Quasi-periodic eruptions from the helium envelope of hydrogen-deficient stars stripped by supermassive black holes

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ABSTRACT

Quasi-periodic eruptions (QPEs), a new kind of X-ray burst with a recurrence time of several hours, have been detected from supermassive black holes (SMBHs) in galactic nuclei. Recently, the two QPEs discovered by the eROSITA showed asymmetric light curves with a fast rise and a slow decline. Current models cannot explain the observational characteristics of QPEs. In this work, we show that QPEs can be generated from the Roche lobe overflows at each periaxis passage of an evolved star orbiting a SMBH. The properties of the companion stars are constrained via analytic estimations. We find that hydrogen-deficient post-AGB stars are promising candidates for exhibiting this phenomenon. We used the Modules for Experiments in Stellar Astrophysics (MESA) stellar evolution code to construct the hydrogen-deficient stars that can fulfill the requirements, as obtained through analytical estimates, to produce the properties of QPEs, including the fast-rise and slow-decay light curves, periods, energetics, and rates. Furthermore, the extreme mass ratio ∼10⁶ between the SMBH and the donor leads to a phenomenon called extreme mass-ratio inspiral (EMRI), producing millihertz gravitational waves. These QPEs would be detected as EMRI sources with electromagnetic counterparts for space-based GW detectors, such as Laser Interferometer Space Antenna (LISA) and Tianqin. These instruments would provide a new way to measure the Hubble constant and further test the Hubble constant tension.

Key words. X-rays: bursts – stars: evolution – black hole physics – accretion, accretion disks

1. Introduction

Quasi-periodic eruptions (QPEs) are a new kind of sudden X-ray brightening originating in the region near the supermassive black holes (SMBHs) of galactic nuclei, both active (Miniutti et al. 2019; Giustini et al. 2020) and quiescent (Arcodia et al. 2021). The peak luminosities of the X-ray bursts are ∼10⁴¹–10⁴⁵ erg in 0.5–2 keV energy band. The average recurrence time of QPEs is ∼2.4 – 18 h with the duty cycle ∼6%–41%. No optical/UV counterpart has been reported for them.

The first QPE was discovered with XMM-Newton in the central region of Seyfert 2 galaxy GSN 069 (Miniutti et al. 2019). The decay of X-ray emission of GSN 069 could be a tidal disruption event (TDE) source according to the long-term evolution of X-ray flux and spectrum (Shu et al. 2018) and the carbon and nitrogen ratio [C/ N] abundance (Sheng et al. 2021). A second QPE system was discovered in RX J1301.9+2747 (Giustini et al. 2020). More recently, two more QPEs, eRO-QPE1 and eRO-QPE2, were discovered with the eROSITA instrument on the Spectrum-Roentgen-Gamma (SRG) space observatory (Arcodia et al. 2021). Follow-up observations by the XMM-Newton X-ray telescope and NICER confirmed the bursting nature of the two sources. The eRO-QPE1 and eRO-QPE2 expand QPE durations and recurrence times towards longer and shorter timescales, respectively. The light curves of them are asymmetric, with a fast rise and a slow decay (Arcodia et al. 2021). The first QPE in GSN 069 also shows asymmetric behavior from its phase-folded light curve.

The QPEs associated with SMBHs have typically low masses. For GSN 069, the mass of SMBH is 4 × 10⁸ M☉ (Miniutti et al. 2019). From the X-ray and UV analysis, the central SMBH mass of RXJ1301.9+2747 is found to be 0.8–2.8 × 10⁶ M☉ (Shu et al. 2017). The total stellar masses are estimated to be 3.8+0.9 −1 × 10⁵ M☉ and 1.01+0.04 −0.01 × 10⁵ M☉ from the optical spectra for eRO-QPE1 and eRO-QPE2 (Arcodia et al. 2021), indicating that the masses of central SMBHs are in the range of 10⁵–10⁷ M☉ based on the scaling relation between central SMBH mass and total galaxy stellar mass (Reines & Volonteri 2015). The properties of the four QPEs are listed in Table 1.

There have been some theoretical models to explain the unusual characteristics of QPEs. For the first QPE, the radiation-pressure accretion disk instability (Janiuk et al. 2002; Merloni & Nayakshin 2006; Janiuk & Czerny 2011; Grzegzorski et al. 2017) was proposed as a way of interpreting this aspect (Miniutti et al. 2019). However, the quiescent galactic nuclei origin of eRO-QPE1 and eRO-QPE2, together with the fast-rise and slow-decay light curves, would challenge this explanation because the stable rise and rapid decay light curves are predicted (Czerny et al. 2009; Wu et al. 2016). The gravitational self-lensing binary SMBH model can generate the flare spacings (Ingram et al. 2021), but cannot reproduce the observed flare profile. Periodic variability is also often associated with a binary orbital period. If the orbital evolution is dominated by gravitational wave emission, the observed properties of eRO-QPE1 and eRO-QPE2 require the mass of the companion to be much smaller than that of the main body (Arcodia et al. 2021),
Table 1. Observed properties of quasi-periodic eruptions events.

