Quantum model of galactic halos with an Navarro–Frenk–White dark matter profile

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Received 17 December 2023 / Accepted 7 April 2024

ABSTRACT

Context. A quantum model of a cold dark matter halo is developed. The model requires specifying the mass and radius of the halo as well as its density profile. The structure of the halo that results from the theory is predicted and its physical properties are determined. Verification of these theoretical predictions by observations is proposed and discussed.

Aims. The model is constructed by analytically solving the governing equation and using its time-independent solutions to determine the internal structure of a galactic halo with an Navarro–Frenk–White cold dark matter density profile.

Methods. The governing equation that is the basis of the developed theory is derived from the irreducible representations of the extended Galilean group. The method of finding the solutions is analytical, even though an Navarro–Frenk–White density profile is used in the calculations.

Results. The theory predicts a halo with a core composed of free dark matter particles that move randomly with frequent collisions. It also predicts an envelope in which the particles are confined to their orbits, which are quantized. Except in the close vicinity of the core, the population of the orbits remains fixed, and physical reasons for the nonexistence of quantum jumps between these orbits are presented.

Conclusions. A quantum model of a cold dark matter galactic halo is developed. The model requires specifying the mass and radius of the halo, as well as its density profile. The model naturally accounts for dark matter being collisionless, and it predicts that dark matter can only emit radiation of one fixed frequency. The values of this frequency are computed for dark matter particles of different masses. A potential observational verification of the theory is also discussed.

Key words. galaxies: halos

1. Introduction

According to the Planck 2018 mission (Planck Collaboration VI 2020), dark matter (DM) constitutes 26.8% of the total mass-energy density of the Universe, which is almost 5.5 times more than the amount of ordinary matter (OM). Of the theories proposed to explain DM (e.g., Rees 2003; Freeman & McNamara 2006; Frieman et al. 2008; Overduin & Wesson 2004; Barbier et al. 2005; Sugita et al. 2008; Arkani-Hamed et al. 2009; Komatsu et al. 2011; Bartone & Hooper 2018; Giagu 2019; Watson & Musielak 2020; Hui 2021; Oks 2021; Musielak 2021, 2022; Chadha-Day & Ellis 2022), some predict the existence of weakly interacting massive particles (WIMPs). However, all attempts to detect WIMPs have failed so far (e.g., Ackermann et al. 2011; Ibarra et al. 2013; Marrodán Undagoitia & Rauch 2016; Hochberg et al. 2022). Recently, Rogers et al. (2023) demonstrated that the large-scale distribution of matter may imply that DM is composed of ultralight axions with masses in the range of $10^{-28}$–$10^{-25}$ eV. A number of experiments (e.g., Crisosto et al. 2020; Jiang et al. 2021; Adair et al. 2022; Chadha-Day 2022) have failed to find any trace of axions.

In order to understand the nature and origin of DM, different relativistic and nonrelativistic theoretical models have been constructed. In this paper, only the latter are considered and discussed. A model of galactic DM halos that is based on the Schrödinger equation, which is known for its correct description of the microscopic structure of OM (e.g., Merzbacher 1998), was proposed by Sin (1994), who suggested that DM particles are extremely light bosons with masses of about $10^{-24}$ eV. It was an interesting idea because it allowed us to solve the Schrödinger equation on the galactic scale because these particles have a very long Compton wavelength. In this model, the gravitational potential added to the equation is calculated by solving the Poisson equation with the DM density as the forcing term. Hu et al. (2000) considered DM particles with masses of about $10^{-22}$ eV and improved the previous models. However, more detailed numerical studies by Spivey et al. (2013, 2015) revealed that the models require the DM particles to have different masses for different galactic halos. This conclusion is difficult to justify physically.

