Propagating a dome-shaped, large-scale extreme-ultraviolet wave in the solar corona

Gottfried Mann¹ and Astrid M. Veronig²,³

¹ Leibniz-Institut für Astrophysik Potsdam, An der Sternwarte 16, 14482 Potsdam, Germany
e-mail: GMann@aip.de
² Institute of Physics, University of Graz, Universitätsplatz 5, 8010 Graz, Austria
³ Kanzelhöhe Observatory for Solar and Environmental Research, University of Graz, Kanzelhöhe 19, 9521 Treffen, Austria

Received 14 December 2022 / Accepted 23 June 2023

ABSTRACT

Context. The first observation of a dome-shaped extreme-ultraviolet (EUV) wave was recorded by the EUVI instrument on board the STEREO-B spacecraft on January 17, 2010. This observation has allowed us to study the three-dimensional propagation of a large-scale EUV wave in the solar corona, which is considered to be a manifestation of a large-amplitude magnetohydrodynamic (MHD) wave.

Aims. These unique observations by EUVI offer the opportunity to compare the theory of large-amplitude MHD waves with observations.

Methods. Nonlinear MHD equations were employed for describing large-amplitude fast magnetosonic waves in terms of so-called simple waves.

Results. Measuring the velocity of the EUV wave across the solar surface allows us to determine the quiet Sun’s magnetic field to be \(\approx 3 \, \text{G} \) (Klassen et al. 2000), with later observations suggesting it may be higher. The wave steepening vanishes at the final state of the evolution of the EUV wave, which is consistent with the observations. Hence, the wave steepening vanishes at the final state of the evolution of the EUV wave, which is consistent with the observations.

Conclusions. The propagation of this dome-shaped EUV wave can be well described by the theory of large amplitude (simple) MHD waves.

Key words. Sun: corona – Sun: flares – Sun: coronal mass ejections (CMEs)

1. Introduction

The Extreme Ultraviolet Imaging Telescope (EIT; Delaboudinière et al. 1995) aboard the ESA/NASA SOHO spacecraft has discovered a new wave phenomenon in the solar corona (Moses et al. 1997; Thompson et al. 1998), hereafter referred to as “EIT waves” (also as “large-scale EUV waves” or “large-scale coronal bright fronts”).

Large-scale EUV waves appear as a bright rim (sometimes nearly circular) propagating as a large-scale disturbance over substantial regions of the Sun. In extreme cases, they can even propagate over the full Sun, like the event associated with the strong X8.2 flare/CME on 10 September 2017 (Liu et al. 2018; Veronig et al. 2018). They are reminiscent of Moreton waves (Moreton & Ramsey 1960), which are observed in the solar chromosphere using filtergrams recorded in the Hα spectral line sensitive to a temperature of about 10^4 K. Moreton waves are often accompanied by solar type II radio bursts (Kai 1970; Warmuth et al. 2004b) as signatures of coronal shock waves (Wild et al. 1959). Because of this relationship, Moreton waves and the type II radio burst related shocked waves are believed to be caused by the same origin, namely by a solar eruption (Uchida 1968; Vršnak et al. 2016). In contrast to Moreton waves, large-scale EUV waves are visible in the EUV spectral lines formed at coronal temperatures in the range of \( T = (1 – 2) \times 10^6 \, \text{K} \) (e.g., Zhukov & Auchère 2004; Vanninathan et al. 2015). Furthermore, the velocities of Moreton waves (e.g., Warmuth et al. 2004a) are (on average) much higher than those of the EUV waves, which lie mostly in the range 200–500 km s\(^{-1}\) (Klassen et al. 2000; Thompson & Myers 2009; Warmuth & Mann 2011; Muhr et al. 2014), though in some cases also speeds of the order of 1000 km s\(^{-1}\) have been reported (Nitta et al. 2013; Liu et al. 2018; Veronig et al. 2018).