<table>
<thead>
<tr>
<th>Source</th>
<th>Redshift</th>
<th>Duration (h)</th>
<th>Recurrence time (h)</th>
<th>$L_p$ (erg s$^{-1}$)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSN 069</td>
<td>0.018</td>
<td>0.54</td>
<td>9</td>
<td>$5 \times 10^{42}$</td>
<td>Miniutti et al. (2019)</td>
</tr>
<tr>
<td>RXJ1301.9+2747</td>
<td>0.02358</td>
<td>-0.3</td>
<td>$\sim 3 - 5$</td>
<td>$1.5 \times 10^{41}$</td>
<td>Giustini et al. (2020)</td>
</tr>
<tr>
<td>eRO-QPE1</td>
<td>0.0505</td>
<td>7.6</td>
<td>18.5</td>
<td>$2 \times 10^{43}$</td>
<td>Arcodia et al. (2021)</td>
</tr>
<tr>
<td>eRO-QPE2</td>
<td>0.0175</td>
<td>0.45</td>
<td>2.4</td>
<td>$10^{42}$</td>
<td>Arcodia et al. (2021)</td>
</tr>
</tbody>
</table>

Fig. 1. Schematic diagram of the mechanism for QPEs. A hydrogen-deficient post-AGB star with He envelope orbits around an SMBH on an eccentric orbit. When the star approaches periastron, it just fills its Roche lobe. The He envelope with low density ends up partially tidally stripped by the SMBH, while the compact core survives the encounter. Finally, the stripped material falls back to the SMBH, resulting in QPEs.

**2. Constraints on the properties of the companion star**

The tidal interaction between the SMBH and the companion can be described via the penetration factor $\beta = R_T/R_p$, where $R_T$ is the tidal radius and $R_p$ is periastron. For $\beta > 1$, the star becomes fully disrupted. The conditions for full tidal disruption can be found in Rosswog et al. (2009) and Gezari (2021). For $0.6 \leq \beta < 1$, partial disruptions are expected to occur (Guillochon & Ramirez-Ruiz 2013). If $\beta < 0.6$, the mass loss of the companion is slow, which is so-called tidal disruption near miss (King 2020). The onset of mass-loss can be assumed when the star just fills the Roche lobe at periastron.

We consider an SMBH with hydrogen-deficient post-AGB star orbiting around it on an eccentric orbit, as illustrated in Fig. 1. It has a compact core surrounded by He envelope. When the star approaches periastron, it marginally fills its Roche lobe, namely, the star’s radius $R_2$ equals its Roche lobe size. The He envelope with low density becomes tidally stripped by the SMBH, while the compact core survives the encounter. The stripped matter loses the orbital energy during the circularization stage. Finally, in the case of highly eccentric orbit, about half of the stripped material falls back to the SMBH, which produces X-ray eruptions. At each periastron passage, mass-transfer onto the SMBH generates QPEs. Mass transfer from the companion onto the SMBH is driven by angular momentum loss caused by gravitational radiation. The system can be analogous to an eccentric version of short-period cataclysmic variable evolution (Paczynski & Sienkiewicz 1981).

The observed recurrence time of QPEs is the orbital period, $P$, in our model, and the orbital semimajor axis is:

$$a = \left( \frac{GM_1P^2}{4\pi^2} \right)^{1/3} = 1.6 \times 10^{12} \text{ cm} \left( \frac{M_1}{10^5M_\odot} \right)^{1/3} \left( \frac{P}{1 \text{ h}} \right)^{2/3}.$$  \hspace{1cm} (1)

In this section, strict constraints of our model are given. In order to produce luminous QPEs with fast rise times, at least three requirements should be satisfied. First, the star’s radius equals its...
Roche lobe size at periapsis. Second, the stars need to be dense enough to make the fast rise timescale of QPEs. Third, the mass-transfer rate driven by gravitational radiation loss should be large enough to generate the luminosities of QPEs.

2.1. The stable radius for mass-loss

The star should fill the Roche lobe at periapsis \( R_p \). The radius of the Roche lobe \( R\text{lobe} \) is taken from Sepinsky et al. (2007). We can get:

\[
R_p = R\text{lobe} \approx 0.46 \left( \frac{M_{\star}}{M_1} \right)^{1/3} R_p. \tag{2}
\]

In this case, \( \beta \approx 0.46 \). We discuss the orbital stability of the onset of mass-loss. For a Schwarzschild BH, the radius of innermost stable circular orbit (ISCO) is:

\[
R_{\text{ISCO}} = \frac{6G M_1}{c^2} \approx 8.86 \times 10^{10} M_{1.5} \text{ cm}, \tag{3}
\]

where \( M_{1.5} = M_1/10^5 M_0 \). For Kerr BHs, the corresponding radius of ISCO are:

\[
R_{\text{ISCO}} = \frac{GM_1}{c^2} \left[ 3 + Z_{\odot} \right] \left( 3 + Z_1 + Z_2 \right)^{1/2},
\]

\[
Z_{\odot} \equiv \left( 1 - 3a_{\odot}^2/M_1^3 \right)^{1/3} \left[ (1 + a_1/M_1)^{1/3} + (1 - a_1/M_1)^{1/3} \right],
\]

\[
Z_1 \equiv \left( 3a_1^2/M_1^3 + Z_1^2 \right)^{1/2},
\]

