How the area of solar coronal holes affects the properties of high-speed solar wind streams near Earth: An analytical model

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ABSTRACT

Since the 1970s it has been empirically known that the area of solar coronal holes affects the properties of high-speed solar wind streams (HSSs) at Earth. We derive a simple analytical model for the propagation of HSSs from the Sun to Earth and thereby show how the area of coronal holes and the size of their boundary regions affect the HSS velocity, temperature, and density near Earth. We assume that velocity, temperature, and density profiles form across the HSS cross section close to the Sun and that these spatial profiles translate into corresponding temporal profiles in a given radial direction due to the solar rotation. These temporal distributions drive the stream interface to the preceding slow solar wind plasma and disperse with distance from the Sun. The HSS properties at 1 AU are then given by all HSS plasma parcels launched from the Sun that did not run into the stream interface at Earth distance. We show that the velocity plateau region of HSSs as seen at 1 AU, if apparent, originates from the center region of the HSS close to the Sun, whereas the velocity tail at 1 AU originates from the trailing boundary region. Small HSSs can be described to entirely consist of boundary region plasma, which intrinsically results in smaller peak velocities. The peak velocity of HSSs at Earth further depends on the longitudinal width of the HSS close to the Sun. The shorter the longitudinal width of an HSS close to the Sun, the more of its “fastest” HSS plasma parcels from the HSS core and trailing boundary region have impinged upon the stream interface with the preceding slow solar wind, and the smaller is the peak velocity of the HSS at Earth. As the longitudinal width is statistically correlated to the area of coronal holes, this also explains the well-known empirical relationship between coronal hole areas and HSS peak velocities. Further, the temperature and density of HSS plasma parcels at Earth depend on their radial expansion from the Sun to Earth. The radial expansion is determined by the velocity gradient across the HSS boundary region close to the Sun and gives the velocity-temperature and density-temperature relationships at Earth their specific shape. When considering a large number of HSSs, the assumed correlation between the HSS velocities and temperatures close to the Sun degrades only slightly up to 1 AU, but the correlation between the velocities and densities is strongly disrupted up to 1 AU due to the radial expansion. Finally, we show how the number of particles of the piled-up slow solar wind in the stream interaction region depends on the velocities and densities of the HSS and preceding slow solar wind plasma.

Key words. solar-terrestrial relations – solar wind – Sun: corona

1. Introduction

Since the 1970s it has been well known that high-speed solar wind streams (HSSs), the fast, Alfvénic component of the solar wind, originate from solar coronal holes (Nolte et al. 1976). Solar coronal holes are regions of reduced density and temperature compared to the ambient solar corona and feature an open-to-interplanetary-space magnetic field topology. Along these open magnetic field lines, solar plasma is accelerated radially away from the rotating Sun, reaching supersonic speeds and forming the large-scale HSSs that transcend our Solar System.
The acceleration process of HSSs, and of the solar wind in general, is one of the large unsolved problems in solar physics (Cranmer 2002). There are two concurring theories for the acceleration. On the one hand, there are wave- and turbulence-driven models, which are based on the observation of slow magneto-acoustic waves traveling along the magnetic field lines upward to larger heights (Banerjee et al. 2009) and their turbulent dissipation by the interaction between outgoing waves and incoming, in the outer corona reflected waves (e.g., review by Cranmer 2009; Hahn et al. 2012; Hahn & Savin 2013). On the other hand, there are loop-opening and reconnection models, which are based on continuous magnetic reconnection processes, for example, with ephemeral magnetic regions within coronal holes (Wang 2020). Although it is not yet clear which of the two processes provides most of the energy needed for the solar wind acceleration, we are sure that the main part of the acceleration takes place at heights <20R⊙, that is, very close to the Sun (Chashei et al. 2005; Wang et al. 2000; Yakovlev & Yakovlev 2019).

The direction of propagation of HSSs throughout the heliosphere is set at similar distances to the Sun. Up to the Alfvénic point, which is located at about 17R⊙ (Kasper et al. 2021), the magnetic energy density dominates both the thermal and kinetic energy density of HSSs. Thus, HSSs mainly follow the global magnetic field configuration at these heights. At the Alfvénic point, the magnetic and kinetic energy density are equal. The magnetic field configuration is affected by the outflowing solar wind, and consequently, magnetic field lines get stretched in the radial direction away from the Sun (Pneuman & Kopp 1971).

Starting from the Alfvénic point, the kinetic energy density becomes dominant, and the solar wind plasma mainly propagates radially away from the rotating Sun. Since the magnetic field is both frozen into the solar wind and anchored in the rotating Sun, the heliospheric magnetic field winds up and forms an Archimedean spiral (Parker 1958). Different coronal sources of the various types of solar wind thereby correspond to different spiral sectors in the Archimedean spiral. Consequently, the interface between HSS plasma and the preceding ambient slow solar wind plasma also forms a spiral. At the back of this interface, the faster HSS plasma runs into the stream interface and becomes thermalized, whereas in the front of this interface, preceding slow solar wind plasma gets piled up, forming a dense, hot stream interaction region (SIR) around the stream interface.

At Earth distance, HSSs usually appear as a gradual increase in velocity across the SIR, an occasionally apparent extended velocity plateau, and a slow decline of velocities back to slow solar wind velocities. Nolte et al. (1977) studied eight HSSs in 1973 at Earth distance and found that most of the HSS plasma during its declining phase seen at Earth originated from a boundary region at the trailing edge of the coronal hole, with a width of about 6°. This approximate size of the boundary regions was further confirmed by a fast-latitude scan of Ulysses, which found a boundary length of 6° for the equator-boundary of an HSS originating from a polar coronal hole (McComas et al. 1998). The velocities and temperatures of undisturbed HSS plasma parcels are generally well correlated, and their velocities and charge states are well anticorrelated, while their densities show only a weak anticorrelation to the velocities (Neugebauer & Snyder 1966; Burlaga & Ogilvie 1970a,b; Ogilvie 1985; Xu & Borovsky 2015).

Besides the abovementioned relationships, there are two well-known relationships between the peak velocity of HSSs near Earth and the decisive properties of their source coronal holes: (1) the relationship to the flux tube expansion factor of coronal holes (Wang & Sheeley 1990) and (2) the relationship to the area of coronal holes (Nolte et al. 1976). The relationship to the flux tube expansion factor is usually interpreted in terms of at which height energy is deposited to accelerate the solar wind plasma close to the Sun (Wang & Sheeley 1990) and is therefore thought to be related to the main acceleration phase of HSSs. Consequently, it is often employed by solar coronal models to provide solar wind properties at a spherical shell at a distance of 0.1 AU from the Sun. These solar wind properties are then used as inner boundary conditions to model the solar wind in the heliosphere, for example, by time-dependent three-dimensional magnetohydrodynamic (MHD) models such as ENLIL (Odstrcil 2003) and EUHFORIA (Pomoell & Poedts 2018).

The relationship of the HSS speed near Earth to the area of coronal holes was first found by Nolte et al. (1976) and later confirmed by Robbins et al. (2006), Vršnak et al. (2007), Abramenko et al. (2009), Karachik & Pevtsov (2011), Rotter et al. (2012), Hofmeister et al. (2018), Garton et al. (2018), and Heinemann et al. (2020). The corresponding Pearson’s correlation coefficients range from 0.4 to 0.96, depending on the periods the data sets cover. Robbins et al. (2006) indicated that the slope of this linear relationship could depend on the latitude of the coronal hole. Hofmeister et al. (2018) show that it is rather the latitudinal separation angle between the center of mass of the coronal hole and the satellite taking the in situ measurements that affects the peak velocity we see in the ecliptic at 1 AU. Garton et al. (2018) find that the peak velocities of HSSs statistically saturate at around 700 km s⁻¹ for coronal holes with a longitudinal width >70° and that they are also well correlated with the longitudinal widths of the coronal holes. And Hofmeister et al. (2020) indicate that the empirical relationship between the peak velocities of HSSs and the area of their source coronal holes is obtained in MHD simulations, which neglect the acceleration phase and purely model the propagation phase of HSSs in the heliosphere. This demonstrates that this empirical relationship describes the propagation phase of HSSs from the Sun to Earth.

In this study, we derive a simplified theoretical model for the propagation of HSSs from the Sun to Earth and show how the area of coronal holes affects the solar wind properties near Earth. This model is not self-consistent but contains all relevant mechanisms to explain the empirical relationships between the HSS properties near Earth and the area of their source coronal holes. The study is divided into five main sections. In Sect. 2 we introduce the approximation that disturbances in HSSs only have local effects, such that disturbances generated at the HSS boundary regions only affect the boundary regions. This also assumes that plasma parcels launched from the HSS core region into a given radial direction travel at a constant speed through interplanetary space before reaching the interface with the preceding slow solar wind. We refer to this approximation as the assumption of freely propagating HSSs. In Sect. 3 we start at the Sun and assume that the cross section of HSSs close to the Sun is similar to the area of their source coronal holes and consists of a core region and a boundary region. We derive how the properties across the cross section translate into temporal distributions in a given radial direction due to the solar rotation. Further, we derive how the plasma parcels evolve with distance from the Sun under the assumption of freely propagating HSSs. In Sect. 4 we relax the assumption of freely propagating HSSs and add the stream interface to our calculations. We study the kinematics of the stream interface between the HSS and the preceding slow solar wind, using the momentum conservation between the impinging HSS plasma at its back and the piling-up slow solar wind plasma at its front. In Sect. 5 we finally postulate that observed HSSs consist of all HSS plasma parcels launched at the

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Sun that have not yet impinged upon the stream interface. We derive the evolution of the main properties of HSSs with distance from the Sun, how their properties at 1 AU depend on one another, and how they depend on the cross section of the HSSs close to the Sun. We also show how the properties of the stream interface relate to the properties of the HSSs, and we deliberate on the limitations of our model. In Sect. 6 we discuss our findings and present our conclusions.

2. Starting assumptions

In this section we discuss the approximation of freely propagating HSS plasma parcels, which is an important concept needed in this study. This approximation implies that disturbances in HSSs only have local effects, and that therefore HSS plasma parcels launched into a given radial direction travel at a constant speed through interplanetary space as long as they do not reach the interface to the preceding slow solar wind.

