1. Introduction

Detonations of sub-Chandrasekhar mass white dwarfs (WDs) in binary systems are a promising scenario for explaining Type Ia supernovae (SNe, Liu et al. 2023). A merger of two WDs could cause a double detonation in which one or both WDs explode (Pakmor et al. 2022).

Here, we consider a stellar binary system, consisting of a WD that is a result of the evolution of a star with initial mass of about $3 M_\odot$, and a companion star with an initial mass of $2 M_\odot$. The companion reaches the end of core hydrogen burning (H) by the time the primary star forms a carbon-oxygen (CO) WD of about $0.6 M_\odot$. The $2 M_\odot$ star becomes a red giant (RG) with a degenerate helium core (He-core), that may, if the initial system was close enough, enter into a common-envelope (CE) phase. Because of dynamical friction and tidal interaction, the WD and the He-core of the companion orbit each other inside the CE and their orbital separation shrinks. The two possible outcomes of this CE interaction are a successful envelope ejection leaving behind a close binary system of the stellar cores or a ‘CE merger’ where the energy release in the orbital decay of the core binary system is not sufficient to drive envelope ejection (Krukow et al. 2021; Röpke & De Marco 2023). In the latter case, the two cores merge inside the part of the CE that is still gravitationally bound to the cores. This is the scenario we explore here. The WD and the He-core of the RG star merge, and the WD explodes as a result of double detonation (see e.g., Fink et al. 2007). This WD detonation happens inside the CE, which will have a distinct effect on the final observational properties. We note that a similar scenario was proposed by Kashi & Soker (2011) and Ilkov & Soker (2012); however, they considered different system parameters. In their ‘core degenerate explosion scenario’, a CO WD merges with the CO core of a massive asymptotic giant branch (AGB) star (see also Ablimit 2021). This is suggested to produce an object consisting of CO material and reaching or exceeding the Chandrasekhar limit. The outcome is hypothesised to be an explosion similar to a SN Ia, but inside the remnant of an unsuccessful CE ejection. This considerably differs from our scenario, where a rather low-mass CO WD interacts with the He-core of a RG star and triggers a detonation via the double-detonation mechanism. Due to the low mass of the exploding sub-Chandrasekhar mass CO WD, very little $^{56}$Ni is produced (Morán-Fraile et al. 2024), and the event arising from a pure and isolated merger of such cores may resemble a calcium-rich transient (Waldman et al. 2011; Kasliwal et al. 2012), although the nature of these objects is still under debate (Polin et al. 2021; Jacobson-Galán et al. 2022; Ertini et al. 2023). However, in the case considered here, this event is buried inside hydrogen-rich CE material.

The extended giant stellar envelope in our scenario in fact represents the second CE episode for this binary since the primary star had to undergo the giant evolutionary phase earlier in the overall binary evolution. Two CE phases in a single system might be relatively rare, though possible, since many explosive events require two CE phases.

The observational properties of a thermonuclear explosion resulting from a merger of two compact cores inside a CE would differ from those of a Type Ia SN.
light curves (LCs) and were used to show the radiative outcome of the highly asymmetric geometry of the product of the merger. The same procedure is used in Blondin et al. (2023).

In Fig. 1, we show the density profiles of all 48 rays in grey and emphasise a few of them as thick coloured lines. The ones we selected are the angle-averaged profile (AA, black), in which all quantities are mass-weighted; case $x$ (or 000000) along the $x$-axis, which points in the direction of a He-core in the orbital plane; case $z$ along the $z$-axis, which is perpendicular to the orbital plane; case 000 040 and case 000 050, both in the $x$–$z$ plane, which is perpendicular to the orbital plane of the progenitor system; and case 045 000 in the orbital plane, pointing 45° from the $x$-axis. The origin of the Cartesian coordinate system (zero point) is the WD centre. To illustrate the 3D geometry of the merger product, we show the ejecta structure with the selected viewing angles in Fig. 2. The detailed description of the ejecta is presented in Morán-Fraile et al. (2024).

The 3D AREPO output at 70 s after thermonuclear detonation of the WD, when all important nuclear reactions have ceased, was mapped into the extended atmosphere of the non-degenerate star. We note that nickel bubbles and, in turn, the nickel-bubble effect, are likely irrelevant in our system. The total amount of radioactive nickel $^{56}\text{Ni}$ contributes only 1.4% of the ejecta mass of the pure merger product, and 0.5% of the total ejecta mass of the merger within the envelope. Furthermore, although nickel is distributed non-isotropically, it does not form clumps; in other words, we do not expect formation of nickel bubbles later than 70 s after the WD detonation, when the merger ejecta reach homologous expansion. The nickel-rich region is confined within 0.9 $M_\odot$ of the ejecta and move equally at the same velocity in different directions; therefore, we exclude development of Kelvin-Helmholtz instabilities at a later time. No further non-radial mixing of the ejecta material is expected, although long-term simulations are required to confirm this assumption. We note that we aim to simulate the merger of the WD and the He-core inside the CE, which is not ejected during the CE phase. We explain the choice of the stellar atmosphere models in Sect. 2.2.

