

# Parameter estimation from the Ly $\alpha$ forest in the Fourier space using an information-maximizing neural network

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## ABSTRACT

**Aims.** Our aim is to present a robust parameter estimation with simulated Ly $\alpha$  forest spectra from Sherwood-Relics simulations suite by using an information-maximizing neural network (IMNN) to extract maximal information from Ly $\alpha$  1D-transmitted flux in the Fourier space.

**Methods.** We performed 1D estimations using IMNN for intergalactic medium (IGM) thermal parameters  $T_0$  and  $\gamma$  at  $z = 2-4$ , and cosmological parameters  $\sigma_8$  and  $n_s$  at  $z = 3-4$ . We compared our results with estimates from the power spectrum using the posterior distribution from a Markov chain Monte Carlo (MCMC). We then checked the robustness of IMNN estimates against deviation in spectral noise levels, continuum uncertainties, and instrumental smoothing effects. Using mock Ly $\alpha$  forest sightlines from the publicly available CAMELS project, we also checked the robustness of the trained IMNN on a different simulation. As a proof of concept, we demonstrated a 2D-parameter estimation for  $T_0$  and H I photoionization rates,  $\Gamma_{\text{HI}}$ .

**Results.** We obtain improved estimates of  $T_0$  and  $\gamma$  using IMNN over the standard MCMC approach. These estimates are also more robust against signal-to-noise deviations at  $z = 2$  and 3. At  $z = 4$ , the sensitivity to noise deviations is on par with MCMC estimates. The IMNN also provides  $T_0$  and  $\gamma$  estimates that are robust against continuum uncertainties by extracting small-scale continuum-independent information from the Fourier domain. In the cases of  $\sigma_8$  and  $n_s$ , the IMNN performs on par with MCMC but still offers a significant speed boost in estimating parameters from a new dataset. The improved estimates with IMNN are seen for high instrumental resolution (FWHM = 6 km s<sup>-1</sup>). At medium or low resolutions, the IMNN performs similarly to MCMC, suggesting an improved extraction of small-scale information with IMNN. We also find that IMNN estimates are robust against the choice of simulation. By performing a 2D-parameter estimation for  $T_0$  and  $\Gamma_{\text{HI}}$ , we also demonstrate how to take forward this approach observationally in the future.

**Key words.** intergalactic medium – quasars: absorption lines – cosmological parameters – diffuse radiation – large-scale structure of Universe

## 1. Introduction

The Ly $\alpha$  forest absorption spectra consist of numerous absorption lines created by the 1s  $\rightarrow$  2p Ly $\alpha$  transition of the intervening neutral hydrogen gas along the lines of sight to distant quasars. These absorption features trace the underlying distribution of matter in the universe, and thus reveal the cosmic web's intricate filamentary structure (Finley et al. 2014; Lee et al. 2018). The large-scale structure traced by the Ly $\alpha$  forest is sensitive to the nature of dark matter and dark energy and allows us to test structure formation processes down to small scales. By studying the clustering and distribution of matter on cosmic scales, one can place constraints on the density fluctuations of dark matter (Croft et al. 1998, 1999, 2002; McDonald et al. 2000; McDonald 2003; Viel et al. 2004a) and the equation of state of dark energy (Viel et al. 2003; Coughlin et al. 2019), along with cosmological parameters (Viel et al. 2004b; McDonald et al. 2000; Viel 2006), warm dark matter models (Viel et al. 2013; Iršič et al. 2017), neutrino mass (Palanque-Desabrouille et al. 2015a,b, 2020; Yèche et al. 2017), and so on governing the constitution of the Universe. In

the astrophysical context, the Ly $\alpha$  forest is also useful in constraining the intergalactic medium (IGM) temperature,  $T_0$ , at the cosmic mean density and slope,  $\gamma$ , of the temperature ( $T$ )-density ( $\Delta$ ) relation ( $T = T_0(\Delta)^{\gamma-1}$ ) (Schaye et al. 1999, 2000; Theuns & Zaroubi 2000; McDonald et al. 2001; Becker et al. 2011; Boera et al. 2014; Gaikwad et al. 2021), cosmic reionization (Fan et al. 2006; Worseck et al. 2018), and the impact of various feedback processes (such as supernova (SN) and active galactic nucleus (AGN)-driven outflows) on the IGM that operate during the formation and evolution of galaxies over cosmic time (Aguirre et al. 2001; Oppenheimer & Davé 2006).

The Ly $\alpha$  forest can also act as a complementary source of information to other cosmological probes like the cosmic microwave background (CMB) and galaxy surveys. Combining data from various sources, including the Lyman forest, allows for more robust and precise cosmological parameter estimation, reducing potential biases and uncertainties (Viel et al. 2004c; Viel 2006; Lesgourgues et al. 2007). In order to extract critical information about the large-scale structure, BAO, redshift space distortions, and the nature of cosmic density fluctuations, astrophysicists have investigated the clustering of the Ly $\alpha$  forest, customarily adopting as a summary statistics the power

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spectrum of the transmitted flux (McDonald et al. 2000, 2006; Croft et al. 2002; Seljak et al. 2006). The advent of high-fidelity spectra also allows one to perform higher-order clustering studies with the Ly $\alpha$  forest (Viel et al. 2004b; Tie et al. 2019; Maitra et al. 2019, 2022a,b).

Moving forward, machine-learning approaches and neural networks (NNs) can enhance the precision and accuracy of parameter estimates from relevant observables, potentially leading to more reliable astrophysical and cosmological models (Gupta et al. 2018; Charnock et al. 2018; Ribli et al. 2019; Nayak et al. 2024). Additionally, a trained NN also offers a substantial boost in computational efficiency when dealing with new datasets (Nygaard et al. 2023), particularly when using the so-called “amortized” method (e.g., Karchev et al. 2022). This feature becomes especially important when estimating parameters from the current age of large astrophysical and cosmological datasets. Parameter estimation using NNs on the real space field information can offer significant constraints on the estimated parameters. However, since NNs are exceptional at picking up nonlinear features in the training data, this makes them susceptible to simulation-specific features in the training data (see Gluck et al. 2023, for example). Thus, robustness against learning unwanted features in the training data can become an issue for inference approaches from the real space field.

An alternative approach is to work in the Fourier domain to filter out such simulation-specific nonlinearities in real space and draw only relevant information from the Fourier space using NNs. In the past, NN approaches have been used to estimate parameters from the Fourier space that were more accurate and robust in comparison to traditional maximum likelihood approaches from the power spectrum (see Villaescusa-Navarro et al. 2022, for example). Some of these applications have also focused on the Ly $\alpha$  forest to extract thermal parameters (Alsing et al. 2018) or detect large-scale power enhancement in the power spectrum due to patchy reionization (Molaro et al. 2022, 2023). Here, we work on extracting maximal information out of the data in the Fourier space using NNs and then compare this approach with traditional likelihood-based approaches using the power spectrum. In particular, we use information-maximizing neural networks (IMNNs, Charnock et al. 2018) to extract maximal information from the Fourier space data and enhance the parameter estimation. The ability of an IMNN to extract almost lossless information for parameter inferences has been demonstrated in Makinen et al. (2021) for correlated normal and lognormal fields with cosmological power spectra. Working in Fourier space, while reducing data dimensionality, also retains relevant information regarding the clustering of the Ly $\alpha$  forest.

In this work, we mainly focus on 1D parameter estimation and its robustness for the thermal parameters,  $T_0$  and  $\gamma$ , and cosmological parameters,  $\sigma_8$  and  $n_s$ , individually. This is because training the NNs for joint parameter estimation in many dimensions requires simulations to have simultaneous parameter variations (similar to Nayak et al. 2024) that we do not have access to at the present time. As a proof of concept, we perform a 2D joint parameter estimation for  $T_0$  and the HI photoionization rate,  $\Gamma_{\text{HI}}$ , which can be varied simply as a post-processing step during the forward modeling of mock Ly $\alpha$  forest spectra without the need to run a new simulation. This is done to demonstrate how to take forward the IMNN approach in the future observationally for a full N-D parameter estimation. While such endeavors will require a vast array of simulations for training the NN, one can draw inspiration from recent works related to generating significantly efficient simulated training examples. For exam-

ple, one way to provide the necessary training examples would be to use fast emulators, like the recently released 21cmEMU (Breitman et al. 2023), which is able to produce summary statistics from a full run of 21cmFAST with a speed-up factor of  $\sim 10^4$ . Other promising approaches to producing cheap simulations rely on Lagrangian deep learning (Dai & Seljak 2021) and its successors (Rigo et al., in prep.).

## 2. Simulations and mock sightlines

For this work, we used the Sherwood-Relics suite of hydrodynamical simulations with a box size of  $40 h^{-1} \text{cMpc}^3$  and  $2 \times 1024^3$  particles (Puchwein et al. 2023; Bolton et al. 2017) to generate mock Ly $\alpha$  forest sightlines to train the NN for parameter estimation. The fiducial simulation used for this work was run with a standard  $\Lambda$ CDM cosmology with cosmological parameters based on Planck Collaboration XVI (2014) ( $\{\Omega_m, \Omega_b, \Omega_\Lambda, \sigma_8, n_s, \text{ and } h\} = \{0.308, 0.0482, 0.692, 0.829, 0.961, \text{ and } 0.678\}$ ). We then used simulations with varied cosmological and astrophysical parameters to train the NN to learn from the associated variations in the Ly $\alpha$  forest and perform parameter estimation. We performed parameter estimation for the cosmological parameters,  $\sigma_8$  and  $n_s$ , and the astrophysical parameters,  $T_0$  and  $\gamma$ , individually.  $T_0$  is the IGM temperature at mean cosmic density and  $\gamma$  is the slope of the IGM temperature,  $T$ , and density,  $\Delta$ , relation ( $T = T_0 \Delta^{\gamma-1}$ ). The fiducial simulation has  $\sigma_8$  and  $n_s$  values of 0.829 and 0.961, as was mentioned earlier, and a redshift-independent  $\gamma$  of 1.3. The temperature evolves with redshift. In the case of  $\sigma_8$ , simulations were used with variation in the parameter as  $\sigma_8 = 0.829 \pm 0.075$ . For  $n_s$ , the parameter variations used were  $n_s = 0.961 \pm 0.04$ . In the case of  $T_0$ , we used simulations that had temperatures 1.5 times higher and lower than the fiducial simulation. For  $\gamma$ , simulations that had  $\gamma = 1.3 \pm 0.3$  were used. In total, we used nine simulations to train the NN to learn parameter estimation for all these parameters individually. All these simulations with variations in parameters were run with the same initial seed density fields. We also used four other simulations that had  $\sigma_8 = 0.804, 0.854$ , and  $n_s = 0.941, 0.981$ . These simulations were used to test the ability of the NN to predict parameter values different from what it has been trained on. Additionally, we also used three other simulations run with fiducial parameters, but with different initial seed density fields on which we performed the testing of the parameter estimation. This was done to check how robust the trained NN is against new unseen data. All the above simulations were run with spatially homogenous photoionization rates and photoheating rates from the fiducial UV background model presented in Puchwein et al. (2019) (see Table D.1). For the 2D parameter estimation of  $T_0$  and the HI photoionization rate,  $\Gamma_{\text{HI}}$ , we varied  $\Gamma_{\text{HI}}$  by 25% as a post-processing step on the Ly $\alpha$  forest spectra by uniformly scaling the optical depth field,  $\tau$ . The list of all the simulations used in this work is given in Table A.1.

