Let us focus on certain key points and questions from the given text:

1. Introduction

Recently, a black hole (BH) of $33 \, M_{\odot}$ was discovered in the Galaxy by Gaia (Gaia Collaboration 2024). The mass of Gaia BH3 is substantially higher than the heaviest Population I BHs, which are $10-20 \, M_{\odot}$ (Belczynski et al. 2010; Miller-Jones et al. 2021). The most logical explanation for the high BH mass in Gaia BH3 is a low metallicity ($Z$). Low $Z$ was also invoked for the first BHs that were detected with gravitational waves (Abbott et al. 2016), as expected from $Z_{\odot}$-dependent winds of Wolf-Rayet (WR) stars (Vink & de Koter 2005), which naturally produce lower BH masses for solar $^1 Z$ (Eldridge & Vink 2006).

It was therefore a surprise that LB-1 was announced to involve a Pop I $70 \, M_{\odot}$ BH (Liu et al. 2019), and while the observational arguments have mostly dissipated (Abdul-Masih et al. 2020; El-Badry & Quataert 2020), the key question of whether it would be theoretically possible to produce such a heavy BH at solar $Z$ is fundamental. In this Letter, we analyse the available knowledge and uncertainties on both the internal mixing in massive stars and their stellar wind mass loss to address the question of the maximum BH mass at $Z_{\odot}$. This is particularly relevant as there appears to be a misconception that the most massive stars produce the heaviest BHs.

Over the last 5 years, we have made a dedicated effort to systematically study the key ingredients relevant for massive star evolution, which vary because of uncertainties in core-boundary mixing (CBM; Higgins & Vink 2019) and wind mass loss (Sabhahit et al. 2022). Mass loss is particularly influential for the most massive stars, where BHs or RSGs are referred to as very massive stars (VMS), as well as other mixing processes, including semi-convection or additional energy transfer in super-adiabatic layers.

As there are many large uncertainties in interior mixing and wind mass loss (see Sect. 2), it is not possible to accurately predict the final stellar or BH mass for a given star of a given initial mass. However, using the relevant observational and theoretical constraints available, we show that it is feasible to compartmentalise these issues, and to provide a realistic maximum BH mass.

We previously performed the analysis for the maximum BH mass at low $Z$, where we used our knowledge of interior mixing and wind mass loss and obtained a blue supergiant (BSG) progenitor for an $85 \, M_{\odot}$ BH, such as was detected in GW 190521 (Vink et al. 2021; Winch et al. 2024). Our studies lead to a maximum BH mass as a function of cosmic time, where at low $Z$ (below $\sim 10% \, Z_{\odot}$), the maximum BH mass is set by pair-instability physics (Woosley 2017; Farmer et al. 2019; Marchant et al. 2019; Renzo et al. 2020), but at high $Z$, where the maximum BH mass is anticipated to be set by wind mass loss, this number still needs to be determined. This is the purpose of this Letter.

In Higgins et al. (2021) we investigated the BH masses during the classical WR stage, but we did not study the evolution into this regime, which we undertook during complementary investigations (Higgins & Vink 2019; Sabhahit et al. 2022). In this Letter, we use our overall knowledge of interior mixing, wind mass loss, and the physics and location of the upper luminosity limit of red supergiants (RSGs) as part of the Humphreys-Davidson (HD) limit (Humphreys & Davidson 1979) to indirectly accommodate for eruptive mass loss to infer the maximum BH mass at $Z_{\odot}$.

2. Stellar evolution models

We computed a detailed set of MESA (Paxton et al. 2011, 2013, 2015, 2018, 2019) stellar evolution models for stars up to $250 \, M_{\odot}$ using the exact same mass-loss implementation and additional physical inputs as for the core H-burning models from Sabhahit et al. (2022). For the main sequence, we employed the

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$^1$ We use solar metallicity ($Z_{\odot}$) when referring to Pop I stars with Galactic metallicity, as the Milky Way also contains stars and BHs that formed at earlier times, when $Z$ was lower.
canonical Vink et al. (2001) wind mass-loss recipe, but above the transition mass-loss rate, we employed the high $\Gamma$ wind relation of Vink et al. (2011). These main-sequence models do not require the usage of MLT++ during core H-burning due to the wind enhancement. Beyond core H-burning, we did employ MLT++ with the same parameters as in Sabbahit et al. (2021), which was calibrated to the upper RSG luminosity of the HD limit of $\log L/L_\odot = 5.5$ (Davies & Beasor 2020; McDonald et al. 2022). Following Vink et al. (2021), we switched to the RSG recipe of de Jager et al. (1988) below 4 kK, while we adopted the new hydrodynamically consistent WR recipe of Sander & Vink (2020) above 100 kK.