In Fig. 3, we show the chemical structure of the merger product without envelope for the selected rays and the AA case. The grey curves in each subplot represent the density profile (scaled for illustrative purposes; for the actual physical density profiles see Fig. 1) to indicate the mass content of given species. For example, the helium fraction is relatively high in the outer part of the $x$-ray (along the axis connecting the WD and the He-core). Helium is less abundant in the outer region in the directions 000 040 and 045 000. The thickness of the helium layer in combination with the $^{56}\text{Ni}$ content influences the rise point of the LC: the lower the helium mass, the shorter the rise time. If $^{56}\text{Ni}$ is present in the same region, the rise time of the LC is even shorter (Piro & Nakar 2013), as $\gamma$-ray photons produced via radioactive decay reach the photosphere earlier, causing an increase in luminosity.

In Table 1, we list the helium mass in each ray and in the AA case for the merger product without envelope. For the mass of helium in the ejecta, we distinguish between the total mass; the mass within the inner 0.6 $M_\odot$ of the merger, which is associated with the WD itself; and the mass in the outer ejecta of the merger. The division is set to help in interpreting the LCs resulting from the pure WD–He-core detonation. It does not mean that the inner 0.6 $M_\odot$ consists of only CO WD; instead, it accounts for the macroscopic mixing happening during the accretion and ignition phase. The latter results in a large fraction of helium in the inner 0.6 $M_\odot$ in almost all directions. We note that we list $4\pi$-equivalent values for the masses in Table 1, which might be overestimated.
We also include the $^{56}\text{Ni}$ masses with the same meaning, particularly to show the effect of the presence of radioactive nickel in the outer ejecta of the merger on the resulting LCs (see Sect. 3.1). In addition, we provide the terminal kinetic energy for each case that corresponds to the total energy flowing in the different directions. The difference in the kinetic energy for different rays is explained by the different density distributions along the rays as velocity profiles tend to be very similar to each other. We do not list the mass of the hydrogen-rich CE in Table 1 because the envelope is equivalent in the sense of mass and composition in all directions. We provide details about the CE in Sect. 2.2.

2.2. One-dimensional MESA simulations of the secondary star

The goal of the study is to investigate the observational implications of a thermonuclear explosion arising from the core merger interacting with CE material. Because we consider a system where CE ejection has not successfully completed and the cores merge instead of forming a close binary system of compact cores, we have to embed the explosion model presented by Morán-Fraile et al. (2024) in a model for the CE material surrounding it. For simplicity, we consider two cases: the unperturbed envelope of the RG star in hydrostatic equilibrium and a perturbed RG envelope that results from the framework of the parametrised one-dimensional CE interaction model described by Bronner et al. (2024).

As discussed in Morán-Fraile et al. (2024), a successful detonation of the primary 0.6 $M_\odot$ CO WD seems to be possible only for He-cores (or He WDs) in a narrow range around 0.4 $M_\odot$. This is due to the He ignition mechanism that ignites the double detonation, relying on the He-core being disrupted relatively close to the CO WD. The tidal forces acting on a significantly less massive core result on a disruption further away from the
Fig. 3. Chemical structure of the individual rays for the pure merger. The grey curves represent scaled density profiles (see details in the text). The labels ‘X’ and ‘Z’ stand for case x and case z in our set of selected directions.

### Table 1. 4π-equivalent parameters of the pure merger without an envelope taken at different rays.

<table>
<thead>
<tr>
<th>Ray</th>
<th>He [M⊙]</th>
<th>56Ni [M⊙]</th>
<th>E_{56}</th>
<th>M_{in}</th>
<th>M_{out}</th>
<th>M_{tot}</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>0.51</td>
<td>0.22</td>
<td>0.29</td>
<td>0.038</td>
<td>0.037</td>
<td>0.001</td>
</tr>
<tr>
<td>Z</td>
<td>0.33</td>
<td>0.15</td>
<td>0.17</td>
<td>0.010</td>
<td>0.010</td>
<td>0.000</td>
</tr>
<tr>
<td>000 040</td>
<td>0.25</td>
<td>0.02</td>
<td>0.23</td>
<td>0.009</td>
<td>0.006</td>
<td>0.003</td>
</tr>
<tr>
<td>000 050</td>
<td>0.46</td>
<td>0.21</td>
<td>0.25</td>
<td>0.043</td>
<td>0.041</td>
<td>0.002</td>
</tr>
<tr>
<td>045 000</td>
<td>0.56</td>
<td>0.29</td>
<td>0.27</td>
<td>0.014</td>
<td>0.014</td>
<td>0.000</td>
</tr>
<tr>
<td>AA</td>
<td>0.32</td>
<td>0.11</td>
<td>0.21</td>
<td>0.014</td>
<td>0.013</td>
<td>0.001</td>
</tr>
</tbody>
</table>

