LETTER TO THE EDITOR

Estimating source distances for high-energy neutrinos: A method for improving electromagnetic follow-up searches

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ABSTRACT

High-energy neutrino telescopes such as ICECUBE or KM3NET issue public alerts describing the characteristics of possible astrophysical high-energy neutrino events. This information, particularly with respect to the arrival direction and the associated uncertainty of the neutrino candidates, is used by observatories to search for possible electromagnetic counterparts. Such searches are complicated by the size of localisation areas, which can be up to tens of squared degrees or more, coupled with the absence of constraints on the distance or nature of the possible source – in contrast to gravitational wave alerts issued by instruments such as LIGO/VIRGO. Here, we describe a method for deriving a probable distance interval for the astrophysical source that may possibly be associated with a high-energy neutrino event, which may then be used in a cross-matching with galaxy catalogues to search for plausible electromagnetic counterparts. This study is intended to serve as a guide for high-energy neutrino followup campaigns.

Key words. astroparticle physics – neutrinos – methods: observational

1. Introduction

High-energy neutrinos (HEN) are the smoking gun of the acceleration of hadronic cosmic rays in astrophysical sources (Halzen 2021), which are produced as a result of their interaction with radiation or matter. Since the discovery of a flux of cosmic HEN of TeV–PeV energies (IceCube Collaboration 2013), a small number of objects have been clearly identified as possible HEN sources, namely: the blazar TXS0506+056, associated with the HEN IC170922A (Abbasi et al. 2018), the tidal disruption event (TDE) AT2019dsq (Reusch et al. 2022) associated with IC191001A, and, more recently, the active galactic nucleus (AGN) NGC 1068 (IceCube Collaboration 2021a). Other potential HEN sources range from short-duration transients, such as gamma-ray bursts (GRBs; Waxman & Bahcall 1997), with electromagnetic (EM) light curves fading after a few hours, to medium-duration transients, such as core-collapse supernovae (CCSN; Murase et al. 2011), which end up fading over a few days or weeks. In long-duration transients, for instance, jetted TDE (Murase et al. 2020), the EM emission can last over a few months or years. The onset and duration of the HEN emission itself depends on the nature of the source.

Telescopes such as ICECUBE or KM3NET (Aiello et al. 2019) issue alerts after the identification of HEN candidates. In the case of ICECUBE, these alerts are classified as ‘gold’ or ‘bronze’ depending on their estimated probability of being of astrophysical origin (IceCube Collaboration 2019). When such a HEN is identified, the arrival direction, plus an uncertainty on the order of $1° – 10°$ (depending on the neutrino type), together with a false-alarm rate and estimated energy, are broadcast in the form of a notice or circular. This information is then relayed to the astronomical community to look for potential transient EM counterparts to the HEN event and to help identify the origin of the detected HEN. This method is already in place for studies of gravitational wave (GW) alerts (Antier et al. 2020) following LIGO/VIRGO notices.

With an observed rate of HEN alerts on the order of one per month for both channels, only a small fraction of these alerts are actually followed up on by public observation reports from observatories, despite the detection of new gamma-ray sources by Fermi-LAT in the 90% uncertainty region (see e.g., IceCube Collaboration 2022b) or the identification of possible counterparts by the Zwicky Transient Factory (see e.g., IceCube Collaboration 2021c). Indeed the HEN error box generally covers an area well below the typical few hundreds of deg$^2$ for GW events (Abbott et al. 2020). With the advent of the O4 observing run of the GW detectors LIGO/VIRGO to start before summer 2023 and its announced rate of one alert per day (Ghosh & De Pietri 2023), it is of paramount importance to improve the information provided to electromagnetic observatories in order for them to efficiently look for potential transients associated with HEN candidates (see e.g., Stein et al. 2023; Necker et al. 2022). The aim of this paper is to show that such a probable distance interval can be derived for HEN sources in the case of a neutrino alert, since this information will facilitate the EM followups of HEN events. This is particularly relevant for short- and medium-duration transients for which a rapid response is critical because of their rapid fading.

The paper is organised as follows. Section 2 describes the fundamental relationships between the energy and distance of
HEN sources, which form the basis for our proposed method. Section 3 shows how the information dispatched by HEN telescopes such as ICECUBE can be used to constrain the search volume for electromagnetic followups, in particular, when they are combined with existing limits on the total energy emitted under the form of HEN. Section 4 presents a possible practical implementation in the ranking of galaxies to be targeted in such searches for electromagnetic counterparts of HEN events.