for the co-rotating and counter-rotating case, respectively, where \( a_1 \) is the dimensionless spin (Bardeen et al. 1972; Jefremov et al. 2015). For \( a_1 = 1 \) and \( a_1 = -1 \), the \( R_{\text{ISCO}} \) is \( 1R_e \) and \( 9R_e \) respectively, where \( R_e = GM_1/c^2 \) is the gravitational radius. The realistic upper limit for \( a_1 \) is 0.998 (Thorne 1974), and the radius of ISCO is \( R_{\text{ISCO}} \approx 1.237 R_e \). The mass–radius relation of WDs can be well approximated by (Zalamea et al. 2010)

\[
R_e = R_{\star} \left( \frac{M_{\odot}}{M_{\star}} \right)^{1/3} \left( 1 - \frac{M_2}{M_{\star}} \right)^{0.447}, \tag{5}
\]

for \( 0.2 M_{\odot} < M_2 < 1.4 M_{\odot} \), where \( M_{\star} \) is the Chandrasekhar mass and \( R_{\star} = 0.013 R_\odot \). From Eqs. (2) and (5), we can get periapsis radius where is the onset of mass-loss for four QPEs (see Fig. 2). The mass of SMHB of GSN 069 is \( M_1 \sim 4 \times 10^5 M_0 \) (Miniutti et al. 2019), and for RXJ1331.9+2747 is \( M_1 \sim (0.8 - 2.8) \times 10^6 M_0 \) (Giustini et al. 2020). Although, the masses of the SMHBs for eRO-QPE1 and eRO-QPE2 are unknown, we can use the BH-to-total stellar mass fraction \( M_1/M_{\star, \text{stellar}} \sim 0.025\% \) (Reines & Volonteri 2015) to estimate the mass of the central SMBH. The total stellar masses of the host galaxies for eRO-QPE1 and eRO-QPE2 are found to be \( 3.8^{+0.9}_{-0.7} \times 10^9 M_0 \) and \( 1.01^{+0.01}_{-0.00} \times 10^9 M_0 \) (Arcodia et al. 2021), respectively. Thus, the mass of the SMBH of eRO-QPE1 and eRO-QPE2 is \( \sim 10^6 - 10^7 M_0 \). The result of King (2020) associated with a low-mass WD companion are shown in black circle. For high-mass SMHBs, the mass of the WD companion should be extreme low (see Sect. 2.4). Therefore, the companion cannot be a WD.

2.2. The mean density of the companion

The light curves of the QPEs have a fast rise and a slow decay (Miniutti et al. 2019; Arcadia et al. 2021). The decay timescale is determined by the viscous timescale of accretion disk, and the rise timescale relates to the radiation region or the interaction between the stripped material and the disk (Shen 2019). The rise time is comparable to the local circularly orbital timescale at periapsis:

\[
t_{\text{ca}}(R_p) = 2\pi \sqrt{\frac{R_p^3}{GM}} \approx 18t_{\text{dyn}}, \tag{6}
\]

where \( t_{\text{dyn}} \approx (\rho_0^{1/2})^{-1/2} \) is the dynamical timescale of the companion star (Shen 2019). The local circularly orbital timescale at periapsis is used to estimate the rise timescale.

For CO WDs \( (t_{\text{dyn}} \approx 3 \text{ s}) \), the rise time of the flux is estimated as \( t_{\text{rise}} \sim 1 \text{ min} \). For He WDs with a mass of \( 0.2 M_2 (t_{\text{dyn}} \approx 12 \text{ s}) \), the rise timescale is \( t_{\text{rise}} \sim 0.06 \text{ h} \). Law-Smith et al. (2017) constructed a He WD with a hydrogen envelope via the MESA stellar evolution code, and they found the dynamical timescale is \( t_{\text{dyn}} \approx 535 \text{ s} \). From the phase-folded light curve of QPEs, the rise timescale is \( -0.2 - 3 \text{ h} \). The rise timescale of QPEs is much longer than that of WDs. For short-duration QPEs (e.g., GSN 069, RXJ1331.9+2747, and eRO-QPE2), the rise timescale is much shorter than He WDs with hydrogen envelopes. From Eq. (6), we can derive \( 52 \text{ g cm}^{-3} < \rho_0 < 4 \times 10^4 \text{ g cm}^{-3} \) from \( 12 \text{ s} \leq t_{\text{dyn}} \leq 535 \text{ s} \). To sum up, the fast rise timescale requires that the stripped material is much denser than Sun-like stars \((\rho \approx 1 \text{ g cm}^{-3})\) or red giants \((\rho \approx 10^{-4} \text{ g cm}^{-3})\).
The MS stars, red giants, and WDs cannot meet the mean density discussed above. The evolved or stripped stars that lost their H envelopes eventually become dense enough. In this work, we focus on the evolved stars. There are two main evolving channels of the hydrogen-deficient stars. For low- or intermediate-mass stars (initial mass $1M_\odot < M < 10M_\odot$), the H envelope was removed in the VLTP phase (Herwig et al. 1999) (see Sect. 3). The other case is the product of the evolution of massive stars (e.g., Wolf-Rayet stars). We did not consider massive stars for the following reasons. First, the massive stars have more obvious optical/UV variability during the circularization stage, however, all the detected QPEs lack the optical/UV counterpart (Miniutti et al. 2019; Giustini et al. 2020; Arcodia et al. 2021). Second, the strong emission bands of helium were found for Wolf-Rayet stars (Hiltner & Schild 1966; Sander et al. 2012), which is inconsistent with the optical observations of eRO-QPE1 and eRO-QPE2 (Arcodia et al. 2021). Third, these kinds of massive stars are rare.