According to Mann et al. (1999a), a flare and/or coronal mass ejection (CME) excites a fast magnetosonic wave in the corona, which is observed as a EUV wave when it propagates along the surface of the Sun. If it enters into the corona, it steepens to a shock wave observed as type II radio bursts. This scenario explains the relationship between EUV waves and solar type II radio bursts as reported by Klassen et al. (2000). Adopting this consideration, a magnetic field strength of \( \approx 3 \, \text{G} \) is deduced for the quiet Sun. The Alfvén-Mach number of the type II radio burst associated shocks is found to be well above 1.6 (Mann et al. 1999a).
Because of the low cadence of 12–15 min of the SOHO/EIT instrument, it has not been possible to properly study the evolution of EUV waves. This limitation was overcome by the Extreme Ultraviolet Imager (EUVI; Howard et al. 2008) on board the twin spacecraft of the Solar-Terrestrial Relations Observatory (STEREO; Kaiser et al. 2008). For studying large-scale coronal waves, the STEREO EUVI observations have the advantage of having a large field of view, high sensitivity, and higher cadence while carrying out simultaneous observations from two well-separated vantage points in space, which have enabled studies of the three-dimensional (3D) evolution of EUV waves and the wave-driver relationship, as well as detailed comparisons with MHD simulations of solar eruptions (e.g., Long et al. 2008; Veronig et al. 2008; Kienreich et al. 2009; Patsourakos & Vourlidas 2009, 2012; Cohen et al. 2009; Downs et al. 2011; Ma et al. 2011; Muhr et al. 2011, 2014; Delanné et al. 2014; Podladchikova et al. 2019). The simultaneous ultra-high-cadence observations (12 s) in six different EUV filters by the Atmospheric Imaging Assembly (AIA; Lemen et al. 2012) on board the Solar Dynamics Observatory (SDO; Pesnell et al. 2012) have brought about new advances in the study of large-scale EUV waves, by allowing for plasma and temperature diagnostics by differential emission measure (DEM) analyses, along with spectroscopic observations from Hinode/EIS and detailed observations and modeling of the interaction (reflection, transmission) of EUV waves through coronal holes and active regions with related seismology of the coronal field, as well as refined stack plot analysis that enable in-depth studies of the speed evolution of the wave and its relation to the driver, namely, the CME and its expanding flanks (e.g., Schrijver et al. 2011; Veronig et al. 2011; Harra et al. 2011; Downs et al. 2012; Cheng et al. 2012; Olmedo et al. 2012; Vanninathan et al. 2015; Liu et al. 2018; Hu et al. 2019; Hou et al. 2022).

The physical nature of EUV waves has been a subject of intense scientific debate and has been summarized from different viewpoints in a number of reviews (Vršnak & Cliver 2008; Wills-Davey & Attrill 2009; Gallagher & Long 2011; Zhukov 2011; Patsourakos & Vourlidas 2012; Warmuth 2015; Long et al. 2017). To search for evidence of potentially different classes of EUV waves, Warmuth & Mann (2011) analysed a sample of 176 EUV wave events with respect to their kinematics. Their study revealed three different classes of EUV waves (see Fig. 8 therein):

1. Their initial velocity is >320 km s\(^{-1}\) and they show a pronounced deceleration during propagation.
2. They propagate with a nearly constant speed of 170–320 km s\(^{-1}\).
3. They have slow speeds (<130 km s\(^{-1}\)) and show a rather erratic motion.

The first class is interpreted as large-amplitude magnetohydrodynamic (MHD) waves or shocks. The propagation speed of these types of events is faster than the local fast magnetosonic speed. Due to the amplitude decrease during propagation, its local speed is decreasing. The dependence of the propagation speed of the wave on its amplitude is a typical nonlinear effect. The second class is considered to be linear waves traveling at the local fast magnetosonic speed. The third class is regarded as disturbances caused by magnetic reconfigurations (Warmuth & Mann 2011). Muhr et al. (2014) found the same basic characteristics of the EUV wave kinematics and speed relations for the first two classes. This suggests that the first and second class of EUV waves represent freely propagating waves in the corona (Warmuth & Mann 2011).

Veronig et al. (2010) reported the first observation of a full dome-shaped EUV wave, which occurred on January 17, 2010 over the time range of 3:52–4:36 UT (see Fig. 1). Further studies on this EUV wave include Grechnev et al. (2011), Zhao et al. (2011), Temmer et al. (2013). Interestingly, this EUV wave-dome event was also associated with one of the first wide-spread solar energetic particle events (SEPs) reported that covered over almost 360° in longitude in the heliosphere (Dresing et al. 2012). Further examples of EUV wave domes are presented in Kozarev et al. (2011) and Francile et al. (2016).

Such observations of the full EUV wave dome are important as they allow us to study the 3D evolution of such a wave in high-cadence EUV imaging, especially with respect to the evolution of its velocity, amplitude, and pulse width. These measurements of a dome-shaped EUV wave therefore provide us with the opportunity to compare the observations with the theory of nonlinear MHD waves. A summary of the properties of the dome-shaped EUV wave of January 17, 2010 is given in Sect. 2.
Because of its kinematics, it belongs to the first and/or second class of EUV waves according to Warmuth & Mann (2011).

Hence, it is considered to be a freely propagating fast mode MHD wave, and its propagation velocity is determined by the fast magnetosonic speed (Mann et al. 1999a). This characteristic speed depends on the magnetic field, the density, and the temperature of the ambient medium. Therefore, models of the magnetic field, the density, and the temperature of the corona are developed in Sect. 3, resulting in a model of the Alfvén speed distribution in the corona. Adopting these results, the spatio-temporal evolution of the dome-shaped EUV wave is studied (Sect. 4). It is well known that fast magnetosonic waves with large amplitudes can change their wave pulse profile due to nonlinear effects such as wave steepening. Thus, the evolution of such a wave pulse is theoretically studied and compared with the measurements in Sect. 5. The results of the paper are summarized in Sect. 6.

