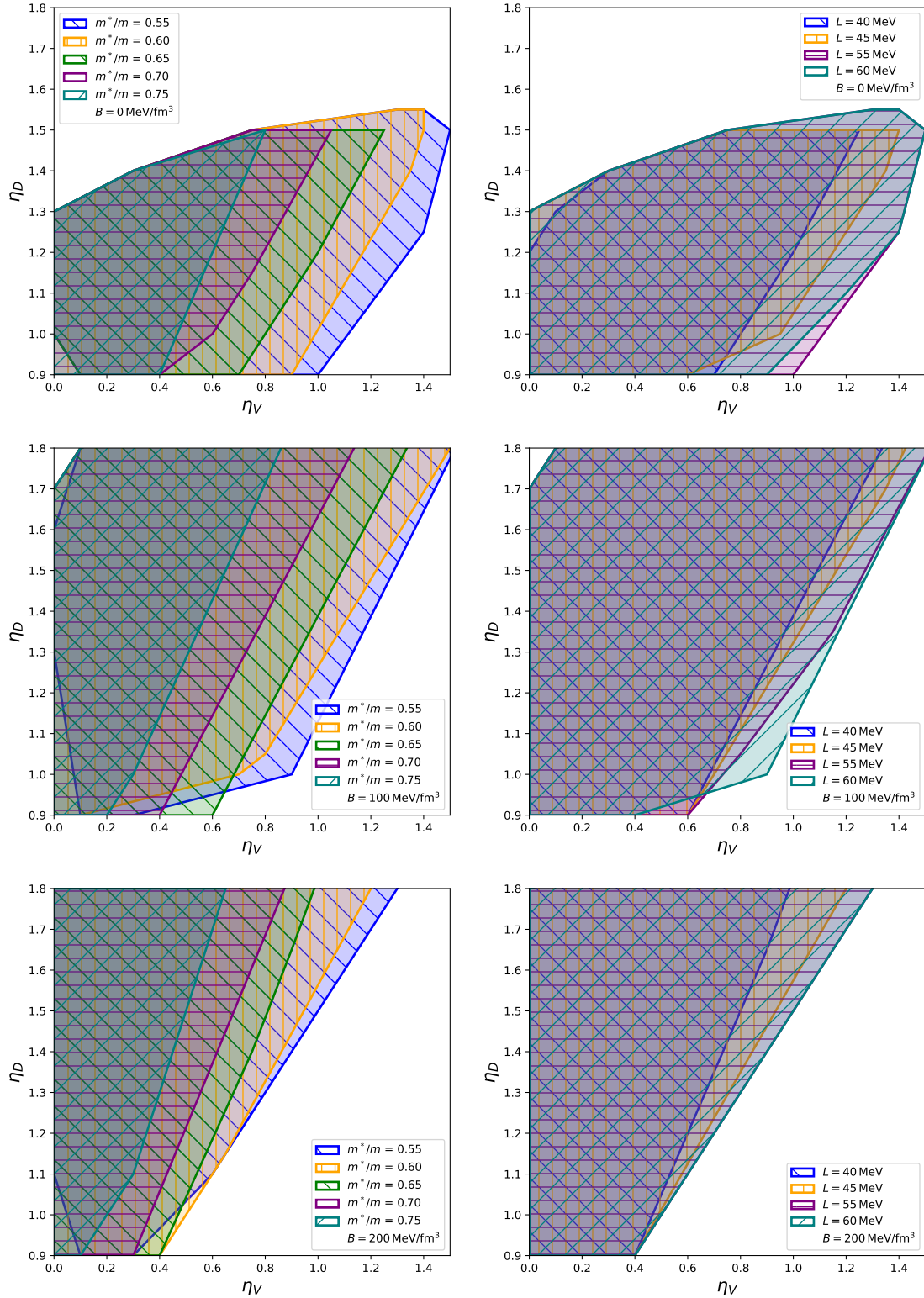


Fig. 1. Quark matter parameter sets that allow matches with the hadronic phase under different conditions. On the left, the effective mass, m^*/m , is kept constant, and n_0 , J , and L are not fixed. The shaded areas represent the region where a hadronic EoS with the given m^*/m can be matched with the corresponding quark matter parameters, η_V and η_D . On the right, the slope parameter, L , of the hadronic phase is kept constant, and n_0 , J , and m^*/m are not fixed. The results of the first row are for a bag constant, $B = 0 \text{ MeV/fm}^3$, which is increased in each descending row by 100 MeV/fm^3 .

phase transition from the mass-radius relation. The PS4 case also includes a kink in the mass-radius relation, but it is much less pronounced and is not resolvable with current observations. Like PS2, it features a phase transition from the hadronic to the

2SC phase, as well as a transition from the 2SC phase to the CFL phase. Finally, the mass-radius relation of PS5 is completely indistinguishable from a purely hadronic EoS, even though its early phase transition would result in exclusively hybrid stars