2. Number of neutrinos versus source properties

The number of HEN events detected in a neutrino telescope depends both on the source emission characteristics and the detector properties.

2.1. HEN emission and detection

The total energy emitted in HEN during a transient emission, $E_{\text{iso}}^{\text{HEN}}$, as measured by the observer, for a source at a redshift, $z$, and luminosity distance, $D_L$, is computed by integrating the neutrino spectrum over an energy range, $[E_{\text{min}}^{\text{obs}} E_{\text{max}}^{\text{obs}}]$, under the assumption that the source is emitting isotropically:

$$\frac{E_{\text{iso}}^{\text{HEN}}}{4\pi D_L(z)^2} = \int_{E_{\text{min}}^{\text{obs}}}^{E_{\text{max}}^{\text{obs}}} E^2 N(E) \frac{dE}{E}.$$  (1)

The corresponding minimum and maximum energies of the HEN emission are:

$$E_{\text{min}}^{\text{min, max}} = E_{\text{min, max}}^{\text{obs}} \times (1+z)$$

because of cosmic dilution. The HEN spectrum here in 1 GeV$^{-1}$ cm$^{-2}$ is generally expressed as a power law because of Fermi acceleration processes, which take the form:

$$\frac{dN}{dE} = \Phi_0 \left(\frac{E}{E_0}\right)^{-\gamma}.$$  (2)

The average expected number of HEN events in a neutrino telescope $\langle N_{\text{HEN}}(\delta) \rangle$ can be estimated by convoluting the observed spectrum with $A_{\text{eff}}(\delta, E)$, namely, the energy and direction-dependent effective area for neutrinos of the telescope:

$$\langle N_{\text{HEN}}(\delta) \rangle = \int_{E_{\text{min}}^{\text{obs}}}^{E_{\text{max}}^{\text{obs}}} A_{\text{eff}}(\delta, E) \frac{dN}{dE}.$$  (3)

Here $E_{\text{min, max}}^{\text{obs}}$ stand for the energy range of the telescope. Using the published effective area for ICECUBE alerts (IceCube Collaboration 2019), this allows us, for instance, to compute for each alert the probability of observing at least one HEN event and the probability of observing exactly one HEN event depending on the direction of the alert:

$$P_{\text{obs}}(N_{\text{HEN}} = 1) = \langle N_{\text{HEN}} \rangle e^{-\langle N_{\text{HEN}} \rangle},$$

$$P_{\text{obs}}(N_{\text{HEN}} > 0) = 1 - e^{-\langle N_{\text{HEN}} \rangle}.$$  (4)

2.2. Relating the source energy and distance

This average number $\langle N_{\text{HEN}} \rangle$ of expected HEN events can then be related to the total emitted energy in the form of HEN:

$$\langle N_{\text{HEN}} \rangle = k_0(\gamma, \delta) \frac{E_{\text{HEN}}}{4\pi D_L}.$$  (5)

The function $k_0(\gamma, \delta)$ depends on the assumed spectral index for the HEN emission and arrival direction especially, because of the direction-dependent effective area. This, in turn, allows us to estimate the probability of observing a number $N_{\text{HEN}}^{\text{obs}}$ of HEN events as a function of source energy and distance – or, alternatively, the number of sources in the universe of a given energy at a given distance that can be detected with a number $N_{\text{HEN}}^{\text{obs}}$ of HEN events. The energy term at the numerator can be corrected for beaming effects with a factor $\delta_{jeh}$, with $\theta_{\text{jet}}$ the HEN emission opening angle.

3. Constraining the observation volume

Using the information provided, for instance, in ICECUBE notices issued after the detection of a candidate HEN, the most probable volume for finding a possible electromagnetic counterpart can be constrained by using the average number of HEN events expected as a function of energy and distance.

3.1. Using the observation of a HEN candidate

The first notice issued after the detection of a HEN track event (see IceCube Collaboration 2023a for an example) yields the false-alarm rate (FAR) for the observed event. Given the total observation duration $T_{\text{obs}}$ (e.g., the time elapsed since the start of the alert system), the average expected number of events due to background only (i.e., atmospheric neutrinos) can be derived, so that an upper limit on $\langle N_{\text{HEN}} \rangle$ can be extracted if an astrophysical origin is assumed. With a minimum FAR of 0.15 events yr$^{-1}$ for past ICECUBE alerts, the computed upper limit using Feldman-Cousins prescriptions is $\langle N_{\text{HEN}} \rangle_{UL} = 3.8$. For a maximum FAR of 4.9 events yr$^{-1}$ as observed, the limit becomes $\langle N_{\text{HEN}} \rangle_{UL} = 0.9$.