2.1. Uncertainties regarding interior mixing

Massive stars have convective cores but radiative envelopes. The size of the core can be extended by CBM (Anders & Pedersen 2023), also known as overshoot. Higher amounts of core overshoot extend the main-sequence lifetime, and thereby the total amount of mass lost in winds.

Until 2010, most modellers employed no overshoot, or relatively small amounts of CBM parametrised in terms of a step overshooting value $\alpha_{ov} \approx 0.1$ in detailed stellar evolution models (Meynet & Maeder 2000; Heger et al. 2000). However, in Vink et al. (2010), we suggested that given the unexpectedly high number of observed B supergiants, stellar models should be explored with $\alpha_{ov}$ up to $\approx$0.5, which widens the main sequence\(^2\).

Ever since, we have taken the pragmatic approach to explore both low values of $\alpha_{ov} = 0.1$ and higher values of $\alpha_{ov} = 0.5$ in our canonical stellar models to accommodate for the full range of mixing uncertainties revealed by asteroseismology Bowman (2020)\(^3\) and eclipsing binaries\(^4\).

While recent advances in simulations of core convection (see Lecoanet & Edelmann 2023 for a review) and Scott et al. (2021) and Herwig et al. (2023) models indicate that more massive stars are expected to have larger amounts of CBM, asteroseismology studies of B stars indicate a roughly equal chance of mixing uncertainties revealed by asteroseismology Bowman (2020)\(^3\) and eclipsing binaries\(^4\).

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From the use of eclipsing binaries in the ML-plane, $\alpha_{ov}$ values (Higgins & Vink 2019) can potentially be obtained by disentangling the effects of interior mixing from wind mass loss.

However, in the upper O-star mass regime of 70–100 $M_\odot$, the stellar HRD and ML-plane locations unfortunately become less sensitive to high and low $\alpha_{ov}$ assumptions. Moreover, due to the larger convective cores, VMS become basically insensitive to the adopted value of $\alpha_{ov}$, and the size of the relative pressure scale-height becomes smaller when moving up the mass range.

2.2. Uncertainties in stellar winds

Stellar winds of massive stars are thought to be driven by radiation pressure on spectral lines (Castor et al. 1975). It is important to realise that in the regimes with the highest uncertainties, the effects of wind mass loss on stellar mass evolution are generally weakest (e.g. in the weak-wind regime; see below).

In domains in which the winds are most dominant, that is, in the regime of the VMS, they probably need to be boosted given both theoretical (Vink et al. 2011) and empirical evidence (Bestenlehner et al. 2014). In Sabbahit et al. (2022) we implemented boosted wind mass-loss above the Vink et al. (2011) kink,

$$\log M = 4.77 \log (L/L_\odot) - 3.99 \log (M_{\text{cur}}/M_\odot).$$

This enhanced mass-loss relation is switched above a specific wind efficiency number $\eta$, which sets the transition mass-loss rate. Alternative approaches have also been used in recent times. For instance, Romagnolo et al. (2024) employed a $\Gamma$-switch from an updated Castor et al. (1975) approach by Bestenlehner (2020), but the value of $\Gamma$ on this mathematical transition has been shown to be insensitive to the CAK $\alpha$ parameter (Sabbahit et al. 2022). Hence, we employed the original physical mass-loss transition point from Vink & Gräfener (2012), determined from the wind efficiency number $\eta$.

We summarise our knowledge and uncertainties in wind mass loss over the full ZAMS mass range below.

- $M_{\text{ZAMS}} \lesssim 20 M_\odot$. In this mass range, stars are in the so-called weak-wind regime (Martins et al. 2005), and it is not yet established whether the cause is theoretical or diagnostic in nature. Nevertheless, even if stellar winds are weaker in this regime, it is important to realise that a huge uncertainty of 1 or 2 orders of magnitude is not expected to have a direct effect on the stellar mass evolution, and uncertainties in interior mixing (Aerts et al. 2019) are expected to dominate the stellar evolution.