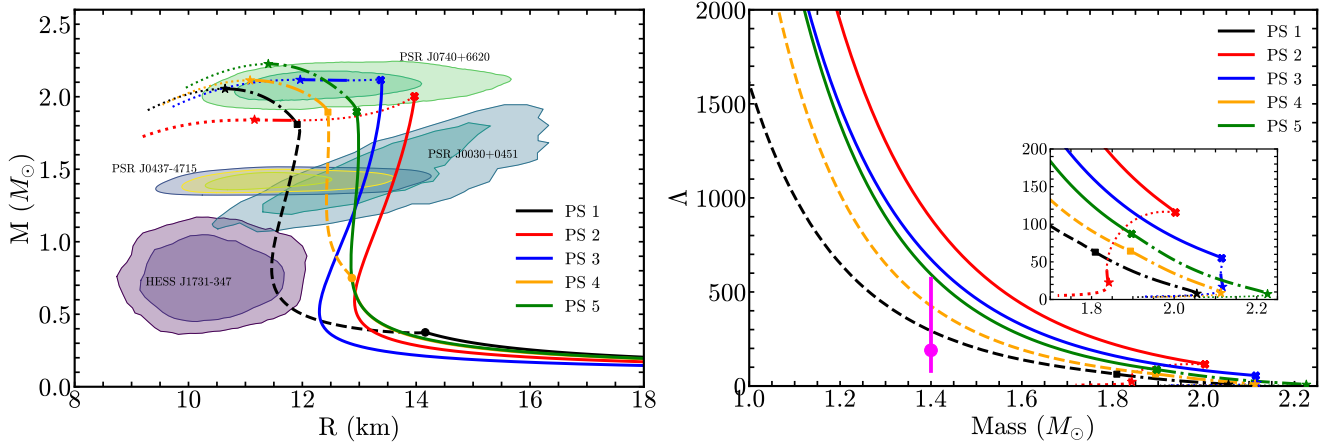


Fig. 2. Left: Mass-radius relation for hybrid stars with different hadronic and quark phase parameters as is displayed in Table 2. Points on solid lines are stars with hadronic matter in the core, while points on the dashed and dash-dotted lines contain 2SC and CFL quark matter in the center, respectively. The star symbol indicates the densest stable star. The solid circle and square symbols mark the onset of the 2SC and CFL phases in the core. The cross symbol corresponds to a direct phase transition from hadronic matter to the CFL phase. The dotted line corresponds to the unstable region. The various shaded areas are 1σ and 2σ credibility regions for mass and radius inferred from the analysis of the pulsar stated in the label. Right: Corresponding dimensionless tidal deformability (Λ)–mass relation. The solid magenta line represents the 90% credibility constraint on the dimensionless tidal deformability at $1.4 M_\odot$ from the GW170817 measurement (Abbott et al. 2018). The high-mass region is shown in more detail.

Table 2. Parameter sets used for Fig. 2.

Parameter	PS1	PS2	PS3	PS4	PS5
m^*/m	0.75	0.55	0.60	0.65	0.65
L [MeV]	60	60	45	60	60
J [MeV]	31	32	30	31	32
n_0 [fm $^{-3}$]	0.15	0.16	0.16	0.16	0.16
η_V	0.90	0.70	1.00	1.00	1.20
η_D	1.80	1.80	1.90	1.80	2.00
B [MeV/fm 3]	50	100	50	50	100
$n_{\text{trans}} [n_0]$	1.3	2.2	2.7	1.75	2.9
GW	Yes	No	No	Yes	Yes
Twins	No	Yes	Yes	No	No
HM-QM transition	2SC	CFL	CFL	2SC	CFL

Notes. The effective mass, m^*/m , vector coupling, η_V , and diquark coupling, η_D , are dimensionless. The symmetry energy, J , and slope parameter, L , are in mega-electronvolts. The saturation density, n_0 , and bag constant, B , are in units of particle per cubic femtometer and mega-electronvolts per cubic femtometer, respectively. The transition density in the hadronic phase, n_{trans} , is given in terms of n_0 . The compatibility with GW170817 is marked with “yes/no”, as is the presence of twin stars. The last row indicates the quark phase at the hadron-quark transition.

being realized in nature. The right side of Fig. 2 shows the tidal deformability as a function of mass for PS1–PS5. The solid magenta line represents the 90% credibility constraint from the GW170817 measurement as $\Lambda_{1.4 M_\odot} = 190^{+390}_{-120}$ (Abbott et al. 2018). Only PS1, PS4, and PS5 are within the constraint, which means that the two cases including twin stars are not consistent with GW170817.

3.2. Applying astrophysical constraints

In the following, we apply constraints from astrophysical data to all the available matches; specifically, the masses and radii of PSR J0030+0451 (Miller et al. (2019),

Riley et al. (2019), Raaijmakers et al. (2019)), PSR J0740+6620 (Miller et al. (2021), Riley et al. (2021), Raaijmakers et al. (2021), Salmi et al. (2024), Dittmann et al. (2024)), and PSR J0437–4715 (Choudhury et al. (2024)) as well as the 90% credibility constraint on the tidal deformability from GW170817 (Abbott et al. (2017, 2019)). We also require the maximum mass configuration of the EoS to support at least $2 M_\odot$ (Demorest et al. (2010), Antoniadis et al. (2013), Fonseca et al. (2016), Cromartie et al. (2019), Romani et al. (2022)). In general, stiff EoSs feature high maximum masses and large radii, compared to soft EoSs at lower masses and smaller radii. Therefore, stiff EoSs are not strongly constrained by the $2.0 M_\odot$ mass constraint or the radius estimate, $R_{2 M_\odot} \geq 11$ km, of a $2 M_\odot$ star by NICER (Miller et al. (2021), Riley et al. (2021), Raaijmakers et al. (2021)). However, the tidal deformability constraint obtained from GW170817 (Abbott et al. (2019)) requires compact solutions, i.e., a high value of compactness, $C = M/R$. Soft EoSs tend to contain more compact neutron stars, as the mass decrease is smaller compared to the radius decrease when going from a stiff EoS to a soft one, which rules out particularly stiff EoSs. In our setup, the purely hadronic EoSs with $m^*/m < 0.65$ are ruled out completely, while only some parameter sets with $m^*/m = 0.65$ are consistent with the tidal deformability constraint.

The way the observations outlined above constrain a given set of parameters is shown for the example of $m^*/m = 0.65$ and $B = 100$ MeV/fm 3 in Fig. 3. The shaded areas correspond to specific constraints. Similarly to Fig. 1, not every parameter set containing the fixed parameters is guaranteed to be compatible with the constraint, but there is at least one parameter set for each point in the shaded area that is compatible. A significant amount of purely hadronic EoSs with $m^*/m = 0.65$ is compatible with GW170817, even without a phase transition. Therefore, the GW170817 constraint (red) does not severely restrict the parameter space. The mass-radius estimate for PSR J0437–4715 (orange) has only slightly more constraining power than tidal deformability. The $2 M_\odot$ mass constraint (cyan) is the most restrictive constraint and decreases the allowed range of η_D for a given η_V substantially. This can be further refined by

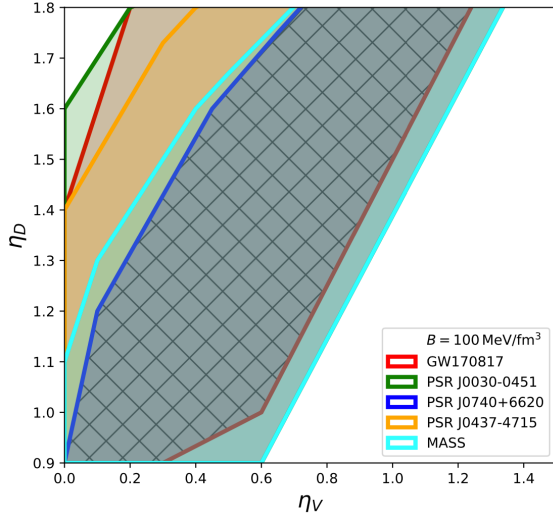


Fig. 3. Values of the diquark coupling (η_D) and vector coupling (η_V) allowed by specific constraints for a hadronic EoS with $m^*/m = 0.65$ and $B = 100 \text{ MeV/fm}^3$ that feature hybrid stars. The band where parameter sets exist that fulfill all constraints is cross-hatched. The specific regions to which η_V and η_D are constrained are indicated with colors that correspond to their respective observables. Red represents GW170817. Green, blue, and orange represent the NICER measurements. Finally, cyan represents the mass constraint. Note that all areas except for GW170817 share a border on the right side, beyond which no stable hybrid stars can be realized.