The snapshots for all these simulations are stored at redshift intervals of  $\Delta z = 0.1$ . We created mock Ly $\alpha$  forest transmitted flux sightlines from these simulations of length  $40 h^{-1} \text{cMpc}$  at redshifts  $z = 2.0, 3.0$ , and  $4.0$ . This was done to test parameter estimation and the constraining power of the Ly $\alpha$  forest at different redshifts. We generated the sightlines by first gridding  $40 h^{-1} \text{cMpc}$  (the length of each sightline) into 2048 grids in wavelength. For the sake of simplicity and the fact that we are not comparing the simulated spectra with observations in this work, we retained the uniform gridding in  $\Delta x = 40/2048 h^{-1} \text{cMpc}$  (corresponding to  $\Delta z \approx 1.96 \times 10^{-5}$ ) inherent in the simulations. We also adjusted the photoionizing

background for each of the varied simulations so that their mean flux matches the fiducial one. To check the effect of instrumental smoothing on the estimation, we also generated sightlines convolved with Gaussian profiles with FWHM = 6, 50, and 150 km s<sup>-1</sup>. However, unless otherwise mentioned, we used the transmitted flux without any Gaussian convolution. Additionally, we added random Gaussian noise to the transmitted flux. We used uniform signal-to-noise (S/N) along a single sightline but varied the S/N levels between different sightlines. The S/N values corresponding to each sightline were drawn uniformly in the logarithmic space in S/N, in the range log S/N ∈ [log 20, log 100]. This ensures that our sample has a larger number of low-S/N sightlines, similar to the observed spectra. We refer to this S/N distribution as S/N<sub>Fid</sub> (for “fiducial”) from now on. We also generated sightlines that have 0.85 × S/N<sub>Fid</sub> to evaluate the sensitivity of the parameter estimation to deviations (in this case, systematic suppression of S/N) in noise levels.

### 3. Parameter estimation with Markov chain Monte Carlo from the power spectrum

For the traditional approach, we used the 1D flux power spectrum,  $P_F(k)$ , computed over mock Ly $\alpha$  forest spectra of length 40 h<sup>-1</sup> cMpc to estimate  $T_0$ ,  $\gamma$ , and  $\sigma_8$  using the maximum likelihood approach. To compute the 1D flux power spectrum, we first Fourier-transformed the 1D flux deviation field,  $\delta_F(x) = (F(x) - \bar{F})/\bar{F}$ , to  $\delta_F(k)$ . The power spectrum was then simply proportional to  $|\delta_F(k)|^2$ . The 1D flux power spectrum was then normalized as

$$\sigma_F^2 = \int_{-\infty}^{\infty} \frac{dk P_F(k)}{2\pi}, \quad (1)$$

where  $\sigma_F^2$  is the variance of the field,  $\delta_F(k)$ . We then binned the power spectra in ten equally spaced logarithmic bins in  $k$  ranging from 0.314 to 31.4 h<sup>-1</sup> cKpc. The smallest scales here correspond to 100 h<sup>-1</sup> ckpc and the largest scale corresponds to 10 h<sup>-1</sup> cMpc. We intentionally went to very small scales to allow the NN described in the following section to extract maximal information from such scales, which are known to be sensitive to noise.

We computed the  $P_F(k)$ s corresponding to the simulation at fiducial parameter values and also for the simulations with parameter variation. The  $P_F(k)$ s corresponding to the fiducial and varied astrophysical parameters ( $T_0$  and  $\gamma$ ) are shown in the top panels of Fig. 1. The error bars shown in the figure are bootstrap error bars computed over 5000 sightlines for a sample size of 50 sightlines with 10 000 bootstrap realizations. In Fig. 2, we plot the difference in  $P_F(k)$  with respect to the fiducial  $P_F(k)$  for variations in the cosmological parameters ( $\sigma_8$  and  $n_s$ ). This was done to highlight the differences better since the variations in the cosmological parameters do not cause appreciable changes in the power spectrum with respect to the error bars shown. Now, to obtain a model for  $P_F(k)$  and its variation with the parameters, we approximated the power spectrum by a Taylor expansion to the first order around the fiducial parameter value,  $\vartheta_0$ , and then modeled  $P_F(k)$  for the varied parameter,  $\vartheta$ , about the fiducial parameter as (check Sect. 4.1 of Viel & Haehnelt 2006, for description)

$$P_F(k, \vartheta) = P_F(k, \vartheta_0) + \left. \frac{\partial P_F(k, \vartheta)}{\partial \vartheta} \right|_{\vartheta=\vartheta_0} (\vartheta - \vartheta_0). \quad (2)$$

Using this, we developed a model to capture the linear variation in  $P_F(k)$  with parameters  $T_0$ ,  $\gamma$ ,  $\sigma_8$ , and  $n_s$  individually. This

modeling was done corresponding to sightlines that have noise levels drawn from S/N<sub>Fid</sub>. We then defined a log-likelihood function of the form

$$-2 \log \mathcal{L}(\vartheta) = (\mathbf{P}_F(\mathbf{k}, \vartheta) - \mathbf{d}_0)^T \mathbf{C}^{-1} (\mathbf{P}_F(\mathbf{k}, \vartheta) - \mathbf{d}_0), \quad (3)$$

where  $\mathbf{d}_0 = \mathbf{P}_F(\mathbf{k}, \vartheta_0)$  are the power spectrum values for the fiducial parameters, arranged in a 10-dimensional vector binned over  $k$ -values in the range of 0.314 to 31.4 h cKpc<sup>-1</sup>, and  $\mathbf{C}$  is the covariance matrix obtained from 10 000 bootstrap realizations of the power spectrum with a sample size of 50 sightlines, as was mentioned before. Using this log-likelihood function, we then used a Markov Chain Monte Carlo (MCMC) with uniform priors of the form

$$p(\vartheta) = \begin{cases} (5 \times \Delta\vartheta)^{-1} & \text{if } |\vartheta - \vartheta_0| \leq 5 \times \Delta\vartheta \\ 0 & \text{if } |\vartheta - \vartheta_0| > 5 \times \Delta\vartheta \end{cases} \quad (4)$$

(where  $\Delta\vartheta$  is the variation in the model parameters for the simulations with varied parameters) to estimate the parameter values from a new set of three simulations run with different initial random seeds at the fiducial parameter values. We used the python package EMCEE (Foreman-Mackey et al. 2013) to run the MCMC. In the middle panels of Figs. 1 and 2, we show the estimated parameter values from these three simulations. This was done using sightlines whose noise levels were drawn from S/N<sub>Fid</sub>. To check the sensitivity of the parameter estimation to the noise levels, we then used the  $P_F(k)$  modeling done using S/N<sub>Fid</sub> sightlines to estimate parameters for 0.85 × S/N<sub>Fid</sub> sightlines. We show the estimated parameters in the bottom panels in Fig. 1. We subsequently compare these results with the NN approach in Sect. 6. It is to be noted that we used Eq. (2) to model individual parameter variations for 1D parameter estimations. In Sect. 8, where we demonstrate a 2D parameter estimation for  $T_0$  and  $\Gamma_{\text{HI}}$ , we model the power spectrum for parameter variations by following a linear interpolation scheme between different simulations.