In the following, we assume an HSS that has a circular cross section at 0.1 AU. Within the HSS, we assume an increased velocity \( v_f \), temperature \( T_f \), and decreased density \( n_f \) as compared to the ambient slow solar wind stream properties \( v_s, T_s, n_s \).

The magnetic flux density, \( B_f \), of the HSS is assumed to be the same as in the ambient slow solar wind, \( B_s \), as the expansion of the coronal hole is governed by the magnetic pressure, which balances the magnetic flux densities. In MHD simulations, providing initial values of

\[
\begin{align*}
  v_f & = 650 \, \text{km} \, \text{s}^{-1}, & n_f & = 150 \, \text{cm}^{-3}, & T_f & = 1.6 \, \text{MK}, & B_f & = 216 \, \text{nT}
\end{align*}
\]

results in HSS velocities of 730 km s\(^{-1}\), densities of 1.5 cm\(^{-3}\), temperatures of 75 kK, and magnetic field strength of about 2 nT near Earth. Initial values of

\[
\begin{align*}
  v_s & = 350 \, \text{km} \, \text{s}^{-1}, & n_s & = 500 \, \text{cm}^{-3}, & T_s & = 400 \, \text{kK}, & B_s & = 216 \, \text{nT}
\end{align*}
\]

lead to slow solar wind velocities of 350 km s\(^{-1}\), densities of 5 cm\(^{-3}\), temperatures of 20 kK, and magnetic field strength of about 2 nT near Earth. Furthermore, the magnetic flux density chosen corresponds to a magnetic flux density of 1 G in the low solar corona.

We use these values as our starting parameters for the HSSs and slow solar wind plasma at 0.1 AU.

Further, throughout the study, we assume (1) that the HSS is a strongly compressible fluid consisting solely of protons, (2) that HSS plasma parcels propagate radially away from the rotating Sun with a constant velocity, (3) that the HSS undergoes adiabatic cooling, and (4) that the magnetic field follows perfectly Parker’s spiral. Within this section we also approximate HSSs to be a steady-state system, which implies conservation of mass flux and magnetic flux within a solid angle as measured from the Sun; however, we will lift this restriction in the following sections, allowing the mass flux within a solid angle to change with distance from the Sun due to compression and dispersion effects. For a stream propagating freely away from the Sun (i.e., experiencing no external forces such as interactions with other streams), these assumptions result in

\[
\begin{align*}
  v(r) & = v_f, & B(r) & = B_f \cdot \frac{r_0^2}{r^2}, \\
  n(r) & = n_f \cdot \frac{r_0^2}{r^2}, & B_s(r) & = B(r) \cdot \cos(\alpha), \\
  T(r) & = T_f \cdot \left( \frac{r_0^2}{r^2} \right)^{\gamma^{-1}}, & B_\perp(r) & = B(r) \cdot \sin(\alpha), \\
  \alpha(r) & = \arctan \left( \frac{r \cdot \omega}{v} \right).
\end{align*}
\]

Here, \( r_0 \) denotes the associated distance from the Sun at which \( n_f \) and \( T_f \) are defined, \( \omega = 14^\circ/d \) is the average rotation rate of the Sun, \( \gamma = 5/3 \) is the adiabatic constant, \( \alpha \) is the angle of the Parker spiral to the radial direction, \( B_r \) is the component of the magnetic field strength in the radial direction, and \( B_\perp \) is the component perpendicular to the radial direction in the solar equatorial plane. In the following, we omit the explicit radial dependence of these variables whenever possible to improve readability.

Next, we divide the propagation of HSSs from the Sun to Earth into different regimes. First, close to the Sun, we set a fast, low-density HSS in a slow, high-density solar wind environment. At the lateral boundaries of the flow, the HSS will be decelerated due to viscous forces and turbulence close to the Sun, resulting in a smooth transition of the velocity profile from the slow solar wind to HSS plasma, and consequently in a velocity profile over the cross section of the HSS at 0.1 AU (Fig. 1 top).

The maximum speed with which any disturbances generated at the lateral boundary can travel from the boundary to the center of the stream is given by the maximum velocity of fast magneto-sonic waves \( v_{ms} \), given by

\[
\begin{align*}
  v_{ms}(r) = \sqrt{v_{so}^2 + v_{A}^2}, & & v_{so}(r) = \sqrt{\frac{\gamma k_B T}{m_p}}, & & v_{A}(r) = \frac{B}{\sqrt{\mu_0 n m_p}},
\end{align*}
\]

where \( v_{so} \) is the sonic speed, \( v_A \) the Alfvén speed, \( m_p \) the proton mass, \( \mu_0 \) the vacuum permeability, and \( k_B \) the Boltzmann constant.

Using the HSS and slow solar wind properties defined above, we find that close to the Sun at 0.1 AU the magneto-sonic speed is 415 km s\(^{-1}\), and strongly reduces to 93 km s\(^{-1}\) at 0.5 AU and 50 km s\(^{-1}\) at 1 AU (Fig. 1 bottom left). Thus, disturbances generated at the boundaries close to the Sun can travel to and affect large parts of the HSS close to the Sun. With increasing distance from the Sun, the wave velocity decreases so that these disturbances are mostly advected with the HSS, affecting only the local plasma
Fig. 1. Velocity profile, wave velocities, and acceleration forces within HSSs at various distances from the Sun. Top: sketch of the formation of the velocity profile across the HSS cross section close to the Sun, which propagates in a channel within a slow solar wind environment. Velocities are delineated as arrows. Bottom left: acoustic, Alfvénic, and magneto-acoustic velocity in an HSS environment defined by Eqs. (1) and (3) versus its distance from the Sun. Bottom right: Acceleration of a plasma parcel in an HSS due to gravitation and pressure gradients as a function of distance from the Sun.

conditions very close to the HSS boundary. Therefore, we may assume that the velocity profile of HSSs is formed mostly close to the Sun, and that with increasing distance from the Sun, the HSS plasma propagates freely in interplanetary space not seeing boundary effects.

The radial acceleration of a freely propagating plasma parcel within the HSS environment can be approximated by the equation of motion, combined with the radial scaling laws for the plasma density, temperature, and magnetic field strength (Eq. (3)). Assuming that the plasma parcel has the same density as the HSS, and with $m_p$ being the proton mass, $G$ being the gravitational constant, and $P$ being the thermodynamic pressure, the acceleration of such a plasma parcel is given by

$$\frac{dv}{dr} = \left( -\nabla P - \nabla \frac{B^2}{2\mu_0} + \frac{(B \cdot \nabla)B}{\mu_0} - \frac{G m_p n M_{\text{Sun}}}{r^2} \right) (m_p n)$$

$$= 2 \gamma k_B m_p^{-1} T_j r_{\odot}^{2(\gamma - 1)} T_j^2 r_{\odot}^2 + \frac{B_j^2}{\mu_0 m_p n_f r_i^2} \left( \sin^2(\alpha) - r \sin \alpha \cos \alpha \frac{d\alpha}{dr} \right) - \frac{G M_{\text{Sun}}}{r^2}. \quad (5)$$

In the bottom right panel of Fig. 1, we plot the acceleration of such plasma parcels versus the distance from the Sun. The acceleration due to the radial gas pressure gradient, which decreases with $r^{2(\gamma - 1)} \approx r^{2.3}$, clearly dominates the total acceleration, with a minor contribution of the gravitation and a negligible contribution due to the magnetic forces. The total acceleration due to the pressure gradients exceeds the deceleration due to the gravitation until at least 1.5 AU, and consequently the plasma is accelerated to at least this distance from the Sun. However, since the acceleration decreases with $\approx r^{2.3}$, the acceleration falls below 0.5 m/s$^2$ already at a distance of 0.2 AU. The associated total velocity increase of a plasma parcel that has a velocity of 650 km/s$^{-1}$ from 0.2 AU to 1 AU is only about 15 km/s$^{-1}$. Therefore, we neglect the radial acceleration of plasma parcels with distance as long as the plasma parcel did not reach the preceding SIR.
3. Freely propagating HSS

In this section we derive a simplified model for the propagation of freely propagating HSSs. We assume a circular coronal hole in the low solar corona, and that the open magnetic field of the coronal hole expands with a constant, arbitrary expansion factor to 0.1 AU, defining the cross section of the associated HSS. This cross section at 0.1 AU consists of a nearly homogeneous core region as characterized by its temperature, density, and radial velocity distribution, and an adjacent boundary region across which the HSS properties adjust to the properties of the ambient slow solar wind. The rotation of the Sun translates these spatial distributions of the properties along the latitudinal slice into temporal distributions at a given radial direction. From there on, we assume that the HSS propagates freely in interplanetary space. Under these assumptions, we derive the velocity distribution of HSSs at a given distance from the Sun by the dispersion of its initial temporal velocity distribution close to the Sun. Further, we derive the evolution of the density and temperature of the associated plasma parcels in interplanetary space by calculating their expansion and presuming adiabatic cooling.

We define the properties of the HSS at 0.1 AU in the following way, as sketched in Fig. 2 (top). We denote the circular diameter of the HSS cross section, which is also the longitudinal width of the HSS, as $\lambda_{HSS}$, and the width of the boundary region as $\lambda_{bd}$. Thereby, $\lambda_{bd}$ is the width over which the HSS properties change by 85% from HSS to slow solar wind properties as measured from the center of the boundary region. Throughout the study, we keep the width of the boundary region fixed at 6°. We assume that the properties across the HSS solely depend on the distance to the center of the boundary region $d$, given by the sigmoid function:

$$\begin{bmatrix}
v_0_{0.1AU}(d)
v_T_{0.1AU}(d)
T_{0.1AU}(d)
\end{bmatrix} = \begin{bmatrix} v_s \\ T_s \\ n_s \\ \end{bmatrix} + \begin{bmatrix} v_f - v_s \\ T_f - T_s \\ n_f - n_s \\ \end{bmatrix} \cdot \frac{1}{2} \left( 1 + \tanh \left( 2.5 \frac{d - \lambda_{bd}/2}{\lambda_{bd}} \right) \right).$$

The tangens hyperbolicus function is chosen over other sigmoid functions for calculation reasons.

