Cosmic-ray-induced $\text{H}_2$ line emission

Astrochemical modeling and implications for JWST observations

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1. Introduction

Low-energy cosmic rays (LECRs), with energies of less than 1 GeV, play a significant role in driving the thermochemistry of the molecular interstellar medium (ISM; Dalgarno 2006; Indriolo & McCall 2013). In regions shielded from ultraviolet (UV) radiation, LECRs are the dominant source of ionization. The ionization they provide drives a rich ion-neutral chemistry, leading to the formation of many astronomically important molecules, as well as the initiation of deuteration (Bayet et al. 2011; Caselli & Ceccarelli 2012; Indriolo & McCall 2013; Bialy & Sternberg 2015; Grenier et al. 2015). Further, LECRs provide an important source of heating and, through the ionization fraction, regulate the impact of nonideal magnetohydrodynamic (MHD) effects such as ambipolar diffusion (Padovani et al. 2020).

Determining the flux of LECRs irradiating molecular clouds is a difficult endeavor. There have been a number of investigations using a range of molecular lines (e.g., Caselli et al. 1998; van der Tak & van Dishoeck 2000; McCall et al. 2002, 2003; Hezareh et al. 2008; Shaw et al. 2008; Ceccarelli et al. 2011, 2014; Hollenbach et al. 2012; Indriolo & McCall 2012; Podio et al. 2014; Vauprê et al. 2014; Clevees et al. 2015; Indriolo et al. 2015, 2018; Le Petit et al. 2016; Fontani et al. 2017; Neufeld & Wolfire 2017; Favre et al. 2018; Bacalla et al. 2019; Barger & Garrod 2020; Bovino et al. 2020; Redaelli et al. 2021) and gas temperature (e.g., Ivlev et al. 2019) to estimate the cosmic-ray ionization rate (CRIR), denoted here as $\zeta$. In diffuse gas, absorption studies of simple molecular ions probe the CRIR. However, dense gas measurements typically rely on astrochemical modeling and thus are prone to a number of degeneracies, in particular the treatment of the CRIR (Gaches et al. 2019).

Recently, Bialy (2020, hereafter B20) proposed a novel method to estimate the LECR flux using $\text{H}_2$ rovibrational line emission. As the primary cosmic-ray (CR) protons penetrate into the cloud, they produce a population of secondary electrons that efficiently excite the rovibrational transitions of $\text{H}_2$ (especially of the first vibrational state, $v = 1$), resulting in $\text{H}_2$ line emission in the near-infrared (NIR). As shown by Bialy et al. (2022) and Padovani et al. (2022, hereafter P22), observations of $\text{H}_2$ rovibrational lines may be used to constrain the spectrum of LECRs that is prevailing in the ISM.

The $\text{H}_2$ lines of interest are shown in Table 1, between 2.22 and 3 $\mu$m. The James Webb Space Telescope (JWST) will be able to observe these lines with the Near InfraRed Spectrograph (NIRSPEC) instrument simultaneously. The unprecedented observations will enable JWST to determine the CRIR.
in dense molecular gas, where absorption measurements are difficult. As such, exploring how different model assumptions impact the line predictions is crucial.

The aforementioned previous calculations assumed a fully molecular one-dimensional slab that enabled parameter-space predictions of the H$_2$ line intensity as a function of the observed H$_2$ column density, $N_{\text{H}_2}$. These calculations assumed fully molecular clouds and did not include the effects of far-ultraviolet (FUV) photodissociation and an inhomogeneous density structure, which result in regions in the cloud that are partially atomic. In addition, as previous models are one-dimensional, they assume that the observed column density along the line of sight (LOS) and the effective column density that attenuates CRs as they penetrate into the cloud are identical. In an inhomogeneous three-dimensional cloud, CRs can penetrate from different directions, along “rays” passing through different density profiles (not only along the direction of the LOS), resulting in strong fluctuations in the local CR ionization and excitation rate. Therefore, the role of density structure and the chemical evolution model (e.g., equilibrium versus nonequilibrium) should be constrained as they will impact the conversion of the local quantity (induced H$_2$ radiation) to an integrated quantity (observed H$_2$ line intensity).