This translates into a condition on the distance of the source:

$$D_L \geq \sqrt{\frac{k_0(\gamma, \delta) \times E_{\text{iso}}^{\text{HEN}}}{4\pi \langle N_{\text{HEN}} \rangle_{UL}}}.$$  (6)

Figure 1 (top) shows the distance lower limits as a function of the HEN energy of the source based on the observation of a single HEN event. Limiting the search for possible counterparts to realistic values of the HEN energy, marked by the colored bands (see for instance Aartsen et al. 2019), already constrains (even if slightly), the volume of universe to be probed, by only targeting galaxies above the distance limit. Maximal HEN energies range from $1.5 \times 10^{51}$ to $3 \times 10^{52}$ erg for GRB-like sources and from $4 \times 10^{48}$ to $2 \times 10^{49}$ erg for supernova-like (SN-like) sources.

3.2. Using the search for additional neutrinos

A second notice (see IceCube Collaboration 2023b for an example) usually reports the search for additional muon neutrino events in the direction of the alert, in time intervals of 1000s and 2 days. The sensitivity $F_{\text{lim}}$ of the search for a given spectral index, equal to the upper limits in the case of no observation, is also provided together with the energy interval which would be valid for 90% of the events that ICECUBE would detect for this particular direction. This allows us to define a lower limit on the distance of the source:

$$D_L \geq \sqrt{\frac{E_{\text{iso}}^{\text{HEN}}}{4\pi F_{\text{lim}} \times f(\gamma, E_{\text{min}}, E_{\text{max}})}},$$  (7)

where $f(\gamma, E_{\text{min}}, E_{\text{max}})$ depends on the spectral index, and where the information on the minimum and maximum energies of the search have to be considered. A beaming correction can also
The rate of transients per unit time is redshift-dependent:

\[ R_{\text{transients}}(z) = \rho(z) \times \frac{dV_c}{dz} \times (1 + z)^{-1}, \]  

where \( \rho(z) \) is the transient density rate expressed in \( 1/\text{Mpc}^3\text{yr}^{-1} \), \( dV_c/dz \) represents the differential comoving volume, and the additional redshift term accounts for cosmic time dilation. Here, we can, for instance, use typical CCSN redshift-dependent rates for \( \rho(z) \), which trace the star formation rate (Madau & Dickinson 2014), or GRB rates (Wanderman & Piran 2010), each with a different redshift dependence, or a source population with no redshift evolution. The many distant and faint sources will dominate over the fewer closer and brighter sources when it comes to detection. An example of rate evolution is shown in Fig. 2.

Convolved with the observation probability of \( N_{\text{HEN}} = 1 \) event which depends on both distance \( D_e \) and energy \( E_{\text{iso}}^\text{HEN} \), a median distance and a 90% interval for the distance can be defined, namely, the distance interval expected to contain 5%-95% of the sources with only one detected HEN. Finally, a 90% containment volume can also be defined, once this distance constraint is combined with the localisation uncertainty of the particular HEN event, which is indicated in the alert notice. The resulting distance interval is shown in the two plots of Fig. 1. It can be seen (top) that the sole information of the observation of a single HEN event is not enough to constrain the observation volume within the 90% interval of the most probable distances. On the contrary, the absence of additional neutrinos and the fluence limit bring strong contraints on the most probable distance of the source (bottom), especially for isotropic energies above \( 10^{52} \text{ erg} \). The fraction of the 90% distance interval of the source accessible within a given reach can be computed: with a 50%-75% probability for a source with \( E_{\text{iso}}^\text{HEN} \leq 3.3 \times 10^{50} \text{ erg} \) with only one observed HEN in ICECUBE, the source must be located within 200 Mpc. This is an important guide for the organisation of searches for transient optical counterparts to HEN events.