including the radius measurement of the $2 M_\odot$ PSR J0740+6620 (dark blue). Only the radius estimate for PSR J0030+0451 (green) limits the parameter space less than GW170817. All NICER constraints share a border on the right side of the plot. This is not a limitation caused by astrophysical observations; instead, none of the parameter sets to the right of this border lead to stable hybrid stars, and therefore none of them are of relevance to this work. Note that a plot similar to Fig. 3 can be made for a variety of parameter sets, but this one is well suited to outline the general trend: GW170817 requires softer hadronic EoSs, which are more difficult to combine with a quark phase. When already considering a soft hadronic EoS, the mass constraint becomes significantly more impactful than the tidal deformability constraint.

Fig. 2 and Table 2 suggest an incongruity between the tidal deformability constraint from GW170817 and twin stars. In Fig. 4, we investigate this thoroughly. The figure is structured identically to Fig. 1. The left side shows the $\eta_V - \eta_D$ parameter space in which constraints set above are met for at least one parameter set containing the effective mass corresponding to the color of the space. For the right side of the plot, L is kept fixed. Only parameter sets that lead to stable hybrid stars are considered. The most noticeable influence on the allowed parameter range comes from the tidal deformability constraint, which rules out nearly all EoSs with $m^*/m \leq 0.65$. For this reason, we only include EoSs with $m^*/m \geq 0.65$ in Fig. 4. However, stiffer EoSs are not completely ruled out. A phase transition to quark matter can, under certain circumstances, increase the compactness of the hybrid stars to such an extent that a previously ruled out hadronic EoS can regain compatibility with the tidal deformability constraint. In our framework, this effect is so small that we only found one combination of η_V and η_D that supports this phenomenon for a given bag constant. This might be caused by a large step size for the coupling strength. Applying a smaller step size should allow us to find a small area rather than the single

parameter combination currently identified. In our present study, the combination for $B = 0 \text{ MeV/fm}^3$ is $\eta_V = 0.80$ and $\eta_D = 1.50$. For these values, some parameter sets with $m^*/m = 0.55$ and $m^*/m = 0.60$ can fulfill all astrophysical constraints. However, stiff EoSs require a low transition density to be compatible with GW170817, in this case only slightly larger than n_0 . Increasing the bag constant, B , increases the transition density, necessitating corresponding adjustments to η_V and η_D . At about $B \simeq 100 \text{ MeV/fm}^3$, the values for $\eta_V = 1.00$ and $\eta_D = 2.00$ are reached, after which no transition at densities higher than $1 n_0$ is possible for $m^*/m \leq 0.60$ EoSs. Since small transition densities are required, all these parameter sets feature a 2SC phase. Conversely, this can be interpreted as a 2SC phase being required to ensure compatibility of an otherwise too stiff hadronic EoS with GW170817, at least in the setup presented here.

Furthermore, we find that by applying astrophysical constraints, the choice of slope parameter, L , loses significance. This is because the mass, radius, and tidal deformability are less sensitive to L than to the effective mass, which gets restricted to such a large extent that all remaining parameter combinations are likely to be matchable no matter the choice of L . However, for very large B , this rule gets softened. We speculate that this is caused by the transition being moved to such large densities that even a slight stiffening in the hadronic EoS can shift the phase transition past the maximum mass configuration of the EoS. In this context, it also becomes clear why higher values of L do not meaningfully change our results. When we increased the slope parameter to values slightly above $L = 60 \text{ MeV}$, the $\eta_V - \eta_D$ parameter space remained nearly unaffected. However, if we change L to significantly larger values the EoS becomes too stiff for the tidal deformability constraint, unless the phase transition takes place at small densities. In this case, however, observables such as mass, radius, and tidal deformability would become in large parts independent of the hadronic EoS and mainly depend on the quark matter EoS.

Fig. 4 also includes an area marked blue with parameter sets that lead to twin star solutions. The topic of twin stars only gains relevance in terms of their detectability. For this reason, we stipulate that observable twin star pairs need to have a mass difference of $\Delta M \leq 1\% M_{\text{twin}}$ and a radius difference of $\Delta R > 1 \text{ km}$, where M_{twin} is the mass of the more massive twin. With this definition of twin stars, we are able to include mass-radius relations with a kink in addition to those with a separate second branch in our analysis. To avoid visual confusion, the blue-shaded areas of Fig. 4 are not further separated into different hadronic parameter sets. Since we already established that only one $\eta_V - \eta_D$ combination is possible for stiff EoSs, we only consider softer ones. In practical terms, this means that every point within the blue-shaded area has at least one parameter set with $m^*/m \geq 0.65$ that leads to twin stars. This also means that points outside of this area cannot lead to twin stars as we defined them. Apart from that, no astrophysical constraints are applied to the twin star area. We find that under these conditions there is no overlap between the twin star area and the area constrained by astrophysical results. If we relaxed our definition of an observable twin star, for example by decreasing the required radius difference, this could be mitigated. However, twin stars are only relevant because they clearly point toward a phase transition. By reducing the required radius difference, one would also reduce their distinguishability from a purely hadronic EoS. One might also consider relaxing the constraint on the tidal deformability. For example, by using the original GW170817 tidal deformability and chirp mass values from Abbott et al. (2017) some parameter sets featuring twin stars, such as PS3 in Fig. 2, would be within