2. Observations

In this section, we provide a summary of the observations of the dome-shaped EUV wave that occurred on 2010 January 17 that are relevant for this study, based on the results from Veronig et al. (2010). The wave was best observed in the 195 Å filter of the STEREO-B/EUVI instrument, which has a peak temperature response at 1.5 MK (Wuelser et al. 2004). The center of origin of the wave is located at a meridional distance of 57° East from the STEREO-B view, and 36° behind the Eastern limb from observatories in the Sun-Earth line (Veronig et al. 2010).

Running difference images were created to determine the distance of the wave front from the center of the wave on the solar surface and, consequently, the kinematics of the wave front (see Fig. 2). The deduced wave velocity is $v_{EUV,\parallel} = 283\pm27\, \text{km s}^{-1}$, as measured along the solar surface. This horizontal propagation of the wave could be followed up to a distance of about 950 Mm from the center of the wave, and was found to be homogeneous, namely, displaying a constant velocity. The upward motion of the wave dome was followed up to a height of ≈1000 Mm above the photosphere by combining measurements from the EUVI instrument and the COR1 white-light coronagraph onboard STEREO-B. The velocity of the upward motion of the wave dome projected onto the plane-of-sky was found to be $v_{EUV,\perp} \approx 650\, \text{km s}^{-1}$, which is much faster than the EUV wave speed along the solar surface, $v_{EUV,\parallel} = 280\, \text{km s}^{-1}$ (see Fig. 2). The horizontal motion was measured at a height of approximately 10 Mm above the photosphere.

Figure 3 shows the spatio-temporal behaviour of the EUV wave in terms of its intensity profile with respect to its horizontal motion. In the early phase, namely, 03:56–04:01 UT, the amplitude is increasing. This can be interpreted, that the wave is externally driven, for instance, by an eruptive magnetic disturbance. After 04:01 UT, the EUV wave behaves as a freely propagating one with an amplitude decreasing during its propagation.

3. Model of the Alfvén speed in the corona

The velocity of a fast magnetosonic wave is given by:

$$V^2 = \frac{(v_A^2 + c_s^2)}{2} + \sqrt{\frac{(v_A^2 + c_s^2)^2}{4} - c_A^2 c_s^2 \cos^2 \theta},$$

(1)
with $\theta$ as the angle between the propagation direction and the ambient magnetic field $B$ (Priest 1982). Here, $v_A = B/(4\pi p)^{1/2}$ and $c_s = (\gamma p/\rho)^{1/2}$ denote the Alfvén- and sound speed, respectively, $(B, \text{magnetic field}; p, \text{mass density}; \rho, \text{pressure}; \gamma, \text{ratio between specific heats}).$ For $\theta = 0^\circ$ and $90^\circ$, the expression (Eq. (1)) for the wave speed reduces to $V = v_A$ and $V = v_{\text{m}} = (c_s^2 + c_e^2)^{1/2}$, respectively. Also, $v_{\text{m}}$ is usually known as the fast magnetosonic speed.

Since both $B$ and $\rho$ are varying in the corona with the distance $r$ from the center of the Sun, the characteristic speeds, namely, $v_A$, $c_s$, and $V$, also depend on $r$. Hence, models of the radial behaviour of the density and the magnetic field are necessary, in order to derive a model of the radial dependence of the Alfvén speed.

An isothermal, gravitationally stratified atmosphere can be described by the stationary momentum equation of MHD

$$0 = -\frac{P}{r} - \rho \cdot \frac{\gamma g M_0^2}{r^2},$$

in spherical symmetry ($\gamma_G$ as the gravitational constant and $M_0$ as the mass of the Sun). The mass density is defined by $\rho = \mu m_0 N$ (where $m_0$ as the proton mass), with $\mu$ as the mean molecular weight and $N$ as the full particle number density. The pressure, $p$, is related to $N$ via the equation of state $p = N k_B T$ ($k_B$, Boltzmann’s constant), with temperature, $T$ (Priest 1982). Assuming that the plasma of the corona consists of electrons, protons, and doubly ionized Helium, $N$ is related to the electron and proton number density by $N = 1.92 N_e = 2.27 N_p$ (Mann et al. 1999b). Here, $\mu = 0.6$ is taken as a typical value for the mean molecular number density in the corona (Priest 1982). The differential equation (Eq. (2)) can be integrated to:

$$N(r) = N_S \cdot e^{-(R_0/r - 1)},$$

with $N_S = N(r = R_0)$ as the full particle number density at the bottom of the corona, and

$$\lambda = \frac{T_0}{T},$$

with $T_0 = \frac{\mu m_0 c^2}{k_B (\gamma + 1)}$, $T = \frac{\mu m_0 c^2}{k_B (\gamma + 1)}$, and $R_0 = 1$ MK, it results in $\lambda_0 = 13.83$. Hence, the barometric scale height $h_{\text{bar}}$ is given by $h_{\text{bar}} = R_0/\lambda$, as it is usually defined. For instance, we would get $h_{\text{bar}} = 70$ Mm for a temperature of 1.39 MK.