In this paper, we present synthetic H$_2$ line emission (two-dimensional plane-of-the-sky) maps of a realistic molecular cloud irradiated by an interstellar CR proton spectrum. We use the three-dimensional astrochemical models presented in Gaches et al. (2022), which include a prescription for CR attenuation and self-consistently formed molecular clouds from the SILCC-Zoom project (Seifried et al. 2017) and the CR excitation rates computed in P22.

2. Methods

We modeled a molecular cloud that is impacted by a flux of CRs and calculated the resulting H$_2$ rovibrational excitation and the consequent NIR line emission from the cloud.

2.1. Incident CR flux

For the CRs that are impinging on the cloud surface, we assumed the interstellar CR proton spectrum from Padovani et al. (2018), with a low-energy spectral slope of $\alpha = -0.8$; this is the “H model,” which provides a good agreement with observations of the CRIR in diffuse clouds.

2.2. CR attenuation

As the CRs penetrate into the cloud, they lose energy through ionization, dissociation, and excitation. We account for this attenuation process by adopting the depth-dependent CRIR, $\zeta(N_{\text{eff}})$, from Table F.1 of Padovani et al. (2018), as well as the depth-dependent H$_2$ excitation rate, $\zeta_{\text{exc}}(N_{\text{eff}})$, from P22 (see their Figs. 5 and 6 and also Fig. 3 in Bialy et al. 2022). Hereafter, we consider a set of cloud models, including one-dimensional slab geometry models and three-dimensional models based on hydro simulations of turbulent clouds, as summarized in Table 2. For our one-dimensional models, $N_{\text{eff}}(z) = \mu N'(z)$, where $N'(z) = \int_{-L/2}^{L/2} ndz$ is the column density from cloud edge to the point of interest at depth $z$ inside the cloud, and $\mu = \cos(\theta)$ is the cosine angle of the $B$-field lines with the cloud normal. We adopted $\mu = 1$. In our three-dimensional models, we utilized the effective column density by accumulating the column along HEALPix rays (Górski et al. 2005),

$$N_{\text{eff}}(x,y,z) = -\frac{1}{2\pi} \ln \left( \frac{1}{N_{\ell}} \sum_{i=1}^{N_{\ell}} e^{-2SN(x,y,z)^{\ell}} \right),$$

(1)

for $N_{\ell} = 12$ rays at the HEALPix $\ell = 0$ level of refinement.

2.3. Density structure

To explore the effect of the cloud structure on the resulting H$_2$ line emission, we considered five models with different density distributions and chemical properties, as summarized in Table 2.

Models 1–3 are one-dimensional slabs. Models 1 and 2 assume a constant density, of $n = 10$ and $10^2$ cm$^{-3}$, respectively. Model 3 has a variable density profile in which the density and column density at each point in the cloud are related through

$$N_{\text{eff}}(x,y,z) = 8.05 \times 10^{19} \exp \left[ 1.6 \left( \frac{n}{\text{cm}^{-3}} \right)^{0.12} \right] \text{ cm}^{-2}.$$

(2)

This relies on the empirical $A_V-n$ relation found by Bisbas et al. (2019) and Bisbas et al. (in prep.) based on a series of turbulent ISM box simulations and galaxy disk simulations. For the densities significant for our results, this relationship accurately reproduces the average densities as a function of effective column density (see Fig. 5 of Gaches et al. 2022). This relationship is for a solar-metallicity gas and can change with metallicity (Hu et al. 2021). Appendix A provides additional details on the one-dimensional astrochemical models.

We also used two three-dimensional density distributions (Models 4 and 5). Model 4 uses the density distribution and astrochemical model from Gaches et al. (2022), which was also previously used in Bisbas et al. (2021). This cloud (called a “dense” cloud) is a subregion from the larger-scale simulations of Wu et al. (2017). It is located in a cube with a uniform resolution of 112$^3$ cells, a side length $L = 13.88$ pc, total mass $M_{\text{tot}} = 5.9 \times 10^4 M_\odot$, and mean H-nucleus density $n = 640$ cm$^{-3}$ (see Wu et al. 2017 and Bisbas et al. 2021 for more details). Model 5 is a molecular cloud from the MHD SILCC-Zoom simulations (Seifried et al. 2017, 2020). These simulations model zoomed-in regions of the stratified disk SILCC (Simulating the LifeCycle of molecular Clouds) simulations (Walch et al. 2015; Girichidis et al. 2016) with the initial Galactic-scale magnetic field set to 3 $\mu$G and use the FLASH 4.3 MHD code (Fryxell et al. 2000). The SILCC-Zoom MHD cloud is located in a cube with a side length of $L = 125$ pc and a total mass of $M_{\text{tot}} = 2.13 \times 10^5 M_\odot$.