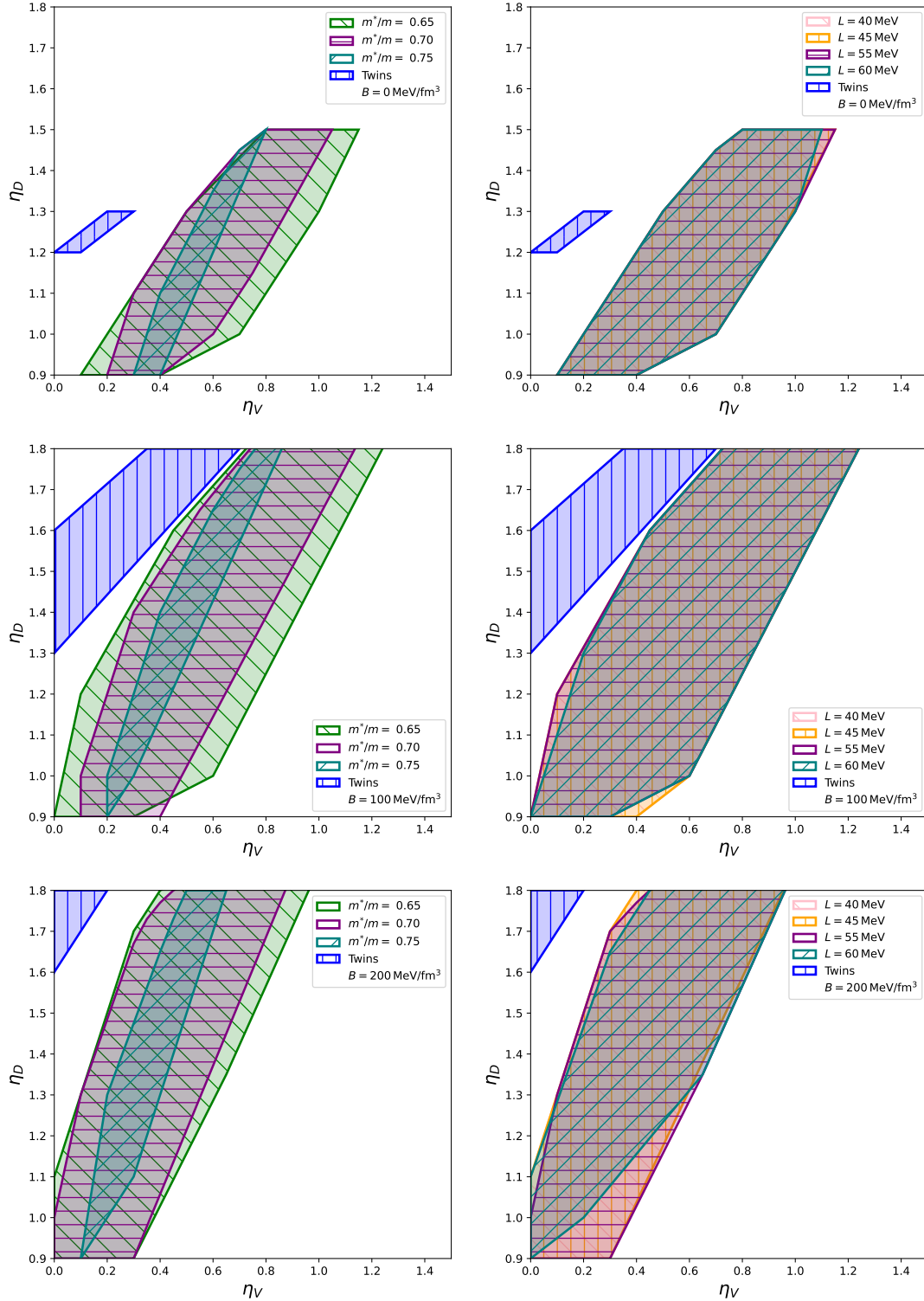


Fig. 4. Constrained areas for given parameter combinations. On the left, the effective mass, m^*/m , is kept constant, and n_0 , J , and L are not fixed. The shaded areas correspond to the parameter sets that generate stars compatible with astrophysical constraints. Additionally, the blue-shaded area shows all parameter sets within the ranges of the used nuclear parameters that lead to twin stars. On the right, the slope parameter, L , of the hadronic phase is kept constant, and n_0 , J , and m^*/m are not fixed. The results of the first row are for a bag constant $B = 0$ MeV/fm³, which is increased in each descending row by 100 MeV/fm³.

the required bounds. However, when applying the updated 90% credibility range from [Abbott et al. \(2019\)](#), this is not the case. In other words, the existence of twin stars in EoSs that reach two solar masses and more is strongly disfavored by GW170817, but not conclusively excluded.

The parameter sets PS1-5 in Table 2 might suggest that a large number of EoSs contain 2SC quark matter, and for the unconstrained EoSs, this is true. However, most EoSs containing a 2SC phase are not compatible with astrophysical constraints. Instead, the majority of the constrained EoSs have a