The associated velocity, temperature, and density distributions along a longitudinal slice through the center of the HSS is shown in the bottom left panel of Fig. 2. Since, in our formulation, the velocity, density, and temperature at 0.1 AU depend only on the distance to the HSS boundary, they are also naturally correlated. This correlation allows two of these quantities to be determined by knowing the third, using the normalized quantities

$$\begin{align*}
v_{0.1AU}(d) - v_s &= T_{0.1AU}(d) - T_s = n_{0.1AU}(d) - n_s. \\
\frac{v_f - v_s}{v_f - v_s} &= \frac{T_{0.1AU}(d) - T_s}{T_f - T_s} = \frac{n_{0.1AU}(d) - n_s}{n_f - n_s}.
\end{align*}$$

This relationship is used later in this study.
At 0.1 AU, this temporal velocity distribution disperses with time and distance from the Sun. As we assume a freely propagating HSS, the overall longitudinal width of the HSS $\lambda_{\text{HSS}}$, the boundary region $\lambda_{\text{bd}}$, the displacement of the slice $\varphi$ to the center of the HSS, the maximum velocity of the HSS $v_f$, and the velocity of the slow solar wind $v_s$.

Table 1. Geometrical relations between the effective longitudinal width of an HSS along a given longitudinal slice $\lambda_{\text{HSS, eff}}$, the associated effective width of the HSS boundary region $\lambda_{\text{bd, eff}}$, the peak velocity in the slice $v_{\text{sl, max}}$, the associated velocity gradient across the boundary region $\frac{\omega}{\lambda}$, the overall longitudinal width of the HSS $\lambda_{\text{HSS}}$, the boundary region $\lambda_{\text{bd}}$, the displacement of the slice $\varphi$ to the center of the HSS, the maximum velocity of the HSS $v_f$, and the velocity of the slow solar wind $v_s$.

<table>
<thead>
<tr>
<th>$\lambda_{\text{HSS, eff}}$</th>
<th>For $0.5 \lambda_{\text{HSS}} - \varphi \geq \lambda_{\text{bd}}$</th>
<th>For $0.5 \lambda_{\text{HSS}} - \varphi &lt; \lambda_{\text{bd}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{\text{HSS, eff}}$</td>
<td>$2 \sqrt{(0.5 \lambda_{\text{HSS}} - \varphi)^2 - \varphi^2}$</td>
<td>$2 \sqrt{(0.5 \lambda_{\text{HSS}} - \varphi)^2 - \varphi^2}$</td>
</tr>
<tr>
<td>$\lambda_{\text{bd, eff}}$</td>
<td>$0.5 \lambda_{\text{HSS, eff}} - \sqrt{(0.5 \lambda_{\text{HSS}} - \lambda_{\text{bd}})^2 - \varphi^2}$</td>
<td>$0.5 \lambda_{\text{HSS, eff}} - \sqrt{(0.5 \lambda_{\text{HSS}} - \lambda_{\text{bd}})^2 - \varphi^2}$</td>
</tr>
<tr>
<td>$v_{\text{sl, max}}$</td>
<td>$v_f$</td>
<td>$v_f + \frac{v_f - v_s}{\lambda_{\text{bd, eff}}} (0.5 \lambda_{\text{HSS}} - \varphi)$</td>
</tr>
<tr>
<td>$\left</td>
<td>\frac{d\varphi}{dt} \right</td>
<td>$</td>
</tr>
</tbody>
</table>

Notes. The left solutions correspond to slices that cross the HSS core region, and the right solution corresponds to slices that lie purely in the northern or southern HSS boundary region.

3.1. Dispersion of the velocity distribution with distance

Next, we derive the temporal HSS velocity distribution at a given distance from the Sun for freely propagating HSSs. We start with the spatial velocity distribution along a longitudinal slice through the HSS close to the Sun (Fig. 2 top and bottom left). Using linear approximations for the velocity increase across the leading (i.e., western) and trailing (i.e., eastern) boundary region, this is given by

$$ v_{0.1\text{AU}}(\lambda) = \begin{cases} \frac{v_f + \lambda \cdot \frac{d\varphi}{dt} + v_s}{v_{\text{sl, max}} + (\lambda_{\text{HSS, eff}} - \lambda) \cdot \frac{d\varphi}{dt}} & \text{for } 0 \leq \lambda \leq \lambda_{\text{bd, eff}}, \\ \frac{v_f + \lambda \cdot \frac{d\varphi}{dt} + v_s}{v_{\text{sl, max}} + (\lambda_{\text{HSS, eff}} - \lambda) \cdot \frac{d\varphi}{dt}} & \text{for } \lambda_{\text{bd, eff}} < \lambda < (\lambda_{\text{HSS, eff}} - \lambda_{\text{bd, eff}}), \\ \frac{v_f + \lambda \cdot \frac{d\varphi}{dt} + v_s}{v_{\text{sl, max}} + (\lambda_{\text{HSS, eff}} - \lambda_{\text{bd, eff}}) \cdot \frac{d\varphi}{dt}} & \text{for } (\lambda_{\text{HSS, eff}} - \lambda_{\text{bd, eff}}) \leq \lambda \leq \lambda_{\text{HSS, eff}}. \end{cases} \quad (8) $$

where $\lambda$ is a location in the slice measured from the leading edge along the slice, $\lambda_{\text{HSS, eff}}$ is the effective longitudinal width of the slice in the HSS, $v_{\text{sl, max}}$ is the peak velocity in the slice, $\lambda_{\text{bd, eff}}$ is the effective width of the associated boundary region along the slice, and $\frac{d\varphi}{dt}$ is the velocity gradient across the effective boundary region. It is important to note that the effective longitudinal width decreases, the width of the effective boundary region increases, and the velocity gradient across the effective boundary region decreases with increasing latitudinal displacement $\varphi$ of the slice to the center of the HSS. Further, also the peak velocity in the slice is decreased if either the entire slice is located in the northern or southern boundary region or if the HSS is so small that the entire HSS consists solely of boundary region plasma. The mathematical relations of these parameters to the latitudinal displacement of the slice, the overall longitudinal width of the HSS, and the width of the boundary region follow from geometrical considerations and are given in Table 1.

As the Sun rotates, this longitudinal velocity distribution $v_{0.1\text{AU}}(\lambda)$ along a longitudinal slice in the HSS maps into a temporal velocity distribution $v_{0.1\text{AU}}(t)$ in a given, fixed heliospheric radial direction. Using $\lambda \rightarrow t = \frac{\lambda}{\omega}$, this results in

$$ v_{0.1\text{AU}}(t) = \begin{cases} v_f + \lambda t \cdot \frac{\omega}{\lambda_{\text{HSS, eff}}} & \text{for } 0 \leq t \omega \leq \lambda_{\text{bd, eff}}, \\ v_f + (\lambda_{\text{HSS, eff}} - \lambda) t \cdot \frac{\omega}{\lambda_{\text{HSS, eff}}} & \text{for } \lambda_{\text{bd, eff}} < t \omega < (\lambda_{\text{HSS, eff}} - \lambda_{\text{bd, eff}}), \\ v_f + (\lambda_{\text{HSS, eff}} - \lambda_{\text{bd, eff}}) t \omega & \text{for } (\lambda_{\text{HSS, eff}} - \lambda_{\text{bd, eff}}) \leq t \omega \leq \lambda_{\text{HSS, eff}}. \end{cases} \quad (9) $$

This temporal velocity distribution disperses with time and distance from the Sun. As we assume a freely propagating HSS, the temporal velocity distribution at an arbitrary distance $r$ from the Sun is related to the temporal velocity distribution at 0.1 AU by

$$ v_{0.1\text{AU}}(t) = v_f \left( t + \frac{r}{v_{0.1\text{AU}}(t)} \right). \quad (10) $$

By substituting $t + \frac{r}{v_{0.1\text{AU}}}$ with $\bar{t}$, this leads to the following dispersed velocity distributions of the leading boundary region, plateau region, and trailing boundary regions of HSSs at a given distance from the Sun.

For the leading boundary region,

$$ v_f(\bar{t}) = \begin{cases} 0.5 \cdot \left( v_f + \bar{t} \lambda \cdot \frac{\omega}{\lambda_{\text{HSS, eff}}} - \sqrt{(v_f + \bar{t} \lambda \cdot \frac{\omega}{\lambda_{\text{HSS, eff}}})^2 - 4 r \lambda_{\text{bd, eff}} \cdot \frac{\omega}{\lambda_{\text{HSS, eff}}}} \right) & \text{for } r < v_f \cdot v_{\text{sl, max}} \left( \frac{\omega}{\lambda_{\text{HSS, eff}}} \right)^{-1} \wedge \frac{\bar{t} \omega}{v_f} \leq \bar{t} \leq \frac{r}{v_{\text{sl, max}}} + \frac{\lambda_{\text{bd, eff}} \omega}{\lambda_{\text{HSS, eff}}}, \\ 0.5 \cdot \left( v_f + \bar{t} \lambda \cdot \frac{\omega}{\lambda_{\text{HSS, eff}}} - \sqrt{(v_f + \bar{t} \lambda \cdot \frac{\omega}{\lambda_{\text{HSS, eff}}})^2 - 4 r \lambda_{\text{bd, eff}} \cdot \frac{\omega}{\lambda_{\text{HSS, eff}}}} \right) & \text{for } r \geq v_f \cdot v_{\text{sl, max}} \left( \frac{\omega}{\lambda_{\text{HSS, eff}}} \right)^{-1} \wedge \frac{r}{v_{\text{sl, max}}} + \frac{\lambda_{\text{bd, eff}} \omega}{\lambda_{\text{HSS, eff}}} \leq \bar{t} \leq \frac{r}{v_f}. \end{cases} \quad (11) $$

We note that the term $v_f + \bar{t} \lambda \cdot \frac{\omega}{\lambda_{\text{HSS, eff}}}$ is associated with the velocity of plasma parcels originating from the leading HSS boundary region at 0.1 AU, $v_{0.1\text{AU}}(t)$. The first line is valid for close distances from the Sun, as long as $r < v_f \cdot v_{\text{sl, max}} \left( \frac{\omega}{\lambda_{\text{HSS, eff}}} \right)^{-1}$, that is, as long as A190, page 6 of 18.
the HSS plasma stemming from the edge to the core region did not overtake the HSS plasma originating from the western edge of the boundary region. The second line is valid for the complementary case, taking place at distances \( r \geq v_r v_{s,\text{max}} \left( \frac{\omega}{d} \right)^{-1} \), that is, when the plasma from the edge to the core region did overtake the plasma from western boundary edge. We note that the second line of this equation, which is marked as a black dotted line in Fig. 2 bottom left, is purely a mathematical solution. These plasma parcels will have impinged upon the stream interface to the preceding slow solar wind plasma, breaking down the assumption of freely propagating plasma parcels.