2.4. Astrochemical models

For Models 1–4, the chemistry is computed with a modified version of the public astrochemistry code 3D-PDR (Bisbas et al. 2012; Gaches et al. 2022). We used a reduced network derived from the UMIST 2012$^1$ chemical network database

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Table 1. H$_2$ transitions and physical constants.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$J_a$</th>
<th>$J_l$</th>
<th>$\lambda$ ($\mu$m)</th>
<th>$E_{ul}$ (eV)</th>
<th>$\alpha_{ul}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1-0)S(0)</td>
<td>2</td>
<td>0</td>
<td>2.22</td>
<td>0.56</td>
<td>0.30</td>
</tr>
<tr>
<td>(1-0)Q(2)</td>
<td>2</td>
<td>2</td>
<td>2.41</td>
<td>0.51</td>
<td>0.36</td>
</tr>
<tr>
<td>(1-0)O(2)</td>
<td>2</td>
<td>0</td>
<td>2.63</td>
<td>0.47</td>
<td>1.00</td>
</tr>
<tr>
<td>(1-0)O(4)</td>
<td>2</td>
<td>4</td>
<td>3.00</td>
<td>0.41</td>
<td>0.34</td>
</tr>
</tbody>
</table>

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$^1$ http://udfa.ajmarkwick.net
Table 2. Physical and chemical models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Density</th>
<th>FUV</th>
<th>CR attenuation</th>
<th>Code</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1D</td>
<td>Constant $n = 10 \text{ cm}^{-3}$</td>
<td>1</td>
<td>✓</td>
<td>3D-PDR</td>
<td></td>
</tr>
<tr>
<td>2D</td>
<td>Constant $n = 10^{3} \text{ cm}^{-3}$</td>
<td>1</td>
<td>✓</td>
<td>3D-PDR</td>
<td></td>
</tr>
<tr>
<td>3D</td>
<td>Variable $N_{\text{eff}} - n$</td>
<td>1</td>
<td>✓</td>
<td>3D-PDR</td>
<td>Following Eq. (2)</td>
</tr>
<tr>
<td>4</td>
<td>Variable simulation</td>
<td>10</td>
<td>✓</td>
<td>3D-PDR</td>
<td>Wu et al. (2017); Bisbas et al. (2021)</td>
</tr>
<tr>
<td>5</td>
<td>Variable simulation</td>
<td>1.4</td>
<td>✓</td>
<td>FLASH</td>
<td>SILCC-Zoom, Seifried et al. (2017)</td>
</tr>
</tbody>
</table>

(McElroy et al. 2013) consisting of 33 species and 330 reactions. The chemistry was then evolved to steady state using an integration time of 10 Myr. Models 1–3 use an external FUV radiation field of $G_{0} = 1$ (normalized to the spectral shape of Habing 1968) to minimize the impact of photochemistry, and Model 4 uses $G_{0} = 10$ to be consistent with previous studies (Bisbas et al. 2021; Gaches et al. 2022).

Model 5 uses nonequilibrium chemistry with a network of seven species (Glover & Mac Low 2007a,b; Glover et al. 2010; Glover & Clark 2012), a constant atomic hydrogen CRIR $\zeta = 3 \times 10^{-17} \text{ s}^{-1}$, and an external FUV radiation field $G_{0} = 1.4$. The effective column density is computed and stored during the simulation as described above using the TREERAY/OPTICAL/DEPTH module (Clark et al. 2012; Walch et al. 2015; Wünsch et al. 2018). Due to the use of a constant CRIR, the chemistry is not entirely self-consistent with our treatment of the excitation rate, as described below. However, the ionization rates we consider are not high enough to greatly impact the H$_{2}$ abundances. Therefore, our main results will not be significantly altered by this assumption.