- 20 $M_\odot \lesssim M_{\text{ZAMS}} \lesssim 60 M_\odot$. In this regime, which we refer to as canonical massive stars, the uncertainties in mass-loss rates are substantial (by a factor of $\sim$3; Krtička & Kuhát 2017; Sundqvist et al. 2019; Vink 2022), and as both mixing and mass loss play a role in the evolution of these stars (Langer 2012), this regime is key for disentangling the separate effects (Higgins & Vink 2019). $M_{\text{ZAMS}} \gtrsim 60 M_\odot$. In this regime, the effect of winds dominates the evolution of interior mixing. While there could still be uncertainty at 60 $M_\odot$, the uncertainty at the mass-loss transition point, which is at $\sim$80–100 $M_\odot$ for ZAMS, the mass-loss rates are the most accurate, within as little as 30% as determined from the Vink & Gräfener (2012) calibration study.

2.3. Uncertainties in the treatment of radiative envelopes

While we have standard ways of including convection via the mixing length theory (MLT), for objects close to the Eddington limit, such as in He-burning supergiants, radiative transport

\(^2\) Alternatively, the cooler objects could be stellar mergers (Menon et al. 2024).

\(^3\) So far, this is only feasible for B-type stars up to $\approx$20 $M_\odot$.

\(^4\) Which are possible at higher masses into the regime of O star, where mass loss becomes a relevant factor.
inside envelopes can become important, and structure models predict strong inflation effects as well as density inversions to maintain hydrostatic equilibrium. Basically, one-dimensional (1D) treatment of radiative envelopes presents an unsolved problem in 1D stellar evolution modelling (Jiang et al. 2015; Moens et al. 2022). In MESA, the envelopes can be treated in the standard MLT approach, or the convective energy transport efficiency can be increased by employing MLT++ in superadiabatic layers (Paxton et al. 2013; Sabhahit et al. 2021). In our analysis of the maximum BH mass, we considered both options.

Moreover, it is not yet known whether episodic LBV-type mass-loss (Grassitelli et al. 2021) or eruptive mass loss (Smith 2014; Cheng et al. 2024) plays a role in addition to that by stellar winds discussed above. For this reason, we discuss the systematic uncertainties in envelope mixing and mass-loss physics for the 30–50 $M_\odot$ range in terms of the HD limit in Sect. 4.

3. Results

We present the results of our MESA evolution models over the entire ZAMS mass range up to 250 $M_\odot$ with the new mass-loss implementation as introduced in Sabhahit et al. (2022), and we showcase a subset of these tracks in the HR diagram of Fig. 1. We computed the stellar and wind evolution for the entire ZAMS mass range until the end of core He-burning.

For a star of a given mass, we need to consider two basic aspects: (i) the duration ($\Delta t$) in a particular evolutionary state, for instance as a blue supergiant (BSG), RSG, or WR star (i.e. the characteristic $T_{\text{eff}}$ of the evolutionary phase), and (ii) the mass-loss prescription in that $T_{\text{eff}}$ range: $\dot{M}(T_{\text{eff}})$, resulting in a total mass loss of

$$\Delta M = \int \dot{M}(T_{\text{eff}})\,\Delta t.$$  

In order not to just provide final masses from our models, but to gain insight into their qualitative behaviour, we first evaluated the mass at the terminal main sequence (TAMS) and then considered the temperature ($T_{\text{eff}}$) during core He-burning.

The ultimate property of interest is the final mass, which we assumed to be equal to the BH mass (Fryer et al. 2012; Fernández et al. 2018), but see Burrows et al. (2023). The initial-final mass results over the entire mass range are shown Fig. 2.

The highest BH mass (the BH peak at 1 in Fig. 2) is not found for the highest ZAMS masses, as might have been expected, but instead, the maximum BH mass (of 30 $M_\odot$) is found for the ZAMS mass range between 35 and 45 $M_\odot$. The 35 $M_\odot$ model looses as little as 4 $M_\odot$ during core H-burning and an additional 1 $M_\odot$ during core He-burning.