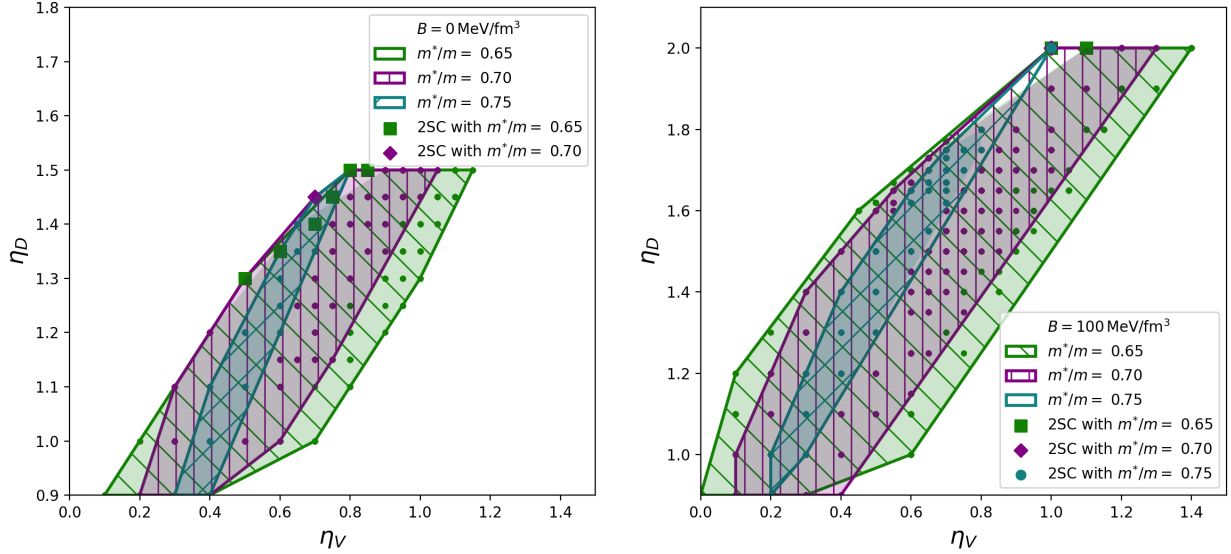


Fig. 5. Constrained parameter space for varied effective masses with the quark phase at the point of the hadron-quark phase transition indicated. The shaded area represents where a transition to the CFL phase is possible for a parameter set containing the effective mass corresponding to the color of the region. The green square marker indicates where at least one hadronic parameter set with $m^*/m = 0.65$ exhibits a phase transition to the 2SC phase. Likewise, the purple diamond marker represents a 2SC transition for $m^*/m = 0.70$ and the teal circle for $m^*/m = 0.75$. The small circles represent the investigated $\eta_V - \eta_D$ combinations within the constraint. The left side of the figure is for $B = 0 \text{ MeV/fm}^3$, the right side for $B = 100 \text{ MeV/fm}^3$.

phase transition from the hadronic phase directly to the CFL phase. Only a small set of η_V and η_D combinations in the upper left edge of the allowed parameter space lead to both a 2SC and a CFL phase being realized within neutron stars. We show this in Fig. 5, where $\eta_V - \eta_D$ combinations that lead to a CFL phase are represented as small circles. The area in which they can be found is shaded, corresponding to the effective mass they need to be combined with, to be within the constrained area. The few points leading to a 2SC phase are indicated with large markers. The absolute values of η_V and η_D and the number of possible combinations can change depending on the specific hadronic parameters, but their location in the allowed space remains the same. We find that the occurrence of a 2SC phase is highly dependent on the stiffness of the hadronic EoS, where stiffer hadronic EoSs exhibit a transition to a 2SC phase more often than softer ones. In contrast, by softening the hadronic EoS, the number of available combinations leading to a 2SC phase shrinks. For the largest effective mass, $m^*/m = 0.75$, no stable 2SC phase is possible if the bag constant is $B = 0 \text{ MeV/fm}^3$. This changes slightly by increasing B . If one disregards stiffness, the trend of increasing the bag constant is a reduction in available 2SC transitions. However, for a fixed stiffness of the hadronic EoS, increasing B also moves the transition density to larger values. This means that some particularly soft EoSs can match the quark phase just above saturation density if B is increased. Such is the case for $m^*/m = 0.75$, which can have a 2SC transition when $B = 100 \text{ MeV/fm}^3$ but not for $B = 0 \text{ MeV/fm}^3$. This dependence on stiffness is the reason why including astrophysical constraints such as GW170817 excludes most EoSs with a 2SC phase. Should future evidence point toward even softer EoSs, a neutron star with a 2SC phase would become even more difficult to construct in our setup.

4. Discussion and outlook

We have performed a comprehensive parameter study for hybrid star EoSs composed of the FB RMF model combined with the

renormalization group-consistent NJL (RG-NJL) model, including color superconductivity for the quark phase, making full use of their respective flexible frameworks. For the FB EoSs, the investigated parameters are the symmetry energy, J , the slope parameter, L , and the effective mass, m^*/m , all of which are fixed at saturation density, n_0 , and varied in ranges compatible with terrestrial experiments, as well as chiral effective field theory calculations. For the RG-NJL model, we varied the vector coupling strength, η_V , diquark coupling strength, η_D , and the bag constant, B , within a large range. In this model, homogeneous phases with color superconductivity of three-flavor quark matter are calculated self-consistently. At high densities, matter is in the CFL phase, while at lower densities, two-flavor color superconductivity (2SC phase) is favored. The hadron-quark phase transition is modeled using a Maxwell transition and the 2SC-CFL transition is calculated self-consistently in the NJL model (neglecting 2SC-CFL mixed phases).

Depending on the parameter set used, we find hybrid stars with either a pure 2SC, a pure CFL, or both phases present in the quark matter core. However, all parameter sets have in common that the most compact (densest) stable star is composed of CFL matter in its center, even if a less dense star of the same parameter set might have a pure 2SC quark core. By applying astrophysical constraints in the form of mass, radius, and tidal deformability, the parameter ranges for possible matches are significantly reduced. For these constrained parameter sets, we found that the EoSs containing only a CFL phase are significantly more abundant than EoSs, which can lead to 2SC quark matter. This is because a phase transition to a 2SC phase in the EoS requires a stiff hadronic EoS; however, stiff EoSs are disfavored by the tidal deformability constraint.

A major component of our analysis was the prospect of detecting the presence of quark matter in neutron stars. For this consideration, we focused on mass and radius as potential indicators. A nearly unambiguous indicator for the presence of quark matter would be a so-called twin star, whereby two stars have the same mass but different radii. A popular approach to investigate

the possibility of quark matter in compact stars is the CSS approach (Alford et al. 2013). It is significantly less microphysically motivated than the framework we employ here, but it can be very useful for generalized investigations, especially when looking for twin stars. Using this approach, Li et al. (2024) find hybrid stars with maximum masses of more than $2 M_{\odot}$ and within NICER, as well as tidal deformability limits. This requires a high speed of sound, $c_s^2 \geq 2/3$, and often features a phase transition at small densities. These small transition densities may limit twin stars to a mass range below the lowest known pulsar mass of about $\sim 1.17 M_{\odot}$ (Martinez et al. (2015) (see also Tauris & Janka 2019)), which makes them unlikely to appear in nature. Christian et al. (2024) found that even under idealized circumstances ($c_s^2 = 1$) only a small set of phase transitions can lead to detectable twin stars. Those transitions usually require a hadronic EoS of moderate stiffness and need to take place at high densities (see also Christian & Schaffner-Bielich 2022). The CSS quark model can also be used for sequential phase transitions, which should more closely resemble the framework employed here. For example, by setting the speed of sound to $c_s^2 = 0.7$ and $c_s^2 = 1$ for the 2SC and CFL phase, respectively, Li et al. (2023) finds configurations in which the 2SC-CFL transition induces another unstable region, resulting in three stars with the same mass but different radii, so-called triplets. However, as was shown in Gholami et al. (2025b), the speed of sound in the RG-consistent NJL model is not a constant quantity but can be non-monotonic at the 2SC-CFL transition. Furthermore, it only reaches values of $c_s^2 \approx 0.5$ – 0.6 at densities in the cores of neutron stars. This reduced speed of sound compared to the CSS approach hinders the construction of twin or triplet stars. As a result, it is not possible to construct twin or triplet stars within the parameter ranges allowed by astrophysical constraints. While twin stars can be constructed outside these constraints by using a combination of smaller values for the vector coupling and larger values of the diquark coupling, triplet stars cannot be constructed at all, despite the 2SC-CFL transition being present in multiple parameter sets.