The white-light observations of the corona by Koutchmy (1994) showed that the radial behaviour of the electron number density, $N_e$, can be fitted by an $\alpha$-fold Newkirk (1961) model as follows:

$$N_e = \alpha \cdot N_0 \cdot 10^{3.32 R_0/r},$$

with $N_0 = 4.2 \times 10^4$ cm$^{-3}$. As presented in Fig. 1 in Koutchmy (1994), the electron number density in the solar corona can vary over a broad range of $10^7 - 10^{10}$ cm$^{-3}$. However, its radial behaviour can be described by $\alpha = 1$, 4, and 10 in an appropriate manner for quiet equatorial regions, dense loops and extremely dense loops, respectively.

Since EUV waves are propagating mostly outside active regions, namely, through the quiet Sun region, a one-fold Newkirk model (i.e., $\alpha = 1$) is a good choice for describing the radial density behaviour in the quiet corona. Comparing Eq. (5) with Eq. (3), the Newkirk model corresponds to a barometric density model with a temperature of $T = 1.39$ MK in the quiet corona. A one-fold Newkirk model corresponds to an electron number density $N_{e1} = 8.775 \times 10^8$ cm$^{-3}$ and a full particle number density of $N_S = 1.687 \times 10^9$ cm$^{-3}$ at the base of the corona.

The temperature of $T = 1.39$ MK leads to a value of 179 km s$^{-1}$ for the sound speed $c_s = (\gamma k_B T/\mu m_p)^{1/2}$ (with $\gamma = 5/3$) in the quiet corona.

As mentioned above, EUV waves are travelling mainly outside active regions, where the magnetic field is substantially lower than in active regions. This quiet Sun’s magnetic field is nearly radially directed (see Fig. 16 in Wedemeyer-Böhm et al. 2009 and the discussion therein). According to the conservation of the magnetic flux, it can be extrapolated from the photosphere into the corona via (see also Mann et al. 2003 and Warmuth & Mann 2005):

$$B(r) = B_0 \cdot \left(\frac{R_0}{r}\right)^2.$$  

By inserting Eqs. (3) and (6) into the definition of the Alfvén speed,

$$v_A = \frac{B}{\sqrt{4\pi \mu m_p N}},$$

we get:

$$v_A(r) = v_{A,S} \cdot \left(\frac{R_0}{r}\right)^2 \cdot e^{-\lambda(R_0/r - 1)/2},$$

with $v_{A,S} = B_0/(4\pi \mu m_p N_0)^{1/2}$. The radial behavior of the Alfvén speed (see Fig. 4) shows a local maximum at $r_{\text{max}} = (\lambda/4) \cdot R_0$. We note that the location of this maximum only depends on the temperature and for increasing temperature shifts towards the Sun. Then, $\lambda = 9.95$ is obtained for a temperature of 1.39 MK. Hence, the maximum appears at $r_{\text{max}} = 1727$ Mm from Sun center or at a height of 1031 Mm above the photosphere (see Fig. 4).

4. Propagation of a fast magnetosonic wave in the corona

The dome-shaped EUV wave observed on January 17, 2010 was initially excited by an eruptive flare or CME event in the vicinity of an active region. Figure 3 shows that after 04:01:02 UT, the EUV wave is a freely propagating wave. At this time, its vertical position is at 576 Mm above the photosphere. Hence, this part of the EUV wave is considered as a fast magnetosonic wave travelling through the corona.

The horizontal motion of the EUV wave appears mainly in quiet Sun regions. At the bottom of the corona, the observed
EUV wave velocity is \( V_{\text{EUV},ij} = 283 \text{ km s}^{-1} \). The temperature in the corona is not isothermal, but the distribution of the differential emission measure of quiet Sun regions shows that the temperature is in the range 1.2–2.0 MK (Brosius et al. 1996). Large-scale EUV waves are best seen in the 195 Å filter of the STEREO-B/EUVI instrument, which has a peak temperature response at 1.5 MK (Wuelser et al. 2004). A coronal temperature of 1.39 MK was derived from the radial behavior of the electron number density (see Koutchmy 1994 and the discussion in Sect. 3). Hence, a temperature of 1.39 MK is justified to be assumed in the corona above quiet equatorial regions, leading to a sound speed of \( c_s = 179 \text{ km s}^{-1} \). The horizontal motion of EUV waves is observed near the bottom of the corona above quiet Sun regions. There, the magnetic field is directed nearly vertical (see Fig. 16 in Wedemeyer-Böhm et al. 2009 and the discussion therein).