### 3.3. Adding population information

The transient rate per unit time is redshift-dependent:

\[ R_{\text{transients}}(z) = \rho(z) \times \frac{dV_c}{dz} \times (1 + z)^{-1}, \]

where \( \rho(z) \) is the transient density rate expressed in \( 1/\text{Mpc}^3\text{yr}^{-1} \), \( dV_c/dz \) represents the differential comoving volume, and the additional redshift term accounts for cosmic time dilation. Here, we can, for instance, use typical CCSN redshift-dependent rates for \( \rho(z) \), which trace the star formation rate (Madau & Dickinson 2014), or GRB rates (Wanderman & Piran 2010), each with a different redshift dependence, or a source population with no redshift evolution. The many distant and faint sources will dominate over the fewer closer and brighter sources when it comes to detection. An example of rate evolution is shown in Fig. 2.

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### 4. Practical implementation for a HEN followup

This information can be used to target galaxies falling inside the 90% probable localisation region, given by the uncertainty on the arrival direction of a particular ICECUBE alert and within the derived 90% distance range. This can be done thanks to a cross-match with a galaxy catalogue to extract the most probable galaxies of origin for the HEN event, for instance, the GLADE+ catalogue (Dálya et al. 2022). A ranking score can be defined for each galaxy of the catalogue for each \( E_{\text{iso}}^\text{HEN} \), for instance, by:

\[ P_{\text{Galaxy}}(E_{\text{iso}}^\text{HEN}) = P_{\text{loc}}(\alpha_{\text{Galaxy}}, \delta_{\text{Galaxy}}) \times P_{\text{D}}(D_{\text{Galaxy}}), \]

where \( P_{\text{loc}}(\alpha_{\text{Galaxy}}, \delta_{\text{Galaxy}}) \approx e^{-\frac{\Delta \theta^2}{\sigma^2}} \), in which \( \Delta \theta \) represents the angular separation between the candidate galaxy and the HEN direction, and \( \sigma \) a proxy for the angular resolution based on the 50% and 90% containment errors provided in the alert. Then, \( P_{\text{D}}(D_{\text{Galaxy}}) \) represents the resulting convolution of the transient rate and detection probability (presented in Fig. 2).
5. Conclusions

Notices issued after the detection of a HEN candidate, currently by ICECUBE or soon by KM3NET (Assal et al. 2021), are used to constrain the volume of universe where a search for a possible associated EM counterpart can be carried out. Additional information, such as the nature of the source, the related source population, and its redshift-dependence, can also constrain the most probable distance of the HEN source. Combined with maximal energies for the HEN emission obtained with previous searches by HEN telescopes, the most probable distance for the astrophysical source can be further constrained. The results of the cross-match with a galaxy catalogue are only slightly dependent on the assumed spectral index for the HEN emission or the source population.

As noticed in Dálya et al. (2022), the GLADE+ catalogue is complete up to =50 Mpc in terms of the cumulative B-band luminosity of galaxies. For $E_{\text{iso}} \leq 10^{50}$ erg, even if the probability for the astrophysical source to lie within 50 Mpc is only 30–70% (depending on the assumed energy), the resulting probability for the source to originate from one of the cross-matched galaxies is still 30–70%. Moreover, the catalogue contains all of the brightest galaxies giving 90% of the total B-band luminosity, half of the total Ks-band luminosity up to ≈130 Mpc, where the probability of finding the astrophysical source is 40–75%. The catalogue contains all of the brightest galaxies giving half of the total B-band (K-band) up to ≈250 Mpc (≈400 Mpc), where the probability of finding the astrophysical source is 50–80%. Finally, even if there is a 15–30% probability that the HEN source lies beyond 800 Mpc where the catalogue completeness falls below 20%, it is still reasonable to look for a possible association with one of the closer catalogued galaxies. We recall that confirmed sources of HEN have been identified at distances between ≈10 Mpc (NGC 1068; IceCube Collaboration 2022a) and ≈1800 Mpc (TXS0506+056; Abbasi et al. 2018), whereas GW170817 at a distance of ≈40 Mpc (Abbott et al. 2017) was an envisaged HEN source (Albert et al. 2017).

Combined with long-term characterisations of existing high-energy gamma-ray sources, for instance, where blazars or AGNs could be present in the 90% containment region of a given HEN alert (at the time of the HEN emission), such distance-constrained cross-matches could be useful in the pursuit of EM counterparts to HEN events, such as those reported in Stein et al. (2023) and Necker et al. (2022).