Since the masses of ultralight axions proposed by Rogers et al. (2023) are even lower than the masses of extremely light bosons considered by Sin (1994), Hu et al. (2000), and Spivey et al. (2013, 2015), we might expect that numerical solutions to the Schrödinger equation for ultralight axions would also reveal that these particles must have different masses for different galactic DM halos. However, this extrapolation may not be valid because a correct description of ultralight axions in the galactic DM halos may require a fully relativistic theory of DM, instead of using its nonrelativistic approximation based on the Schrödinger equation with the gravitational potential term.
The previous attempts to formulate nonrelativistic theories of DM using the Schrödinger equation have failed. Therefore, a different governing equation may be needed to formulate a new DM theory. An equation like this was recently derived (Musielak 2021) by using the irreducible representations (irreps) of the so-called extended Galilean group of the metric, which includes rotations, translations, and boost in Galilean space and time (Lévy-Leblond 1967, 1969; Kim & Noz 1986). The new equation is nonrelativistic and asymmetric in space and time derivatives, and it was recently used to describe the propagation of classical waves (Musielak 2023a) and to construct a quantum theory of DM (Musielak 2022, 2023b). It is known (e.g., Musielak & Fry 2009) that the Schrödinger equation can also be derived from the same irreps, which means that both equations are complementary to each other.

The main physical implication of this complementarity is that the basic quantum rules (e.g., Merzbacher 1998) can be applied to them, and each equation can be used to formulate a quantum theory. However, while the Schrödinger equation correctly describes the atomic structure and its unitary evolution, the new equation naturally accounts for its interactions with macroscopic measuring devices. In other words, a quantum theory based on the new equation can be used to describe the quantum measurement problem (Musielak 2024), which violates the unitary evolution represented by the Schrödinger equation (e.g., Merzbacher 1998).

In this paper, the new equation is used to develop a quantum model of a galactic cold dark matter halo with a given Navarro–Frenk–White (NFW) density profile (Navarro et al. 1996, 2010). The model predicts a quantum structure of the halo that is significantly different than that proposed by any previous DM model (e.g., Sin 1994; Hu et al. 2000; Spivey et al. 2013, 2015; Schive et al. 2014; Zhang et al. 2017). The model also naturally accounts for DM being collisionless, and it predicts that DM can emit radiation of only one frequency, whose values are computed for DM particles of different masses. The paper suggests potential observational verification of the theory.

This paper is organized as follows: The basic equations and a method for constructing galactic halo models are presented in Sect. 2. A quantum model of a dark matter halo is described in Sect. 3. The developed quantum model is applied to a halo with an NFW density profile in Sect. 4. The dark matter characteristic frequency and its observational detection is discussed in Sect. 5. Section 6 is finally devoted to our conclusions.

\[ \left[ i \frac{\partial}{\partial t} + \frac{\hbar}{2m} \nabla^2 \right] \Psi(t, x) = 0, \]  
\[ \left( i \frac{\partial^2}{\partial \omega^2} + \frac{\hbar}{2m} k \cdot \nabla \right) \Phi(t, x) = 0, \]

which is the Schrödinger equation (e.g., Merzbacher 1998), and a new asymmetric equation (Musielak 2021),

2. Basic equations and galactic halo models

In Galilean Relativity, space and time are separated and represented by two different metrics that are invariant with respect to all translations, rotations, and boosts that form the Galilean group of the metric (Lévy-Leblond 1967, 1969; Kim & Noz 1986). According to Inönü & Wigner (1952) and Bargmann (1954), the irreducible representations (irreps) of the Galilean group are known, and they can be used to derive the basic equations of physics that are allowed to exist in Galilean space and time. A scalar wavefunction $\Phi(t, \mathbf{x})$ transforms as one of the irreps if the following eigenvalue equations $i \hbar \partial_\omega \Phi = \omega \Phi$ and $-i \nabla \Phi = k \Phi$ are satisfied, where $\partial_\omega = \partial / \partial \omega$, and $\omega$ and $k$ are the eigenvalues that label the irreps (Musielak & Fry 2009).