The reason for the seemingly counter-intuitive behaviour of the constant BH mass tail (4 in Fig. 2) is that we included a $\Gamma$-dependent VMS mass loss, which takes hold above the 80–100 $M_\odot$ transition ZAMS mass, and which causes the inflection point in the TAMS mass behaviour at point 3 in Fig. 2, although it should be noted that the switch to high-$\Gamma$ winds does not automatically imply very strong mass loss, but that this takes some time to build up. For the highest-mass stars, the mass-loss rates are sufficiently high to not only remove the H envelope, but to even remove mass from the stellar core. The BH masses from these stripped WR stars are 10–15 $M_\odot$ (Higgins et al. 2021) at most at Z$_\odot$. For the mass range below 50 $M_\odot$, core He-burning does not take place in the WR phase, but during the (blue, yellow, or red) supergiant phase.

The post-He-burning lifetime comprises just 1% of the stellar evolution, so this mass loss only becomes relevant if the mass-loss rate after core He burning were to be 100 times more powerful than during the main sequence, or 10 times more powerful than during core He-burning phases (see Eq. (2)). As we cannot exclude such enhanced mass loss phases, for instance via episodic mass loss, or even eruptions, we employ an alternative upper supergiant limit approach below.
4. Discussion of the maximum BH mass

Figure 2 was computed using one particular set of physical input choices (e.g., an intermediate amount of overshooting, α_{ov} ~ 0.3; Brott et al. 2011), no rotation, and Ω-dependent mass-loss rates. However, there are many uncertainties in both mass loss and mixing (in particular, α_{ov} and MLT/MLT++), and these decide whether a star stays in the blue (for negligible α_{ov} and MLT++) or becomes red. The maximum BH mass in our models is reached when the star is kept in the blue. If no such low mixing were to occur in nature and the star were to move over to the red, however, strong RSG mass loss may potentially occur. In this case, this would imply that the BH mass decreases, it will not increase.

Given that stellar evolution is non-linear and has many inter-dependent physical quantities related to both mixing and mass loss, we require an alternative strategy to infer the maximum BH mass from the supergiant phase. In other words, we need to consider the maximum stellar mass that stays below the observed maximum luminosity limit (HD-limit) for the Milky Way. According to McDonald et al. (2022), this is found to be at log (L/L⊙)_{HD} = 5.5.

We experimented using negligibly low amounts of additional mixing (α_{ov} = 0) and non-boosted standard Vink et al. (2001) mass-loss rates, which both helped to keep the stars blue by preventing potential high mass-loss episodes as an RSG. Basi-cally, our pragmatic approach of considering BH progenitor stars below the HD limit simulates what occurs for stars when they approach the Eddington limit, and where they may be expected to undergo LBV-type mass loss, or more general Ω-dependent mass loss, independent of its underlying effective temperature (Vink & Sahhahit 2023). As the α_{ov} parameter cannot become negative, 30 M⊙ is indeed the maximum BH mass for the upper RSG luminosity of Davies & Beasor 2020; McDonald et al. 2022. We also tested the same models including rotation, and while moderate rotation (Ω of 40% critical) increased the initial ZAMS luminosity somewhat, its further evolution was almost identical to those shown in Fig. 1.

To summarise, the amounts of mixing in terms of α_{ov} and Ω are irrelevant for setting numbers 3 and 4 in Fig. 2, and whilst they do affect the numerical values of 1 and 2, the shape of these additional features 1 and 2 is also invariant to mixing.

The low Ω Vink et al. (2001) mass-loss recipe includes a bistability jump, where the mass-loss rate is predicted to increase by a factor of about 5, but which is empirically still elusive (Crowther et al. 2006; Markova & Puls 2008). On balance, we may have over- rather than underestimated the mass-loss rates in this mass regime (Petrov et al. 2016; Krtička & Kubát 2017; Sundqvist et al. 2019). The overall mass loss is already modest. It drops from 35 M⊙ on the ZAMS to just 31 M⊙ on the TAMS and 30 M⊙ by the end of core He-burning; that is, 5 M⊙ were removed from its initial value. We therefore consider the uncertainty of stationary wind mass-loss rates on the maximum BH mass to be ~5 M⊙. In order to provide an exact error bar on our maximum BH mass, we would need to perform a N-dimensional parameter search, which is unfeasible with current machines, as the number of physical parameters of mixing and mass loss is manifold.