2.3. Orbital eccentricity

The tidal stripping process is stable for an EMRI binary system lasting for $\sim 100 000$–$100 000$ years when the star fills the Roche lobe (Hameury et al. 1994; Dai & Blandford 2013). The star should fill the Roche lobe at periapsis. From Eqs. (1) and (2), we can get the relation between the star parameters and the eccentricity:

$$
0.46 \left( \frac{G M_1^2}{4 \pi^2} \right)^{1/3} M_2^{1/3} (1 - e) = R_p^2,
$$

or in terms of the average density:

$$
0.46 \left( \frac{G \rho_1^2}{3 \pi} \right)^{1/3} \tilde{\rho}_2^{1/3} (1 - e) = 1.
$$

Obviously, the orbital eccentricity must be positive ($e > 0$), thus

$$
\tilde{\rho}_2 > 1.1 \times 10^5 \text{ g cm}^{-3} \left( \frac{P}{1 \text{ h}} \right)^{-2}.
$$

From the $R_p > R_{\text{ISCO}}$ for a Schwarzschild BH, the eccentricity needs to meet the following terms:

$$
1 - e > 0.055 \left( \frac{M_1}{10^5 M_\odot} \right) \left( \frac{P}{1 \text{ h}} \right)^{1/2}.
$$

The periapsis radii, $R_p$, as a function of eccentricity for four QPEs are shown in Fig. 2. For GSN 069, $e = 0.94$ is taken from King (2020), which is shown as black circle. For eRO-QPE1 and eRO-QPE2, $M_1$ is estimated by 0.025% $M_{\text{stellar}}$ (Reines & Volonteri 2015). For high eccentricity orbits, the mass transfer is not stable, namely, the WD companion are to be swallowed by the SMBH ($R_p < R_{\text{ISCO}}$).

2.4. Mass-transfer rate

The accretion luminosity of the stripped material falling back to the SMBH is $L = e M^2 c^2$, where $e$ is the radiative efficiency. We take a typical value $e = 0.1$ which has been widely used, such as King (2020). In order to generate luminous QPEs, the cycle-average mass-accretion rate is:

$$
M_2 \eta = \eta \frac{L_p}{e c^2} \simeq 3.5 \times 10^{-5} M_\odot \text{ yr}^{-1},
$$

where $\eta = \Delta P/P$ is the average duty cycle.

Similar to the process of CV binary evolution (Paczynski & Sienkiewicz 1981), the mass transfer is driven by angular-momentum loss caused by gravitational radiation. The gravitational wave radiation is efficient due to the short period and it goes on to drive the mass transfer near periapsis. The orbital angular momentum lost caused by gravitational radiation is (Paczynski & Sienkiewicz 1981):

$$
J = \frac{32 G^3 M_1 M_2 M}{5 \pi^5} \frac{a^5}{\dot{a}^2} f(e),
$$

where $M = M_1 + M_2$ is the total mass, and

$$
f(e) = \frac{1}{1 - e^2} \left( 1 - \frac{1}{2} e^2 + \frac{7}{16} e^4 \right).
$$

In the case of extreme eccentricity, $e \rightarrow 1$, $f(e) \approx 2^{-7/2} (1 + 73/24 + 37/96) (1 - e^{-7/2})$. As discussed by King (2020), periapsis separation is almost constant. Therefore, the variation of the orbital angular momentum is mainly resulted from the mass transfer of the secondary star $J = M_1/M_1 + M_2/M_2 = M_2/M_1 (1 - M_2/M_1) = M_2/M_1$ (King 2020). Therefore, we can get the mass transfer rate due to gravitational wave radiation as:

$$
\dot{M}_2 \approx 5.4 \times 10^{-5} M_\odot \text{ yr}^{-1} \left( \frac{M_1}{10^5 M_\odot} \right)^{2/3} \left( \frac{M_2}{1 M_\odot} \right)^{2} \left( \frac{P}{1 \text{ h}} \right)^{-1} (1 - e)^{-7/2}.
$$

The mass-transfer timescale is $t_{GW} = -M_2/M_2 \sim 10^5$ yr. If the post-AGB stars which still have burning He shells are captured by the SMBHs, the onset of RLOF would also be caused by nuclear evolution. The nuclear timescale of He shell burning is $10^5 - 10^6$ yr, which is similar to $t_{GW}$. The ongoing RLOF process caused by thermodynamical conditions of the convective envelope is also discussed in Sect. 3, and we find the mass transfer rate is mainly determined by gravitational wave radiation.

From Eqs. (8) and (14), we find that the mass-transfer rate is:

$$
\dot{M}_2 \approx 2.1 \times 10^{-5} M_\odot \text{ yr}^{-1} \left( \frac{M_1}{10^5 M_\odot} \right)^{2/3} \left( \frac{M_2}{1 M_\odot} \right)^{2} \times \left( \frac{P}{1 \text{ h}} \right)^{-1} \left( \frac{\tilde{\rho}_2}{200 \text{ g cm}^{-3}} \right)^{2/3},
$$

and the luminosity is

$$
L \approx 1.2 \times 10^{31} \text{ erg s}^{-1} \left( \frac{e}{0.1} \right) \left( \frac{M_1}{10^5 M_\odot} \right)^{2/3} \left( \frac{M_2}{1 M_\odot} \right)^{2} \times \left( \frac{P}{1 \text{ h}} \right)^{-1} \left( \frac{\tilde{\rho}_2}{200 \text{ g cm}^{-3}} \right)^{2/3}.
$$

For a MS star (e.g., the Sun) and a red giant (e.g., $M_2 = 1 M_\odot$, $R_2 = 25 R_\odot$), the characteristic accretion luminosities are $3.6 \times 10^{38}$ erg s$^{-1}$ and $6.3 \times 10^{33}$ erg s$^{-1}$, respectively. Thus, they are much lower than those of observed QPEs.