By using the de Broglie relation (e.g., Merzbacher 1998), the eigenvalues can be determined (Musielak 2021). Then, the eigenvalue equations give the following two equations that are asymmetric in space and time derivatives:

\[ \left[ i \frac{\partial}{\partial t} + \frac{\hbar}{2m} \nabla^2 \right] \Psi(t, x) = 0, \]  
\[ \left( i \frac{\partial^2}{\partial \omega^2} + \frac{\hbar}{2m} k \cdot \nabla \right) \Phi(t, x) = 0, \]

where $\omega$ represents quanta of energy, and $\mathbf{r} = r \mathbf{e}_r$; there is no dependence of the angle on $\theta$ nor on $\phi$. This is the governing equation used herein to develop a quantum theory of a galactic DM halo.
3. Quantum model of a dark matter halo

Solutions of Eq. (4) were found by separating the variables \( \Phi(t,r) = \chi(t)\psi(r) \), which gives the following time-independent equation for \( \eta(r) \):

\[
\frac{d\eta}{\eta} = i \frac{2m}{\hbar}\left[\mu^2 - \Omega_h^2(r)\right]dr,
\]

(5)

where \( \mu^2 \) is the separation constant to be determined. In general, the solutions of this equation are complex (Musielak 2023b). Their real part can be written as

\[
\eta(r) = \eta_0 \cos \left( \frac{2Gm^2[\mu^2 - L_n(r)]}{c_o^2\hbar^2}\right),
\]

(6)

where \( L_n(r) = \int_0^r \Omega_h^2(\tilde{r})d\tilde{r} \), and \( \hbar k_o = k_o\hbar \), with \( k_o = 1/\lambda_o = Gm^2/c_o = \text{const.} \)

To formulate a quantum theory, it is required that its quantum orbits be labeled by positive integers (the principle quantum numbers), which limits the values of the cosine function to be either the maxima (±2\(n\pi\)) or minima (±(2n + 1)\(\pi\), where \( n = 0, 1, 2, 3, \ldots \), and the plus and minus signs correspond to the orbits inside and outside the halo, respectively. By choosing \( n = 0 \) to label the orbit at the edge of the halo \( r = R_h \), all the remaining quantum orbits inside and outside the halo must be labeled by using ±2\(n\pi\pi\) because ±(2n + 1)\(\pi\) makes the labeling inconsistent with the selected reference orbit at \( r = R_h \). In the remaining parts of this paper, only the orbits inside the halo are considered.

Thus, the separation constant is given by

\[
\mu^2 = 1 - \frac{L_n(r) + n\pi\omega_o^2\hbar^2}{Gm^2r^2}.
\]

(7)

Finding \( \mu^2 \) requires \( r = R_h \), \( \hat{r} = \hat{R}_h \), and evaluating \( L_n(R_h) \), which gives

\[
L_n(R_h) = \int_0^{R_h} \Omega_h^2(\tilde{r})d\tilde{r} = C_o R_h \Omega_h^3,
\]

(8)

where \( C_o \) is a dimensionless constant that depends on the density profile of the halo, and \( \Omega_h^2 \) is the orbital frequency at the edge of the halo, given as \( \Omega_h^2 = GM_h/R_h^3 \). It must be noted that \( \Omega_h^2 \) corresponds to the orbital velocity \( v_o^2 \), which is twice lower than the escape velocity \( v_e^2 \) at the edge of the halo.

Taking \( \mu^2 = C_o \Omega_h^2 \), where \( \Omega_h^2 \) represents the quantized orbital frequencies. The quantization condition is

\[
\Omega_h^2 = \Omega_o^2 + n\pi\kappa_o\omega_o^2,
\]

(9)

where

\[
\kappa_o = \frac{\hbar^2}{Gm^2C_o\rho_s R_h}
\]

(10)

does not have a dimensionless constant. Multiplying Eq. (9) by \( \hbar^2 \), the spectrum of quantum energies that corresponds to Eq. (9) is obtained:

\[
E_n^2 = E_o^2 + n\pi\kappa_o\omega_o^2.
\]

In the above derivation of \( \Omega_o^2 \) and \( E_o^2 \), it is assumed that \( \langle \hat{k}_o \cdot \hat{R}_h \rangle = 1 \). This is justified by the fact that for spherically symmetric halos, the unit vector \( \hat{R}_h \) is not restricted and can always be aligned with the unit vector \( \hat{k}_o \). However, for the case of asymmetric DM halos, \( \langle \hat{k}_o \cdot \hat{R}_h \rangle \neq 1 \), and its value would account for deviations from perfect spherical symmetry, making the evolution of the wavefunction also dependent on direction.