2.5. H$_{2}$ excitation and line emission

In steady state, the flux of secondary electrons becomes independent of the local density (see Ivlev et al. 2021). Thus, we can use the calculation of the excitation rate $\xi_{\text{exc},\alpha}$ from P22 for $\nu = 1$, $J_{\alpha} = 0.2$. For the "$H$" CR flux model, the excitation rate varies from $10^{-15}$ to $10^{-16} \text{ s}^{-1}$ between the cloud surface and interior (see Fig. 5 in P22). This calculation uses the CR energy loss function assuming a fully molecular gas. In practice, the loss function should account for a mix of atomic and molecular hydrogen; however, as we show in the appendix, this has a marginal impact on our results for $N_{\text{obs}} > 10^{21} \text{ cm}^{-2}$ (see Fig. A.1).

Given $\xi_{\text{exc},\alpha}$, the emissivity for a specific H$_{2}$ line is

$$\epsilon_{\text{H}_2} = \alpha_{\text{H}_2} \frac{E_{\text{H}_2}}{4\pi} \xi_{\text{exc},\alpha} n(\text{H}_2),$$  

(3)

where $\alpha_{\text{H}_2}$ is the probability that the excitation of state $(\nu_{\alpha}, J_{\alpha})$ will be followed by radiative decay to state $(\nu_{1}, J_{1})$, $E_{\text{H}_2}$ is the transition energy, and $n(\text{H}_2)$ is the H$_{2}$ number density. The $\alpha_{\text{H}_2}$ factor does not include collisional quenching as the densities we consider lie below the critical density (e.g., $n_{\text{crit}} \approx 10^{11} \text{ cm}^{-3}$ at 100 K; Bialy 2020). Our models assume H$_{2}$ is entirely in the para state, which is applicable for the dense regions we are primarily concerned with (Flower et al. 2006, see our Appendix A for an exploration of the impact of a different ortho-to-para ratio).

The LOS-integrated line intensity is then

$$F_{\text{H}_2}(x,y) = \int_{0}^{L} \epsilon_{\text{H}_2}(x, y, z)e^{-\sigma_{\text{H}_2} L}(x, y, z) dz',$$  

(4)

where $N'(x, y, z) = \int_{0}^{L} n(x, y, z') dz'$ is the cumulative column density along the LOS, from the cloud edge to a point inside the cloud (at depth $z$), $L$ is the cloud size, and $\sigma_{\text{H}_2} = 4.5 \times 10^{-23} \text{ cm}^{2}$ is the NIR dust absorption cross section per hydrogen nuclei (B20; Draine 2003). Hereafter, we also define the total column density integrated along the LOS, $N_{\text{obs}} \equiv \int_{0}^{L} n(x, y, z') dz'$. Similarly, $N_{\text{obs}}$(H$_{2}$) is the LOS-integrated column density of H$_{2}$.

3. Results

Figure 1 shows the line intensity of the denoted H$_{2}$ line, seen along the z axis, for Models 4 (top) and 5 (bottom). The observed fluctuations in the line intensities correspond to density fluctuations in the cloud, as well as variations in the effective column density. The emission saturates at high column densities due to the obscuration of dust. We note that these clouds formed through different processes: the Model 4 cloud is the product of a cloud-cloud collision, and the Model 5 cloud is likely the result of supernova shells interacting and contains structures on larger length scales, and thus also more diffuse gas.

Figure 2 shows the line intensities as a function of the total integrated column of hydrogen nuclei, $N_{\text{obs}}$, for our three-dimensional models (Models 4 and 5) and the $N_{\text{eff}} - n$ one-dimensional model (Model 3). We find that these models rapidly diverge for $N_{\text{obs}} < 10^{22} \text{ cm}^{-2}$. The divergence is caused by the substantial differences in the H–H$_{2}$ chemical structure. In particular, Model 5 is more diffuse than Model 4, and while both Models 3 and 5 evolve the chemistry to steady state, Model 4 evolves the chemistry with a nonequilibrium solver. As such, the models exhibit different H$_{2}$ abundance distributions, driving the divergence at low column densities.

The impact of different H$_{2}$ abundance distributions can be factored out by comparing the H$_{2}$ line intensity versus the H$_{2}$ column density. Figure 3 shows the logarithmic column-density bin-averaged line intensity as a function of $N_{\text{obs}}$(H$_{2}$) to investigate whether the differences in chemistry evolution, and thus the abundance profiles, are a dominant factor. We find that now the...
agreement between Models 3, 4, and 5 is strong. We also compare our results to the P22 model (solid curve), which assumes a constant-density, purely molecular slab. Despite the different treatments of the chemistry and density distributions in the various models, there is a good agreement on the line intensity as a function of $N_{\text{obs}}$(H$_2$). We also show the observational upper limits from B20, which are consistent with the various models presented.