### 4. Information-maximizing neural network

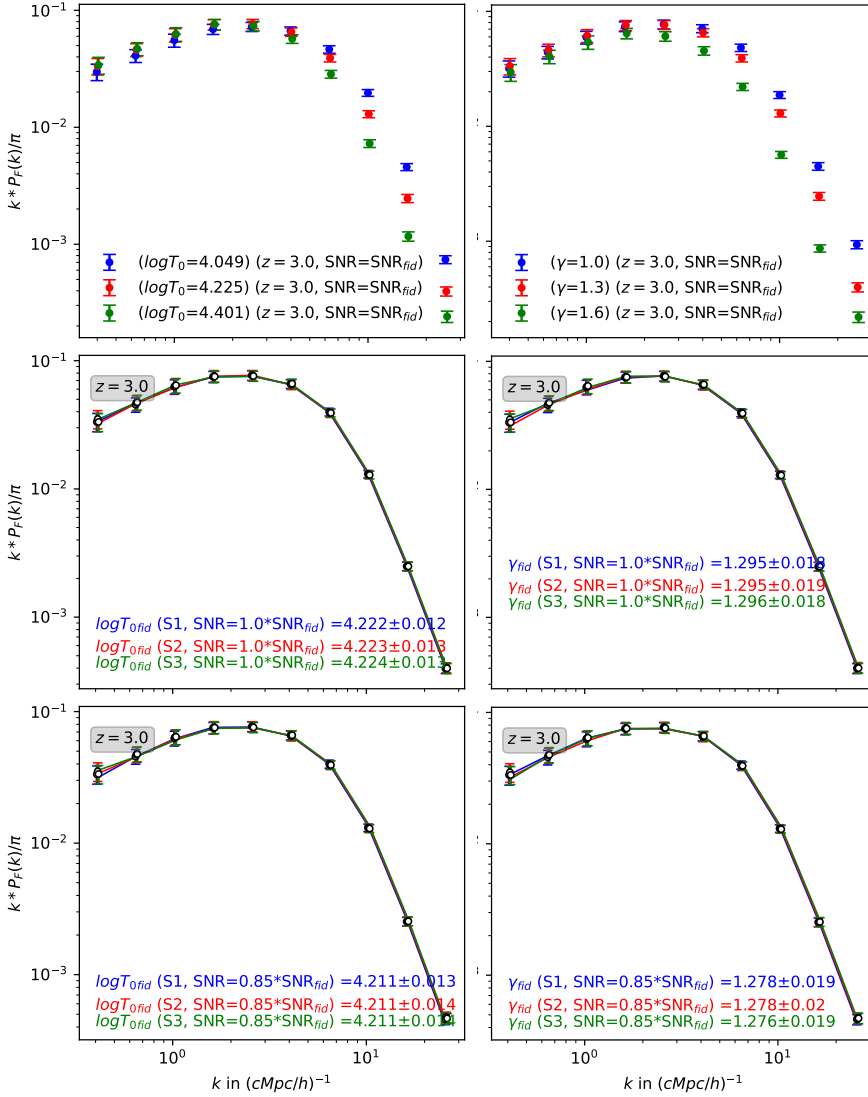
Summarizing large datasets in a collection of sufficient summary statistics (e.g., mean flux, power spectrum, etc.) is becoming a necessary approach to deal with current cosmological and astronomical data. The aim is to reduce the data into the smallest number of summary statistics with minimum loss of information. Massively optimized parameter estimation and data (MOPED; Heavens et al. 2000) is a popular approach to summarizing the data, wherein the summaries are linear combinations of the data, reducing the number of data points down to the number of model parameters describing the data. MOPED is entirely lossless, under the assumption that the noise is independent of the model parameter and the likelihood, at least up to the first approximation, is Gaussian. However, using a linear combination of the data for the compression might not be the most optimal approach. Using machine-learning, the IMNN provides a more convenient and informative way of compressing the data into nonlinear summaries (check Prelogović & Mesinger 2024, for a recent work showing the constraining power of several 21 cm summary statistics using IMNN).

Drawing motivation from the MOPED algorithm, IMNN aims to find some transformation,  $f: \mathbf{d} \rightarrow \mathbf{x}$ , which maps the data ( $\mathbf{d}$ ) to the compressed summary ( $x_\alpha$ , for the model parameter  $\alpha$ ) (check Charnock et al. 2018, for reference).

It transforms the original likelihood into the form

$$-2 \ln \mathcal{L}(\mathbf{x}|\vartheta) = (\mathbf{x} - \mu_f(\vartheta))^T \mathbf{C}_f^{-1} (\mathbf{x} - \mu_f(\vartheta)), \quad (5)$$





**Fig. 1.** Plots showing the modeling of the variation of Ly $\alpha$  forest flux power spectrum with astrophysical parameters,  $T_0$  and  $\gamma$  at  $z = 3$ . The top panels show the plots of the Ly $\alpha$  forest flux power spectrum corresponding to fiducial and varied  $T_0$  and  $\gamma$ . We added Gaussian noise to the sightlines with an S/N distribution ranging from 20 to 100 (S/N distribution is uniform in the log scale, making the distribution have more low S/N sightlines). We plot the power spectrum corresponding to the fiducial simulation (red) and the varied parameters (blue and green) corresponding to mock Ly $\alpha$  with this S/N distribution. The error bars correspond to bootstrapping errors computed over 5000 sightlines for a sample size of 50 sightlines with 10000 bootstrap realizations. The power spectra corresponding to the simulation runs (with similar S/N distribution) with fiducial parameters but different initial seed density fields are plotted with colored hollow points in the middle panels. We modeled the power spectrum based on the curves in the top panels. We then used the posterior distribution by running an MCMC with flat priors (see Eq. 4) to estimate the parameter values (see Sect. 3). In the middle panels, hollow blue, green, and red points correspond to the fiducial simulations with different seeds, and the posterior estimates of the parameters (and the posterior standard deviation) are given in the plots. In the bottom panels, we show the sensitivity of the parameter estimates based on power spectrum to noise levels when we lower the S/N distribution ( $0.85 \times \text{S/N}_{\text{fid}}$ ) of the sightlines and use the same power spectrum modeling to estimate the parameters.

where

$$\mu_f(\vartheta) = \frac{1}{n_s} \sum_{i=1}^{n_s} \mathbf{x}_i^s \quad (6)$$

is the mean of  $n_s$  summaries.  $\vartheta$  is the set of model parameters and  $C_f$  is the covariance matrix. The modified Fisher information matrix can be expressed as

$$\mathbf{F}_{\alpha\beta} = \text{Tr}[\mu_{f,\alpha}^T \mathbf{C}_f^{-1} \mu_{f,\beta}], \quad (7)$$

where  $\mu_{f,\alpha}$  is the partial derivative of  $\mu_f$  with respect to the model parameter. Since the model parameters appear only in simulations, numerical differentiation was done to compute this. The numerical differentiation was performed using three different simulations, one at the fiducial parameter value and the other two at some small deviations from the fiducial parameter. We then used a neural network to find this mapping function with the Fisher information as the reward function (which maximizes the Fisher information). This ensured a mapping that preserves maximal information. After this mapping, one can then get model parameter estimates,  $\vartheta_\alpha$ , for the compressed testing data using score-compression,

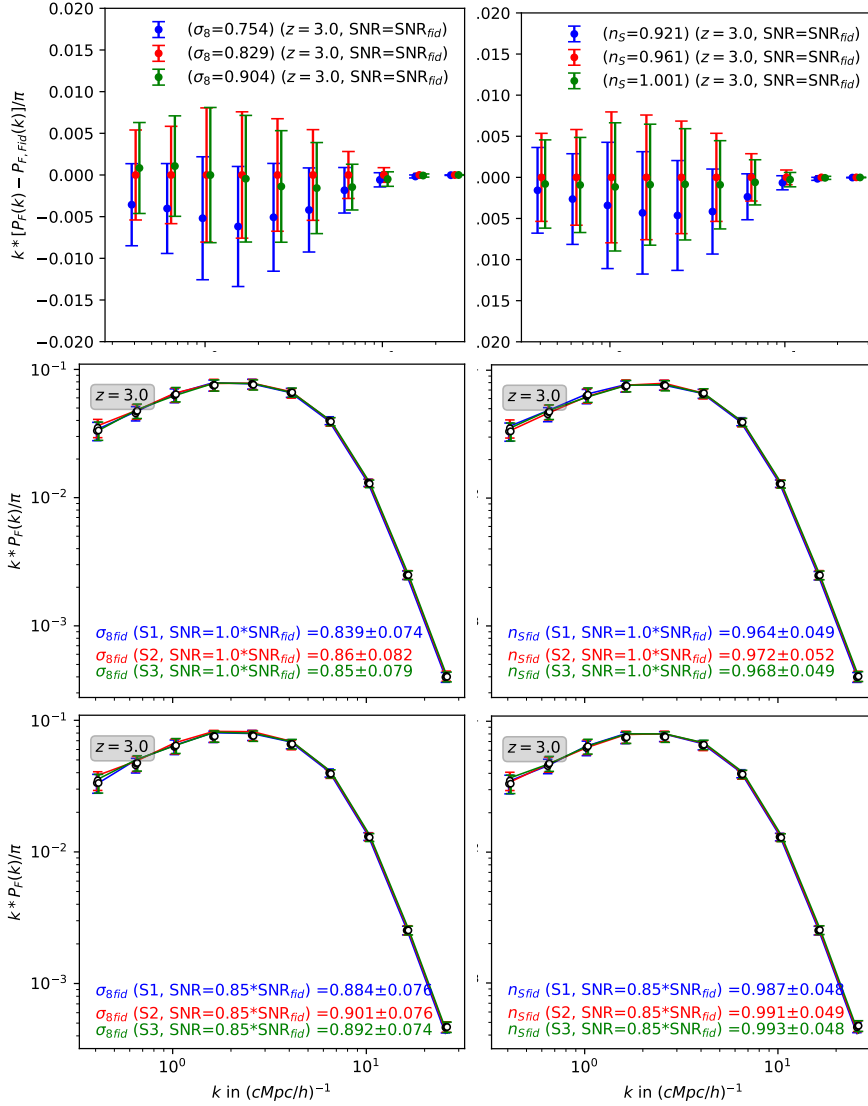
$$\vartheta_\alpha = \vartheta_\alpha^{\text{fid}} + \mathbf{F}_{\alpha\beta}^{-1} \mu_{f,\beta}^T \mathbf{C}^{-1} (x - \mu_f). \quad (8)$$

In this work, we used the IMNN approach to extract maximal information out of Ly $\alpha$  forest transmitted flux in the Fourier domain and performed 1D model parameter estimation on astrophysical parameters,  $T_0$  and  $\gamma$ , and cosmological parameters,  $\sigma_8$  and  $n_s$ . We did this individually for each parameter, using three simulations for each of them, since training the IMNN for an N-D parameter estimation requires training sets in which the parameters are varied simultaneously and we currently do not have access to such simulations (in Sect. 8, we demonstrate a 2D parameter estimation case for  $T_0$  and  $\Gamma_{\text{HI}}$ , where  $\Gamma_{\text{HI}}$  has been varied for all  $T_0$  values as a post-processing step without the need to run new simulations). Using IMNN, we first compressed the training set to a summary statistic and then used it to get parameter estimates from the testing data set using Eq. (8).