The location \( r_o \) of the quantized orbits \( \Omega_h \) inside the halo can be calculated by using the conservation of angular momentum (Musielak 2023b), and the result is

\[
r_o = R_h \sqrt{\frac{\Omega_o}{\Omega_h}},
\]

(11)

where \( \Omega_o = \sqrt{\Omega_o^2 + n\pi\kappa_o\omega_o^2} \), and for the quantized orbits inside the halo, \( r_o \leq R_h \).

The developed model demonstrates that the halo has its quantum structure, which resembles atoms with their available energy levels. The model is now applied to a galactic halo with a given NFW density profile (Einasto \& Haud 1989; Navarro et al. 1996, 2010; Merritt et al. 2006).

4. Halo with an NFW density profile and its quantum structure

To construct a quantum model of a galactic halo, the density profile for the halo must be specified. Of the different halo models simulated by Navarro et al. (2010), the NFW density profile given by

\[
\rho(r) = \frac{\rho_s}{(\frac{r}{r_s}) (1 + \frac{r}{r_s})^2},
\]

(12)

was considered, with its parameters \( \rho_s \) and \( r_s \) being specified as the numerical model Aq-A-3 in Table 2 by Navarro et al. (2010). Then, \( r_s = r_{200} = 1.1 \times 10^3 \) (kpc \( h^{-1} \)) and \( \rho_s = 0.65 \left( M_{200}/10^{12} M_{\odot} h^{-1}\right)^{-1} \) and \( 3.0 \times 10^8 \) (\( 10^{10} h^2 M_{\odot} \text{Mpc}^{-3} \)), where \( h \) is the dimensionless Hubble parameter. It must be noted that \( \rho \to \infty \) when \( r = 0 \), but in the same limit, the term \( \rho_r r^2 \to 0 \). After the numerical model was selected, the term of Table 1 in Navarro et al. (2010) were used to find the total mass of the halo, which is \( M_h = 200 M_{200} = 1.3 \times 10^{12} M_{\odot} h^{-1} \), and its radius \( R_h = r_{200} = 1.8x10^6 \) (kpc \( h^{-1} \)).

The NFW density profile given by Eq. (12) can be used to obtain the resulting mass distribution \( M(r) \),

\[
M(r) = 4\pi \rho_s r_s^3 \ln \left( 1 + \frac{r}{r_s} \right) \frac{r}{r + r_s},
\]

(13)

which shows that \( M(r \to 0) = 0 \). Taking \( r = R_h \), the total mass \( M_h = M(R_h) \) was calculated, and its value was consistent with \( M_h \) given in Table 1 of Navarro et al. (2010). Similarly, \( M_s = M(r = r_s) \) can also be determined, and its value is \( M_s = 0.76\pi \rho_s r_s^3 \). Thus, the ratio of these two masses of the halo is \( M_s/M_h \approx 10 \), which means that the amount of DM mass inside the radius \( r_s \) is ten times lower than the total mass \( M_h \) of the halo.

After we obtained \( M(r) \), \( M_h \), and \( M_s \), Eq. (3) was used to find the orbital frequencies corresponding to the masses \( M_h \) and \( M_s \). Their values are \( \Omega_h \approx 3.0 \times 10^{-12} \) (Hz \( h^{-1} \)) and \( \Omega_s \approx 3.5 \times 10^{-15} \) (Hz \( h^{-1} \)). The orbital velocities of DM particles moving on the orbits with these frequencies are \( v_h = 1.8 \times 10^7 \) (km s\(^{-1}\)) and \( v_s = 1.1 \times 10^5 \) (km s\(^{-1}\)), respectively.

Construction of the quantum model of the galactic DM halo with an NFW density profile requires evaluation of the integral given by Eq. (8). However, since \( \rho(r) \) diverges as \( r \to 0 \), so does \( \Omega(r) \) and the integral \( I(R_h) \). By changing the lower limit of the integration in Eq. (8), a core inside the halo was introduced. The existence of such a core has been verified for other density profiles (Musielak 2023b). Since the NFW density profile directly depends on \( r_s \), let \( r_s \) be the lower limit of the integral, and thereby the radius of the core. However, if the core is
smaller, and its radius $r_c < r_s$, then $r_c$ would be the lower limit of the integration, with $r_s$ to be determined from the theory.