In summary, we find large parameter ranges for both hadronic and quark parameters that are compatible with all astrophysical constraints. This includes parameter sets with various transition densities, as well as parameter sets featuring both 2SC and CFL quark matter. Despite this, we do not find any clear mass-radius indicators that could help differentiate between hybrid and purely hadronic stars (Alford et al. 2005). Furthermore, we find that the occurrence of a 2SC phase correlates strongly with a stiff hadronic EoS. Consequently, any future observational evidence indicating a softer EoS would significantly restrict the possibility of sequential phase transitions.

Our work can be extended in various ways. For example, there are additional degrees of freedom that could be included in our setup, such as hyperons (Bonanno & Sedrakian (2012), Ranea-Sandoval et al. (2017)), which might become relevant at densities comparable to the ones of the hadron-quark phase transition. However, we note that their inclusion tends to soften the EoS, which would make the combination of hyperons and quark matter in the same EoS even more difficult to realize than the setup presented here. In the present study, we concern ourselves with the allowed parameter ranges for a potential phase transition. However, we do not consider whether or not a phase transition is likely. For this reason, a full Bayesian analysis might elevate our work and provide a likelihood function for the presence of quark matter in neutron stars (Pfaff et al. (2022), Annala et al. (2023), Albino et al. (2024), Kurkela et al. (2024)). Furthermore, we suggest looking

more strongly at alternative ways of detecting the presence of a phase transition such as cooling behavior (de Carvalho et al. (2015), Lyra et al. (2023), Mendes et al. (2022, 2025)), oscillation modes (Ranea-Sandoval et al. (2018), Pradhan et al. (2024), Rather et al. (2025)), or thermal twins (Hempel et al. (2016), Carlomagno et al. (2024)).

Acknowledgements. The authors thank Michael Buballa, Alexander Haber, and Jürgen Schaffner-Bielich for helpful discussions. JEC was funded by the European Research Council (ERC) Advanced Grant INSPIRATION under the European Union’s Horizon 2020 research and innovation programme (Grant agreement No. 101053985). IAR acknowledges support from the Alexander von Humboldt Foundation. HG acknowledges support from the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through the CRC-TR211 ‘Strong-interaction matter under extreme conditions’ project number 315477589 – TRR 211. MH is supported by the GSI F&E.

References

- Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2017, *Phys. Rev. Lett.*, **119**, 161101
 Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2018, *Phys. Rev. Lett.*, **121**, 161101
 Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2019, *Phys. Rev. X*, **9**, 011001
 Abbott, R., Abbott, T. D., Abraham, S., et al. 2020, *ApJ*, **896**, L44
 Adhikari, D., Albataineh, H., Androic, D., et al. 2021, *Phys. Rev. Lett.*, **126**, 172502
 Adhikari, D., Albataineh, H., Androic, D., et al. 2022, *Phys. Rev. Lett.*, **129**, 042501
 Alaverdyan, G. 2021, *Symmetry*, **13**, 124
 Alaverdyan, G. B. 2022, *Astrophysics*, **65**, 278
 Albino, M., Malik, T., Ferreira, M., & Providência, C. 2024, *Phys. Rev. D*, **110**, 083037
 Alford, M. G., & Sedrakian, A. 2017, *Phys. Rev. Lett.*, **119**, 161104
 Alford, M. G., Rajagopal, K., & Wilczek, F. 1998, *Phys. Lett. B*, **422**, 247
 Alford, M., Rajagopal, K., & Wilczek, F. 1999a, *Nucl. Phys. B*, **537**, 443
 Alford, M. G., Berges, J., & Rajagopal, K. 1999b, *Nucl. Phys. B*, **558**, 219
 Alford, M., Braby, M., Paris, M., & Reddy, S. 2005, *ApJ*, **629**, 969
 Alford, M. G., Han, S., & Prakash, M. 2013, *Phys. Rev. D*, **88**, 083013
 Alford, M. G., Burgio, G., Han, S., Taranto, G., & Zappalà, D. 2015, *Phys. Rev. D*, **92**, 083002
 Annala, E., Gorda, T., Kurkela, A., & Vuorinen, A. 2018, *Phys. Rev. Lett.*, **120**, 172703
 Annala, E., Gorda, T., Katerini, E., et al. 2022, *Phys. Rev. X*, **12**, 011058
 Annala, E., Gorda, T., Hirvonen, J., et al. 2023, *Nat. Commun.*, **14**, 8451
 Antoniadis, J., Freire, P. C., Wex, N., et al. 2013, *Science*, **340**, 6131
 Bailin, D., & Love, A. 1979, *J. Phys. A: Math. Gen.*, **12**, L283
 Bailin, D., & Love, A. 1984, *Phys. Rep.*, **107**, 325
 Baldo, M., Buballa, M., Burgio, F., et al. 2003, *Phys. Lett. B*, **562**, 153
 Barrois, B. C. 1977, *Nucl. Phys. B*, **129**, 390
 Baym, G., Pethick, C., & Sutherland, P. 1971, *ApJ*, **170**, 299
 Baym, G., Hatsuda, T., Kojo, T., et al. 2018, *Rep. Prog. Phys.*, **81**, 056902
 Blaschke, D., & Alvarez-Castillo, D. E. 2016, *AIP Conf. Proc.*, **1701**, 020013
 Blaschke, D., Alvarez-Castillo, D. E., Ayriyan, A., et al. 2020, *Astrophysical Aspects of General Relativistic Mass Twin Stars*, 207
 Blaschke, D., Shukla, U., Ivanytskyi, O., & Liebving, S. 2023, *Phys. Rev. D*, **107**, 063034
 Boguta, J., & Bodmer, A. R. 1977, *Nucl. Phys. A*, **292**, 413
 Boguta, J., & Stöcker, H. 1983, *Phys. Lett. B*, **120**, 289
 Bonanno, L., & Sedrakian, A. 2012, *A&A*, **539**, A16
 Braun, J., Leonhardt, M., & Pawłowski, J. M. 2019, *SciPost Phys.*, **6**, 056
 Buballa, M., Neumann, F., Oertel, M., & Shovkovy, I. 2004, *Phys. Lett. B*, **595**, 36
 Burgio, G. F., Baldo, M., Sahu, P. K., & Schulze, H. J. 2002, *Phys. Rev. C*, **66**, 025802
 Carlomagno, J. P., Contrera, G. A., Grunfeld, A. G., & Blaschke, D. 2024, *Phys. Rev. D*, **109**, 043050
 Chatziioannou, K. 2020, *Gen. Relat. Grav.*, **52**, 109
 Chen, W.-C., & Piekarewicz, J. 2014, *Phys. Rev. C*, **90**, 044305
 Chen, H., Baldo, M., Burgio, G., & Schulze, H.-J. 2011, *Phys. Rev. D*, **84**, 105023
 Choudhury, D., Salmi, T., Vinciguerra, S., et al. 2024, *ApJ*, **971**, L20
 Christian, J.-E., & Schaffner-Bielich, J. 2020, *ApJ*, **894**, L8
 Christian, J.-E., & Schaffner-Bielich, J. 2022, *ApJ*, **935**, 122
 Christian, J.-E., Zacchi, A., & Schaffner-Bielich, J. 2018, *Eur. Phys. J. A*, **54**, 28
 Christian, J.-E., Zacchi, A., & Schaffner-Bielich, J. 2019, *Phys. Rev. D*, **99**, 023009