For the plateau region,

\[
v_r(i) = v_{s,\text{max}} \, \frac{r}{v_{s,\text{max}}} + \frac{\dot{A}_{\text{bd,eff}}}{\omega} < i < \frac{r}{v_{s,\text{max}}} + \frac{\dot{A}_{\text{HSS,eff}} - \dot{A}_{\text{bd,eff}}}{\omega_{\text{sun}}}.
\]

(12)

This equation expresses that the peak velocity of the HSS plateau at any distance to the Sun is given by the maximum velocity in the slice, as long as one assumes freely propagating HSSs (i.e., neglecting compression effects near the preceding slow solar wind in the SIRs and any acceleration effects).

And for the trailing boundary region,

\[
v_r(i) = 0.5 \left( \dot{A}_{\text{HSS,eff}} - \dot{A}_{\text{bd,eff}} \right) \frac{dv}{di} + \sqrt{v_r + (4 \, r \, \omega \frac{dv}{di})^2} + r \, \omega \frac{dv}{di}
\]

\[
\leq i \leq \frac{r}{v_r} + \frac{\dot{A}_{\text{HSS,eff}} - \dot{A}_{\text{bd,eff}}}{\omega}.
\]

(13)

We note that the term \( v_r + (\dot{A}_{\text{HSS,eff}} - \dot{A}_{\text{bd,eff}}) \frac{dv}{di} \) is associated with the velocity of plasma parcels originating from the trailing HSS boundary region at 0.1 AU, \( v_{0.1\,\text{AU}}(t) \).

### 3.2. Density and temperature of freely propagating HSS plasma parcels

Next, we estimate the density and temperature of HSS plasma parcels at a distance \( r \) from the Sun by deriving their expansion of volume with distance from the Sun, \( \frac{V_{0.1\,\text{AU}}(t)}{V_r(i)} \), and presuming adiabatic cooling,

\[
n_r(i) = n_{0.1\,\text{AU}}(t) \left( \frac{V_{0.1\,\text{AU}}(t)}{V_r(i)} \right)^{1-\frac{\omega}{\omega_{\text{sun}}} - 1}.
\]

(14)

\[
T_r(i) = T_{0.1\,\text{AU}}(t) \left( \frac{V_{0.1\,\text{AU}}(t)}{V_r(i)} \right)^{1-1}.\]

(15)

The density and temperature of plasma parcels at 0.1 AU, \( n_{0.1\,\text{AU}} \) and \( T_{0.1\,\text{AU}} \), can be derived using their correlation to the velocity at 0.1 AU using Eq. (7). The expansion of the volume of plasma parcels propagating radially away from the Sun is given by their lateral and radial expansion. As we assume HSSs propagating radially away from the Sun, the lateral expansion follows an \( r^2 \) law. The radial expansion within plasma parcels can be determined by their intrinsic radial velocity gradient. This leads to the expansion of the volume,

\[
\frac{V_{0.1\,\text{AU}}(t)}{V_r(i)} = \frac{r_0^2}{r_2^2} \left( 1 + \frac{dv_{0.1\,\text{AU}}(t)}{dr} \right)^{-1}.\]

(16)

To evaluate this equation, a smooth function for the plasma velocity from the slow solar wind to HSS plasma is needed. Consequently, we approximate the velocity distribution across the leading and trailing HSS boundaries at 0.1 AU again by sigmoidal functions,

\[
v_{0.1\,\text{AU}}(t) = \left\{ \begin{array}{ll}
\frac{v_r + \frac{v_{\text{max}} - v_r}{2}}{2} & \text{for } \lambda < \frac{\dot{A}_{\text{HSS,eff}}}{2}, \\
\frac{v_r - \frac{v_{\text{max}} - v_r}{2}}{2} & \text{for } \lambda > \frac{\dot{A}_{\text{HSS,eff}}}{2},
\end{array} \right.
\]

(17)

whereby the first line describes the leading HSS boundary region, and the second line the trailing HSS boundary region. Taking the derivative leads to the increase in volume depending on the distance from the Sun, given by:

\[
\frac{V_{0.1\,\text{AU}}(t)}{V_r(i)} = \frac{r_0^2}{r_2^2} \left( 1 + \frac{5 \frac{\omega}{d} (r - 0.1 \, \text{AU}) \frac{v_{\text{max}} - v_{0.1\,\text{AU}}(t)}{v_{s,\text{max}} - v_{0.1\,\text{AU}}(t)} \frac{\dot{A}_{\text{HSS,eff}}}{2 \omega} \right)^{-1}
\]

\[
\leq \frac{r}{v_r} < \frac{r}{v_{\text{max}}} + \frac{\dot{A}_{\text{HSS,eff}}}{2 \omega} \leq \frac{r}{v_r} + \frac{\dot{A}_{\text{bd,eff}}}{\omega}.
\]

(18)

In the bottom right panel of Fig. 2, we present the resulting temporal density and temperature behavior for the HSS of the top panel in Fig. 2 at Earth distance. The dotted lines, which correspond to the leading edge of the HSS at 0.1 AU and which are derived by the first line of Eq. (18), are purely mathematical solutions since these plasma parcels will have impinged upon the stream interface. Following the plateau region of the HSS, the temperature decreases fast to slow solar wind values, whereas the density shows a small drop for an extended time, and then increases fast to slow solar wind values. We note that this drop in density is an effect of the radial expansion of plasma parcels with distance from the Sun. It therefore depends on the chosen sigmoid function we used to model the velocity gradient across the boundary region and on the assumed boundary width. Using less steep sigmoid
functions or wider boundary regions, respectively, will soften or even remove this density drop. In solar wind plasma measurements in the heliosphere, this density drop will only be observable if the HSS has a steep velocity gradient across its trailing boundary region close to the Sun, and if the HSS plateau region is very extended so that the reference density level before the drop is well defined.

Further, in Fig. 3, we show the distribution of densities and temperatures of HSS plasma parcels at 1 AU versus their velocities. These are derived for an HSS having a longitudinal width of 22°, and slices through that HSS at latitudinal displacements in steps of 1° to the HSS center. For comparison, the dashed diagonal lines give the temperature and density distributions for the case of purely lateral expanding plasma parcels, i.e., where the radial expansion is neglected. At the endpoint of each line (i.e., \( v_r = v_{d,max} \)), the plasma parcel originates from the center of the slice where the radial velocity gradient along the slice is \( \approx 0 \text{ km s}^{-1} \). Therefore, the plasma parcels do not expand into the radial direction, and the solutions for considering and neglecting the radial expansion of plasma parcels with distance coincide. The same is valid for plasma parcels originating from pure slow solar wind plasma. For all other points along the lines, the plasma parcels originate from within the boundary region, where the radial velocity gradient along the slice is \( \neq 0 \text{ km s}^{-1} \). These plasma parcels expand in the radial direction with distance from the Sun, additionally reducing their densities and temperatures compared to a purely lateral expansion.

4. Kinematics of the stream interface

In this section we derive the kinematics of the stream interface between the HSS and the preceding slow solar wind plasma, and estimate its arrival time at Earth. Due to the very high magnetic Reynolds numbers, the HSS and slow solar wind plasma are confined within the magnetic field and cannot mix; therefore, the HSS accumulates at the back of the stream interface, and the preceding slow solar wind is piled up at its front. By presuming the stream interface to be a discrete entity, we derive the kinematics of the stream interface by momentum conservation between the stream interface, the impinging HSS plasma, resulting in a different radial expansion. The numbers give the latitudinal displacement of the slice to the center of the HSS; slices >5° lie entirely within the northern boundary region. For comparison, the dashed diagonal line shows the solution for the case in which the radial expansion of plasma parcels is neglected.

At a given time, \( t \), and velocity of the stream interface, \( v_{SI} \), the number of accumulating HSS particles per time per area is given by \( n_r(t) (v_r(t) - v_{SI}) \, dt \), impinging with a velocity \( (v_r(t) - v_{SI}) \). Analogously, the number area density of the piling-up slow solar wind plasma is given by \( n_s (v_{SI} - v_s) \, dt \) impinging with a velocity \( (v_{SI} - v_s) \). With \( N_{SI} \) being the number area density of the stream interface, the kinematics of the stream interface is therefore determined by

\[
\frac{dN_{SI}(t)}{dt} = n_s (v_{SI} - v_s) + n_r (v_r - v_{SI}) - N_{SI} (v_r(t) - v_{SI}),
\]

When we assume constant properties of the HSS and slow solar wind plasma impinging upon the stream interface per solid angle, the set of differential equations is solved by

\[
N_{SI}(t) = \sqrt{t} \, n_s (v_r(t) - v_s) + 2 \, t \, N_0 \, (n_r (v_r(t) - v_0) + n_s (v_0 - v_s)) + N_0^2,
\]

\[
v_{SI}(t) = \frac{\frac{d}{dt} N_{SI}(t) + n_s v_s - n_r v_r}{n_s - n_r},
\]

whereby \( v_0 \) and \( N_0 \) are the velocity and number area density of the stream interface at the time \( t_0 \). When we further assume that the stream interface starts with a velocity of 0 km s\(^{-1}\), that the number area density of the stream interface at the start, related to a solid
angle, is $0/(^\circ)^2$, and that the HSS velocity is equal to $v_{d,\text{max}}$, the equations simplify. With $n_{d,\text{min}}$ being the density at the center of the longitudinal slice within the HSS, which can be derived by Eq. (7), the mass increase rate of the stream interface, its velocity, and its distance traveled at a time $t$ become

$$\frac{d}{dt} N_{SI} = \sqrt{n_{d,\text{min}} n_s (v_{d,\text{max}} - v_s)}, \quad (22)$$

$$v_{SI} = \frac{v_s \left(n_s - \sqrt{n_{d,\text{min}} n_s} \right) + v_{d,\text{max}} \left(\sqrt{n_{d,\text{min}} n_s} - n_{d,\text{min}} \right)}{n_s - n_{d,\text{min}}}, \quad (23)$$

$$r_{SI}(t) = t \cdot v_{SI}. \quad (24)$$

We note that the velocity of the stream interface is independent on time. This means that the velocity immediately adjusts so that the accumulated HSS mass in its back balances the piling-up slow solar wind mass at its front, and it is constant as long as the impinging HSS and piling-up slow solar wind properties stay constant.