## 5. Training the neural network

For 1D parameter estimation of  $T_0$ ,  $\gamma$ ,  $\sigma_8$ , and  $n_s$ , we used the `NumericalGradientIMNN`<sup>1</sup> subclass of IMNN, which uses the derivatives of the network outputs with respect to the physical

<sup>1</sup> <https://docs.aquila-consortium.org/imnn/main/pages/examples/subclasses/NumericalGradientIMNN/NumericalGradientIMNN.html>



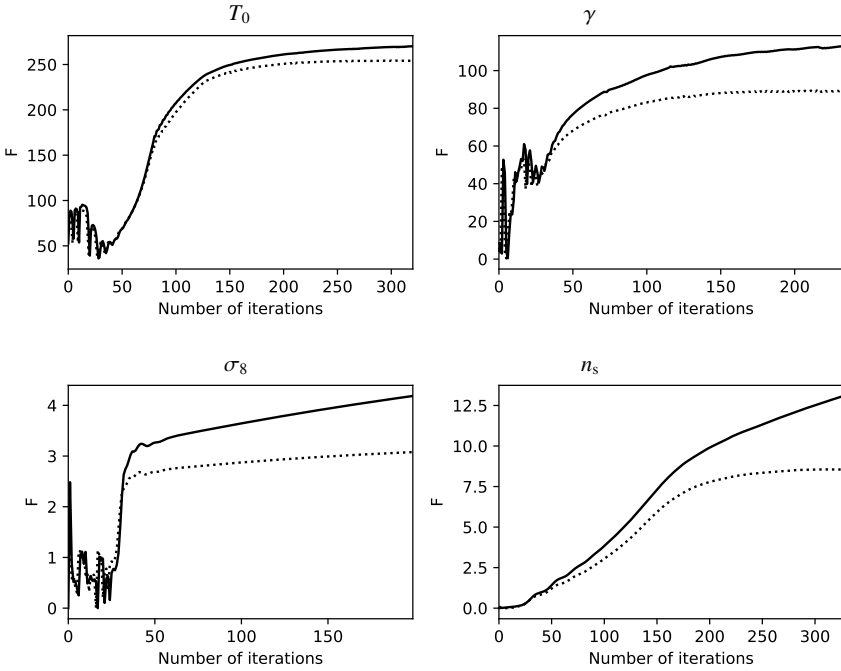
**Fig. 2.** Same as Fig. 1 but with cosmological parameters  $\sigma_8$  and  $n_s$ . In the top panels, however, we plot the difference between the power spectrum at a certain parameter value to the one at the fiducial parameter. This was done for a better visualization, since the variations in  $\sigma_8$  and  $n_s$  do not cause an appreciable change in the power spectrum within the error bars shown.

model parameters necessary to fit an IMNN. The simulations at parameter values above and below the fiducial values were used for this. To train the IMNN, we generated 5000 sightlines in total. We used 3000 of these as the training set for the NN. The remaining 2000 sightlines were used as the validation set for the training. The validation set was used at each training epoch to validate the NN trained on the training set and then adjust the network hyperparameters accordingly. As an input to the IMNN, we used the 1D field  $\sqrt{k}\delta_F(k)$ , where  $\delta_F(k)$  is the Fourier-transformed flux deviation field and  $k$  is the corresponding wave number, ranging linearly from  $0.314$  to  $31.4 h \text{ kpc}^{-1}$  (the same range as for the power spectrum analysis using MCMC) and containing 197 entries.

We used three different simulations, one at the fiducial parameter value,  $\vartheta_0$ , and the other two at  $\pm\Delta\vartheta$ , with  $\vartheta$  corresponding to each of the parameters  $\{T_0, \gamma, \sigma_8, n_s\}$ , to compute the variation in the mean summary statistic with respect to the model parameters essential for calculating the Fisher information (Eq. 7) and for model parameter estimation (Eq. 8). We did this separately for each of the redshifts ( $z = 2, 3$  and  $4$  for  $T_0$  and  $\gamma$ ). In the case of  $\sigma_8$  and  $n_s$ , the effect of varying the parameters on the Fourier space grows weaker with decreasing redshift. We find that the training doesn't converge with the amount of train-

ing data at hand for  $z = 2$  and that it requires a larger simulated volume to train properly. So, for  $\sigma_8$  and  $n_s$ , we stuck to  $z = 3$ , and  $4$  only. The IMNN was then trained over 3000 sightlines for each parameter value at each redshift value separately. For each of these sightlines, the Gaussian noise added to the transmitted flux was derived from the  $S/N_{\text{Fid}}$  distribution mentioned earlier. Since the Fourier-transformed Ly $\alpha$  spectra are quite noisy individually, we performed a running mean over 5 pixels in the Fourier profile. We find that this makes the training much more stable without the loss of significant information. We trained a separate network for each parameter and each redshift.

Various training parameters, such as learning rates and the structure of the network, are mentioned in Table B.1. The choice of the training parameters was based on trial and run. We also ran the training with different initial random seeds (30 in total) for the NN to make the exercise more robust and then used all the trained neural estimates on our testing set to estimate the parameters. We find that 30 random seeds for the training is sufficient enough to sample the variance in the network outputs for different random initializations. The parameter estimation was done using the combined results of all the trained NNs. Examples of a single training instance at  $z = 3.0$  and the evolution of Fisher information with the number of iterations in training are



**Fig. 3.** IMNN training example (using NumericalGradientIMNN subclass of IMNN) corresponding to the astrophysical parameters,  $T_0$  and  $\gamma$  (top panels), and cosmological parameters,  $\sigma_8$  and  $n_s$  (bottom panels), for  $z = 3.0$ . The plots show the evolution of Fisher information during training with the training epoch. The solid lines correspond to the training sample and the dotted lines correspond to the validation sample. The training was stopped at the maximum iteration values shown in the x axis of the above plots. This choice was based on the training epoch when the Fisher information extracted from the validation sample reaches a saturation value and shows no further improvement with additional training epochs.

shown in Fig. 3. Currently, we have been running the training on a CPU, and each training instance takes under 1 CPU-hour. We will eventually utilize GPUs to train the IMNN over a multiple-parameter space, in which we expect it to be dramatically faster.

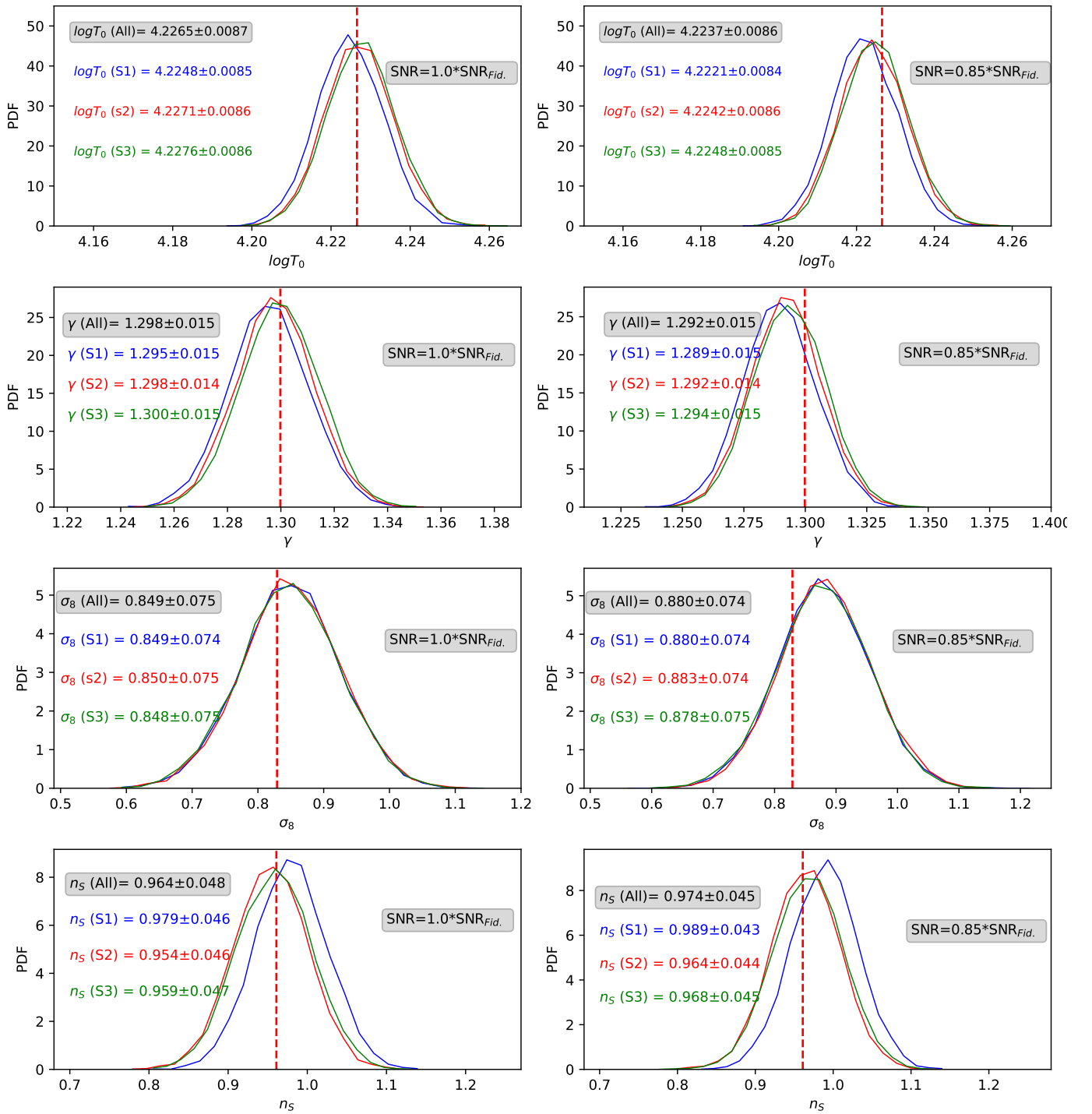
## 6. Parameter estimation with the information-maximizing neural network

As a testing set for the parameter estimation, we used three different simulations run with the same fiducial parameter values used for training, but with different initial random seeds, and therefore showing the network sightlines that it never saw in training. Currently, we do the testing at the fiducial values of parameters since we only have access to simulations that have different initial seeds from the ones we trained the NN on. In the left panels of Fig. 4, we show the distribution of the estimated parameters at  $z = 3.0$  computed using Eq. (8) for the testing set of sightlines with S/N values derived from the  $S/N_{\text{Fid}}$  distribution. Each realization in the distribution of the estimated parameters is a bootstrap realization of a sample size of 50 mock spectra over the entire validation set. The distributions of the parameters were generated with 10 000 bootstrap realizations for each parameter. The mean and the associated standard deviation of the estimated parameters are shown in Table 1 for all three simulations. The mean and associated error estimates over all three simulations (taking into account the effect of cosmic variance on the error estimates) are also quoted in the plots, along with Table 1. Comparing these estimates with those from the MCMC approach using the power spectrum, we find that the error estimates with IMNN are 1.52 times smaller in the case of  $\log T_0$  and 1.27 times in the case of  $\gamma$ . However, we do not see any significant improvements in the  $\sigma_8$  and  $n_s$  estimates with IMNN. The reason is that varying  $\sigma_8$  primarily changes the total power in the Fourier space, and thus the overall amplitude of the power spectrum. In this case, the MCMC approach with the power spectrum only performs well enough to extract nearly maximal information from the Fourier space. In Fig. 5, we have also shown that the IMNN works well in estimating  $\sigma_8$  and  $n_s$  parameter values which is different than the ones that it has been trained on.