Hence, the wave propagates nearly perpendicularly to the ambient magnetic field, namely, \( \theta \approx 90^\circ \). In this case, \( V \) is reduced to \( V_{\text{rms}} \) (see Eq. (1)). Consequently, the EUV wave velocity can be identified with the fast magnetosonic speed, that is:

\[
V_{\text{EUV},ij} = V_{\text{rms}} = (\varepsilon_{\text{rms}} + c_s^2)^{1/2}.
\]

Because of \( V_{\text{EUV},ij} \approx 283 \text{ km s}^{-1} \) and \( c_s = 179 \text{ km s}^{-1} \), it leads to \( \varepsilon_{\text{rms}} = 219 \text{ km s}^{-1} \) as the Alfvén speed at the bottom of the corona. Adapting for \( N_5 = 1.687 \times 10^8 \text{ cm}^{-3} \) (see Sect. 3), one finds \( B_8 = 3.19 \text{ G} \) for the quiet Sun magnetic field at the bottom of the corona. This is a typical value for this quantity (Schrijver & De Rosa 2003). The radial behaviour of the local Alfvén speed in the quiet corona is given completely by Eq. (8), with \( N_5 = 1.687 \times 10^8 \text{ cm}^{-3} \) and \( B_8 = 3.19 \text{ G} \). This is illustrated in Fig. 4. As can be seen, it has a maximum of 680 km s\(^{-1}\) at a height of 1031 Mm above the photosphere.

The employment of Eq. (2) requires a stationary atmosphere. This is justified for regions well below the critical radius \( r_c \) according to Parker (1958) solar wind model. The critical radius \( r_c \) is defined by \( r_c = \gamma c M_0 / 2 c_s^2 \) with the critical velocity \( v_c = (k_B T / \mu m_p)^{1/2} \), which is the sound speed for an isothermal plasma (Priest 1982). According to this approach, the solar wind speed exceeds the speed \( v_c \) at the critical radius \( r_c \). For a temperature of \( T = 1.39 \text{ MK} \), the critical radius \( r_c \) is 4.94 \( R_\odot \). Since the location of the maximum of the local Alfvén speed (see Fig. 4) is well below the critical radius, Eqs. (2) and (8) are appropriate to describe the behaviour of the density and Alfvén speed in the solar corona, respectively.

In the following, the vertical motion of the dome-shaped EUV wave is considered. It travels with the speed \( V \) given in Eq. (1). Then, the radial propagation of the EUV wave can be described by:

\[
t - t_0 = \int_{R_\odot + h(t)}^{R_\odot + h_0} \frac{dr}{V(r)}
\]

(9)

with \( h = h(t) = t_0 \). Since the vertical motion happens above an active region, the magnetic field topology can vary between a parallel and a perpendicular one. Therefore, in Eq. (9) a mean value of \( \varepsilon_A \) and \( \varepsilon_{\text{rms}} \) is employed for the speed \( V \), namely, \( V = (\varepsilon_A + \varepsilon_{\text{rms}})/2 \).

To compare the kinematic description (Eq. (9)) with the observations, \( h = h(t) = 576 \text{ Mm} \) and \( t_0 = 675 \) are chosen. In Fig. 5, \( t = 0 \) corresponds to 03:50:18 UT in Fig. 2. Here, we consider the wave is a freely propagating one after 04:01:02 UT, as argued in Sect. 2. At this time, the wave front is located at a height of 576 Mm above the photosphere.

The numerical solution of Eq. (9) that is found in this way is presented in Fig. 5. The crosses show the vertical motion of the dome-shaped EUV wave derived from the observations (see Fig. 3). Figure 5 shows that the theoretical description agrees well with the observations for heights above 500 Mm. That can be explained by the following manner: The EUV wave is initially excited by a sudden energy release (flare or CME) in an active region. There, the magnetic field is strong and the density is enhanced. Nevertheless, the motion of the EUV wave can be described well by means of the behaviour of the Alfvén speed above quiet Sun regions (see Eq. (8)). Hence, the influence of the active region can be neglected with respect to the density and magnetic field for heights \( > 500 \text{ Mm} \) above the photosphere. For illustration, the Alfvén speed in the height range 500–1000 km above the photosphere is about 600 km s\(^{-1}\) (see Fig. 4). That results in \( \varepsilon_{\text{rms}} = 626 \text{ km s}^{-1} \) with \( c_s = 179 \text{ km s}^{-1} \) and \( V = (\varepsilon_A + \varepsilon_{\text{rms}})^{1/2} = 613 \text{ km s}^{-1} \), namely, \( V \approx \varepsilon_A \). Hence, \( V \) is not crucially depending on the angle \( \theta \) and, hence, on the local magnetic field topology. This means that the magnetic field of the quiet Sun dominates at heights above 500 Mm, and the local Alfvén speed can appropriately be described by Eq. (8) in the corona at heights above 500 Mm.