**Notes.** Total mass of helium and radioactive nickel 56Ni in M⊙ (total mass M_{tot}, mass within the inner 0.6 M⊙, and mass in the outer 0.4 M⊙), terminal kinetic energy as a representative of explosion energy E_{56} in the units of 10^{56} erg.

At the end of the simulation, the star has a radius of 75 R⊙ and a mass of 1.9 M⊙. Then, we perturbed the envelope of the secondary to mimic a CE evolution following the model described in Bronner et al. (2024). This model integrates the orbits of the two stars in the CE by assuming a drag force (Kim 2010) and injects the released orbital energy as heat into the envelope. We chose \(a_{\text{ini}} = 70 R_\odot\) for the initial separation and \(C_d = 0.25\) and \(C_b = 2.0\) for the two free parameters. The numerical values for \(C_d\) and \(C_b\) were inspired by the CE simulations of a 1 M⊙ AGB star described in Bronner et al. (2024). The CE simulation lasts for 77 d, during which the envelope expands to about 330 R⊙. The calculation with the unperturbed envelope was used as a reference point to illustrate the effect of an inflated envelope around the detonating merger. As our primary interest lies along the perturbed case, we name this kind of envelope ‘env1’, while we keep the unperturbed case as a reference with the name ‘env2’.

The hydrogen-rich atmosphere of the giant star with the extracted He-core has a mass of 1.5 M⊙. In order to append the atmosphere, we link the profiles of both cases of the envelope to selected profiles of the merger product (1 M⊙), so that the junction is smooth density-wise.

### 2.3. One-dimensional radiative-transfer STELLA simulations

The profile representing the AA case and profiles in selected directions were mapped into the 1D radiation-hydrodynamics code STELLA (Blinnikov et al. 2006)\(^3\). STELLA is capable of modelling hydrodynamics, including shock propagation and its interaction with the medium, as well as the radiation field evolution; in other words, it self-consistently computes the LCs, the spectral energy distribution, and the resulting broad-band magnitudes and colours. We used the standard parameter settings, which are explained in many papers involving STELLA simulations (see e.g., Tsvetkov et al. 2021; Moriya et al. 2020). The thermalisation parameter is set to 0.9, as recommended by the recent study of Kozyreva et al. (2020). The profiles in different

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\(^3\) The version of STELLA used in the current study is the private version and not the one implemented in MESA (Paxton et al. 2018).
directions are considered as $4\pi$-equivalent spherically symmetric models, and cannot represent the entire picture of a full 3D radiative transfer simulations as can be done with ARTIS, SEDONA, SuperNu, and other sophisticated spectral synthesis codes (Blondin et al. 2022), which, however, lack models for the interaction of hydrodynamics and radiation.

3. Results and discussion

3.1. Light curves

In Fig. 4 we present bolometric LCs for the pure merger product without envelope seen from different viewing angles and the AA case. We calculate and show these LCs as a reference to demonstrate how different the LCs for the merger with envelope look. The LCs resemble LCs of SNe Ia which means that they are powered by the radioactive decay of $^{56}$Ni and $^{56}$Co. The peak luminosity of the nickel-powered LC is expected to be connected to the mass of $^{56}$Ni following the relation (Arnett 1979)

$$L_{\text{peak}} \sim M_{\text{Ni}} \varepsilon(t_{\text{peak}}),$$

where $\varepsilon(t_{\text{peak}})$ is the decay function of nickel and cobalt. The shape of these LCs also depends on the distribution of radioactive $^{56}$Ni within the ejecta and the ejecta mass. The shallower the $^{56}$Ni distribution, the earlier and the lower the main peak compared to the centrally concentrated $^{56}$Ni distribution. For instance, the presence of radioactive $^{56}$Ni in the outer layers in the 000 040 case shortens the rise time (Piro & Nakar 2013, 2014), and even produces a bump in the early LC (Noebauer et al. 2017; Magee et al. 2020). We note that the $^{56}$Ni masses for individual rays listed in Table 1 are considered as $4\pi$-equivalents and do not necessarily correspond to the actual mass of this isootope. As discussed in Sect. 2, the rise time of the LC also depends on the mass of material lying on top of the $^{56}$Ni-enriched layers, particularly the amount of helium in the outer part of the ejecta. Helium that is mixed with a small fraction of radioactive nickel $^{56}$Ni is non-thermally ionised and produces higher opacity, which in turn increases the rise time (Dessart et al. 2012). Thus, the 000 040 LC has the shortest rise to the peak as this input model has the lowest helium content and some amount of $^{56}$Ni in the outer layers. Another impact of the high degree of mixing of $^{56}$Ni is the redder colours of the resulting LCs (Yoon et al. 2019) because nickel (similar to iron and other iron-group elements) has a high line opacity, and efficiently redistributes blue flux into redder wavelengths (Kasen 2006).