However, it is worth noting that even in the cases in which IMNN performs on par with the standard MCMC approach, it offers a significant boost in speed. Once trained, summarizing a new dataset and estimating the model parameters from it is almost an instantaneous process in comparison to MCMC. For instance, in estimating the model parameter  $\sigma_8$ , generating a power spectrum from 5000 sightlines takes about 30 CPU-seconds, and then subsequently estimating the parameter using MCMC takes about 35 CPU-seconds. In comparison, estimating parameters using a trained NN on 5000 sightlines takes only about 0.1 CPU-seconds, corresponding to a speed-up factor of  $(30 + 35)/0.1 = 650$  with respect to the MCMC approach. Thus, using an NN-based approach provides a substantial advantage in dealing with large astrophysical or cosmological datasets.

### 6.1. Effect of noise levels

Next, we checked the robustness of our method against variations in noise levels. Similar to what we did for the MCMC approach with the power spectrum, we used the NN trained on sightlines with noise levels derived from the  $S/N_{\text{Fid}}$  distribution and then tested it on sightlines with  $0.85 \times S/N_{\text{Fid}}$ . The corresponding estimated parameter values are shown in the right panels of Fig. 4 and also quoted in Table 1. Comparing the deviations with those from the MCMC approach, we find that IMNN estimates are 6.25 and 10.0 times (ratio of the amount of deviation in parameter estimates at  $0.85 \times S/N_{\text{Fid}}$  in comparison to  $S/N_{\text{Fid}}$ ) less sensitive to variations in noise levels for  $\log T_0$  and  $\gamma$  at  $z = 2$ , respectively. At  $z = 3$ , the IMNN estimates are less sensitive to noise variations by a factor of 4.17, 2.86, 1.41, and 2.2 times in the case of  $\log T_0$ ,  $\gamma$ ,  $\sigma_8$ , and  $n_s$ , respectively. At  $z = 4$ , the dependence on noise variations is almost similar for the MCMC and IMNN approaches. In the case of  $n_s$ , though it might seem like there is an improvement (with an S/N deviation ratio of 1.5), it might be worth noting that there is not much variation seen in the  $n_s$  estimate itself with noise. In short, we can conclude that the IMNN estimates are more robust against noise variations in comparison to the MCMC approach at  $z = 2$  and 3, with the approach being more robust at lower redshifts.



**Fig. 4.** Estimation of  $T_0$ ,  $\gamma$ ,  $\sigma_8$ , and  $n_s$  using score-compression from IMNN on the Fourier-transformed Ly $\alpha$  forest transmitted flux for three simulations run at fiducial parameter values with different initial random seeds. Each realization in the distribution of the estimated parameters is a bootstrap realization of a sample size of 50 mock spectra over the entire testing set. The distribution of the parameters was generated with 10 000 bootstrap realizations for each parameter. On the left panels, we plot the estimation for simulated sightlines that have the S/N distribution  $S/N_{\text{fid}}$  with IMNN trained with sightlines that have  $S/N_{\text{fid}}$ . On the right panels, we plot the estimation for  $0.85 \times S/N_{\text{fid}}$  with IMNN trained with  $S/N_{\text{fid}}$  sightlines. This shows the sensitivity of the trained NNs to noise level deviations in the data.

## 6.2. Effects of instrumental smoothing

We investigated the effects of instrumental smoothing in the parameter estimation procedure by training the NNs with sightlines convolved with a Gaussian profile. We used Gaussian profiles that have FWHM = 6, 50, and 150  $\text{km s}^{-1}$  to simu-

late the usual convolutional scales of high-, medium-, and low-resolution Ly $\alpha$  forest spectra. In Table 2, we present the results of the effects of instrumental smoothing on the estimation of parameters with the IMNN approach and compare it with the results from the MCMC approach. For the parameters  $T_0$  and  $\gamma$ , we find that the estimation with IMNN is

**Table 1.** Estimation of parameters (for different redshift ranges).

Parameter	$z$	Input parameter	MCMC estimates ( $S/N_{\text{Fid}}$ )	MCMC estimates ( $0.85 \times S/N_{\text{Fid}}$ )	IMNN estimates ( $S/N_{\text{Fid}}$ )	IMNN estimates ( $0.85 \times S/N_{\text{Fid}}$ )	S/N deviation Ratios ( $\frac{\text{MCMC}}{\text{IMNN}}$ )	Estimate error deviation Ratios ( $\frac{\text{MCMC}}{\text{IMNN}}$ )
$\log(T_0)$ ( $4.040 \pm 0.176$ )	2.0	4.040	$4.0355 \pm 0.0246$	$4.0189 \pm 0.0254$	$4.0394 \pm 0.0130$	$4.0367 \pm 0.0125$	6.25	1.89
$\gamma$ ( $1.3 \pm 0.3$ )	2.0	1.3	$1.294 \pm 0.027$	$1.274 \pm 0.028$	$1.299 \pm 0.018$	$1.297 \pm 0.0185$	10.0	1.50
$\log(T_0)$ ( $4.225 \pm 0.176$ )	3.0	4.225	$4.223 \pm 0.013$	$4.2109 \pm 0.0134$	$4.2265 \pm 0.0087$	$4.2237 \pm 0.0086$	4.17	1.52
$\gamma$ ( $1.3 \pm 0.3$ )	3.0	1.3	$1.295 \pm 0.019$	$1.278 \pm 0.019$	$1.298 \pm 0.015$	$1.292 \pm 0.015$	2.86	1.27
$\sigma_8$ ( $0.83 \pm 0.075$ )	3.0	0.83	$0.850 \pm 0.079$	$0.892 \pm 0.076$	$0.849 \pm 0.075$	$0.880 \pm 0.074$	1.41	1.05
$n_s$ ( $0.961 \pm 0.040$ )	3.0	0.961	$0.968 \pm 0.050$	$0.990 \pm 0.049$	$0.964 \pm 0.048$	$0.974 \pm 0.045$	2.2	1.04
$\log(T_0)$ ( $4.060 \pm 0.176$ )	4.0	4.060	$4.0615 \pm 0.0138$	$4.0569 \pm 0.0137$	$4.0631 \pm 0.0114$	$4.0584 \pm 0.0114$	0.98	1.21
$\gamma$ ( $1.3 \pm 0.3$ )	3.0	1.3	$1.306 \pm 0.029$	$1.293 \pm 0.030$	$1.298 \pm 0.022$	$1.285 \pm 0.022$	0.93	1.32
$\sigma_8$ ( $0.83 \pm 0.075$ )	4.0	0.83	$0.825 \pm 0.048$	$0.843 \pm 0.049$	$0.830 \pm 0.043$	$0.848 \pm 0.045$	1.0	1.12
$n_s$ ( $0.961 \pm 0.040$ )	3.0	0.961	$0.957 \pm 0.018$	$0.96 \pm 0.019$	$0.955 \pm 0.016$	$0.957 \pm 0.016$	1.5	1.13

**Table 2.** Estimation of parameters for different instrumental smoothing in both training and testing set (Gaussian;  $z = 3.0$ ).

Parameter	Input parameter	Output parameter (Error corresponding to 30 sightlines)			
		No smoothing	FWHM = $6 \text{ km s}^{-1}$	FWHM = $50 \text{ km s}^{-1}$	FWHM = $150 \text{ km s}^{-1}$
$\log(T_0)$ (IMNN) ( $4.225 \pm 0.176$ )	4.225	$4.2265 \pm 0.0087$	$4.2237 \pm 0.0088$	$4.2230 \pm 0.0360$	$4.3322 \pm 0.4573$
$\log(T_0)$ (MCMC) ( $4.225 \pm 0.176$ )	4.225	$4.2228 \pm 0.0131$	$4.2226 \pm 0.0135$	$4.2277 \pm 0.0325$	$4.698 \pm 0.2260$
$\gamma$ (IMNN) ( $1.3 \pm 0.3$ )	1.3	$1.298 \pm 0.015$	$1.297 \pm 0.015$	$1.297 \pm 0.040$	$1.268 \pm 0.168$
$\gamma$ (MCMC) ( $1.3 \pm 0.3$ )	1.3	$1.295 \pm 0.019$	$1.297 \pm 0.020$	$1.301 \pm 0.041$	$1.279 \pm 0.162$
$\sigma_8$ (IMNN) ( $0.83 \pm 0.075$ )	0.83	$0.849 \pm 0.075$	$0.850 \pm 0.075$	$0.835 \pm 0.084$	$0.861 \pm 0.085$
$\sigma_8$ (MCMC) ( $0.83 \pm 0.075$ )	0.83	$0.850 \pm 0.079$	$0.844 \pm 0.080$	$0.848 \pm 0.085$	$0.848 \pm 0.099$
$n_s$ (IMNN) ( $0.961 \pm 0.040$ )	0.961	$0.964 \pm 0.048$	$0.966 \pm 0.047$	$0.966 \pm 0.049$	$0.972 \pm 0.70$
$n_s$ (MCMC) ( $0.961 \pm 0.040$ )	0.961	$0.968 \pm 0.050$	$0.969 \pm 0.050$	$0.963 \pm 0.058$	$0.966 \pm 0.064$

better with high-resolution spectra that have  $\text{FWHM} = 6 \text{ km s}^{-1}$ . The estimates with  $\text{FWHM} = 6 \text{ km s}^{-1}$  are roughly similar to what we obtained with unsmoothed sightlines. In the case of  $\text{FWHM} = 50$  and  $150 \text{ km s}^{-1}$ , we do not see any improvements in the parameter estimates using IMNN over MCMC with power spectrum. In fact, the parameter estimation fails for  $T_0$  and  $\gamma$  at  $\text{FWHM} = 150 \text{ km s}^{-1}$ , as can be seen from the large error bars. This was expected as the thermal effects

on Ly $\alpha$  forest are small-scale effects that alter the broadening of Ly $\alpha$  absorption lines. Large-scale smoothing introduced by low-resolution spectrographs washes away this small-scale information. In conclusion, the improvements that we observed with IMNN over MCMC with the power spectrum come from enhanced extraction of small-scale information by the IMNN, which can only be realized with high-resolution spectra.