5. Nonlinear MHD waves

Figure 3 shows the evolution of the intensity profile of the EUV wave during its horizontal evolution, which was derived from the EUVI-A 195 Å filtergrams. As mentioned above, after 04:01 UT the EUV wave is a freely propagating one. The wave profile shows an initial steepening of the leading edge and a subsequent (i.e., after 04:16 UT) decay during its further evolution. The maximum intensity enhancement is \( I_{\text{max}}/I_0 = 1.45 \), which corresponds to a maximum density enhancement of \( \approx 1.2 \) for optically thin emission lines (Veronig et al. 2010). Wave steepening is a typical property of nonlinear waves. Therefore, nonlinear MHD wave theory (Mann 1995) must be employed.

In order to study the evolution of the wave profile, for instance, wave steepening, the approach of so-called “simple waves” (Landau & Lifshitz 1987) is employed. The disturbances associated with the wave are travelling along the \( x \)-axis. All varying quantities are assumed to be depending only on the spatial and temporal coordinates \( x \) and \( t \). The ideal MHD equations are conveniently written in terms of Lagrangian coordinates (see Eq. (3) in Mann 1995). In the framework of simple waves, there are well defined relationships between the disturbed quantities.

Fig. 5. Height–time diagram (full line) of the radial motion of a fast magnetosonic wave according to Eq. (9) by adopting \( \varepsilon_A = 219 \text{ km s}^{-1} \) and a coronal temperature of 1.39 MK (i.e., \( \lambda = 9.88 \)). The crosses show the radial motion of the dome-shaped EUV wave from measurements according to Fig. 2. Here, \( t = 0 \) corresponds to 03:50:18 UT in Fig. 2.
Eq. (11) is reduced to
\[ x = b \beta \theta \]
where \( b \) provides the ambient magnetic field at the bottom of the corona, the angle \( \beta \) between the ambient magnetic field and the factor-axis by \( b_x = \cos \theta \).

Since EUV waves are propagating nearly perpendicular to the ambient magnetic field at the bottom of the corona, the angle \( \theta = 90^\circ \) has to be chosen leading to \( b_{xa} = 0 \). Then, Eq. (12) provides \( V_x^2 = c_s^2 + c_P^2 \), and after inserting Eq. (14) into Eq. (11), Eq. (11) is reduced to
\[ \frac{db}{dN} = \frac{b_x^2}{N} \]

With the initial conditions, this leads to \( b(N) = N \). Finally, Eq. (12) can be written as
\[ V_x = \frac{N}{2} \cdot \beta_0 \cdot N^{(\gamma - 1)} \]

By means of Eqs. (17), (10) can then be expressed as:
\[ \frac{dv}{dN} = \frac{1}{N} \cdot \sqrt{N + \frac{2}{\gamma} \cdot \beta_0 \cdot N^{(\gamma - 1)}} \]

Figure 6 shows the numerical solutions of Eqs. (17) and (18) with \( \beta_0 = 0.8 \) (i.e., \( \beta_0 = 8 \pi N b_0 T_0 / B_0^2 \)). Then, \( N = N_S = 1.687 \times 10^3 \mathrm{cm}^{-1} \), \( T_0 = 1.39 \mathrm{MK} \), and \( B_0 = B_S = 3.19 \mathrm{G} \) and \( \gamma = 5/3 \). Both, \( V \) and \( v_x \) are functions monotonically increasing with \( N \).

In MHD, it is well known that a fast magnetosonic wave is accompanied with a density enhancement. However, a density enhancement is also related to an increase in the temperature:
\[ \frac{T}{T_0} = N^{(\gamma - 1)} \]

The EUV wave under study is related to a density enhancement of 1.2 and hence, to a maximum temperature of 1.57 MK according to Eq. (19). Here, the value of \( T_0 = 1.39 \mathrm{MK} \) has been used. This may explain why the EUV wave is best seen in the EUVI 195 Å passband. However, it also indicates that \( \approx 1.4 \mathrm{MK} \) is a good choice for the temperature of the quiet corona.