Let $x = r/r_s$ be a new variable that allows us to write the integral $I(r_s, R_h)$ in the following form:

$$I_c(r_s, R_h) = 4\pi G \rho \int_1^{R_h/R_s} \left[ \ln(1 + x) - \frac{x}{1 + x} \right] \frac{dx}{x^3} \tag{14}$$

After using the values of $R_h, r_s, \rho$, the integration gives $I(r_s, R_h) \approx 1.1 \times 10^{-11} \text{ (m}^2 \text{s}^{-2})$. This result can be compared to that of Eq. (8). With $R_h, \Omega_n^2 = 6.11 \times 10^{-22} \text{ (m}^2 \text{s}^{-2})$, the dimensionless constant is $C_p = 1.8 \times 10^{10}$ for a halo with an NFW density profile.

The developed quantum model of a galactic halo with an NFW density profile predicts that the halo has two components, namely, the core of radius $r_s$ and density $\rho_0(r \leq r_s) = \rho_s = \text{const}$, and an envelope that extends from $r = r_s$ to $r = R_h$. The physical properties of the core and envelope are described below.

The core in the above halo model extends from $r = 0$ to $r = r_s$, which means that its radius is 11 kpc, and it occupies 6% of the size of the halo. Moreover, the core contains 10% of the total mass of the halo. Since the density is constant inside the core, the orbital frequency $\Omega$ has the same value at every point in the core, $0 < r < r_s$, and as a result, the core has no quantized orbits. Instead, the DM particles are allowed to move randomly and to collide frequently with each other. The core may also contain a large number of free quanta of energy $\varepsilon^\text{q} = \hbar \omega^\text{q}$, which can be exchanged between the DM particles inside the core. The quanta of energy are called dark gravitons (e.g., Musielak 2022, 2023b). The sea of dark gravitons in the core may contribute to the gravitational wave background (e.g., Romano & Cornish 2023b). The sea of dark gravitons in the core may contribute to the gravitational wave background (e.g., Romano & Cornish 2023b). The sea of dark gravitons in the core may contribute to the gravitational wave background (e.g., Romano & Cornish 2023b).

The envelope that surrounds the core extends from the edge of the core $r = r_s$ to the edge of the halo at $r = R_h$. It therefore occupies 94% of the size of the halo and contains 90% of the total halo mass. The density in the envelope is described by an NFW profile. As a result, the orbital frequencies $\Omega_n$ in the envelope range from $\Omega_1$ (lowest) to $\Omega_h$ (highest), and they are quantized according to the rule given by Eq. (9). The location of the quantized orbits $r_n$ in the envelope can be calculated using Eq. (11). The quantization rule shows that the number of orbits increases rapidly when $r \rightarrow r_s$ and that density of the orbits in the vicinity of the core is high, so that DM particles on orbits like these may directly interact with the particles and dark gravitons of the core, causing the exchange of some particles.

However, away from the core, DM particles are confined to their quantized orbits because the differences $\Delta \Omega_n = \Omega_{n+1} - \Omega_n$ are not exact multiples of $\omega_0$, as they also depend on $k_0$ given by Eq. (14). Since the DM particles may only emit or absorb the characteristic frequency $\omega_0$, which is fixed for DM, there are no quantum jumps of the particles between the quantized orbits (see Sect. 5). Instead, the DM particles are spinless and zero-charged and are confined to their orbits without any other quantum restrictions imposed on the population of these orbits. In other words, the theory presents physical reasons, which are confinement of the DM particles to their quantum orbits, for DM to be collisionless that is typically assumed while constructing models of DM halos, such as the NFW density profile used in this paper.

Clearly, the described quantum structure of a galactic DM halo with an NFW density profile resembles an atom whose size is enormous as it exists on the galactic scale. Similar general conclusions were presented by Musielak (2022, 2023b) and were obtained for a simple linear density profile of DM, which is not realistic for typical galactic DM halos (e.g., Einasto & Haud 1989; Navarro et al. 1996, 2010). In this paper, the same conclusion is drawn from the results obtained with an NFW density profile, which is commonly used in studies of galactic DM halos. This implies that the predicted atomic structure of galactic DM halos is independent from density profiles. The structure ceases to exist for the constant density profile (Musielak 2023b), however. In the following, the developed model with an NFW density profile is used to make theoretical predictions that can be verified by astronomical observations.