In realistic 3D simulations the LC will depend on the viewing angle and should be a sum of the photons streaming in different rays. With our 1D simulations we show the scatter of the final LCs the observer can see from different viewing angles, which depends on the position of the asymmetric merging system relative to a point of view. The LCs computed with STELLA agree reasonably well with those simulated with 3D version of ARTIS (Morán-Fraile et al. 2024).

When embedded in the hydrogen-rich envelope, the original shape of a SN Ia-like LC of the pure merger (see Fig. 3 in Morán-Fraile et al. 2024) is significantly erased, as shown in Fig. 5. The resulting LCs exhibit a pronounced plateau, which resembles that of a hydrogen-rich SN II. The luminosity on the plateau and its duration vary within 0.3 dex and depend on the effective explosion energy according to the scaled relations of Popov (1993): $\log L_{\text{plateau}} \sim 0.8 \log E_{\text{exp}}$, where the dependence on mass and radius is irrelevant for our case because radius and mass are the same for all viewing angles. The duration of the plateau varies between 30 days and 45 days for the cases of low (ray 000 040, ray 045 00, z-ray, and AA case) and high (x-ray and ray 000 050) $^{56}$Ni mass (see Table 1), respectively. The LCs for the x-ray and the ray 000 050 show the delayed $^{56}$Ni-powered contribution to the overall LCs, extending the plateau by ten more days. The influence of $^{56}$Ni in extending and lifting the plateau is also present in usual SNe IIP (Kasen & Woosley 2009; Goldberg et al. 2019; Kozyreva et al. 2019). The larger radius of the extended envelope env1, 330 $R_\odot$, causes shallower decline in LCs after shock breakout (see Fig. 5), according to the time $t_{\text{sec}} \sim R_{\odot}^{1.76}$, when recombination settles in the outer ejecta (Shussman et al. 2016). In contrast, the LCs of the merger in the compact envelope env2 ($75 R_\odot$) drop quickly after shock breakout.

As seen in Fig. 5, there are two extreme cases among all the considered models in our study. Therefore, we picked the x-ray and the AA case as boundary examples to show the broad-band LCs in the $U$, $B$, $V$, and $R$ bands in Fig. A1.1. Interestingly, the luminosity of the low-energy (0.6 foe) transient considered in our study is similar to plateau luminosity found for average SNe IIP. Particularly, the models including env1 (extending to 330 $R_\odot$) have up to $\sim 17$ mag in $U$-band, $\sim 16.3$ mag in $B$-band, $\sim 16.4$ mag in $V$-band, and $\sim 16.3$ mag in $R$-band, which is seen in SN 1999em (Elmhamdi et al. 2003). However, the distinct difference of our transients is the very short plateau, lasting for only 30–45 days, while a usual duration of the plateau in SNe IIP is from 100 days to 150 days.

3.2. Dependence on metallicity

As described in Sect. 2.2, the chemical composition of the modelled stellar envelopes does not include iron. However, iron plays an important role in radiative transfer simulations, in particular for shaping the X-ray, and $U$- and $V$-band LCs. Therefore, we calculated an additional set of models for two cases, AA and ray 000 050 for both cases of the envelope, extended (env1) and compact (env2), with solar metallicity abundance of iron ($X/Fe = 1.46 \times 10^{-3}$, Lodders 2003). The resulting comparison plots are presented in Figs. 6 and B.1. We note that the main influence of iron in the hydrogen-rich envelope is line blanketing in the bluer wavelengths, while it does not affect redder wavelengths ($V$-band and beyond). The bolometric LCs are also not heavily affected by iron, similarly to the $V$- and $R$-bands, since the flux in these bands mostly contributes to overall luminosity,
Fig. 5. Bolometric LCs for the selected rays and the AA case embedded into extended (env1, left) and compact (env2, right) envelopes. The labels ‘X’ and ‘Z’ stand for case x and case z in our set of selected directions.