**Table 3.** Effects of continuum variation on the estimation of astrophysical parameters,  $\log T_0$  and  $\gamma$ , at  $z = 3.0$ . In the case of cosmological parameters,  $\sigma_8$  and  $n_s$ , IMNN fails to extract continuum-independent information out of the Fourier space.

Parameter	Input parameter	Output parameter (Error corresponding to 30 sightlines)	
		Continuum ( $\mu = 1.0, \sigma = 0$ )	Continuum ( $\mu = 0.9, \sigma = 0.1$ )
$\log(T_0)$ (IMNN) ( $4.225 \pm 0.176$ )	4.225	$4.224 \pm 0.009$	$4.224 \pm 0.010$
$\log(T_0)$ (MCMC) ( $4.225 \pm 0.176$ )	4.225	$4.223 \pm 0.013$	$4.204 \pm 0.014$
$\gamma$ (IMNN) ( $1.3 \pm 0.3$ )	1.3	$1.300 \pm 0.015$	$1.301 \pm 0.015$
$\gamma$ (MCMC) ( $1.3 \pm 0.3$ )	1.3	$1.295 \pm 0.019$	$1.265 \pm 0.021$

On the other hand,  $\sigma_8$  and  $n_s$  primarily affect the global amplitude of the power spectrum, as was mentioned earlier. So, we do not find any appreciable change in the parameter estimates by introducing a Gaussian convolution with FWHM = 6, 50, and  $150 \text{ km s}^{-1}$ . The IMNN approach works on par with the MCMC using the power spectrum in all cases.

### 6.3. Effect of continuum uncertainty

In this section, we check the effectiveness of IMNN in extracting continuum-independent information from the Fourier space for parameter estimation. For this, we first generated a training set of mock Ly $\alpha$  sightlines with their continuum levels altered. We generated random continuum levels for each sightline from a Gaussian distribution of mean  $\mu = 1$  and standard deviation  $\sigma = 0.2$ , and then normalized the transmitted flux for each sightline to these random continuum levels. The validation set was also treated in the same way. The NN was then trained on these continuum-altered Ly $\alpha$  transmitted flux sightlines. We then estimated the parameters using these trained NNs for sightlines that have continuum levels of ( $\mu = 1, \sigma = 0$ ) and ( $\mu = 0.9, \sigma = 0.1$ ). The estimated astrophysical parameters,  $\log T_0$  and  $\gamma$ , are presented in Table 3 at  $z = 3$ . We also compare our results with the standard MCMC approach for parameter estimation using the power spectrum. We find that training the IMNN on sightlines that have varying continuum levels makes it able to extract continuum-independent information. The estimated  $\log T_0$  values for ( $\mu = 1, \sigma = 0$ ) and ( $\mu = 0.9, \sigma = 0.1$ ) sightlines using IMNN are  $4.224 \pm 0.009$  and  $4.224 \pm 0.010$ , respectively, as opposed to  $4.223 \pm 0.013$  and  $4.204 \pm 0.014$  using the standard MCMC approach. In the case of  $\gamma$ , the estimated parameters using IMNN are  $1.300 \pm 0.015$  and  $1.301 \pm 0.015$  as opposed to  $1.295 \pm 0.019$  and  $1.265 \pm 0.021$  using the MCMC approach. So, IMNN clearly outperforms the MCMC approach when considering robustness against continuum uncertainties in estimating  $T_0$  and  $\gamma$ .

The situation is different in the case of the cosmological parameters,  $\sigma_8$  and  $n_s$ , which are known to be sensitive to the global amplitude of the power spectrum, as opposed to the astrophysical parameters,  $T_0$  and  $\gamma$ , which are more sensitive to the small-scale information. As was expected, the IMNN trained on ( $\mu = 1.0, \sigma = 0.2$ ) sightlines doesn't estimate the parameters accurately for sightlines that have ( $\mu = 1.0, \sigma = 0.0$ ) and ( $\mu = 0.9, \sigma = 0.1$ ). In the case of  $\sigma_8$ , the estimated parameters for these two cases are  $0.891 \pm 0.090$  and  $0.887 \pm 0.097$ , respectively. In the case of  $n_s$ , the estimated parameters are  $0.995 \pm 0.050$  and  $0.991 \pm 0.062$ .

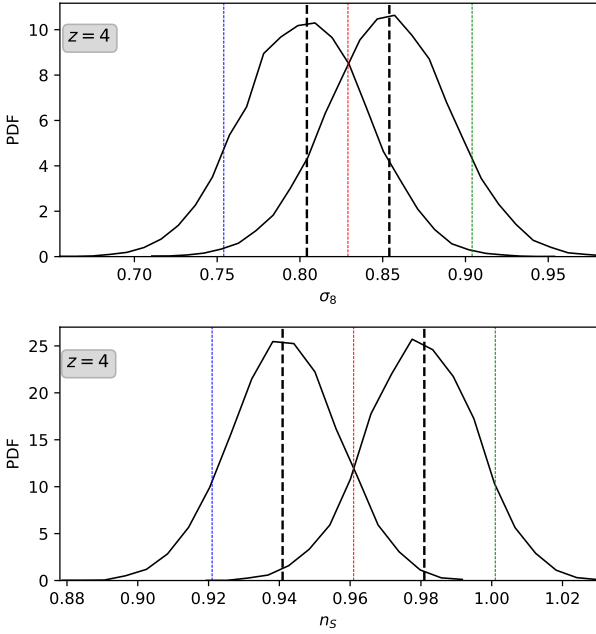
## 7. Robustness check with a different simulation

To check the robustness of parameter estimation using IMNN on Ly $\alpha$  forest spectra in the Fourier domain, we used alternate simulations to estimate parameters based on the neural trained with the Sherwood-Relics simulations. For this, we used mock Ly $\alpha$  forest spectra from the CAMELS project (Villaescusa-Navarro et al. 2021), generated using a  $(25 h^{-1} \text{ cMpc})^3$  IllustrisTNG simulation box at  $z = 2$ . We did not perform this exercise at  $z = 3$  since IGM temperatures are elevated in the case of Sherwood-Relics simulations due to He II reionization, which makes the trained NNs unfit for testing on IllustrisTNG simulations that do not incorporate this. The cosmological parameters used in this simulation are ( $\{\Omega_m, \Omega_b, \Omega_\Lambda, \sigma_8, n_s, h\} = \{0.30, 0.049, 0.70, 0.84, 0.9624, 0.6711\}$ ). We first linearly interpolated the spectra to the grid size of our simulation and then added random Gaussian noise with S/N levels drawn from the  $S/N_{\text{Fid}}$  distribution mentioned before. We then matched the mean flux of the sightlines to the mean flux of our fiducial simulation. We simultaneously checked the parameter estimation for  $\log T_0$  using the standard MCMC approach from the power spectrum with the IMNN approach.

In Fig. 6, we have shown the IMNN estimates of  $\log T_0$  and  $\gamma$  for the CAMELS simulations with the IMNN that have been trained with Sherwood-Relics simulations. The expected values of  $\log T_0$  and  $\gamma$  in the CAMELS simulations are approximately 3.97 and 1.26, respectively. In the case of the MCMC with the power spectrum, the  $\log T_0$  predicted is  $3.997 \pm 0.036$ . With IMNN, the value predicted is  $3.9898 \pm 0.0154$ . We see that the estimated values are consistent with each other as well as the expected values from CAMELS within  $1\sigma$  error bars, with the IMNN predicting it with  $\approx 2.34$  times better accuracy in the case of  $\log T_0$  and  $\approx 1.73$  times better accuracy in the case of  $\gamma$ . We thus conclude that this NN approach is relatively robust and does not learn simulation-specific features from the Fourier-transformed Ly $\alpha$  forest spectra.

## 8. Parameter inference in two dimensions

In the previous sections, we used the IMNN approach to estimate thermal and cosmological parameters individually. However, in realistic scenarios, such estimations would be performed jointly over several parameters, and then individual parameter estimates would be obtained by marginalizing over the other parameters. In this section, we demonstrate a 2D parameter estimation. Ideally, we would have liked to do this for the thermal parameters,  $T_0$  and  $\gamma$ . However, we find that IMNN doesn't properly learn to perform 2D parameter inference and lift the degeneracy between



**Fig. 5.** Estimation of  $\sigma_8$  and  $n_s$  at  $z = 4$  for parameter values (two dashed black lines) different from the parameter values with which the IMNN has been trained (dashed blue, red, and green lines). The black curves show the PDF of the estimated parameter values (at two different values). Each realization in the distribution of the estimated parameters is a bootstrap realization of a sample size of 50 mock spectra over the entire testing set. The distributions of the parameters were generated with 10 000 bootstrap realizations for each parameter.

these two degenerate parameters, due to the lack of training samples in which both  $T_0$  and  $\gamma$  are varied simultaneously. Since we currently lack such simulations in the SHERWOOD suite, as a proof of concept we demonstrate 2D parameter inference for the two parameters,  $T_0$  and the HI photoionization rate,  $\Gamma_{\text{HI}}$ . The 2D parameter inferences,  $T_0$  and  $\gamma$ , using several parameter variation instances, similar to Nayak et al. (2024), will be addressed in a future work.

For this work, we used our original three simulations at  $z = 3$  with  $\log T_0 = 4.225 \pm 0.176$  and simply generated sightlines corresponding to  $\Gamma_{\text{HI}} = \Gamma_{\text{HI}}/1.25, \Gamma_{\text{HI}}$  and  $1.25 \times \Gamma_{\text{HI}}$  to generate nine simulations in total with varied  $T_0$  and  $\Gamma_{\text{HI}}$ . We used the SimulatorIMNN<sup>2</sup> subclass for 2D parameter inference as we find that it is more suited to simultaneous parameter variations. It is also suited to future exercises in which we shall train the NN over simulations with different parameters in several dimensions. Unlike the NumericalGradientIMNN used in the previous sections – which uses the numerical gradient between the training samples corresponding to the varied parameter values – SimulatorIMNN requires a function to generate the training samples based on input parameter values. We did this by defining a function that linearly interpolates the Fourier-transformed field between the nine simulations with varied  $T_0$  and  $\Gamma_{\text{HI}}$ . Such an interpolation scheme becomes possible in our case since we used simulations that have the same initial density fields and differ only in the parameters  $T_0$  and  $\Gamma_{\text{HI}}$ . This means that we can easily interpolate in the parameter space between sightlines, since they essentially trace similar density fields that differ only in  $T_0$  and  $\Gamma_{\text{HI}}$ . Currently, to keep the exercise simple, we avoid modeling the noise effects on the Fourier-transformed fields.