Next, we split the travel time of the stream interface from the Sun to Earth into two regions. First, we estimate the time and distance from the Sun at which the HSS plasma overtakes the preceding slow solar wind plasma, that is, where the SIR is formed. Up to this time, the stream interface roughly propagates with the velocity of the preceding slow solar wind as no momentum has yet been transferred from the HSS to the stream interface. Within the first-order approximation of linearly increasing velocities across the leading boundary region, all plasma parcels originating from the leading boundary region run into the stream interface at nearly the same time. Evaluating this condition for plasma parcels originating from the western and eastern edge of the leading boundary region, that is,

$$v_s t_1 = v_{d,\text{max}} (t_1 - \lambda_{bd,\text{eff}}/\omega), \quad (25)$$

one finds for the impingement time $t_1$, and for the corresponding distance $r_1$ at which the SIR is formed,

$$t_1 = v_{d,\text{max}} \left(\omega \frac{dv}{dt}\right)^{-1}, \quad (26)$$

$$r_1 = v_s v_{d,\text{max}} \left(\omega \frac{dv}{dt}\right)^{-1}. \quad (27)$$

From there on, we neglect the following acceleration phase of the stream interface, assume that it is instantaneously accelerated to its equilibrium velocity given by Eq. (24), and approximate it to further propagate with this constant equilibrium velocity. The arrival time of the stream interface $t_{SI,\text{r}}$ at a distance $r$ is then given by the time at which the SIR is formed plus the remaining travel time of the stream interface at its equilibrium speed up to the distance $r$,

$$t_{SI,\text{r}} = v_{d,\text{max}} \left(\omega \frac{dv}{dt}\right)^{-1} + \frac{r - v_s v_{d,\text{max}} \left(\omega \frac{dv}{dt}\right)^{-1}}{v_{SI}} \quad \text{for } r > r_1. \quad (28)$$

In the following, we apply these equations to the HSS plasma defined in Sect. 3. We assume that the HSS has longitudinal widths of 22° close to the Sun and evaluate the HSSs and associated stream interface properties at latitudinal displacements of 0°, 4°, 6°, 8°, and 10° to the HSS center. First, we derive the numerical solution for the kinematics of the stream interface by using its defining set of differential equations combined with the full time-dependent solution of the impinging HSS plasma. For each time step, we derive iterative the HSS properties velocity and density at the location of the stream interface using Eqs. (11)–(13) and Eq. (18), assuming the preceding slow solar wind to have constant properties, defined by Eqs. (1) and (2), and update the velocity, number density, and location of the stream interface using Eq. (19). In the top right panel of Fig. 4, we show the resulting velocity of the stream interface versus its distance from the Sun. Starting at the Sun, the velocity of the stream interface first coincides with the velocity of the preceding slow solar wind. Next, the velocity increases sharply to the steady-state solution given by Eq. (24), which is related to the formation of the SIR by the impinging HSS plasma. Finally, the velocity of the stream interface slowly declines again, which is related to the reduced velocity of HSS plasma parcels originating from the trailing boundary region of the HSS. The much slower decrease in the velocity of the stream interface as compared to its sharp increase is explained by its larger, accumulated area density and momentum at larger distances, which is shown in the bottom left panel of Fig. 4. While the accumulated particles per area are close to zero near the Sun, they roughly increase linearly as soon as the SIR is formed. In the bottom right panel of Fig. 4, we compare the estimated arrival times at 1 AU derived by Eq. (28) versus the exact arrival times derived by solving the differential Eq. (19) numerically. The estimated arrival times match the exact arrival times within an error of 1 hour; they are therefore good analytical proxies and are used as such in the next section.
which results in freely propagating HSSs (Eqs. (12) and (13)) evaluated at the arrival time of the stream interface at the given distance (Eq. (28)), originating from the trailing boundary region that did not run into the stream interface yet. These cases can be approximated by the velocity of the fastest parcel is equal to the velocity of the HSS core region. Else, the velocity is given by the fastest plasma parcel in the stream interface versus its distance from the Sun. Bottom left: total mass of the stream interface per solid angle, dependent on its distance from the Sun. Bottom right: numerical arrival time of the stream interface at 1 AU versus the analytically approximated arrival time derived by Eq. (28).

5. Characteristic properties of HSS and the stream interfaces in the inner heliosphere

In this section, we postulate that the HSS plasma observed by satellites consists of all HSS plasma parcels launched from the Sun that did not run into the stream interface at the satellite’s distance. Accordingly, we study the properties of HSSs by evaluating only freely propagating HSS plasma parcels, derived in Sect. 3, that arrive later than the stream interface at the given distance, derived in Sect. 4. We first derive the characteristic properties of HSSs, which we define as the properties of the fastest HSS plasma parcel, as a function of the distance from the Sun. In case that not the entire HSS core region has impinged upon the stream interface, the velocity of the fastest parcel is equal to the velocity of the HSS core region. Else, the velocity is given by the fastest plasma parcel originating from the trailing boundary region that did not run into the stream interface yet. These cases can be approximated by freely propagating HSSs (Eqs. (12) and (13)) evaluated at the arrival time of the stream interface at the given distance (Eq. (28)), which results in

\[v_p(r) = \begin{cases} v_{sl,\text{max}} & \text{for } t_{SI} \leq \frac{\lambda_{HSS,\text{eff}} - \lambda_{bd,\text{eff}}}{\omega} + \frac{r}{v_{sl,\text{max}}} , \\ 0.5 \left( v_s - v_{sl,\text{max}} + \frac{\lambda_{HSS,\text{eff}}}{v_{sl}} - \frac{\omega}{v_{sl}} \frac{dv}{dt} + \lambda_{HSS,\text{eff}} \frac{dv}{dt} + \sqrt{(v_s - v_{sl,\text{max}} + \frac{\lambda_{HSS,\text{eff}}}{v_{sl}} - \frac{\omega}{v_{sl}} \frac{dv}{dt} + \lambda_{HSS,\text{eff}} \frac{dv}{dt})^2 + 4 r \frac{dv}{dt}} \right) & \text{for } t_{SI} > \frac{\lambda_{HSS,\text{eff}} - \lambda_{bd,\text{eff}}}{\omega} + \frac{r}{v_{sl,\text{max}}} . \end{cases} \]  

(29)

Therefore, the peak velocity is dependent on the properties of the slow solar wind and of the HSS, the velocity of the stream interface, the effective longitudinal width of the HSS, the velocity gradient across the HSS boundary close to the Sun, and the distance at which the equation is evaluated. We note that the effective longitudinal widths of the HSS and the longitudinal velocity gradient across the HSS boundary are also implicitly dependent on the latitudinal position of the longitudinal slice within the HSS (Table 1).

The temperature and density of the plasma parcels associated with the peak velocity can be determined by first deriving the plasma temperature and density of these parcels close to the Sun using the correlation between the velocity, temperature, and density (Eq. (7)) and then deriving their expansion and associated cooling (Eqs. (14), (15), and (18)):

\[n_{HSS}(v_p) = \left( \frac{0.1 \text{ AU}}{r} \right)^2 \left( \frac{1}{1 + \frac{5 \omega \left( r - 0.1 \text{ AU} \right) (v_s - v_p) (v_{sl,\text{max}} - v_p) \beta_{bd,\text{eff}}}{v_{sl} (v_{sl,\text{max}} - v_p)}} \right)^{-1} \text{ for } t_{SI} \leq \frac{\lambda_{HSS,\text{eff}} - \lambda_{bd,\text{eff}}}{\omega} + \frac{r}{v_{sl,\text{max}}} , \]

\[n_{HSS}(v_p) = \left( \frac{0.1 \text{ AU}}{r} \right)^2 \left( \frac{1}{1 + \frac{5 \omega \left( r - 0.1 \text{ AU} \right) (v_s - v_p) (v_{sl,\text{max}} - v_p) \beta_{bd,\text{eff}}}{v_{sl} (v_{sl,\text{max}} - v_p)}} \right)^{-1} \text{ for } t_{SI} > \frac{\lambda_{HSS,\text{eff}} - \lambda_{bd,\text{eff}}}{\omega} + \frac{r}{v_{sl,\text{max}}} . \]  

(30)
interface, and thus an earlier onset of the decrease in the peak velocities with distance. At latitudinal displacements of 6
° along the slice. A smaller core region is equivalent to less time until all plasma parcels of the core region have run into the stream
Eqs. (29)–(31) at latitudinal displacements of 0
° plasma parcels versus the distance from the Sun. The HSS properties have been derived at latitudinal displacements of 0
° (blue), 4
° (orange), 6
° (green), 8
° (red), and 10
° (purple) to the center of the HSS. The dashed vertical lines mark the distance at which the plasma parcels start to
originate from the trailing boundary region.