From Eqs. (8), (11), and (14), we derived the relation between star parameters ($\tilde{\rho}_2$ and $M_2$) and BH mass for four QPEs (see Fig. 3). The white solid lines refer to constraints from the rise time. The infeasible range is in blue. The yellow vertical solid line (for GSN 069 shown in panel (a) and shading regions – for the other three shown in panels (b), (c), and (d) –
Fig. 3. Contours of allowed average density for different values of the SMBH mass, $M_1$, and the companion star mass, $M_2$. The white solid lines refer to constraints from the rise time. The excluded region is shown in blue. The gray region shows different densities of the companion star. The yellow vertical solid line (for panel a) and shading regions (for panel b–d) represent the mass of SMBHs. The allowed parameter space is shown in yellow. For GSN 069, the star with mass of 0.6 $M_\odot$ and 1.1 $M_\odot$ are shown in red circle and square in panel a, respectively. For the other three shown in panel b–d, the stars with a mass of 0.6 $M_\odot$ and 1.1 $M_\odot$ are shown in red solid and dashed lines, respectively. The average density of stars to explain the properties of QPEs is listed in Table 2, which is between 52 g cm$^{-3}$ and 3642 g cm$^{-3}$. Only low- and intermediate-mass stars whose H envelopes are lost in the VLTP phase (Herwig et al. 1999) can fulfill these requirements. The mass of hydrogen-deficient post-AGB stars are almost the same as the final stellar evolution product WDs, but the density is lower because the existence of He envelopes.

**Table 2. Required average density of post-AGB stars to produce QPEs.**

<table>
<thead>
<tr>
<th>Source</th>
<th>$M_1$ (10$^3$ $M_\odot$)</th>
<th>$M_2$ ($M_\odot$)</th>
<th>$\bar{\rho}_2$ (g cm$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSN 069</td>
<td>4</td>
<td>0.6</td>
<td>899.26</td>
</tr>
<tr>
<td>RXJ1301.9+2747</td>
<td>8–28</td>
<td>0.6</td>
<td>91.81–187.85</td>
</tr>
<tr>
<td>eRO-QPE1</td>
<td>4.75–10.5</td>
<td>0.6</td>
<td>2314.79–3642.22</td>
</tr>
<tr>
<td>eRO-QPE2</td>
<td>1.28–2.55</td>
<td>0.6</td>
<td>538.98–800.92</td>
</tr>
</tbody>
</table>

represent the mass of the SMBH. The allowed parameter space is shown in yellow. It is obvious that MS stars and red giants with higher mass and lower density are located in the infeasible range. Only low and intermediate mass stars whose H envelopes have been removed in the post-AGB phase can fulfill them. The mass of hydrogen-deficient post-AGB stars are almost the same as the final stellar evolution product WDs (0.5 – 1.4$M_\odot$), but the density is lower because of the existence of He envelopes. In Sect. 3, we show the evolved low- and intermediate-mass stars (1$M_\odot$ < $M$ < 10$M_\odot$) that have lost their H envelopes in the post-AGB phase, which can explain all the observation properties. Taking the hydrogen-deficient post-AGB stars constructed in Sect. 3 as examples, the required averaged density is depicted in Fig. 3. For GSN 069, the stars with masses of 0.6 $M_\odot$ and 1.1 $M_\odot$ are shown as a red circle and square in panel (a), respectively. For the other QPEs shown in panels (b–d), the stars with masses of 0.6 $M_\odot$ and 1.1 $M_\odot$ are shown in red solid and dashed lines, respectively. The average density of stars required to explain the properties of QPEs is listed in Table 2, which is between 52 g cm$^{-3}$ and 3642 g cm$^{-3}$.

### 3. Hydrogen-deficient post-AGB stars

For a low- or intermediate-mass star (initial mass of 1$M_\odot$ < $M$ < 10$M_\odot$), the mass loss becomes important in the AGB phase (Blooree 1995). After hydrogen envelopes are lost in the VLTP phase (Herwig et al. 1999), a hydrogen-deficient star forms. The hydrogen-deficient post-AGB stars have a compact core with He envelopes of several times 10$^{-2}M_\odot$. The mass of hydrogen-deficient post-AGB stars is almost the same as the final stellar evolution product WDs (0.5 – 1.4$M_\odot$), but the density is lower because of the existence of He envelopes. When it evolves to the phase that average density satisfying the range given above, it is captured by the SMBH. It fills the Roche-lobe and donates to the SMBH, generating QPEs.