**Fig. 1.** Line-of-sight line intensities of the 3D-PDR model (top; Model 4) and the SILCC-Zoom MHD cloud (bottom; Model 5) for the four different H$_2$ lines in Table 1.

**Fig. 2.** Logarithmic column-density bin-averaged line intensities versus the total H-nucleus column density, $N_{\text{obs}}$, for the H$_2$ lines in Table 1. The blue and red lines correspond to Models 4 and 5, respectively, with the filled regions representing ±2σ. The dashed-dot black line shows the results for Model 3. The shadowed region shows the JWST sensitivity with a signal-to-noise ratio of 3 with 1.25 h of integration and 50 shutters (P22).

**Fig. 3.** Same as Fig. 2 but plotted against the LOS H$_2$ column density, $N_{\text{obs}}$(H$_2$). The solid black line corresponds to the calculation in P22. Black triangles show the observational upper limits on the (1-0)S(0) line from Bialy et al. (2022).

**4. Conclusions**

The results presented in this paper demonstrate that the assumptions regarding geometry, chemical evolution, and density distribution do not play a significant role when the H$_2$ column density is used along with the H$_2$ emission lines for constraining the CRIR. However, if the H$_2$ column density is not constrained, and the total hydrogen column density, $N_{\text{obs}}$, is used instead, then the various models diverge in their prediction of the H$_2$ line intensity, especially at low columns, $N_{\text{obs}} < 10^{22}$ cm$^{-2}$. This...
is because the various models assume different density distributions and chemical evolution, which result in different H$_2$ abundances.

There are a few chemical effects not included in this study that will be investigated in future work. First, we focused here primarily on the role of CRs. However, X-rays can also excite the H$_2$ lines through secondary electrons produced by X-ray ionizations, in a similar manner as CRs. Second, the H$_2$ excitation rate and emission assume the H$_2$ is primarily in the para-H$_2$ state. However, at low column densities, particularly in regions with enhanced radiation, this assumption may begin to break down (Flower et al. 2006). As a result, at low column densities, there may be variations in the H$_2$ line depending on the ortho-to-para ratio (OPR) and whether or not state-specific chemistry is included (see Appendix A for a model using an approximate treatment of the OPR). Third, we have neglected additional H$_2$ excitation processes, such as FUV photo-excitation at the cloud envelopes and H$_2$ formation pumping. As shown in B20 (see their Fig. 1 and Eqs. (10) and (11)), this assumption is valid for clouds sufficiently high in CRIRs or low in FUV irradiation.

A comprehensive three-dimensional model that includes these additional H$_2$ excitation mechanisms will be presented in a future work.

To summarize, we have presented synthetic CR-induced H$_2$ line emission maps of four key emission lines (Table 1) for two simulated three-dimensional molecular clouds and several one-dimensional models. These lines are of particular importance: B20 and P22 predicted that they trace the CRIR in dense gas and can be simultaneously observed using NIRSPEC on the JWST. We find that the H$_2$ line intensity as a function of the H$_2$ column density is relatively insensitive to the assumed density distribution or chemical model. Due to this insensitivity, we recommend the use of the H$_2$ lines in Table 1 for constraining the CRIR in dense gas, in particular when using the JWST.

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References

Bialy, S. 2020, Commun. Phys., 3, 32
Appendix A: One-dimensional astrochemical models

We used three different density distributions: constant \( n = 10, 10^2 \text{ cm}^{-3} \) and one following the \( N_{\text{eff}} - n \) relation of Eq. (2). Cosmic-ray attenuation is included following the prescription given in Gaches et al. (2022), where the hydrogen nucleus column density, \( N_{\text{eff}} \), is equated with the attenuating column and the ionization rate follows the polynomial fit of the \( \mathcal{H} \) model from Padovani et al. (2018), as described above.