Fig. 6. Metallicity dependence for the extended envelope. The U (magenta), B (blue), and R (red) LCs show the difference between the case of the iron-free envelope (dashed) and iron-polluted envelope (solid) for the AA case (left) and the ray 000050 (right).

except the length of the plateau. The plateau is five days longer in the case of non-zero abundance of iron, since iron contributes to the overall opacity, making the optical depth and diffusion time longer for the hydrogen-rich envelope. The latter can be seen, for example, in the R-band LCs. The U-band LCs decline faster in the case of non-zero iron abundance in the envelope in comparison to the case of the iron-free envelope, and the luminosity at the end of the plateau is more than 1 mag lower for the iron-enriched case.

4. Looking for the possible observed candidates

While looking for the possible observed candidates of our modelled events, we applied the following criteria: the luminosity on the plateau should be a little bit lower than usual for SNe IIP (−16 mag) and, the most important criterion, the duration of the plateau should not exceed 40 days. These criteria are strict and difficult to match. However, we found a few possible SNe with similar properties in a few observational sets such as Anderson et al. (2014)\(^4\) and Martinez et al. (2022)\(^5\). We limited our comparison to the V-band because in many cases data in other broad bands are unavailable and because detailed comparison between models and observations is beyond the scope of the current study. The observational data are from the Open Supernova Catalogue (Guillochon et al. 2017)\(^6\).

In total, we found five examples among the observed SNe II with low to intermediate luminosity on the plateau and short plateau duration. The best matching synthetic LCs are from the following models: X-ray, and 040000 and 045000 rays, and AA in the extended envelope; and z-ray in the compact envelope. In Fig. 7 we compare the V-band magnitude for our selected rays and the AA case to data from SN 2004dy, SN 2005af, SN 2005hd, SN 2007aa, and SN 2008bu. The cases of SN 2004dy, SN 2005hd, SN 2007aa, and SN 2008bu display V-band LCs similar to our models in the extended envelope, while SN 2005af\(^7\) might be considered close to the z-ray in the

\(^4\) The list of transient surveys used can be found in the publication.

\(^5\) Carnegie Supernova Project-I.

\(^6\) https://github.com/astrocatalogs/supernovae

\(^7\) The explosion epoch for SN2005af is uncertain. Hence, Filippenko & Foley (2005) report that the explosion happened ‘perhaps a month before’ their acquired spectrum. Furthermore, Kotak et al. (2006) claim that ‘the explosion epoch is probably uncertain by up to a few weeks’. Therefore, 2005af should be considered as a candidate for our merger with some caution.
As we note above in Sect. 3.1, the luminosity on the plateau is not a strong criterion for looking for possible candidates. Our models within the extended envelope have $-16.3 \text{ mag}$ to $-17 \text{ mag}$ in $U$, $B$, $V$, and $R$ broad bands, which is close to normal values for SNe IIP, even though slightly lower. The low-luminosity SNe IIP have even lower luminosities on the plateau (about $-14 \text{ mag}$, for example SN 2020cdx and SN 2005cs; see Pastorello et al. 2009; Yang et al. 2021; Valerin et al. 2022), but the duration of the plateau is 150 days, which is mainly the signature of a relatively low-energy explosion. Therefore, the study by Kozyreva et al. (2022) shows that these SNe can be easily explained by neutrino-driven core-collapse explosions of low-mass massive stars in the initial mass range around $9 \, M_\odot$. The very low-luminosity event SN 1997D with a $V$-band magnitude of $-14.65 \text{ mag}$ suffers from a significant uncertainty in the explosion epoch, although the best estimate based on the spectral synthesis and colour evolution shows that plateau duration of SN 1997D is longer than 50 days (Turatto et al. 1998; Chugai & Utrobin 2000; Benetti et al. 2001; Zampieri et al. 2003), meaning that it might not be a good match for our models. The recently reported luminous short-plateau events SNe 2006Y, 2006ai, and 2016eg have plateaus of 50–70 days, and their plateau luminosity is relatively high, $-17$ to $-17.5 \text{ mag}$ (Hiramatsu et al. 2021), which is more likely to be explained by relatively high for CCSNe explosion energies (Popov 1993).

We conclude that our simulated merger product, which is surrounded by an extended stellar atmosphere of a giant star, can be seen as a very short-plateau SN IIP. However, the host galaxy of this SN should contain an old stellar population, or the SN location should be associated with the old population regions of a galaxy. The rate of this kind of explosion can reach up to a few per cent that of all SNe IIP, if considering the observations by Anderson et al. (2014). Nevertheless, the detection of transients matching the predictions of the models in our study is complicated because of their short 30–45 day plateau; this means that their detectability is lower in comparison to the 100-day SNe IIP, as some of these short-living transients can be missed. In reality, the probability of the possible candidates for our models, specifically those within a compact stellar atmosphere, is even lower because of relatively low luminosity, which depends on the total amount of detected SNe IIP. Next-generation transient facilities such as the James Webb Space Telescope (JWST), Euclid Telescope, Roman Space Telescopes, and Rubin Observatory (LSST) will provide a deeper observations and cover a larger volume of the Universe. As a result, a larger number of low-luminosity transients will be detected, potentially including events that can be explained by our models.