For the stars with the initial mass 1$M_\odot$ < $M$ < 8$M_\odot$, the degenerate CO cores form after core He burning, and the remnants are CO WDs after the residual burning. If the initial stellar
mass is higher ($8M_{\odot} < M < 10M_{\odot}$), the core carbon will be ignited. In this work, we take the evolution of the CO WD with $\sim 0.6M_{\odot}$ and one WD $\sim 1.1M_{\odot}$ as examples. The initial mass is $3.1M_{\odot}$ and $10M_{\odot}$, respectively. The MESA stellar evolution code (Paxton et al. 2011, 2013, 2015, 2018) is used to construct the hydrogen-deficient post-AGB stars.

In Fig. 4, the evolution of the average density of hydrogen-deficient post-AGB stars is shown. For the star with an initial mass of $3.1M_{\odot}$, the degenerate CO core (preformed CO WD) is formed in the final phase of AGB evolution. The black line in panel (a) represents the evolution of the post-AGB phase after the H shell extinct in VLTPs. The star has a burning He envelope and finally cools down as a CO WD with the mass of $\sim 0.6M_{\odot}$. However, for the star with an initial mass of $10M_{\odot}$, it becomes a super-AGB star that is massive enough to ignite C after the giant branch. The black line in panel (b) represents the evolution in C burning phase after the H envelope was removed due to the strong stellar wind. Finally, the star becomes a one WD with the mass of $\sim 1.1M_{\odot}$. The shading regions represent the conditions to produce QPEs given in Table 2, which can last for several times $10^3$ years and $10^4$ years for $M_2 = 0.6$ and $M_2 = 1.1$, respectively. The age and average density corresponding to Fig. 5 are represented by red stars, and the stellar properties are listed in Table 3. The regions of degenerate CO core and He envelope are shown in blue and orange, respectively.

The density profile of the hydrogen-deficient post-AGB stars is at periapsis and donates material to SMBHs, producing QPEs. Due to nuclear evolution or gravitational wave radiation, the star marginally fills its Roche lobe before the post-AGB phase. Finally, the star becomes a ONe WD with the strong stellar wind. The shading regions represent the conditions to produce QPEs given in Table 2, which can last for several times $10^3$ years and $10^4$ years for $M_2 = 0.6$ and $M_2 = 1.1$, respectively. The star properties are the same as those in Table 3. The subsequent evolution of the hydrogen-deficient post-AGB stars can be ignored because the life span of QPEs are quite short. Finally, the He envelopes are all stripped, the compact objects are swallowed by the SMBHs.

4. Event rate

The standard formation channel of EMRIs is the capture of a compact object (WD, NS, or BH) by an SMBH (Sigurdsson & Rees 1997; Amaro-Seoane et al. 2007; Amaro-Seoane 2018). Its rate is about a few percent of the

\[ M_0 = \frac{2\pi}{\sqrt{\nu}} \left( \frac{\beta T_2^3}{\mu} \right)^{3/2} \frac{R_1^3}{G M_2} \rho_{ph} F(q) \]

\[ = 7.66 \times 10^{-9} M_\odot \text{ yr}^{-1} \left( \frac{T_2}{10^5 \text{ K}} \right)^{3/2} \left( \frac{\mu}{4} \right)^{-3/2} \times \left( \frac{R_2}{0.1 R_\odot} \right)^3 \left( \frac{M_2}{0.6 M_\odot} \right)^{3/2} \left( \frac{\rho_{ph}}{10^{-2} \text{ g cm}^{-3}} \right) F(q), \]

where $\beta$ is the gas constant, $T_2$ is the effective temperature, $\mu$ is the mean molecular weight, $\rho_{ph}$ is the photosphere density, and $F(q) \ll 1$ for our interest (e.g., the mass ratio $q = M_1/M_2 \sim 10^3$). The characteristic stellar parameters of hydrogen depleted post-AGB stars are given in Table 3. This mass-transfer rate is negligible compared with that caused by gravitational radiation.

If the star fills its Roche lobe at periapsis, the stripped matter falls back to the black hole and generate QPEs. The characteristic parameters of hydrogen depleted post-AGB stars can be ignored because the life span of QPEs are quite short. Finally, the He envelopes are all stripped, the compact objects are swallowed by the SMBHs.

\[ \frac{\tau_{QPE}}{\tau_{AGB}} = 1428.5 \text{ yr} \left( \frac{M_1}{0.03 M_\odot} \right)^{3/2} \left( \frac{M_2}{1 M_\odot} \right)^{-2} \times \left( \frac{1}{1 \text{ h}} \right) \left( \frac{\rho_2}{200 \text{ g cm}^{-3}} \right)^{-2}. \]

For $M_1 = 10^5 M_\odot$ and $P = 1$ h, the life span of QPEs for the post-AGB star donor is $\sim 2721.57$ yr and $\sim 768.40$ yr for $M_2 = 0.6 M_\odot$ and $M_2 = 1.1 M_\odot$, respectively. The star properties are the same as those in Table 3. The subsequent evolution of the hydrogen-deficient post-AGB stars can be ignored because the life span of QPEs are quite short. Finally, the He envelopes are all stripped, the compact objects are swallowed by the SMBHs.