Figure A.1 shows the hydrogen nucleus number density and \( \text{H}_2 \) abundance, \( x(\text{H}_2) = n(\text{H}_2)/n \), as a function of hydrogen nucleus column density, \( N_{\text{eff}} \). The models exhibit a transition from predominantly atomic \( \text{H} \) to molecular \( \text{H}_2 \), gas with increasing column density, as the photodissociating FUV radiation is absorbed in the \( \text{H}_2 \) lines and in the dust (for \( n = 10^3 \text{ cm}^{-3} \), this transition occurs at small columns, beyond the \( x \)-axis lower limit), (Sternberg et al. 2014; Bialy & Sternberg 2016; Bialy et al. 2017).

Figure A.2 shows that, as a function of the total hydrogen column density, the line intensity shows significant variation between the models. Figure A.3 shows the line flux as a function of the \( \text{H}_2 \) column density for the one-dimensional astrochemical models and exhibits far less variation. The constant-density model with \( n = 10^3 \text{ cm}^{-3} \) is fully molecular and matches the P22 predictions best, which assumed a fixed \( \text{H}_2 \) abundance, \( x(\text{H}_2) = 0.5 \). The \( N_{\text{eff}} - n \) model line intensities are dimmer than the \( n = 10 \text{ cm}^{-3} \) model at low column densities due to the lower \( \text{H}_2 \) abundances in this limit. As functions of the \( \text{H}_2 \) column density, all one-dimensional models are in agreement to within an order of magnitude and are consistent with the Bialy et al. (2022) upper limits.

![Fig. A.1. Total gas density (top) and \( \text{H}_2 \) abundance (bottom) versus total hydrogen nucleus column density, \( N_{\text{eff}} \), for three different one-dimensional 3D-PDR models.](image)

In our fiducial models, we assume a metallicity of \( Z = 1.0Z_\odot \) and that the \( \text{H}_2 \) is entirely in the para-\( \text{H}_2 \) spin state. We ran an additional set of four models using the \( N_{\text{eff}} - n \) density distribution. Three models used different metallicities, \( Z = 0.1Z_\odot \), \( Z = 0.5Z_\odot \), and \( Z = 2.0Z_\odot \), and the fourth used \( Z = 1.0Z_\odot \) but assumed the \( \text{H}_2 \) OPR is in thermal equilibrium (Flower et al. 2006),

\[
\text{OPR} = 9 \exp(-170.5/T_g),
\]

(A.1)

![Fig. A.3. Line intensity as a function of \( \text{H}_2 \) column density for the four \( \text{H}_2 \) lines considered in this work. The dotted blue line corresponds to the calculation in P22. Black triangles show the upper limits on the (1-0)S(0) line from Bialy et al. (2022).](image)
where $T_g$ is the gas temperature computed by 3D-PDR. The OPR ratio is applied in post-processing when computing the $\text{H}_2$ emission since 3D-PDR does not include spin chemistry. This relationship deviates from the expected asymptotic OPR of 3 at high temperatures, but the gas we consider is generally cool ($T < 100$ K), and this relationship still produces an OPR of less than 3. However, the use of this approximation will provide a first indication of the importance of the OPR in determining these line intensities. The $\text{H}_2$ line emissivity is then modified as

$$e_{ul} = \frac{\alpha_{ul}}{4\pi} \frac{E_{ul}}{\zeta_{exc,n}} \left( \frac{n(\text{H}_2)}{1 + \text{OPR}} \right).$$

(A.2)

The dust opacity and $\text{H}_2$ formation rates are linearly scaled with metallicity. Further, the metallicity impacts the heating and cooling due to the changes in the abundances of metals and dust.

Figure A.4 shows the $\text{H}_2$ line intensities versus $\text{H}_2$ column density for these four different models along with the fiducial $N_{\text{eff}} - n$ model. At low $\text{H}_2$ column densities, there is little deviation, although the “thermal OPR” model shows a slight decrease in line intensity at low column densities (and thus higher temperatures) due to $\text{H}_2$ also being in the ortho-$\text{H}_2$ spin state. At high column densities, the line intensities asymptote to different values due to lower (higher) metallicities and have their $\tau_d = 1$ surface deeper (shallower) in the clouds.

![Figure A.4](image)

**Fig. A.4.** Same as Fig. A.3 but plotting the $N_{\text{eff}} - n$ model for different metallicities and a model using an OPR in thermal equilibrium. The vertical lines in the top-left plot denote the $\text{H}_2$ column density where $e^{-\tau_d} = 0.5$. 

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B. A. L. Gaches et al.: Cosmic-ray-induced $\text{H}_2$ emission