Fig. 5. Evolution of HSS properties with distance from the Sun. Top left: schematic cross section of an HSS at 0.1 AU with a longitudinal width of 22
°. Top right: peak velocity of the HSS versus its distance from the Sun. Bottom: temperature (left) and density (right) of the associated HSS plasma parcels versus the distance from the Sun. The HSS properties have been derived at latitudinal displacements of 0
°, 4
°, 6
°, 8
°, and 10
° to the center of the HSS. The dashed vertical lines mark the distance at which the plasma parcels start to
originate from the trailing boundary region.

\[ T_{\text{eff}}(r) = T_{0.1 \text{AU}}(v_p) \left( \frac{0.1 \text{ AU}}{r} \right)^{2(\gamma-1)} \cdot \left( 1 + \frac{5 \omega (r-0.1 \text{ AU})(v_{p}\text{max}-v_r)}{v_{\lambda,\text{eff}} v_r^2 (v_{p,\text{max}}-v_r)} \right)^{1-\gamma} \]

\[ T_{\text{ff}1} \leq \frac{\rho_{\text{HSS,s}} - \rho_{\text{bd}}}{\omega} + \frac{\rho_{\text{max}}}{\omega} \quad \text{for} \quad T_{\text{ff}1} > \frac{\rho_{\text{HSS,s}} - \rho_{\text{bd}}}{\omega} + \frac{\rho_{\text{max}}}{\omega} \]

(31)

In the next sections we analyze the behavior of these properties with distance from the Sun and their interrelations in more
detail.

5.1. Evolution of the velocity, density, and temperature with distance from the Sun

First, we present how the velocity, density, and temperature of HSSs, represented by the plasma parcels having the peak velocity, behave with distance from the Sun. We again assume an HSS having a longitudinal width of 22
° close to the Sun, and evaluate Eqs. (29)–(31) at latitudinal displacements of 0
°, 4
°, 6
°, 8
°, and 10
° to the HSS center, as sketched in the top left panel of Fig. 5.

In the top right panel of Fig. 5, we show the peak velocity of the HSS versus its distance from the Sun. In the center of the HSS (blue line), the peak velocity is constant with 650 kms
−1 up to a distance of 0.92 AU, and decreases from thereon continuously. Therefore, up to 0.92 AU, all these plasma parcels originate from the core region of the HSS. Starting from 0.92 AU, the peak velocity is given by the fastest plasma parcel from the trailing boundary region that did not run into the stream interface yet. As more “fastest” plasma parcels impinge upon the stream interface, the peak velocity decreases with distance. The rate of decrease thereby mostly depends on the velocity gradient across the HSS boundary region close to the Sun, and the velocity with which the stream interface propagates away from the Sun (Eq. (29)). At a latitudinal displacement of 4
°, the longitudinal slice still crosses the HSS core region. Accordingly, the maximum velocity in the longitudinal slice, and thus the peak velocity of the HSS, is still 650 kms
−1. However, the effective longitudinal width of the HSS is reduced, which results in an effective smaller HSS core region along the slice. A smaller core region is equivalent to less time until all plasma parcels of the core region run into the stream interface, and thus an earlier onset of the decrease in the peak velocities with distance. At latitudinal displacements of 6
°, 8
°, and 10
°, the longitudinal slice through the HSS crosses only the boundary region of the HSS. This results in a reduced maximum velocity within the slice, and consequently in smaller peak velocities already close to the Sun. As the velocity gradient across the leading HSS boundary region is smaller at larger latitudinal displacements to the HSS center, the time at which the SIR forms, increases (see Eq. (28)). Up to that distance, the HSS peak velocity is given by the maximum velocity within the longitudinal slice. Right afterward, the peak velocities start to decrease as there is no HSS plateau region.

In the bottom panels of Fig. 5, we show the associated densities and temperatures of the plasma parcels representing the peak velocities versus the distance from the Sun, in a double-logarithmic plot. As long as all the plasma parcels originate either from the HSS core region or from the longitudinal center of the HSS (i.e., where the velocity gradient along the longitudinal slice at 0.1 AU is zero), the plasma parcels only expand laterally. Consequently, the density decreases with \( r^{-2} \) and the temperature with \( r^{2(\gamma-1)} \). As soon as the HSS peak velocity gets associated with plasma parcels originating from the trailing boundary region (i.e., where the longitudinal velocity gradient across the boundary region is \( \neq 0 \text{ km s}^{-1} \)), the plasma parcels also expand radially. Due to the radial
Fig. 6. Dependence of the properties of HSSs at 1 AU on their area and longitudinal width close to the Sun. Top: peak velocity of HSSs at 1 AU versus their longitudinal widths (left) and versus the areas of their source coronal holes at the solar surface (right). Bottom: temperatures (left) and densities (right) of the HSS plasma parcels that have the peak velocity at 1 AU versus the longitudinal widths of the HSSs. The data were derived for latitudinal displacements of 0°, 3°, 6°, 9°, and 12° to the center of the HSS.

expansion, the densities of these plasma parcels decrease faster than $r^{-2}$, and the temperatures decrease faster than $r^2(\gamma - 1)$. Therefore, in general, it should not be assumed that the densities in HSSs decrease with $r^{-2}$, with the exception of plasma parcels originating from the HSS core region.

5.2. Dependence of the properties of HSSs at 1 AU on the HSS geometry close to the Sun

Here, we show how the properties of HSSs at 1 AU depend on each other, and on the geometry of the HSS close to the Sun. According to Eq. (29), the peak velocity of HSSs is dependent on the longitudinal width of the HSS close to the Sun. The longitudinal width of the HSS determines the spatial distance between the leading edge of the HSS, which coincides with the stream interface at this distance, and the trailing boundary region of the HSS. Consequently, it also affects the time and the distance from the Sun at which a plasma parcel originating from the trailing boundary region catches up with the stream interface. The larger the distance between the leading edge and trailing boundary region, the fewer plasma parcels from the trailing boundary region have run into the stream interface at a given distance from the Sun, and the higher is the peak velocity of the HSS at this distance. This explains the dependence of the peak velocity of HSSs on the longitudinal widths of the HSSs. In the top left panel of Fig. 6, we show the peak velocities of HSSs at a distance of 1 AU versus the longitudinal width of HSSs close to the Sun, at latitudinal displacements of 0°, 3°, 6°, 9°, and 12° to the center of the HSS. Dotted lines are associated with plasma parcels that originate from the HSS core region. Dashed-dotted lines represent plasma parcels originating from the trailing boundary region within a slice that crosses the HSS core region. And solid lines describe plasma parcels that originate from a slice that lies entirely in the northern or southern boundary region. The peak velocities are roughly linearly dependent on the longitudinal width of the HSSs and have similar slopes for all latitudinal displacement; solely the $x$-intersections vary. The $x$-intersection gives the minimal latitudinal extent of an HSS that is needed so that the HSS crosses the longitudinal slice, and thus that an HSS appears in the data.

In the top right panel of Fig. 6, we show the HSS peak velocities versus the areas of their solar source regions (i.e., coronal holes). The areas are thereby derived by $A_{CH} = \pi (\lambda_{HSS}/2)^2$ in square degrees, and subsequently projected to the solar sphere having a radius of 1 $R_\odot$ to derive their areas in km². This associates the areas of the HSSs with the areas of coronal holes at the solar surface. This functional relationship between longitudinal width and area results in that the lines get slightly skewed, dependent on the latitudinal displacement $\varphi$.

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In the bottom panels of Fig. 6, we show the dependence of the temperature and density of the plasma parcels at 1 AU that are associated with the peak velocity versus the longitudinal width of the HSS. With increasing longitudinal width, the temperature increases and the density decreases, and finally saturates. The overshoot in the density decrease is related to the radial expansion of plasma parcels originating from the trailing HSS boundary. Presuming a less steep sigmoidal velocity function across the HSS boundary or a wider HSS boundary region, respectively, reduces the density overshoot.

We note that the kinks in the relationships between the longitudinal widths and areas to the velocities (at velocities of 550–600 km s⁻¹), in the temperatures (at 30–50 kK), and in the densities (at about 1 cm⁻³) are artifacts due to the approximation we used.
Fig. 7. Temperatures (left column) and densities (right column) versus the velocities of HSS plasma parcels for three subsets. First row: plasma parcels that have the peak velocities of HSSs. Here, we considered all HSSs with longitudinal widths of 0–40° at latitudinal displacements of the slice of 0°, 3°, 6°, 9°, and 12° to the center of the HSS. Second row: HSS plasma parcels originating from an HSS with a longitudinal width of 22°, for latitudinal displacements of 0–11° (labeled on top of the lines). Third row: all HSS plasma parcels derived from all HSSs with longitudinal widths of 0–40° at latitudinal displacements of 0–12°.

for modeling the HSS boundary regions. Different sigmoidal functions and definitions of the size of the boundary region will lead to slightly different results for the strength of the kinks. In purely numerical solutions of our initial equations, the kinks are smoothed out.

5.3. Interrelation of HSS properties at 1 AU

In this section we show the interrelation between the velocity, density, and temperature of HSS plasma parcels at 1 AU. We investigate the interrelation for three cases: (1) the distributions of plasma parcels that are related to the peak velocity of HSSs, (2) the distribution of HSS plasma parcels originating from a single longitudinal slice within an HSS, and (3) the statistical distribution of plasma parcels from large HSS data sets.

In the top panels of Fig. 7, we show the temperature and density of the plasma parcels representing the peak velocities at 1 AU versus the peak velocities. For most of the distribution, the temperature of these plasma parcels increases and the densities decrease with increasing peak velocities. This is a consequence of their assumed (anti) correlation close to the Sun, but amended by the radial expansion of the plasma parcels on their way to 1 AU. We note that for the longitudinal slice through the center of the HSS (i.e., at \( \varphi = 0° \)), we find small densities even for the small peak velocities. As the overall HSS has to be very small to provide small peak velocities in the center of HSS, and as thereby the chance that a satellite takes measurements right in the center of these HSSs becomes negligible, these low densities can probably be never observed. Therefore, from observational HSS data sets that analyze only the plasma parcels that have the peak velocities, we expect a correlation between the velocity and temperature and an anticorrelation between the velocity and density.
In the mid panels of Fig. 7, we show the temperature and density distributions of HSS plasma parcels versus their velocity for an HSS having a longitudinal width of 22°. Each line thereby represents plasma parcels originating from a different longitudinal slice through this HSS. For each slice, the dependence between the HSS density and temperature to the velocity has a unique curvature. This curvature is related to the radial expansion of the plasma parcels on their way from the Sun to 1 AU, and thus to the velocity gradient across the trailing HSS boundary within that slice. The expansion is largest for plasma parcels originating from the center of the trailing boundary region and smaller for parcels that arise from closer to the western and eastern edge of the trailing boundary region (i.e., from the transition to the HSS core region or to the subsequent slow solar wind plasma).