\[ \tau_{QPE} = \frac{M_{He}}{M_2} \left( \frac{M_{He}}{0.03 M_\odot} \right)^{3/2} \left( \frac{M_1}{10^5 M_\odot} \right)^{-2} \left( \frac{M_2}{1 M_\odot} \right)^{-2} \times \left( \frac{1}{1 \text{ h}} \right) \left( \frac{\rho_2}{200 \text{ g cm}^{-3}} \right)^{-2}. \]
TDE rate. Some other processes include the tidal separation of compact binaries or the formation or capture of massive stars in accretion disks (Amaro-Seoane et al. 2007; Maggiore 2018). In addition, “fake plunges” can serve as high-eccentric EMRIs with a rate about 30 times larger than the typical rate of EMRIs (Amaro-Seoane et al. 2013). Considering the stars injected on high-eccentric orbits in the vicinity of the SMBH due to the Hills binary disruption, the EMRI rate can approach the TDE rate if the binary fraction at the SMBH affecting radius is close to unity (Sari & Fragione 2019). Interestingly, the fraction of binaries is larger than 50% based on the observations. Therefore, we safely assume that the total EMRI rate is on the same order as the TDE rate. Below, we follow the method proposed by our previous work (Wang et al. 2019) to estimate the QPE rate. The mass of SMBHs can be approximated by the $M_{BH} - \sigma$ relation:

$$M_{BH} = M_{BH,\sigma} \left( \frac{\sigma}{\sigma^*} \right)^{4/3},$$

where $\sigma$ is the spheroid velocity dispersion. The $M_{BH} - \sigma$ relation also applies for low-mass SMBH ($<10^5 M_\odot$), considering the uncertainties (Xiao et al. 2011). Hence, this is the relation used in this work. Combined with the constraints from galaxy luminosity functions and the $L-\sigma$ correlation (Aller & Richstone 2002), the BH mass function is (Gair et al. 2004):

$$\frac{dN}{dM_{BH}} = \phi_\sigma \left( \frac{\sigma}{\sigma^*} \right)^{\gamma} \left( \frac{M_{BH}}{M_{BH,\sigma}} \right)^{3/2} \exp \left( - \left( \frac{M_{BH}}{M_{BH,\sigma}} \right)^{1/2} \right),$$

where $\sigma = 3.08/\lambda, \phi_\sigma$ is the total number density of galaxies, and $\Gamma(\gamma)$ is the gamma function. The spatial density of BHs can be estimated from the parameters of low-mass SMBHs ($<10^5 M_\odot$) (Aller & Richstone 2002):

$$\frac{dN}{dM_{BH}} \sim 2 \times 10^{-3} h_{70}^2 \text{Mpc}^{-3},$$

where $h_{70} = H_0/70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is the dimensionless Hubble constant. Then, for solar-type stars, the disruption rate per galaxy is (Wang & Merritt 2004):

$$R = 6.5 \times 10^{-4} \text{yr}^{-1} \left( \frac{M_\odot}{M_\odot} \right)^{-1/3} \left( \frac{R_\odot}{R_\odot} \right)^{1/4} \left( \frac{M_{BH}}{10^5 M_\odot} \right)^{-1/4}.$$

The number ratio of 1–10 $M_\odot$ stars to solar-type stars is about 0.48 using the Salpeter initial mass function, while the lifetime ratio is estimated to be 2% on average. Since the density of the He envelope in our scenario is $10^5$ to $10^6$ times larger than that of solar-type stars, this results in a smaller tidal radius. We reduce the rate by a factor of $1 \times 10^{-2}$. Last but not least, our model requires the star to be located in the He main sequence, whose duration is roughly 0.1 times that of the H main sequence. Integrating Eq. (22) over $1 M_\odot < M < 10 M_\odot$ and $20 R_\odot < R < 60 R_\odot$ and then combining all the aforementioned factors gives the event rate of $N_{QPE} = 1.5 \text{ Gpc}^{-3} \text{ yr}^{-1}$ for $M_{BH} = 5 \times 10^5 M_\odot$. Therefore, the observed number of QPEs can be calculated as $N_{QPE} \sim N_{QPE} V \tau_{QPE}$, where $V$ is the searching volume, and $\tau_{QPE}$ is the active lifetime of QPEs. The co-moving volume within the redshift $z = 0.0505$ of the most distant QPE event (eRO-QPE1) is $V \sim 0.04 \text{Gpc}^3$. In our model, the range of $\tau_{QPE}$ is $100 \sim 1000 \text{yr}$ (Eq. (18)). Hence, $N_{QPE}$ is between 6 – 60. It has been estimated that eROSITA would discover up to about 10 or 15 QPEs by the end of 2023 (Arcodia et al. 2021), which is fully consistent with our estimation.

### 5. Gravitational wave signal detection

The mass-loss systems involving MS and SMBH can be the GW sources (Linial & Sari 2017). In our model, these QPEs are also promising GW sources for space-based GW detectors, such as LISA (Amaro-Seoane et al. 2017; Amaro-Seoane 2018) and TianQin (Luo et al. 2016). The Keplerian orbital frequency of QPEs is about $f_{orb} \sim 10^{-4} \text{Hz}$. The compact core with helium envelopes inspiral into the SMBH produces EMRI signals at a frequency of $f = 2f_{orb}$, which can be detected by LISA and TianQin. The current LISA mission, planned to be launched in 2030s, has an arm-length of $2.5 \times 10^9 \text{m}$ and is sensitive to low frequency bands ($10^{-4} \sim 1 \text{Hz}$). The TianQin has a similar scientific goal, but uses the Earth orbit instead of the heliocentric orbit. Generally, it takes several years for the compact core to plunge into the SMBH after entering the GW detection bands. Therefore, $10^4$ to $10^5$ circles can be recorded by detectors to build up the signal-to-noise ratio (S/N) with a hierarchical matched filtering method, which divides data into short segments and adds their power incoherently.