### References


### Table 1. Distance and ranking, $R$, of galaxies from the GLADE+ catalogue for the cross-matching with HEN candidate IC230306A, using $P_D$ for $E_{\text{iso}} = 10^{50}$ erg, $10^{52}$ erg, or without for $E_{\text{iso}} = 3.3 \times 10^{50}$ erg.

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<tr>
<th>$D_{\text{L}}$ (Mpc)</th>
<th>$R$</th>
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Notes. Only the first ten galaxies are shown. In bold we highlight those galaxies that are not ranked in the top five with no distance information. For $E_{\text{iso}} = 10^{50}$ erg, we show the galaxies found with or without taking into account the fluence limit provided by the ICECUBE notice. The symbol (*) indicates the galaxies present in both the low- and high-energy scans. The average (R) uses $E_{\text{iso}} = 10^{50}$ erg–$10^{52}$ erg with distance information without a fluence limit. The last two columns include the fluence limit.

4.1. Example of ICECUBE-230306A

Taking the example of ICECUBE-230306A (IceCube Collaboration 2023c,d), which is labelled as a ‘gold’ event detected with an energy of 176 TeV on 6 March 2023, the probability density functions $P_{\text{iso}}$ and $P_D$ can be determined for a given energy, for instance, $E_{\text{iso}} = 3.3 \times 10^{50}$ erg, 1% of a typical GRB energy. The galaxy ranking combining the distance and localisation information is obviously different from the one that is based only on the localisation, as can be seen in Table 1 (first and second columns). For $E_{\text{iso}} = 10^{52}, 10^{54}$ erg, the selected galaxies are identical. With increasing values of $E_{\text{iso}}$, the selected galaxies are increasingly distant. Given the ranges of energies expected for most HEN sources, it is safe to assume that $E_{\text{iso}} \leq 10^{52}$ erg. For $E_{\text{iso}} = 10^{52}$ erg, galaxies beyond 250 Mpc must be discarded from the selection because of the fluence limit reported in the second notice. When including this distance threshold in the cross-matching with the GLADE+ catalogue, new galaxies are found. Their distances are shown in the fifth column of Table 1 for $E_{\text{iso}} = 10^{52}$ erg. For lower values of $E_{\text{iso}}$, the distance lower limit does not bring any constraint on the selection. On the other hand for $E_{\text{iso}} > 10^{52}$ erg, only a handful of galaxies are selected (last column of Table 1): the GLADE+ catalogue is not sufficient, given the fluence limit at this energy yielding a distance lower limit of ≈800 Mpc. When considering the different populations and spectrums, the selected galaxies do not change.

4.2. Method for galaxy-targeted HEN followup

Finally, we propose organising the search for EM transient counterparts of HEN events as follows:

1. At $T_0$ (first notice a few minutes after the detection of a HEN event), the galaxies resulting from the cross-match with distance information are targeted, based on the average ranking obtained for $E_{\text{iso}} \approx 10^{52}$ erg$–10^{53}$ erg. This could save valuable time in the localisation of the potential counterpart.

2. At $T_1 \approx T_0 + 1.5$ days (second notice reporting the fluence limit), a low-energy ranking ($E_{\text{iso}} = 3.3 \times 10^{49}$ erg) and a high energy ranking ($E_{\text{iso}} = 10^{52}$ erg) are performed to target the selected galaxies in the search for associated EM counterparts.
Ghosh, S., & De Pietri, R. 2023, Low-latency Update for Open LVEM, LIGO
Document G2300151-v2
Halzen, F. 2021, Ann. Phys., 533, 2100309
IceCube Collaboration 2013, Science, 342, 1242856
IceCube Collaboration 2019, IceCube High-Energy Neutrino Track Alerts
IceCube Collaboration 2021, GCN Circulars Related to IceCube-210629A
IceCube Collaboration 2022b, GCN Circulars Related to IceCube-220822A
IceCube Collaboration 2022a, GCN Circular, 33244
IceCube Collaboration 2023b, GCN Circular, 33256
IceCube Collaboration 2023c, GCN Circular, 33403
IceCube Collaboration 2023d, GCN Circular, 33409
LIGO Scientific Collaboration & Virgo Collaboration 2021, Skymaps and GCN
Circulars S200225q
Murase, K., Thompson, T. A., Lacki, B. C., & Beacom, J. F. 2011, Phys. Rev. D,
84, 043003
221101