The larger uncertainty in our maximum BH assessment is related to the empirical HD limit, which could indirectly account for episodic LBV-type mass loss (Grassitelli et al. 2021). We have employed the McDonald et al. (2022) value of log(L/L⊙)_{HD} = 5.5. Had we used the original higher value of log(L/L⊙)_{HD} = 5.8 (Humphreys & Davidson 1979) that was in use over the last four decades, our maximum BH mass could be a high as 40 M⊙ (Higgins & Vink 2019, 2020). On the lower maximum BH side, if all uncertainties in both mixing and mass loss were to conspire to very high mass-loss rates inside the canonical O star 20–60 M⊙ range, the maximum BH mass would simply be given by this lower limit of ~20 M⊙ where mass-loss rates are negligibly low and envelopes are not removed. When all relevant factors we discussed above are taken into account, the realistic maximum BH mass for Z⊙ is M_{BH} = 30 ± 10 M⊙.

5. Final words

The analysis performed above implies that it would be highly unlikely that for instance a 70 M⊙ BH would be discovered at solar Z. This is not to say that a 70 M⊙ BH could not exist in the Milky Way, but it should have formed at an earlier epoch when its Z was still below ~10% of Z⊙ (Vink et al. 2021). As shown in Fig. 3 of Vink et al. (2021) we anticipated the maximum BH mass at solar metallicity to be set by wind mass loss. As shown by Eq. (2) above, the total mass lost in stationary winds depends on both the temperature-dependent mass-loss rate and on the evolutionary duration (∆t). For this reason, we considered both the effects of wind mass loss and interior mixing, but we concluded that the maximum BH mass for Z⊙ is set by the HD limit. Therefore, the error bars on our maximum BH mass could potentially be refined by future analyses of the HD limit at Z⊙.

At face value, our results for the maximum BH mass at Z⊙ appear to be similar to those of Bavera et al. (2023) and Romagnolo et al. (2024), but there are fundamental qualitative, as explained in Fig. 2, as well as quantitative differences. Firstly, while Romagnolo et al. (2024) included a boosted Ω-dependent mass loss, they employed a mathematical Ω transition point, which we do not subscribe to (see the discussion in Sect. 2.2). This appears to be the main reason for the divergent results at high M_{ZAMS} and the prime reason for the constant BH mass tail we find (4 in Fig. 2). Instead, Romagnolo et al. (2024) reported an increasing BH mass with ever-increasing ZAMS mass, which is similar to our fiducial models that have no mass-loss boost (the dashed lines in Fig. 2).

Secondly, it is interesting that the shape of our initial mass-final-mass relation (Fig. 2) shows qualitative resemblances to the shapes reported by Bavera et al. (2023), although they employed a simple constant LBV-like mass loss while we employed an exponentially increasing Ω-dependent mass loss. While the peak of the mass function is reached at a similar BH mass, the initial mass to which this BH mass corresponds is twice as high in Bavera et al. (2023). We attribute this to our strategy of employing the empirical HD limit.

We would like to raise one potential caveat to our analysis. For our results on the maximum BH mass at solar metallicity, we used enhanced mass-loss rates for VMS, which led to our conclusion that VMS do not yield the maximum BH mass at solar Z. If we have over-estimated mass-loss rates for VMS using the Sahhahit et al. (2022) implementation, then our analysis would be notably affected. The reason we nonetheless should have confidence in the high mass-loss rate implementation for VMS is that this solution alone naturally explains the almost constant effective temperatures of VMS at various metallicities, whereas lower mass-loss rates would need to be tweaked for each and every VMS mass bin value to either avoid redward inflation resulting from mass-loss rates that are too low or a blueward evolution due to high mass loss (see Sahhahit et al. 2022 for a detailed discussion).
We would like to conclude by reflecting on the fact that there are still many uncertainties in both interior mixing and stellar wind mass loss, but using key observational and theoretical constraints, we have shown that it is possible to provide a realistic maximum BH mass. If a BH were on day discovered at solar Z that is substantially higher than $30-40 \, M_\odot$, we would need to seriously reconsider mass-loss rate implementations for VMS because we have exhausted all other relevant physical uncertainties in detailed stellar evolution modelling.

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