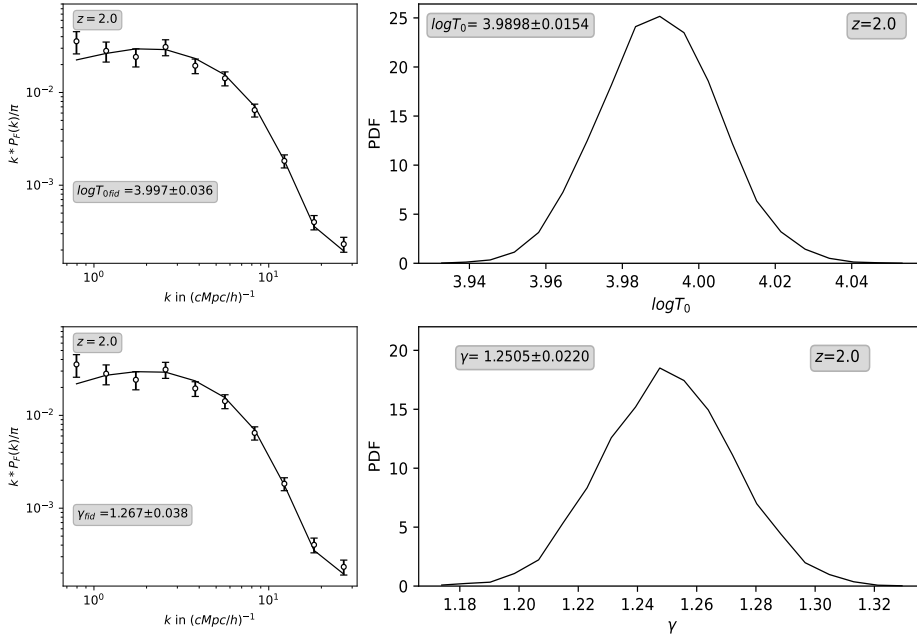
We used sightlines that have a uniform noise distribution with S/N = 50. In order to compare the IMNN approach with the standard MCMC approach using power spectra, we adopted the same interpolation scheme to model the power spectrum as a function of  $T_0$  and  $\Gamma_{\text{HI}}$ , which we used subsequently in the likelihood of the MCMC analysis.

For the training of IMNN, we used 5000 sightlines corresponding to each parameter value shot from the nine simulation boxes. For SimulatorIMNN, the training does not require a validation set. The network architecture and training hyperparameters used are mentioned in Table B.1. We tested the trained NN on 5000 sightlines, each corresponding to the three simulation boxes run with fiducial parameters but with different initial seed density fields. The training was done at  $z = 3$  and the evolution of the Fisher information with the number of iterations in training is shown in the left panel of Fig. 7. In the right panel of Fig. 7, we show the 68% contour enclosing 68% of the highest-density estimates for the 2D histogram of the  $\log T_0$  and  $\log \Gamma_{\text{HI}}/\Gamma_{\text{HI}}$ , Fid estimates based on the IMNN approach, compared with the equivalent posterior from the standard MCMC approach. Similar to the previous exercises, each estimate corresponds to bootstrap realizations over 50 sightlines for both the IMNN and MCMC approaches. For the 2D estimation, the distribution of the parameter values was obtained with 50 000 bootstrap realizations. We also plot the distribution functions for both these parameters individually by marginalizing over the other parameters. The average estimates along with the associated error (computed as the width of 68% interval of the distribution function) are quoted along with the plots. We find that both approaches predict the fiducial parameters ( $\log T_0 = 4.225$  and  $\log \Gamma_{\text{HI}}/\Gamma_{\text{HI}}, \text{Fid} = 0.0$ ) well within the error bars. For  $\log \Gamma_{\text{HI}}/\Gamma_{\text{HI}}, \text{Fid}$ , the errors are similar between the two approaches, with the MCMC error bars being about 13% smaller than the IMNN error bars. This was expected, as IMNN gives similar estimates as MCMC when the associated parameter has a more global effect (such as  $\Gamma_{\text{HI}}$ , which uniformly alters the photoionizing background at all scales). The IMNN estimates get significantly better for  $\log T_0$ , whose effect is on small scales, with MCMC error bars being 95% larger than the IMNN error bars. This exercise here demonstrates a case for 2D parameter estimation using IMNN, which outperforms the standard MCMC approach in extracting small-scale information from the Ly $\alpha$  forest in Fourier space. This also sets a pathway for performing a full N-D parameter estimation for Ly $\alpha$  forest using IMNN. The scalability of the method to a larger number of dimensions will be the subject of future work. For starters, one needs a sufficiently large number of simulations over which to train the network, which isn't trivial to achieve unless one resorts to semi-analytical approximations or fast emulators for the purposes. Makinen et al. (2021) only addresses a 2-dimensional parameter space, and addresses scalability only in the sense of the large numerosity of data points, rather than the number of dimensions of the underlying parameter space. Prelogović & Mesinger (2024) do investigate a larger number of parameters in the context of the 21 cm signal, and train their IMNN with about 11 000 simulations of lightcones obtained with 21cmFAST, at a computational cost of 20 k GPUh. This is only achievable thanks to the fast nature of the simulator. In the near future, we expect to be able to take advantage of similarly accelerated simulators based on the Lagrangian Deep learning framework (Rigo et al., in prep.).

## 9. Summary and discussion

With the current advent of large astrophysical and cosmological data, reducing data dimensionality is now of the utmost

<sup>2</sup> <https://docs.aquila-consortium.org/imnn/main/pages/modules.html#simulatorimnn>



**Fig. 6.** Estimation of  $T_0$  and  $\gamma$  using MCMC with power spectrum and IMNN on the Fourier-transformed Ly $\alpha$  forest transmitted flux for CAMELS simulation at  $z = 2$ . The expected parameter values are  $\log T_0 = 3.97$  and  $\gamma = 1.26$ . The power spectrum modeling for the MCMC approach and the training for the IMNN approach were done with previous Sherwood-Relics simulations. We checked the robustness of both approaches by testing them on a different simulation.

importance. Reducing the entire dataset to a set of summary statistics relevant to the problem at hand with minimum loss of information is one way of achieving that. One popular approach of summarizing large datasets is MOPED (Heavens et al. 2000), which reduces the entire dataset to the number of model parameters describing the data. One can also use NNs to obtain a mapping function that maps the dataset to a set of nonlinear summaries with minimal loss of information.

In this work, we primarily performed 1D parameter estimation for model parameters  $T_0$  (IGM temperature at mean cosmic density),  $\gamma$  (slope IGM temperature density,  $\Delta$ , relation  $T = T_0 \Delta^{\gamma-1}$ ),  $\sigma_8$ , and  $n_S$  using the IMNN (Charnock et al. 2018). We performed a 1D estimation since we find that training the NN to do N-D parameter estimation requires the training set to have simulations in which the parameters are varied simultaneously. We currently do not have access to such simulations for the model parameters mentioned, but we did perform a 2D parameter estimation for  $T_0$  and the HI photoionization rate,  $\Gamma_{\text{HI}}$ , to demonstrate how one will be able to use IMNN to perform N-D parameter estimation in the future. For the 1D parameter estimation, we used IMNN to summarize the Ly $\alpha$  forest data in the Fourier space as a single summary value with minimal loss of information. We obtained these summary values individually. One can then use the summary values to obtain the model parameter estimates. We then compared this approach with a standard technique of maximum likelihood estimation (MCMC) from the Fourier space using the power spectrum. We find that the IMNN approach leads to a significant enhancement in the parameter estimation for  $T_0$  and  $\gamma$ . For  $\log T_0$ , the enhancements are by a factor of 1.89, 1.52, and 1.21 times at  $z = 2, 3$ , and 4, respectively. For  $\gamma$ , the enhancements are by a factor of 1.5, 1.27, and 1.32. Besides enhancing the estimation of these parameters, we also find that the NNs are more robust against variation in noise levels in the spectra. We confirmed this by performing an exercise in which we estimated the parameters using a test set of spectra that have an S/N distribution that is 0.85 times lower than the fiducial distribution of S/N. At  $z = 2$  and 3, the NNs were less sensitive to this variation in noise levels than the standard MCMC approach by a factor of 6.25 and 4.17 times in the case