5. How to distinguish thermonuclear Type II SNe

In the previous sections we show that the LC resulting from the merger of a low-mass WD and a He-core of a RG star happening inside the CE is similar to a SN II-P, although with some distinct properties. The mechanism providing the energy deep inside the hydrogen-rich envelope, however, is a thermonuclear explosion instead of a gravitational collapse of a stellar core. This leads to the question of how to distinguish this scenario from the common gravitational collapse-induced SNe II. In the following we discuss how electromagnetic observables can break this degeneracy, and how non-electromagnetic observables can provide a solid distinction between the two explosion scenarios.
After the onset of the explosion the photosphere reaches a layer that moves at 3000 km s$^{-1}$ and slower. The photospheric phase turns to the nebular phase; the spectra become dominated by distinct lines, which looking at the line width, because of a similar explosion energy of 0.6 foe (Ertl et al. 2016, 2020).

After day 40, the photosphere moves into the inner part of the ejecta (which include the actual merger product) and the inner mechanism of the SN event is revealed to electromagnetic observation. We expect, at this time, that there will be a transition to the nebular phase for our event, which is earlier than in Type II SNe (day 150) since the photospheric phase is over. The spectra should show lines of He, O, Ca, Si, and other heavy elements in addition to the typical spectra of Type II SNe, namely hydrogen lines, because the presence of a hydrogen-rich envelope cannot be hidden and will contribute to the spectra even after the average photosphere moves to the inner ejecta (Jerkstrand et al. 2012). An attempt to model a thermonuclear explosion of a Chandrasekhar-mass WD (with 0.3$M_\odot$ of $^{56}$Ni) within a circumstellar material (which means CE with the total mass of 13$M_\odot$) is proposed in Jerkstrand et al. (2020). The nebular spectra of their transient were a mixture of spectra of a WD explosion and a strong Hz line.

To make a more educated guess about the resulting spectra, we analyse the total yields of our merger in comparison to yields for core-collapse explosions and three models of thermonuclear WD explosions. The yields for CCSNe are: electron-capture SN e88 with the initial mass of 8.8$M_\odot$ (Kozyreva et al. 2021), s9 (9$M_\odot$), and s15 (15$M_\odot$) progenitors from Sukhbold et al. (2016), and the set of models ww11–ww25 (Woosley & Weaver 1995, their models s11A–s25A, respectively). Thermonuclear explosions include an explosion of a sub-Chandrasekhar mass WD (Model 1 of Woosley & Weaver 1994), the W7 model of an explosion of a Chandrasekhar-mass WD (Nomoto et al. 1984), and the Chandrasekhar-mass WD explosion model DDC10 (Blondin et al. 2013). We show yields of the selected species in Fig. 9. The yields of carbon, oxygen, and $^{56}$Ni in our merger model are closer to the yields of core-collapse events in low-mass star range (9–12$M_\odot$). The yields in neon, magnesium, silicon, sulphur, and argon yields are similar to both core-collapse events in stars above 15$M_\odot$ and thermonuclear explosions of WDs, while the calcium yield is clearly more compatible with the thermonuclear explosion DDC10. Therefore, the spectra of the merger might contain features that can indicate a core-collapse of a massive star in an entire range of CCSN progenitors, or a WD explosion at the same time. However, it is difficult to assess the strength of the spectral lines of these species in our merger in comparison to core-collapse SNe (CCSNe) and WD explosions since ionisation states can differ significantly. Based on the nebular spectral synthesis by Jerkstrand et al. (2020), we estimate that spectra after about day 40 will appear as a blend of Type I SNe with a pronounced Hα line, which is a diagnostic for the transients analysed in this study. For a preliminary investigation of the spectra without the hydrogen-rich envelope, we refer to Morán-Fraile et al. (2024). The lines of the elements resulting from the thermonuclear burning are expected to be narrower, in contrast to the typical photospheric velocity of a SN Ia about 10000 km s$^{-1}$ during the first 60 days after the explosion.

The nebular spectra for one of our possible observed candidates, namely SN 2005af, were published by Kotak et al. (2006).

Among the spectral features usual for a SN IIP, SN 2005af shows strong [Ar II] and [Ni II] lines, which are claimed in Kotak et al.