The GW emission power evolution is (Peters 1964):

$$\dot{E} = -\frac{32}{5} G^{3/2} M_1^2 M_2 M \left( \frac{R}{R_{orb}} \right)^{5/2} f^2,$$

and the Keplerian orbital evolution is:

$$\dot{a} = -\frac{64}{5} G^{3/2} M_1 M_2 M a^2 f.$$  

The characteristic strain of the GW emission from the proper distance $D$ away from the detector is (Amaro-Seoane 2018; Maggiore 2018):

$$h_c(f) = \frac{2 f \tilde{h}(f)}{f^2} = \frac{2 f^{1/2}}{f} \tau_{orb} = \frac{(2f/\dot{E})^{1/2}}{\pi D},$$

where $h_c$ is an instantaneous root-mean-square amplitude. Unlike Chen et al. (2021), GW radiation in harmonic frequencies is not considered. The characteristic strains of different BH masses are shown in Fig. 6, where we set $z = 0.02, M_2 = 0.6 M_\odot, a = 5 \times 10^{12} \text{cm}$, and $e = 0.9$. These GW signal of these QPEs is well above the sensitivity curves of LISA (blue line) and TianQin (purple line). These sources with EMRI signals and electromagnetic counterparts are important for cosmological purposes, such as measuring the Hubble constant (Abbott et al. 2017; Chen et al. 2018; Yu et al. 2018) and the peculiar velocity (Wang et al. 2018; Palmese & Kim 2021).

### Table 3. Properties of hydrogen-deficient post-AGB stars with initial masses $3.1 M_\odot$ and $10 M_\odot$.

<table>
<thead>
<tr>
<th>$M_{init}$</th>
<th>$\text{Age}$</th>
<th>$M_{core}$</th>
<th>$R_{core}$</th>
<th>$M$</th>
<th>$R$</th>
<th>$\log T_{eff}$</th>
<th>$\log \rho_{ph}$</th>
<th>$\log \dot{\rho}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.1$</td>
<td>$430.31$</td>
<td>$0.57$</td>
<td>$0.017$</td>
<td>$0.60$</td>
<td>$0.15$</td>
<td>$4.97$</td>
<td>$-7.67$</td>
<td>$2.37$</td>
</tr>
<tr>
<td>$10$</td>
<td>$22.33$</td>
<td>$1.10$</td>
<td>$0.027$</td>
<td>$1.14$</td>
<td>$0.18$</td>
<td>$5.18$</td>
<td>$-8.27$</td>
<td>$2.47$</td>
</tr>
</tbody>
</table>
Fig. 6. Diagram of the characteristic strains of EMRIs for different BH masses. We set $z = 0.02$, $M_2 = 0.6M_\odot$, $a = 5 \times 10^3$ cm, and $e = 0.9$. The sensitivity curves of LISA (blue line) and Tianqin (purple line) are also plotted for comparison. These QPEs are promising EMRI sources for LISA and Tianqin.

6. Summary

In this paper, we propose a hydrogen-deficient post-AGB star orbiting the SMBH as the generation mechanism of QPEs. The whole picture of this scenario is as follows: a star with an initial mass between $1\sim10 M_\odot$ evolves into a post-AGB phase and then it is captured by a SMBH to form an elliptic orbit. When it passes the periapsis, the star fills its Roche lobe, leading to a mass transfer. When it is accreted at the SMBH, this accretion of the mass by the SMBH leads to the production of QPEs.

According to the rise time, the orbit stability, and the luminosity of QPEs, we find the average density of the companion ranges from tens to thousands of g cm$^{-3}$. The average density of the donor of GSN 069, RXJ1301.9+2747, eRO-QPE1 and eRO-QPE2 is expected to be 899.26 g cm$^{-3}$, 91.81$\sim$187.85 g cm$^{-3}$, 2314.79$\sim$3642.22 g cm$^{-3}$, and 538.98$\sim$800.92 g cm$^{-3}$ for $M_1 = 0.6M_\odot$, respectively. For $M_1 = 1.1M_\odot$, the required average density is 318.13 g cm$^{-3}$, 52.66$\sim$46 g cm$^{-3}$, 818.92$\sim$1288.53 g cm$^{-3}$, and 190.68$\sim$283.35 g cm$^{-3}$, respectively. The properties of hydrogen-deficient post-AGB stars are consistent with these constraints.

The MESA stellar evolution code is used to construct the evolutions of low and intermediate-mass stars. We find that when they lost the H envelopes in the VLTP phase and evolve to hydrogen-deficient post-AGB stars, they lost the H envelopes and then are evolved into a post-AGB phase, then it is captured by a SMBH to form an elliptic orbit. When it passes the periapsis, the star fills its Roche lobe, leading to a mass transfer. When it is accreted at the SMBH, this accretion of the mass by the SMBH leads to the production of QPEs.

The sensitivity curves of LISA (blue line) and Tianqin (purple line) are also plotted for comparison. These QPEs are promising EMRI sources for LISA and Tianqin.