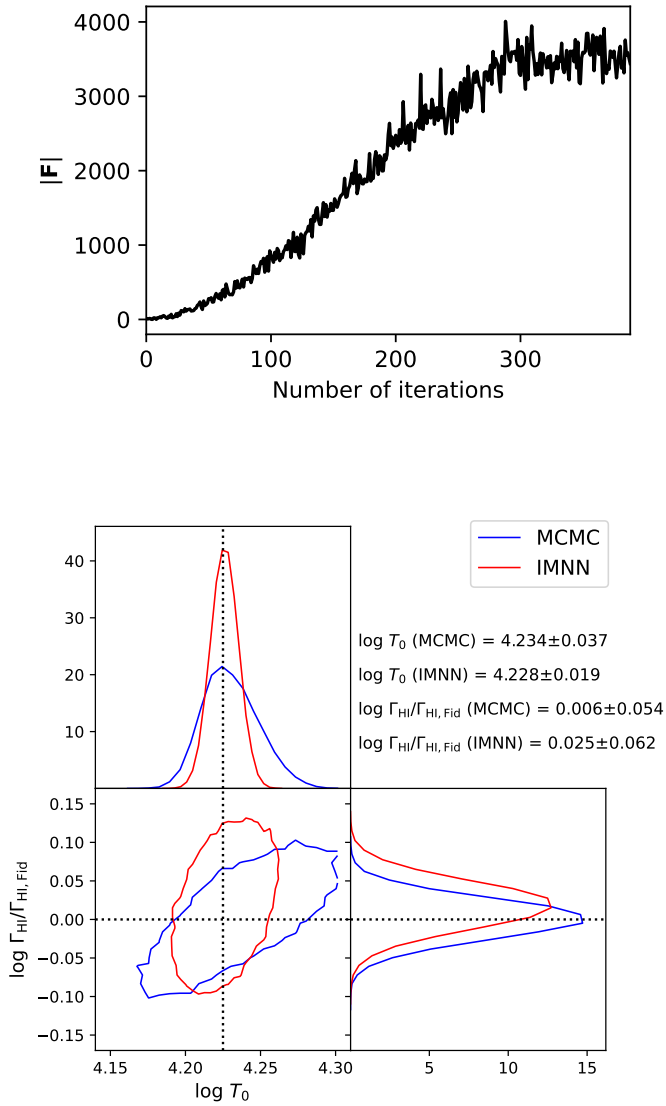
of  $\log T_0$  and by a factor of 10.0 and 2.86 times for  $\gamma$ . However, at  $z = 4$ , the sensitivity to noise variations was more or less on par with the MCMC approach. We also performed the parameter estimation using IMNN with the cosmological parameters  $\sigma_8$  and  $n_S$ . We find that the estimates and their sensitivity to noise variations were on par with the MCMC estimates. We think that this might be because varying  $\sigma_8$  or  $n_S$  primarily varies the total power in the Fourier space, and thus the global amplitude of the power spectrum. Hence, the MCMC approach with the power spectrum is good enough to extract maximal information present in the Fourier space. However, it is worth emphasizing here that even in cases in which a standard MCMC approach with power spectrum works well enough, a trained NN can offer a significant boost in speed in summarizing a new dataset and estimating the model parameters from that. This becomes especially important in the current era of large astrophysical and cosmological datasets.

We also find that IMNN can provide parameter estimates for  $T_0$  and  $\gamma$  that are more robust against continuum uncertainties in comparison to the standard MCMC estimates. It does this by extracting small-scale continuum-independent information from the Fourier domain for these parameters. It fails, however, to do this for the cosmological parameters  $\sigma_8$  and  $n_S$ , which are more sensitive to the global amplitude of the power spectrum.

Additionally, we also checked the parameter estimation process with different instrumental smoothing scales with FWHM = 6, 50, and 150 km s $^{-1}$ , corresponding to typical high-, moderate-, and low-resolution spectra. Interestingly, we find that the improvements seen in estimating  $T_0$  and  $\gamma$  are also seen for FWHM = 6 km s $^{-1}$ , but not for 50 and 150 km s $^{-1}$ . In fact, they perform on par with the MCMC estimates. Based on this, we can infer that the improvement we see in the case of IMNN comes mostly from additional small-scale information. In conclusion, IMNN works efficiently in extracting the maximal small-scale information in comparison to the MCMC approach. In doing so, it not only does not compromise robustness against noise level variations but rather improves it.

We also demonstrated a 2D parameter estimation for model parameters  $T_0$  and  $\Gamma_{\text{HI}}$  using IMNN. In doing so, we find that the





**Fig. 7.** 2D parameter estimation of  $T_0$  and  $\Gamma_{\text{HI}}$  using IMNN. Top: IMNN training plot (using SimulatorIMNN subclass of IMNN) corresponding to thermal parameter,  $T_0$ , and HI photoionization rate,  $\Gamma_{\text{HI}}$ , for  $z = 3$ . The plot shows the evolution of Fisher information during training with the training epoch. Bottom: Contour plots showing the 68% credible region for  $\log T_0$  and  $\log \Gamma_{\text{HI}}/\Gamma_{\text{HI,Fid}}$  estimates based on the IMNN approach and with MCMC. It is to be noted that in the case of MCMC, the contour plot corresponds to a posterior distribution drawn from running MCMC using a flat prior (see Eq. 4), while in the case of IMNN it corresponds to the distribution of the estimates. Accompanying this are the plots for the distribution functions (for MCMC and IMNN, respectively) for both the parameters  $\log T_0$  and  $\log \Gamma_{\text{HI}}/\Gamma_{\text{HI,Fid}}$  individually by marginalizing over the other parameters.

IMNN estimation works on par with the MCMC approach using the power spectrum for  $\Gamma_{\text{HI}}$  (with the MCMC estimates having errors about 13% smaller), but we see significant improvements in  $\log T_0$  estimates, wherein the errors are about 84% smaller. This behavior is similar to what we have seen before. Varying  $\Gamma_{\text{HI}}$  uniformly rescales the optical depth field of Ly $\alpha$  forest, and hence induces a global effect. For such parameters, the MCMC approach with the power spectrum can extract maximal information. The IMNN approach outperforms the MCMC approach for  $T_0$  (which induces small-scale effects on the Ly $\alpha$  forest) by extracting additional small-scale information.

In our upcoming work, we shall implement this on observational data comprising high-resolution Ly $\alpha$  forest spectra obtained from publicly available surveys like KODIAQ (O’Meara et al. 2017) based on the KECK/HIRES spectrograph and the SQUAD survey (Murphy et al. 2019) based on VLT/UVES. This will also be complemented with high-resolution spectra from ESPRESSO to improve upon the current estimates of the thermal evolution of the IGM.

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## Appendix A: Sherwood-Relics simulations used

Table A.1 shows the list of all Sherwood-Relics simulations used in this work with varied astrophysical and cosmological parameters for training the neural network, as well as simulations having fiducial parameters but with different initial seed density fields for testing the trained neural network.

## Appendix B: Network architecture

The details of the network parameters including the number of layers and nodes in each layer, the learning rate and regularization parameters ( $\lambda$  and  $\epsilon$ ) are given in Table B.1. We have used an additional layer in case of  $\log T_0$  using which we input the log of the  $\sqrt{k}\delta_F(k)$ . We find that this gives a better estimation of  $T_0$ . Also, we consider the summary covariances over a batch of 2000 sightlines in case of  $\log T_0$  and  $\gamma$  and 3000 in case of  $\sigma_8$ . As a stopping criterion of the training, we choose a patience value of 20, which is the number of iterations where there is no increase in the value of the determinant of the Fisher information matrix.

**Table A.1.** Sherwood-Relics simulations used.

Simulation Model	Purpose	Seed	Temperature	$\gamma$	$\sigma_8$	$n_s$	$\Gamma_{\text{HI}}$
40-1024 (Fiducial)	Training	181170	Fiducial	1.3	0.829	0.961	<a href="#">Puchwein et al. (2019)</a>
40-1024-cold	Training	181170	Fiducial/1.5	1.3	0.829	0.961	<a href="#">Puchwein et al. (2019)</a>
40-1024-hot	Training	181170	Fiducial*1.5	1.3	0.829	0.961	<a href="#">Puchwein et al. (2019)</a>
40-1024-g10	Training	181170	Fiducial	1.0	0.829	0.961	<a href="#">Puchwein et al. (2019)</a>
40-1024-g16	Training	181170	Fiducial	1.6	0.829	0.961	<a href="#">Puchwein et al. (2019)</a>
40-1024-s754	Training	181170	Fiducial	1.3	0.754	0.961	<a href="#">Puchwein et al. (2019)</a>
40-1024-s804	Training	181170	Fiducial	1.3	0.804	0.961	<a href="#">Puchwein et al. (2019)</a>
40-1024-s854	Training	181170	Fiducial	1.3	0.854	0.961	<a href="#">Puchwein et al. (2019)</a>
40-1024-s904	Training	181170	Fiducial	1.3	0.904	0.961	<a href="#">Puchwein et al. (2019)</a>
40-1024-n921	Training	181170	Fiducial	1.3	0.829	0.921	<a href="#">Puchwein et al. (2019)</a>
40-1024-n941	Training	181170	Fiducial	1.3	0.829	0.941	<a href="#">Puchwein et al. (2019)</a>
40-1024-n981	Training	181170	Fiducial	1.3	0.829	0.981	<a href="#">Puchwein et al. (2019)</a>
40-1024-n1001	Training	181170	Fiducial	1.3	0.829	1.001	<a href="#">Puchwein et al. (2019)</a>
40-1024 (lowered $\Gamma_{\text{HI}}$ )	Training	181170	Fiducial	1.3	0.829	0.961	<a href="#">Puchwein et al. (2019)</a> /1.25
40-1024 (elevated $\Gamma_{\text{HI}}$ )	Training	181170	Fiducial	1.3	0.829	0.961	<a href="#">Puchwein et al. (2019)</a> ×1.25
40-1024-seed001	Testing	965431	Fiducial	1.3	0.829	0.961	<a href="#">Puchwein et al. (2019)</a>
40-1024-seed002	Testing	126642	Fiducial	1.3	0.829	0.961	<a href="#">Puchwein et al. (2019)</a>
40-1024-seed003	Testing	140516	Fiducial	1.3	0.829	0.961	<a href="#">Puchwein et al. (2019)</a>

**Table B.1.** Training parameters used for each network.

Parameter	Redshift	No. summaries	Network description	Learning rate	$\lambda, \epsilon$
$\log(T_0)$ [1D]	2.0	1	$3 \times 256$	0.005	10.0, 0.1
$\gamma$ [1D]	2.0	1	$2 \times 128$	0.01	10.0, 0.1
$\log(T_0)$ [1D]	3.0	1	$2 \times 256$	0.01	10.0, 0.1
$\gamma$ [1D]	3.0	1	$2 \times 128$	0.01	10.0, 0.1
$\sigma_8$ [1D]	3.0	1	$2 \times 512$	0.005	10.0, 0.1
$n_s$ [1D]	3.0	1	$4 \times 256$	0.0001	10.0, 0.1
$\log(T_0), \Gamma_{\text{HI}}$ [2D]	3.0	2	$2 \times 512$	0.001	10.0, 0.1
$\log(T_0)$ [1D]	4.0	1	$2 \times 256$	0.005	10.0, 0.1
$\gamma$ [1D]	4.0	1	$3 \times 128$	0.001	10.0, 0.1
$\sigma_8$ [1D]	4.0	1	$2 \times 256$	0.01	10.0, 0.1
$n_s$ [1D]	4.0	1	$3 \times 256$	0.001	10.0, 0.1