5. Characteristic frequency and its detection

According to Eq. (9), the quantized orbital frequency $\Omega_n$ depends on the orbital frequency $\Omega_0$ at the edge of the halo, the dimensionless parameter $k_0$, and the DM characteristic frequency $\omega_0$. For the considered model of a galactic halo with an NFW density profile, $\Omega_0 \approx 3 \times 10^{-17}$ (Hz h). The parameter $k_0$ is a sensitive function of the mass $m$ of DM particles, which is currently unknown. When the values of the universal constants $h$ and $G$, the value of $\Omega_0$, and $C_p = 1.8 \times 10^{10}$ are used for a halo with an NFW density profile, then the constant given by Eq. (10) becomes

$$k_0 = 1.6 \times 10^{-90} \text{ m}^{-3},$$

and it allows us to write the quantization condition given by Eq. (9) as

$$\Omega_n^2 = 1.1 \times 10^{-13} + 1.6 \times 10^{-89} m \pi \omega_0^2.$$  

This value shows that the quantization rule for $\Omega_n^2$ may be used to estimate the DM characteristic frequency $\omega_0$ when the value of $m$ is specified. In the considered halo model, $\Omega_n^2$ can range from $\Omega_1^2$ to $\Omega_h^2$, which are the lowest and highest frequencies, respectively. Thus, the quantized orbits in the halo envelope and their number depend on the values of $\omega_0$ and $m$. Neither value is known, however. When one of these two quantities is specified, then the other can be estimated (see Table 1).

When $\Omega_n$ is known, the location of the quantized orbits in the envelope can be calculated by finding $r_n$ from Eq. (11), whose explicit form for a galactic halo with an NFW density profile is given by

$$r_n \approx 1.8 \times 10^2 \sqrt{\frac{\Omega_n}{\Omega_h}} \text{ (kpc h}^{-1}).$$

By finding $r_n$, the number of quantized orbits located in the envelope between $r = r_s$ and $r = R_h$ can be calculated. The number of quantized orbits is very sensitive to the mass (m$^{-3}$) of DM particles; in general, the higher the mass, the more quantized the orbits. Our model predicts that most quantized orbits are located near the core and that the separation between these orbits is small and decreases toward the core. Since the DM particles are confined to the quantized orbits, the dense population of the orbits near the core contains a large amount of the total mass of the halo. On the other hand, the outer orbits contain less mass, and their separation increases with the distance from the core. This structure of quantum orbits makes the entire galactic halo a very stable and long-lasting object.

The developed quantum model of galactic DM halos with an NFW density profile predicts that the core has a radius of 11 kpc and that its mass is $1.3 \times 10^{11} M_{\odot}$, with most of the total mass of
the halo confined to many quantized orbits located near the core. The size and mass of a typical galaxy located at the center of the halo can be similar to (or larger and higher than) those of the core. Moreover, the surroundings of the core is full of quantum orbits with a high population of DM particles. Thus, the observed rotational curves of different galaxies can be explained by this inner structure of the quantum halo. However, the structure may lead to small wiggles on otherwise flat rotational curves at the edge of the central galaxy. These wiggles might be caused by the quantum shells of DM near the core and away from it.

When we assume that the developed quantum theory correctly describes a galactic halo with an NFW density profile and that quantized orbits exist in the envelope, then the value of $\omega_0$ can be estimated for a given $m$. In the following, these estimates are made by specifying the mass $m$ of DM particles and expressing it in terms of the proton mass $m_p = 0.938\,\text{GeV} = 1.7 \times 10^{-27}\,\text{kg}$. The estimates were made using Eq. (16), and the results are presented in Table 1.