In the bottom panels of Fig. 7, we show statistical distributions of the temperatures and densities versus velocities, by superposing all HSS plasma parcels originating from all longitudinal slices for all HSSs up to a longitudinal width of 40°. The color gives the associated average latitudinal displacement of the origin of the plasma parcels to the center of the HSS. In this statistical data set, the temperature increases with velocity, resulting from the original correlation of temperature and velocity across the HSS cross section close to the Sun. Again, the radial expansion of plasma parcels originating from the trailing boundary region results in additional cooling, which counteracts the temperature increase. In contrast, the velocity-density distribution is much less confined. The decrease in density with increasing velocity originates from the anticorrelation of the density and velocity across the HSS cross section close to the Sun. Though, the radial expansion of plasma parcels additionally enhances the density decrease. Consequently, the densities can decrease very fast from slow solar wind to HSS values, dependent on the velocity gradient across the trailing boundary region, and thus strongly dependent on the latitudinal origin of the plasma parcels with respect to the center of the HSS. This amplification for the density decrease increases the range at which densities appear at given velocities.

We note that the curvatures in each of the plots depend on the radial expansion of plasma parcels from the Sun to Earth, and thus on the widths of the boundary region and the approximation we have used for the velocity profile across the boundary region. Larger widths, or less steep velocity gradients, respectively, result in smaller curvatures in these plots.

5.4. Properties of the stream interface at 1 AU

Finally, we show how the properties of the stream interface at 1 AU relate to the properties of HSSs at 1 AU, and on the geometry of the HSSs close to the Sun. The velocity of the stream interface depends on the momentum transferred from the HSS to the stream interface, and consequently on the underlying velocity profile along the longitudinal slice through the HSS. As the velocity profile depends on the longitudinal width of the HSS and the latitudinal displacement of the slice to the center of the HSS, the velocity of the stream interface is also dependent on these quantities, shown in the top left panel of Fig. 8. Furthermore, since both the peak velocity of the HSS and the velocity of the stream interface depend on the longitudinal width of the HSS, they are naturally correlated. However, it is important to note that this is a result of the mutual dependence on the longitudinal width of the HSS, and not a causal dependence.

The accumulated mass of the stream interface is dependent on the velocity difference between the stream interface, the preceding slow solar wind, and the subsequent HSS (Eq. (19)). A higher velocity of the stream interface implicates that less mass from the HSS accumulates, but also that more preceding slow solar wind plasma gets piled up. As the preceding slow solar wind plasma...
has a higher density than the HSS plasma, the slow solar wind term dominates. Consequently, the total accumulated mass increases with increasing velocity of the stream interface, which is shown in the bottom right panel of Fig. 8. Since the velocity of the stream interface is again dependent on the longitudinal width of the HSS, so is the accumulated mass of the stream interface as shown in the top panels of Fig. 8.

5.5. Limitations

In this study, we made a number of simplifications so that the model was analytically solvable. These simplifications should not strongly affect the relationships between HSS properties derived throughout the study. However, some simplifications affect the absolute values of the properties near Earth. First, we implicitly assumed that coronal holes extend radially into interplanetary space, forming the cross section of HSSs at 0.1 AU. In reality, each coronal hole has its unique expansion factor, and thus, the relationship between the area of coronal holes in the low corona to the area of the associated HSSs at 0.1 AU is not trivial. Second, we assumed that HSSs are propagating radially away from the Sun. HSSs are being bent toward the ecliptic very close to the Sun. Our calculations do not require radially propagating HSSs as assumed here for simplicity, but only that the HSS plasma propagate along a guiding line, which does not affect its kinematics. Bent magnetic field lines provide such guiding lines for plasma, and thus do not affect our calculations regarding the HSS velocities. Third, we neglected a further acceleration of the solar wind in interplanetary space. Although this acceleration after 0.1 AU is comparably small, it will still shift all velocities to slightly larger values. Fourth, we assumed that the impingement of HSS plasma into the preceding slow solar wind plasma happens instantaneously, neglecting the spatial extension of the SIR. This assumption might work well as long as the SIR has a limited extension compared to the spatial scales of the overall system; but it will break down at the latest when the backward wave steepens to a shock wave and decouples from the stream interface in the outer Solar System. Fifth, we neglected viscosity in the solar wind plasma, which results in a slower dispersion of the velocity distribution than assumed by our calculations. Sixth, our model is only one-dimensional for the propagation of plasma parcels from the Sun to Earth, and therefore neglects a possible redistribution of plasma along the two-dimensional SIR. This could affect the momentum balance between the HSS plasma and the preceding slow solar wind plasma at the stream interface, and thus the velocity of the stream interface and the amount of piled-up slow solar wind plasma. Seventh, the assumption of radially propagating HSSs, affect the temperatures we derive at Earth distance. MHD simulations show that HSSs originating from low and medium-latitude coronal holes usually are bent toward the ecliptic, whereby their expansion is inhibited toward higher latitudes. Thus, the plasma expands less into the latitudinal direction with distance to the Sun, which results in less cooling. Eighth, we neglected waves in the plasma, which heat HSSs in interplanetary space, and could also stabilize HSS plasma, counteracting velocity gradients. And ninth, adiabatic cooling by expansion, as assumed in our calculations, requires that work has to be done that transfers the energy away from the cooled gas. However, it is not clear to us what expansion work HSSs do during their propagation away from the Sun. This question is particularly interesting for plasma parcels in the HSS tail, as the preceding HSS plasma parcels are faster and thus run away so that no compression work should be involved. We note that points seven and nine primarily affect the temperature; as the temperature is derived independently from the velocity and density in our calculations, this does not strongly affect the other main findings of our study. All above points will lead to deviations from the results presented here, and require more detailed studies. But they are not expected to change the physical explanation of how the area of solar coronal holes affects the solar wind properties near Earths.

6. Summary, discussion, and conclusions

In this study we assumed that four main effects determine the properties of HSSs in interplanetary space at a given distance from the Sun. First, a velocity profile forms across the lateral HSS boundary close to the Sun. Due to the solar rotation, this spatial velocity profile maps into a temporal velocity profile in a given radial direction. Second, HSS plasma parcels propagate freely (i.e., with a constant radial velocity) in interplanetary space as long as they do not reach the SIR with the preceding slow solar wind. Third, the leading HSS plasma parcels run into the preceding slow solar wind plasma, thermalize, and drive the SIR. And fourth, the characteristic properties of HSSs observed at a given distance from the Sun are given by the fastest HSS plasma parcels that have not yet impinged upon the SIR.

By building an analytical model on these assumptions, we found that:

– The peak velocity of HSSs at Earth distance is dependent on the effective longitudinal widths of HSSs at 0.1 AU (i.e., the length of a longitudinal slice through the HSS at 0.1 AU that represents the foot points of the HSS plasma seen at Earth). The shorter the effective longitudinal widths, the more plasma from the HSS core region and trailing boundary region has already impinged upon the stream interface at 1 AU. The more “fastest” plasma parcels from the trailing boundary region have impinged upon the stream interface, the smaller the velocity of the leading plasma parcel is, which has not yet run into the stream interface at 1 AU, and thus the smaller the peak velocity of the HSS seen at 1 AU is.

– The lengths of the effective longitudinal width are dependent on the longitudinal widths of the overall HSS at 0.1 AU and on the latitudinal displacement of the associated slice representing the foot points to the center of the HSS. When we approximate the longitudinal widths from the area of the HSS (by assuming that the HSS has a circular cross section), and further assume that the area of HSSs at 0.1 AU statistically depends on the area of their source coronal holes, we can explain the long-known empirical relationship between the area of solar coronal holes and the peak velocities of the associated HSSs near Earth.

– The temperature and density of HSS plasma parcels near Earth depend on their original temperature and density close to the Sun, and thus on their initial location within the HSS core or boundary region close to the Sun.

– Furthermore, their density and temperature near Earth depend on their expansion on their way from Sun to Earth. Plasma parcels originating from the boundary region of HSSs expand not only in the longitudinal and latitudinal direction, but also in the radial direction on their way from the Sun to Earth. The radial expansion thereby depends on the strengths of the radial velocity...
gradient within the parcels close to the Sun, which is again set by the initial location of the plasma parcels within the HSS core or boundary region and, in particular, by the widths of the HSS boundary region.

- As we assumed a correlation between the solar wind temperature and velocity close to the Sun, the radial expansion of the plasma parcels reducing the temperature counteracts this correlation. However, the velocity and temperature are still well correlated at Earth distance.

- As we assumed an anticorrelation between the solar wind density and velocity close to the Sun, the radial expansion enhances the decrease in density with increasing velocity at larger distances from the Sun. This leads to a fast drop in density with increasing velocity in the associated scatter plot and nearly destroys the initial anticorrelation between the solar wind density and velocity at Earth distance.

In the following, we briefly discuss some consequences of our results. (1) How our calculations help in understanding from where within coronal holes the HSS plasma seen at Earth stems, (2) how the correlation between HSS velocity, density, and temperature evolves from the Sun to Earth, and (3) how our calculations explain the empirical relationship between the HSS peak velocity at Earth and the area of coronal holes and why this relationship is similar to the distance to the coronal hole boundary (DCHB) model.

### 6.1. Where within coronal holes the solar wind plasma that we see at Earth stems from

Based on our calculations, an extended temporal velocity plateau of HSSs at Earth indicates that not all plasma stemming from the HSS core close to the Sun has impinged upon the stream interface yet. Consequently, the plasma from the plateau region at Earth stems from the inner region of coronal holes, away from the coronal hole boundaries, and the peak velocity of the HSS at Earth is similar to the peak velocity of the HSS close to the Sun. Supposing an extended plateau region at Earth is not apparent, this indicates that all the plasma from the HSS core region has already run into the stream interface. Consequently, the plasma seen at Earth stems mainly from the trailing boundary region of the coronal hole. The more plasma from the trailing boundary region has already run into the stream interface at Earth distance, the slower the peak velocity of the HSS at Earth will be. Thereby, the amount of plasma that has already impinged upon the stream interface depends on the longitudinal extension of the HSS close to the Sun and the velocity gradient across the trailing boundary region. The velocity gradient again depends on the latitudinal distance of the foot points of the solar wind plasma in the coronal hole from its center. This interpretation goes along with that of Nolte et al. (1977), who studied eight HSSs at Earth distance in 1973. By ballistic back-mapping, they found that all the HSS plasma within the declining velocity tail at Earth distance stems from the trailing boundary region of the coronal holes.