8 More precisely, the width of lines corresponds to the quantity $\sqrt{E_{\text{exp}}/M_\odot}$.
to be stronger than in usual SNe IIP. Their estimates for the lower limits of the Ne, Ar, and stable Ni yields are $10^{-3} M_{\odot}$, $2.2 \times 10^{-3} M_{\odot}$, and $3.7 \times 10^{-3} M_{\odot}$, respectively, although these estimates are almost ten times lower than those for our merger model. The total mass of radioactive nickel $^{56}$Ni, as estimated in Kotak et al. (2006), is 0.027 $M_{\odot}$, which is higher than that produced by our merger (0.014 $M_{\odot}$). However, the detected degree of polarisation, which corresponds to 20% asphericity of the SN ejecta (Pereyra et al. 2006) may favor the ‘merger-CE’ origin of SN 2005af. The spectra for another candidate, SN 2004dy, show a mixture of a SN IIP and a SN Ib/c, and therefore it was identified as a peculiar Type II SN (IAUC 8404 and IAUC 8409)\(^9\). It has been suggested that the presence of the strong He I 5876 Å (which is found to be much stronger than usual in SNe IIP) requires a high helium abundance; however, no quantitative analysis of the helium mass in this SN is available in the literature. For comparison, the helium yield is 0.72 $M_{\odot}$ in our merger with CE material included, while a 15 $M_{\odot}$ CCSN progenitor yields about 5 $M_{\odot}$ helium, although the conditions for forming lines can be different. There is an ongoing debate about the amount of hidden helium in the ejecta in different types of SNe (Hachinger et al. 2012; Williamson et al. 2021). We note that helium produces a distinct emission line when being non-thermally excited (if it is located within one mean-free path of $\gamma$-rays from radioactive nickel $^{56}$Ni; Dessart et al. 2012). The latter is realised in our merger product, as seen in Fig. 3, and hence the strong He I line in SN 2004dy might favour the merger-CE origin of this SN. In addition, the photospheric velocity in SN 2004dy estimated via a minimum of PCyg of Fe II line are around 4000 km s\(^{-1}\) at day 30, which is close to the photospheric velocities of our merger at a corresponding epoch (see Fig. 8).

5.2. Non-electromagnetic observables

In addition to the electromagnetic signal, multi-messenger observations could in principle lead to a clear distinction between a thermonuclear explosion and a core-collapse event. In particular the expected gravitational wave (GW) signal and neutrino radiation are expected to be fundamentally different. It has recently been suggested that nearby CE events can produce a GW signal detectable by the upcoming space-based GW observatory LISA\(^10\) and future planned deci-hertz observatories such as DECIGO (Kawamura et al. 2019) and BBO (Harry et al. 2007) if they lead to a core merger (Ginat et al. 2020; Morán-Fraile et al. 2023) or to a tight enough orbit (Renzo et al. 2021). The binary system studied in the present paper is similar to that considered in Morán-Fraile et al. (2023), releasing GW radiation on a broad frequency range during the CE event ($f_{GW} \sim 10^{-2}$ Hz) until the core merger ($f_{GW} \sim 10^{-2}$ Hz), and should be detectable under the same conditions. This signal will only diverge from that presented in Morán-Fraile et al. (2023) in the post-merger stage as the thermonuclear explosion taking place shortly after the disruption of the core will avoid the emission of most of the high-frequency components.

The GW signal of our merger will be fundamentally different to GW radiation produced by CCSNe, as the GW signal from CCSNe spreads in a very different frequency range between a few hundred and a few thousand Hz (Andresen et al. 2017; Vartanyan et al. 2023). Therefore, a multi-messenger detection would help to identify the explosion mechanism of such a transient.

The event considered in our study would release neutrino radiation similar to other thermonuclear explosions (Seitenzahl et al. 2015). In our simulations the treatment of neutrinos is very coarse and should be improved for a more detailed prediction of observables. It only takes into account thermal neutrino losses via a cooling term and it misses the neutrinos produced in weak reactions. At the point of carbon ignition, the merger releases $5 \times 10^{52}$ erg in neutrinos, which is much lower than the typical total neutrino energy of $10^{53}$ erg in neutrino-driven core-collapse explosions (Fischer et al. 2010; Ertl et al. 2016; Kresse et al. 2021). Hence, the lack of a detectable neutrino signal could be another diagnostic to distinguish between the thermonuclear and the core-collapse explosion scenarios, assuming that the explosions happen at close enough distances.

6. Summary and conclusions

In the present study we conducted radiative-transfer simulations for a merger of a white dwarf (WD) of 0.6 $M_{\odot}$ and a degenerate He-core of a red giant star (Morán-Fraile et al. 2024). We show that the light curves of a pure merger are low luminosity, as expected, because of the low yield of radioactive nickel $^{56}$Ni (0.014 $M_{\odot}$).