- Christian, J.-E., Schaffner-Bielich, J., & Rosswog, S. 2024, *Phys. Rev. D*, **109**, 063035
- Coelho, J., Lenzi, C., Malheiro, M., Marinho, R. M., & Fiolhais, M. 2010, *Int. J. Mod. Phys. D*, **19**, 1521
- Cromartie, H. T., Fonseca, E., Ransom, S. M., et al. 2019, *Nat. Astron.*, **4**, 72
- de Carvalho, S. M., Negreiros, R., Orsaria, M., et al. 2015, *Phys. Rev. C*, **92**, 035810
- Demorest, P., Pennucci, T., Ransom, S., Roberts, M., & Hessels, J. 2010, *Nature*, **467**, 1081
- Dietrich, T., Coughlin, M. W., Pang, P. T. H., et al. 2020, *Science*, **370**, 1450
- Dittmann, A. J., Miller, M. C., Lamb, F. K., et al. 2024, *ApJ*, **974**, 295
- Doroshenko, V., Suleimanov, V., Pühlhofer, G., & Santangelo, A. 2022, *Nat. Astron.*, **6**, 1444
- Duerr, H.-P. 1956, *Phys. Rev.*, **103**, 469
- Essick, R., Tews, I., Landry, P., & Schwenk, A. 2021, *Phys. Rev. Lett.*, **127**, 192701
- Fattoyev, F. J., Piekarewicz, J., & Horowitz, C. J. 2018, *Phys. Rev. Lett.*, **120**, 172702
- Fonseca, E., Pennucci, T. T., Ellis, J. A., et al. 2016, *ApJ*, **832**, 167
- Gao, B., Yuan, W.-L., Harada, M., & Ma, Y.-L. 2024, *Phys. Rev. C*, **110**, 045802
- Gärtlein, C., Ivanytskyi, O., Sagun, V., & Blaschke, D. 2023, *Phys. Rev. D*, **108**, 114028
- Gärtlein, C., Sagun, V., Ivanytskyi, O., Blaschke, D., & Lopes, I. 2025, *Phys. Rev. D*, **111**, 123021
- Gholami, H., Rather, I. A., Hofmann, M., Buballa, M., & Schaffner-Bielich, J. 2025a, *Phys. Rev. D*, **111**, 103034
- Gholami, H., Hofmann, M., & Buballa, M. 2025b, *Phys. Rev. D*, **111**, 014006
- Ghosh, S., Chatterjee, D., & Schaffner-Bielich, J. 2022, *Eur. Phys. J. A*, **58**, 37
- Glendenning, N. K., & Kettner, C. 2000, *A&A*, **353**, L9
- Hempel, M., Pagliara, G., & Schaffner-Bielich, J. 2009, *Phys. Rev. D*, **80**, 125014
- Hempel, M., Heinemann, O., Yudin, A., et al. 2016, *Phys. Rev. D*, **94**, 103001
- Hornick, N., Tolos, L., Zacchi, A., Christian, J.-E., & Schaffner-Bielich, J. 2018, *Phys. Rev. C*, **98**, 065804
- Itoh, N. 1970, *Prog. Theor. Phys.*, **44**, 291
- Ivanenko, D. D., & Kurdgelaidze, D. F. 1965, *Astrophysics*, **1**, 251
- Ivanytskyi, O. 2025, *Phys. Rev. D*, **111**, 034004 [Erratum: *Phys. Rev. D*, **111**, 079904 (2025)]
- Ivanytskyi, O., & Blaschke, D. 2022a, *Phys. Rev. D*, **105**, 114042
- Ivanytskyi, O., & Blaschke, D. B. 2022b, *Particles*, **5**, 514
- Johnson, M. H., & Teller, E. 1955, *Phys. Rev.*, **98**, 783
- Kämpfer, B. 1981, *J. Phys. A*, **14**, L471
- Klähn, T., Blaschke, D., Sandin, F., et al. 2007, *Phys. Lett. B*, **654**, 170
- Klähn, T., Lastowiecki, R., & Blaschke, D. B. 2013, *Phys. Rev. D*, **88**, 085001
- Kurkela, A., Rajagopal, K., & Steinhorst, R. 2024, *Phys. Rev. Lett.*, **132**, 262701
- Landry, P., Essick, R., & Chatziioannou, K. 2020, *Phys. Rev. D*, **101**, 123007
- Lattimer, J. M. 2023, *Particles*, **6**, 30
- Li, J. J., Sedrakian, A., & Alford, M. 2023, *ApJ*, **944**, 206
- Li, J. J., Sedrakian, A., & Alford, M. 2024, *ApJ*, **967**, 116
- Lim, Y., & Schwenk, A. 2024, *Phys. Rev. C*, **109**, 035801
- Lyra, F., Moreira, L., Negreiros, R., Gomes, R. O., & Dexheimer, V. 2023, *Phys. Rev. C*, **107**, 025806
- Martinez, J. G., Stovall, K., Freire, P. C. C., et al. 2015, *ApJ*, **812**, 143
- Masuda, K., Hatsuda, T., & Takatsuka, T. 2013, *ApJ*, **764**, 12
- Mendes, M., Fattoyev, F. J., Cumming, A., & Gale, C. 2022, *ApJ*, **938**, 119
- Mendes, M., Christian, J.-E., Fattoyev, F. J., & Schaffner-Bielich, J. 2025, *Phys. Rev. D*, **111**, 063007
- Miller, M. C., Lamb, F. K., Dittmann, A. J., et al. 2019, *ApJ*, **887**, L24
- Miller, M. C., Lamb, F. K., Dittmann, A. J., et al. 2021, *ApJ*, **918**, L28