The length between the top of the EUV wave and the wings is derived from the observations as \( \Delta S_0 = 60 \mathrm{Mm} \) (see Fig. 3). The relative velocity between the top and the wing of the EUV wave is given by \( \Delta W = W_{top} - W_{wing} \). For \( N_{max} = 1.2 \), inspection of Fig. 3 provides \( W_{top} = 1.65 \) and \( W_{wing} = 1.29 \) leading to a relative velocity of \( (1.65-1.29) \times v_A = 78 \mathrm{km \cdot s}^{-1} \). Since the evolution of the EUV wave during its propagation along the solar surface is studied here, \( v_A = 219 \mathrm{km \cdot s}^{-1} \) must be taken for the Alfvén velocity (see Sect. 3). Then, a wave steepening should appear if the top of the wave pulse overtakes the wave of the leading edge. That should occur after a time \( t_{steepening} = \Delta S/\Delta W \) = 60 Mm/78 km s\(^{-1}\) = 769 s \((\approx 13 \mathrm{min})\). Yet this does not happen, as seen in Fig. 3. In this estimation, the amplitude at the top of the wave pulse is regarded to be constant, but this is not the case for real EUV waves. Since these waves travel as nearly circular waves over the Sun, their amplitude is decreasing, in the typical way of a circular or spherical wave. Consequently, the velocity \( W_{max} \) at the top of the wave pulse is also decreasing, leading to a prevention of the wave steepening.
The quiet Sun magnetic field is the dominant one there. Consequently, the EUV wave is fully developed and the wing with \( \delta = 1 \) (dashed line) and \( \delta = 2 \) (dotted line).

In order to investigate this subject, the motion of the top \( s_{\text{top}} \) and the wing \( s_{\text{wing}} \) of the wave pulse is considered according to:

\[
s_{\text{top}} = W(N_{\text{max},i}) \cdot v_{\text{A},0},
\]

and

\[
s_{\text{wing}} = W_0 \cdot v_{\text{A},0} + \Delta s_0,
\]

with \( W_0 = W(N = 1) = 1.29, v_{\text{A}} = 219 \text{ km s}^{-1}, N_{\text{max}} = 1.2, \) and \( \Delta s_0 = 60 \text{ Mm} \). The evolution of the density is described by an iterative manner. At the time \( t_i \), the top of the wave is located at \( r_{i} \). There, it has an enhanced particle number density \( N_{\text{max},i} \). Hence, it propagates with the velocity \( W(N_{\text{max},i})v_{\text{A},i} \). After the time step \( \Delta t \), the top of the wave travels up to \( r_{i+1} = r_{i} + W(N_{\text{max},i})v_{\text{A},i} \cdot \Delta t \). During this time step, the density at the top is diminished to \( N_{\text{max},i+1} \) according to:

\[
N_{\text{max},i+1} = 1 + \frac{(N_{\text{max},i} - 1)}{1 + \frac{W(N_{\text{max},i})v_{\text{A},i} \cdot \Delta t}{r_{i}}},
\]

with \( \delta = 1 \) and 2 for a circular and spherical wave, respectively. As already mentioned, the EUV wave is fully developed at 4:01:02 UT. At that time, the top of the EUV wave pulse is 357 Mm away from the center of the wave (see Fig. 3). This region is sufficiently far away from the active region, so that the quiet Sun magnetic field is the dominant one there. Consequently, \( r_0 \) is taken to be 357 Mm. Now, Eq. (20) can numerically be solved by means of Eq. (22) in an iterative manner. \( \Delta t = 100 \text{ s} \) and \( N_{\text{max},0} = 1.2 \) are chosen.

In Fig. 8, the result is presented in terms of \( \Delta s = s_{\text{top}} - s_{\text{wing}} \). Complete wave steepening occurs, if the top of the wave pulse overtake its leading wing, namely, \( \Delta s = 0 \). This is the case after 1100 s (\( \approx 18 \text{ min} \)) for \( \delta = 1 \), as seen in Fig. 8. Within this time-frame, the density at the top diminishes from 1.2 to 1.094. Then, a shock with a density compression of 1.094 and an Alfvén Mach number 1.461 can be established. However, it is a weak shock that is formed. For the other case, that is, \( \delta = 2 \), the dotted line approaches continuously zero, but it will never cross zero (see Fig. 8). This means that the wave initially steepens and, subsequently, the steepening stops due to the decrease of the amplitude at the top of the wave pulse. Such an evolution can indeed be observed, as demonstrated in Fig. 3.

Figure 9 plots the decrease of the maximum density enhancement (\( N - 1 \)) at the top of the wave pulse during its evolution for the two cases, namely, a circular (\( \delta = 1 \)) and a spherical (\( \delta = 2 \)) wave. It shows that in the initial phase the horizontal motion of the EUV wave can be aptly described as a circular wave, whereas for the later phase, a spherical wave is a better description. This finding can be explained as follows: on the one hand, the dome-shaped EUV wave under study should be regarded as a spherical wave (i.e., \( \delta = 2 \)); on the other hand, the density in the corona is radially stratified, namely, the highest density is at the bottom of the corona. Thus, the dome-shaped EUV wave is in reality a mixture between a circular and a spherical wave. The observations show that during the evolution of the EUV wave its maximum intensity, \( I_{\text{max}} \), is decreasing according to \( \propto r^{-2.5 \pm 0.3} \) (Veronig et al. 2010).