We considered the system in which the merger occurs inside the atmosphere of a giant star, a binary undergoing the common-envelope (CE) episode. In this system, the CE is not fully ejected under certain conditions, but expands and results in an extended envelope. We consider two possibilities for CE extension, to the radius of 75 $R_{\odot}$ and 330 $R_{\odot}$, without and with perturbations, respectively. These two possibilities can be interpreted as follows. The unperturbed envelope is a default atmosphere of a red giant star with an initial mass of 2 $M_{\odot}$ at the end of the hydrogen core burning, while the perturbed envelope is a result of an evolution with the additional injected energy that mimics the heat from the orbital energy coming from the inspiraling of a WD and a He-core.

A WD–He-core merger happening inside the hydrogen-rich atmosphere of a giant star will have observational properties different to a SN Ia transient. Instead, the predicted transient LC resembles a short 30–45 day plateau of Type II SNe. Spectroscopically the transient is supposed to resemble a hydrogen-rich SN during the plateau phase as the atmosphere of a giant star contains 1 $M_{\odot}$ of hydrogen. Depending on the viewing angle, the plateau lasts 30–45 days. The luminosity on the plateau is intermediate to low: $L_{bol} = 41.7–42$ erg s\(^{-1}\) (or \(-15.4\) mag to \(-16.4\) mag in V-band).

We found five SNe with the short 30–40 day plateau among the observed SNe IIP available in the literature that can be candidates for our models. These are SN 2004dy, SN 2005af, SN 2005hd, SN 2007aa, and SN 2008bu. However, our analysis was limited because of a lack of a full observational set for these events.

Our models at later phases will have spectra that are supposed to be a mixture of both Type Ia and Type II SNe, and low photospheric velocities of about 2500 km s\(^{-1}\). Hence, late time observations, later than 40 days after the first detection, will help to distinguish events similar to the models in our study. The merger inside the CE will also have a certain GW signal, which is different to WD–WD background noise and very different to the GW signal from core-collapse SNe, and very low-luminosity
neutrino radiation different to the neutrino radiation released by core-collapse explosions.

In some binary stellar systems entering CE evolution it is expected that the envelope ejection is not successful, and that a CE-merger event of the two stellar cores inside the envelope ensues. We predict that the resulting transient will resemble SNe II, although the physical origin is a thermonuclear explosion of the merging cores. An observational identification of the transients resulting from such a scenario based on the predicted synthetic observables will shed light on the CE physics.

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Appendix A: Broad-band LCs of the selected models: AA and \( x \)-ray cases

In Figure A.1 we present \( U \), \( B \), \( V \), and \( R \) broad-band LCs for the AA case (black) and the \( x \)-ray case (blue) in the extended env1 (solid) and compact env2 (dashed) envelopes. These plots show that the models within the compact envelope have intermediate to low luminosity during the plateau phase, \(-15\) mag to \(-16\) mag. This low magnitude in combination with the very short plateau of about 30 days (AA; black) and 45 days (\( x \)-ray; blue) makes it difficult to find a suitable candidates among observed SNe IIP (Anderson et al. 2014; Faran et al. 2014; Martinez et al. 2022). However, we succeeded with this task and found five possible candidates, as discussed in Section 4. We conclude that the case of the merging WD–He-core system within a compact envelope is very rare in nature. In our search, one SN out of five candidates was found. The case of the same system being embedded in an extended envelope is more probable; based on the current observational record, we conclude that at least some SNe IIP with normal to low luminosity with an extremely short plateau of 30–40 days can be a result of the evolution of a binary system in which a WD engulfed into the CE merges with the He-core of a red giant and detonates.

Fig. A.1. \( U \), \( B \), \( V \), and \( R \) broad-band LCs for the AA (black) and \( x \)-ray (blue) cases in the extended env1 (solid) and compact env2 (dashed) envelopes. The label ‘\( X \)’ stands for case \( x \) in our set of selected directions.
Appendix B: Dependence on metallicity

In Figure B.1 we provide plots showing the dependence on metallicity for the case of the compact envelope of the giant star in addition to the problem discussed in Section 3.2. The resulting LCs display behaviour that is similar to the case of the extended envelope. Specifically, the models with the iron polluted envelope with the iron fraction corresponding to the solar metallic-

ity have $U$-band LCs declining faster during the plateau phase, and end up with the $U$-band magnitude 1 mag lower. The $R$-band LCs serve as a representative of the bolometric LCs showing that the plateau lasts five days longer than those LCs for the iron-free models. The latter is explained by the higher opacity in the hydrogen-rich envelope, as the line opacity of the iron-group elements is the highest among all the species.

**Fig. B.1.** Metallicity dependence for the compact envelope. The $U$ (magenta), $B$ (blue), and $R$ (red) LCs show the difference between the case of iron-free envelope (zero abundance of iron; dashed) and iron-polluted envelope (iron abundance corresponding to solar metallicity; solid) for the AA case (left) and the ray 000 050 case (right).