- Montana, G., Tolos, L., Hanauske, M., & Rezzolla, L. 2019, *Phys. Rev. D*, **99**, 103009
- Mueller, H., & Serot, B. D. 1996, *Nucl. Phys. A*, **606**, 508
- Negele, J. W., & Vautherin, D. 1973, *Nucl. Phys. A*, **207**, 298
- Pagliara, G., & Schaffner-Bielich, J. 2008, *Phys. Rev. D*, **77**, 063004
- Paschalidis, V., Yagi, K., Alvarez-Castillo, D., Blaschke, D. B., & Sedrakian, A. 2018, *Phys. Rev. D*, **97**, 084038
- Pfaff, A., Hansen, H., & Gulminelli, F. 2022, *Phys. Rev. C*, **105**, 035802
- Pradhan, B. K., Chatterjee, D., & Alvarez-Castillo, D. E. 2024, *MNRAS*, **531**, 4640
- Raaijmakers, G., Riley, T. E., Watts, A. L., et al. 2019, *ApJ*, **887**, L22
- Raaijmakers, G., Greif, S. K., Hebeler, K., et al. 2021, *ApJ*, **918**, L29
- Ranea-Sandoval, I. F., Orsaria, M. G., Han, S., Weber, F., & Spinella, W. M. 2017, *Phys. Rev. C*, **96**, 065807
- Ranea-Sandoval, I. F., Guilera, O. M., Mariani, M., & Orsaria, M. G. 2018, *JCAP*, **12**, 031
- Rapp, R., Schäfer, T., Shuryak, E. V., & Velkovsky, M. 1998, *Phys. Rev. Lett.*, **81**, 53
- Rather, I. A., Kumar, A., Das, H. C., et al. 2020, *Int. J. Mod. Phys. E*, **29**, 2050044
- Rather, I. A., Usmani, A. A., & Patra, S. K. 2021, *J. Phys. G*, **48**, 085201
- Rather, I. A., Marquez, K. D., Thakur, P., & Lourenço, O. 2025, *Phys. Rev. D*, **112**, 023013
- Rehberg, P., Klevansky, S. P., & Hufner, J. 1996, *Phys. Rev. C*, **53**, 410
- Riley, T. E., Watts, A. L., Bogdanov, S., et al. 2019, *ApJ*, **887**, L21
- Riley, T. E., Watts, A. L., Ray, P. S., et al. 2021, *ApJ*, **918**, L27
- Romani, R. W., Kandel, D., Filippenko, A. V., Brink, T. G., & Zheng, W. 2022, *ApJ*, **934**, L18
- Salmi, T., Choudhury, D., Kini, Y., et al. 2024, *ApJ*, **974**, 294
- Schäfer, T. 2000, *Nucl. Phys. B*, **575**, 269
- Schäfer, T., & Wilczek, F. 1999, *Phys. Rev. D*, **60**, 114033
- Schaffner-Bielich, J., Hanauske, M., Stöcker, H., & Greiner, W. 2002, *Phys. Rev. Lett.*, **89**, 171101
- Schertler, K., Greiner, C., Schaffner-Bielich, J., & Thoma, M. H. 2000, *Nucl. Phys. A*, **677**, 463
- Serot, B. D., & Walecka, J. D. 1986, *Adv. Nucl. Phys.*, **16**, 1
- Shahrbaf, M., Antić, S., Ayriyan, A., Blaschke, D., & Grunfeld, A. G. 2023, *Phys. Rev. D*, **107**, 054011
- Shovkovy, I. A., Ruester, S. B., & Rischke, D. H. 2005, *J. Phys. G*, **31**, S849
- Sieniawska, M., Turczanski, W., Bejger, M., & Zdunik, J. L. 2019, *A&A*, **622**, A174
- Son, D. T. 1999, *Phys. Rev. D*, **59**, 094019
- Steiner, A., Prakash, M., & Lattimer, J. M. 2000, *Phys. Lett. B*, **486**, 239
- Tan, H., Noronha-Hostler, J., & Yunes, N. 2020, *Phys. Rev. Lett.*, **125**, 261104
- Tauris, T. M., & Janka, H.-T. 2019, *ApJ*, **886**, L20
- Todd-Rutel, B. G., & Piekarewicz, J. 2005, *Phys. Rev. Lett.*, **95**, 122501
- Typel, S., Röpke, G., Klähn, T., Blaschke, D., & Wolter, H. H. 2010, *Phys. Rev. C*, **81**, 015803
- Walecka, J. D. 1974, *Ann. Phys. (N.Y.)*, **83**, 491
- Wetterich, C. 1993, *Phys. Lett. B*, **301**, 90
- Xie, W.-J., & Li, B.-A. 2021, *Phys. Rev. C*, **103**, 035802
- Yasutake, N., Lastowiecki, R., Benic, S., et al. 2014, *Phys. Rev. C*, **89**, 065803
- Yuan, W.-L., Gao, B., Yan, Y., & Xu, R. 2025, *Phys. Rev. D*, **112**, 023019
- Zacchi, A., Hanauske, M., & Schaffner-Bielich, J. 2016, *Phys. Rev. D*, **93**, 065011
- Zacchi, A., Tolos, L., & Schaffner-Bielich, J. 2017, *Phys. Rev. D*, **95**, 103008
- Zdunik, J., & Haensel, P. 2013, *A&A*, **551**, A61
- Zhang, Z., & Chen, L.-W. 2023, *Phys. Rev. C*, **108**, 024317
- Zhang, C., & Mann, R. B. 2021, *Phys. Rev. D*, **103**, 063018
- Zhao, T., Lin, Z., Kumar, B., Steiner, A. W., & Prakash, M. 2024, ArXiv e-prints [arXiv:2406.05267]