The developed quantum model of DM predicts the quantum structure of galactic halos described in Sect. 4. According to this theory, DM may emit (absorb) radiation of only one fixed frequency $\omega_0$. Now, if DM is described by the theory, then there is a relation between the mass of DM particles and the characteristic frequency of radiation that DM may emit and absorb. The relation is given by Eq. (16), which requires that the term $10^{-30}\,\text{m}^3\,\text{s}^{-2}\omega_0^2$ is approximately on the same order as $10^{-33}$ because $\Omega_\Lambda^2$ is restricted to the interval $[0, 1]$. If this condition is valid, then the quantum rule given by Eq. (16) can be applied to galactic halos, and it predicts its quantum structure. The estimates were made by using the relation, and the obtained results are presented in Table 1. These preliminary estimates only allow a prediction an order of magnitude.

The results of Table 1 show that the characteristic DM frequency is a very sensitive function of the DM particle mass. The presented theory cannot estimate the mass, and thus it must be determined by experiments. There are no experimental or observational constraints on this value to date. The masses of DM particles we used in the estimates in Table 1 span many orders of magnitude, from approximately the mass of the Higgs boson to the mass of ultralight axions suggested by Rogers et al. (2023). The results given in Table 1 show that higher masses give higher frequencies $\omega_0$ and shorter wavelengths $\Lambda_\omega$ of radiation that can be emitted and absorbed by DM particles. In general, the resulting frequencies are very low, and they become extremely low for extremely light bosons and axions, but then the masses were only included in the estimates for comparison. An interesting result is that when the mass of DM particles is increased by one order of magnitude, the resulting DM frequency increases by two orders of magnitude. This relation between the mass of DM particles and the characteristic DM frequency is important as it allows us to determine the frequency when the mass is determined experimentally, or to determine the mass when DM radiation is detected.

After predicting the values of the characteristic DM frequency $\omega_0$ for different masses of DM particles, the possibilities of a detection remain to be explored. The first question is what type of radiation can be emitted by DM. There is no observational evidence so far for any radiation associated with DM (e.g., Planck Collaboration VI 2020). The quantum theory of DM presented in this paper only accounts for gravitational interactions between the DM particles and the halo, but it does not account for any interaction between the particles. The quanta of energy $\omega_0 = \hbar \omega_0$ that DM emits and absorbs are called dark gravitons (see Sect. 4) to emphasize that they are quanta of gravitational interactions of DM particles.

The dark gravitons in the halo envelope, where the orbits are quantized, are limited by the fact that the DM particles are confined to their orbits because there are no quantum jumps between the orbits (see Sect. 4). However, dark gravitons may be abundant in the halo core, where orbits are not quantized and the DM particles move randomly and emit and absorb dark gravitons, which carry the quanta of energy $\omega_0$. The abundance of dark gravitons in the core may contribute to the gravitational wave background (GWB; e.g., Romano & Cornish 2017), which was recently discovered by the NANOGrav detector (Agazio et al. 2023; Caprini 2024).

The GWB has two main components: a cosmological and an astrophysical component. This is based on their origin or on the sources that generate it. The two components are characterized by different frequency ranges (in Hz): from $10^{-16}$ (or lower) to 1 for the former, and from $10^{-10}$ to $10^2$ for the latter (after the NASA Beyond Einstein Program). Comparing these frequencies to $\omega_0$ given in Table 1, we observe that for the masses ranging from the electron mass to the mass of the Higgs boson, the characteristic frequency $\omega_0$ of dark gravitons may also contribute to the observed GWB. However, the mass range that corresponds to extremely light bosons and axions gives $\omega_0$, whose wavelengths $\Lambda_\omega$ exceed the radius of the Universe by many orders of magnitude. They are therefore not observable.