In the case of optically thin EUV line emission, the intensity of the emitted radiation is depending on both the temperature, \( T \), and the density, \( N \), in the source region (see Eqs. (2.8.6) and (2.8.7) in Aschwanden 2005). In case that the temperature variation is small in large-scale EUV waves (e.g., Vanninathan et al. 2015), the density dependence can be approximated by \( I_{\text{max}}/I_0 \propto N^2 / (I_0, \text{intensity of the background}) \). Hence, in the event under study, the density, \( N \), is decreasing according to \( \propto r^{-1.25} \). This means that the EUV wave under study is indeed a mixture between a circular and spherical wave.

As already mentioned, Fig. 6 shows that the propagation velocity depends on the density compressions of \( N \) that accompany the wave, which is a purely nonlinear property. In the case of \( N \rightarrow 1 \), the propagation velocity continuously approaches the linear value. As discussed above, the EUV wave starts with the velocity \( W_{\text{top}} = 1.65 \text{ (361 km s}^{-1} \)) at the top, whereas the wings have the velocity \( W_{\text{wing}} = 1.29 \) (i.e., \( 283 \text{ km s}^{-1} \)). In Sect. 3, the linear wave velocities (see Eq. (1)) are employed to derive plasma properties from the velocities of the EUV wave under discussion. The error of that is of the order of 12% (because of \( W_{\text{top}} - W_{\text{wing}})/(W_{\text{top}} + W_{\text{wing}}) \)). Such an error can be accepted, since the EUV measurements are also connected with errors and the model of simple MHD waves is a simple model, that is to say that it neglects the large scale inhomogeneity of the corona, for instance. Nevertheless, the presented approach is able to describe

---

1 A density compression is also connected with an adiabatically enhancement of the temperature as discussed with Eq. (19).
the propagation of a dome-shaped, large-scale EUV wave in the corona in a quantitative manner.

6. Summary

The observation of a dome-shaped EUV wave by the EUVI instrument onboard STEREO-B on January 17, 2010 (Veronig et al. 2010) allows us to study the 3D evolution of such a wave in the corona. This EUV wave can be regarded as a freely propagating, large-amplitude fast magnetosonic wave. These complete and comprehensive observations of the full wave dome offer a unique opportunity to link the nonlinear MHD wave theory with observations.

Although large-scale EUV waves are initially excited by a solar eruption (e.g., Veronig et al. 2008, 2018; Patsourakos & Vourlidas 2009; Kienreich et al. 2009; Shen & Liu 2012), they mostly travel outside active regions, namely, in quiet Sun regions. The measurements of the spatio-temporal behavior of the 17 January, 2010 EUV wave reveal that the quiet Sun’s magnetic field at the bottom of the corona can be estimated to be $\approx 3.2$ G. This magnetic field is extrapolated into the corona by using the conservation of the magnetic flux. With this knowledge of the radial behavior of the magnetic field and the assumption of a barometric model for the density behaviour, the radial behavior of the local Alfvén speed for the quiet corona can be derived (see Fig. 2). It shows a local maximum of $\approx 680$ km s$^{-1}$ at a height of $\approx 1030$ Mm above the photosphere. With this characteristic of the Alfvén speed, the vertical motion of the EUV wave can be calculated. The resulting height-time diagram agrees well with the observations, as can be seen in Fig. 5. This agreement demonstrates that the presented approach aptly describes the kinematics of a radially propagating EUV wave and that the models of the density, the magnetic field, and Alfvén speed are an appropriate choice for the quiet solar corona.

The evolution of large-amplitude MHD waves can be described in terms of simple MHD waves as developed by Mann (1995). This theoretical study shows that the wave is initially steepening. Subsequently, this steepening stops due to the decrease of the wave amplitude during its further evolution. Such a decrease of the amplitude occurs for a circular and/or spherical wave during its propagation (see Figs. 8 and 9). We emphasize that this behavior, namely, the initial wave steepening and subsequent decay, is indeed observed in the case of the dome-shaped EUV wave under study (see Fig. 3).

These results confirm that nonlinear MHD wave theory provides a valid description of the 3D evolution of freely propagating, large-amplitude EUV waves in the corona.

Acknowledgements. The authors express their thanks to Hakan Önel for his assistance for the numerical evaluation of Eqs. (17) and (18) by mathematica. A. V. acknowledges the Austrian Science Fund (FWF): P24092-N16.

References


Koutchmy, S. 1994, AdSpR, 14, 29

A144, page 8 of 9