We note that these calculations have two interesting consequences. First, if almost all of the HSS plasma originating from a small- to medium-sized coronal hole has already run into the stream interface at a given distance from the Sun, we will see plasma that still has the composition of HSSs, but only slow speeds. This “slow Alfvénic solar wind” is described by D’Amicos & Bruno (2015) and Stansby et al. (2020) as a distinct solar wind class, and they relate their small velocities compared to usual HSSs to differing magnetic field geometries in the solar corona. We note that some of these slow Alfvénic solar wind streams could also be usual HSSs where almost all of the plasma of the coronal hole has already impinged upon the stream interface. Second, in the very rare case of very small velocity gradients across the longitudinal HSS boundary region, it can happen that none of the HSS plasma has yet impinged upon the stream interface at Earth distance (i.e., that the stream interface has not even formed). This case most likely happens if the plasma stems from very close to the latitudinal edge of a coronal hole, where the velocity is also close to the adjacent slow solar wind velocity. In this case, one could observe almost undisturbed plasma stemming from leading boundary regions of coronal holes even at Earth distance.

### 6.2. Evolution of the solar wind speed, density, and temperature with distance from the Sun

In our calculations, we assumed a velocity, density, and temperature gradient across the HSS boundary region close to the Sun, dependent on the distance to the HSS boundary. Consequently, the pairs of these three parameters have been assumed to be (anti) correlated to one another close to the Sun. Thereupon, we determined the expansion of the plasma parcels with distance from the Sun by their longitudinal and latitudinal expansion, which is related to the spherical geometry, times their radial expansion, which is related to the radial dispersion due to radial velocity gradients. Finally, the temperature was derived under the assumption of adiabatic cooling and the negligence of wave heating.

We assumed that the radial plasma velocity across the HSS core region is roughly constant and that, consequently, the associated radial velocity gradient is roughly zero. When we follow an isolated plasma parcel originating from the HSS core region while it propagates away from the Sun, its volume does not expand in the radial but only in the longitudinal and latitudinal directions. Therefore, its density follows an $r^2$ law, and its temperature follows an $r^{2(\Gamma-1)}$ law. In contrast, plasma parcels originating from the boundary region of HSSs inherit a radial velocity gradient. Therefore, these plasma parcels naturally also expand in the radial direction while propagating away from the Sun. The rate of radial expansion depends on the velocity gradient and, thus, on their initial location within the HSS boundary. Therefore, its density decrease as a function of distance will in general not follow an $r^2$ law, and consequently the temperature will not follow an $r^{2(\Gamma-1)}$ law. Furthermore, we note that the radial expansion of these plasma parcels, in combination with keeping their mean velocity constant, implies that the mass flux is in general not conserved. Thus, the functional relationship of the density and temperature of individual plasma parcels with distance from the Sun, and whether the mass flux is a conserved quantity, depends on the initial location of the plasma parcels within the HSS close to the Sun.

Furthermore, when one compares the plasma properties of HSSs at various distances from the Sun, the properties around the peak velocity or within the velocity plateau are usually used. With increasing distance to the Sun, the fastest plasma parcels will have impinged upon the stream interface, and thus the peak velocity will be represented by another plasma parcel originating from closer to the HSS trailing edge. Consequently, we should be careful when we compare the properties of HSSs at various distances from the...
Sun and interpret the results physically using thermodynamics, since we might be comparing the properties of distinctly different plasma parcels within the HSS. Interpreting the evolution of HSS properties with simple thermodynamics as done by Perrone et al. (2019) might work well for HSSs that have a clear, extended velocity plateau (i.e., where all plasma parcels originate from the HSS core region). However, if such an extended velocity plateau is not apparent, the plasma parcels analyzed at the various distances will originate from different locations within the trailing boundary region close to the Sun, and the method will inevitably fail.

Our calculations further explain reasonably well why the velocities of HSS plasma parcels at Earth distance are still well correlated to their temperatures but only weakly anticorrelated to their densities (e.g., Neugebauer & Snyder 1966; Burlaga & Ogilvie 1970a,b; Ogilvie 1985; Xu & Borovsky 2015). The additional decrease in temperature by the radial expansion of plasma parcels counteracts the original assumed correlation between the velocities and temperatures close to the Sun. This introduces a slope into the temperature-velocity distribution at Earth distance, but the temperature-velocity distribution remains well defined. In contrast, the radial expansion further reduces the densities of the plasma parcels and thereby enhances the decrease in densities with increasing velocities we assumed close to the Sun. The result of this enhancement is that the densities can drop very fast with increasing velocities to the minimum value in the case where the associated radial velocity gradients close to the Sun have been large. This fast drop, superposed with the regular slower decreases in plasma parcels that have smaller velocity gradients, results in the worse-defined density-velocity distribution shown in Fig. 7.

6.3. The empirical relationships of the solar wind speed at Earth to the area of coronal holes, to the distance to the coronal hole boundary, and to the flux tube expansion factor

The empirical relationship between the areas of solar coronal holes and the peak velocities of HSSs near Earth, as found by Nolte et al. (1976), Robbins et al. (2006), Vršnak et al. (2007), Abramenko et al. (2009), Karachik & Pevtsov (2011), Rotter et al. (2012), Hofmeister et al. (2018), Garton et al. (2018), and Heinemann et al. (2020), consists of three parts: (1) a roughly linear relationship between the areas of coronal holes and the peak velocities near Earth for small- to medium-sized coronal holes, (2) a saturation of the peak velocities of large coronal holes, and (3) smaller peak velocities for coronal holes of a given area located at higher latitudes.

In our calculations, we explained the roughly linear relationship between the areas of solar coronal holes and the peak velocities of the associated HSSs near Earth as an effect of the reduction in peak velocities by the continuous absorption of the fastest HSS plasma parcels by the stream interface, combined with the decreasing starting velocities of HSS plasma parcels across the HSS boundary region close to the Sun. We note that this also includes very small HSSs that do not have a core region and consist purely of boundary region plasma, so that the plasma velocity is even decreased in the center of the HSS. After all the plasma originating from the leading boundary region and the HSS core region has impinged upon the stream interface, the fastest plasma parcel originating from the trailing boundary region, which has not yet run into the stream interface, determines the peak velocity. The amount of plasma that has not yet run into the stream interface depends on the longitudinal widths of the coronal holes, or on their areas, and on the displacement of the longitudinal slice to the center of the coronal hole. Further, the saturation of peak velocities near Earth for very elongated coronal holes arises from the fact that for very elongated coronal holes not all plasma parcels originating from the HSS core region have run into the stream interface at Earth distance. Thus, the peak velocities are given by plasma parcels originating from the HSS core region, which all have similar velocities. Finally, a larger latitudinal separation angle between the satellite measuring the HSS properties at Earth distance to the center of the coronal hole allows us to measure the HSS properties farther in its latitudinal flanks. Farther in the latitudinal flanks, the effective longitudinal widths of the HSS are reduced compared to the overall longitudinal widths of the HSS in its center. A smaller effective longitudinal width is again related to the fact that more plasma from the trailing boundary region has already impinged upon the stream interface at Earth distance, which results in smaller measured peak velocities at Earth distance.

Moreover, our results relate the empirical coronal hole area--HSS peak velocity relationship to the DCHB model (Riley et al. 2001) and the Wang-Sheeley-Arge (WSA) model (Arge et al. 2003). The DCHB model, which depends solely on the distance to the coronal hole boundary, and the WSA model, which depends on the inverse flux tube expansion factor and the distance to the coronal hole boundary, are usually used to derive an HSS velocity, density, and temperature profile above coronal holes at 0.1 AU. Both have in common that the velocities are large in the center of the HSS and decrease to the ambient slow solar wind values at the HSS boundary. These are then used as input for MHD simulations to propagate the HSSs from the Sun to Earth. In this study, we showed that the empirical relationship between the area of coronal holes and the peak velocities near Earth has implicitly incorporated the formation of the velocity profile above the coronal hole close to the Sun and the propagation phase from the Sun to Earth. The main difference in these three models is the assumed physical origin of the velocity profile close to the Sun. In the DCHB model, the acceleration of HSS plasma is assumed to depend on the distance to the coronal hole boundary, which is interpreted by diffusion across or loop-opening processes at the coronal hole boundary. In the WSA model, the solar wind acceleration depends on the distance to the coronal hole boundary and a modeled flux tube expansion factor, whereby flux tubes near the boundary of coronal holes naturally show a larger expansion. A larger expansion is again interpreted with slower solar wind velocities. Within this study on the relationship between the coronal hole areas and HSS peak velocities at Earth, it does not matter where, or by which physical processes, the HSS boundary region forms; it is only important that it exists. Despite the different interpretations involved, all of these models describe a similar idea: that a velocity profile above the coronal hole forms, which sets the HSS properties close to the Sun, and that these HSSs are propagated from the Sun to Earth, taking the interaction with the preceding slow solar wind into account.

Finally, we would like to add that the ongoing Parker Solar Probe and Solar Orbiter missions (Fox et al. 2016; Müller et al. 2020) will soon allow us to confront our model with more detailed observations. Parker Solar Probe’s measurements during its perihelions close to the Sun will give us more insight into the distribution of the HSS properties across their cross section near the Sun, while Solar Orbiter’s unique orbit will allow us to analyze the radial evolution of HSSs and the associated SIRs in the inner
These measurements will further improve our understanding of how HSSs evolve in the inner heliosphere and how, correspondingly, the geometry of coronal holes on the Sun affects the properties of HSSs near Earth.

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