According to the quantum model of galactic DM halos we presented, the contributions to the GWB should mainly come from the halo cores, where dark gravitons are abundant; the contributions from halo envelopes should be significantly smaller. Since the cores are much smaller than the envelopes, the core contributions should be highly localized. In other words, the predicted low-frequency emission should be enhanced at the center of galactic halos, which can also be identified with the centers of their host galaxies. This localized nature of the predicted

<table>
<thead>
<tr>
<th>Specified mass in terms of $m_p$</th>
<th>Mass of DM particle [kg]</th>
<th>Parameter $\kappa_h$</th>
<th>Frequency $\omega_0$ [Hz]</th>
<th>Wavelength $\Lambda_\omega$ [light-year]</th>
<th>Reference to known masses</th>
</tr>
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<tbody>
<tr>
<td>$10^{-25}$</td>
<td>$1.7 \times 10^{-25}$</td>
<td>$3.2 \times 10^{-16}$</td>
<td>$\sim 10^{-8}$</td>
<td>$\sim 10^1$</td>
<td>Higgs boson</td>
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<td>$10^{-26}$</td>
<td>$1.7 \times 10^{-26}$</td>
<td>$3.2 \times 10^{-13}$</td>
<td>$\sim 10^{-10}$</td>
<td>$\sim 10^2$</td>
<td>10 Protons</td>
</tr>
<tr>
<td>$1$</td>
<td>$1.7 \times 10^{-27}$</td>
<td>$3.2 \times 10^{-10}$</td>
<td>$\sim 10^{-12}$</td>
<td>$\sim 10^4$</td>
<td>Proton</td>
</tr>
<tr>
<td>$5.4 \times 10^{-4}$</td>
<td>$9.1 \times 10^{-31}$</td>
<td>2.0</td>
<td>$\sim 10^{-17}$</td>
<td>$\sim 10^8$</td>
<td>Electron</td>
</tr>
<tr>
<td>$5.8 \times 10^{-32}$</td>
<td>$1.0 \times 10^{-38}$</td>
<td>$1.6 \times 10^{94}$</td>
<td>$\sim 10^{-59}$</td>
<td>$\sim 10^{51}$</td>
<td>Extremely light boson</td>
</tr>
<tr>
<td>$5.8 \times 10^{-35}$</td>
<td>$1.0 \times 10^{-61}$</td>
<td>$1.6 \times 10^{97}$</td>
<td>$\sim 10^{-62}$</td>
<td>$\sim 10^{54}$</td>
<td>Ultralight axion</td>
</tr>
</tbody>
</table>

References. (**Hu et al. (2000) and Rogers et al. (2023).**)
emission should be distinguishable from the other sources of the GWB that contribute more uniformly to it. Therefore, it is possible that a localized enhancement of the GWB near the centers of galactic DM cores can be detected. Observationally, the NANOGrav detector (Agazie et al. 2023), future missions devoted to studies of the GWB, or more specifically, to the detection of very low frequency gravitational waves, could be used to validate the halo models we presented in this paper.

6. Conclusions

A quantum model of DM was developed based on a new equation that is complementary to the Schrödinger equation. The new quantum theory predicts that DM emits and absorbs radiation of one fixed frequency, whose quanta are called dark gravitons, and that DM is collisionless as a result of quantum effects. Moreover, application of the model to a galactic DM halo with an NFW density profile showed that a halo is composed of a core and envelope that have different physical properties. In the core, the DM particles move randomly and frequently collide with each other. However, in the envelope the particles are confined to their quantized orbits. The population of orbits in the envelope remains fixed as there are not quantum jumps between them, except in the close vicinity of the core, where the density of orbits is high. The quantum structure of the halo is significantly different from those proposed by previous models of DM (e.g., Sin 1994; Hu et al. 2000; Spivey et al. 2013, 2015; Schive et al. 2014; Zhang et al. 2017).

The new quantum model predicts relations between the quantum structure of the halo and its global physical parameters, such as mass and radius. It requires its density profile to be specified. Moreover, the quantization rule gives a relation between the mass of DM particles and the characteristic frequency of radiation that DM may emit and absorb. Since neither the mass of DM particles nor the frequency of DM radiation are currently known, such as mass and radius. It requires its density profile to be specified. Moreover, the quantization rule gives a relation between the mass of DM particles and the characteristic frequency of radiation that DM may emit and absorb. Since neither the mass of DM particles nor the frequency of DM radiation are currently known, it is possible that a localized enhancement of the GWB near the centers of galactic DM cores can be detected. Observationally, the NANOGrav detector (Agazie et al. 2023), future missions devoted to studies of the GWB, or more specifically, to the detection of very low frequency gravitational waves, could be used to validate the halo models we presented in this paper.

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