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The star catalogue of Wilhelm IV, Landgraf von Hessen-Kassel

Accuracy of the catalogue and of the measurements★

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ABSTRACT

We analyse a manuscript star catalogue by Wilhem IV, Landgraf von Hessen-Kassel from 1586. From measurements of altitudes and of angles between stars, given in the catalogue, we find that the measurement accuracy averages 26″ for eight fundamental stars, compared to 49″ of the measurements by Brahe. The computation in converting altitudes to declinations and angles between stars to celestial positions is very accurate, with errors that are negligible with respect to the measurement errors. Due to an offset in the position of the vernal equinox, the positional error of the catalogue is slightly worse than that of Brahe's catalogue; however, when a correction is made for the offset, which was known to 17th century astronomers, the catalogue is more accurate than that of Brahe by a factor of two. We provide machine-readable tables of the catalogue.

Key words. astrometry – history and philosophy of astronomy

1. Introduction

It is well known that the measurements by Tycho Brahe of the positions of stars and planets in the sky were an order of magnitude more accurate than those of earlier astronomers, such as Ptolemaios (or Hipparchos) and Ulugh Beg. As a result, Brahe's catalogue of positions of the fixed stars became the new standard, with editions in 1598 (manuscript), 1602 (in Astronomiae Instauratae Progymnasmata), and in 1627 (by Kepler, Tabulae Rudolphinae). Machine-readable versions of these catalogues and an analysis of their accuracy are provided by Verbunt & van Gent (2010, 2012).

It is less well known that Brahe was preceded by Wilhelm IV, Landgraf von Hessen-Kassel, whose measurements were equally accurate. As an aspiring astronomer, Brahe visited the already experienced observer Wilhelm IV in April 1575 and studied his instruments. In the correspondence that followed, Wilhelm IV in turn profited from suggestions for improved measurement accuracy by Brahe (Hamel 2002).

The star catalogue of Wilhelm IV was mostly based on work by Wilhelm IV himself and by two people he employed: the mathematician-astronomer Christoph Rothmann and the brilliant clock-maker, instrument maker, and mathematician Jost Bürgi. All three of them participated in the observations, with instruments made by Bürgi. The computational work was presumably done by Rothmann and Bürgi. We describe the work at the observatory in Kassel and the historical context in a separate paper (Schrimpf & Verbunt 2021). The manuscript of the catalogue was almost ready for print. It is available as a scan via the on-line portal of the library of Kassel University (Wilhelm IV 1587). Rothmann also wrote a handbook for astronomers of his time, describing observational and computational methods that were used to prepare the star catalogue. The Latin manuscript for this handbook is available via the on-line portal of the library of Kassel University (Rothmann Handbook), and it has been printed by Granada et al. (2003), with an introduction in German.

The catalogue only appeared in print, edited by Curtz in 1666, long after the dissemination of Brahe's catalogue, and as a result it had a rather limited impact, even though the catalogue was reprinted (with adaptations) by Hevelius (1690) and by Flamsteed (1725). Hamel (2002), borrowing extensively from Wolf (1878) and Repsold (1919), has written a monograph on the astronomical research in Kassel under Wilhelm IV, in which he mentions three earlier star catalogues, with 58, 58, and 121 stars, respectively. We intend to compare these different catalogues, which mark the progress to the higher accuracy, in a future paper. In the current paper we study the manuscript version of the final catalogue from 1587, which contains 387 entries.

Rothenberg (in Hamel 2002) compared this final catalogue with the SAO star catalogue (SAO 1966), identified 361 stars securely, found a possible counterpart for seven, and was unable to identify 14 stars. Not included in these numbers are three repeated stars and two clusters of stars (see Table 2). Assuming an offset in right ascensions of 6′, he found systematic + random errors 0.′22 + 1.′2 in right ascension and 0.′82 + 1.′5 in declination, respectively. No details are provided about individual (non-)identifications. We refer to the final catalogue below as *Manuscript*, identify all of its entries with use of the HIPPARCOS Catalogue (ESA 1997), analyse the results in detail, and provide

★ Full Tables 5–7 are only available at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via http://cdsarc.u-strasbg.fr/viz-bin/cat/J/A+A/649/A112

Table 1. Constellations in *Manuscript* and numbers of stars in them.

C		N	W	C		N	W
1	UMi	7	1	24	Tau	16	218
2	UMa	18	8	25	Gem	13	234
3	Dra	18	26	26	Cnc	7	247
4	Cep	10	44	27	Leo	9	254
5	Boo	13	54	28	Vir	12	263
6	CrB	4	67	29	Lib	2	275
7	Her	10	71	30	Sco	6	277
8	Lyr	8	81	31	Sgr	7	283
9	Cyg	12	89	32	Cap	4	290
10	Cas	11	101	33	Aqr	9	294
11	Per	20	112	34	Psc	16	303
12	Aur	8	132	35	Cet	10	319
13	Oph	10	140	36	Ori	17	329
14	Ser	7	150	37	Eri	1	346
15	Sge	5	157	38	Lep	4	347
16	Aql	4	162	39	CMi	2	351
17	Atn	4	166	40	CMa	7	353
18	Del	5	170	41	Nav	3	360
19	Equ	3	175	42	Hya	17	363
20	Peg	12	178	43	Crt	3	380
21	Tri	3	190	44	Crv	4	383
22	And	16	193	45	PsA	1	387
23	Ari	9	209		total	387	

Notes. For each constellation, the columns give its sequence number C, the abbreviation we use, the number N of stars in the constellation, and the sequence number W of the first star in the constellation. Atn indicates Antinous, and Nav indicates Navis Argo.

Table 2. Non-stellar, added, and repeated entries in *Manuscript*.

Entry	Cluster	Entry	Added star
W 112 (Per 1)	h&χ Per	W 39 (Dra 14)	RR UMi
W 247 (Cnc 1)	Praesepe	W 232 (Tau 15)	Alcyone
		W 233 (Tau 16)	Atlas
		W 999 (Aql –)	ϵ Aql
First entry	**Repeated entry**	**Star**	
W 60 (Boo 7)	W 80 (Her 10)	ν^2 Boo	
W 139 (Aur 8)	W 227 (Tau 10)	β Tau	
W 302 (Aqr 9)	W 387 (PsA 1)	α PsA	

Table 3. Frequency of different combinations of information given in *Manuscript*, after the removal of three repeated entries and five reverse repeated angles.

	0 ϕ	1 ϕ	2 ϕ	3 ϕ	#	# ϕ
0 h	2	2	37	0	41	76
1 h	1	257	53	0	311	363
2 h	0	16	15	1	32	49
#	3	275	105	1	384	
# h	1	289	83	2		375\488

Notes. Columns 2–5 sort the entries with respect to the number of angular distances, rows 2–4 sort the entries with respect to the number of altitudes (or equivalently, equatorial coordinates). Thus Col. 3, row 3 indicates that there are 257 entries with one angular distance and one altitude. Columns 6 and 7 give the total number of entries and angular distances, respectively, from Cols. 2–5; rows 4 and 5 provide the total number of entries and altitudes from rows 2–4. W 999, for which only two angular distances are given, is not included in the numbers.

a machine-readable version of catalogue and analysis in the following three data files: *WilhelmIV* (see Table 5), *WilhelmIV-Dist* (Table 6), and *WilhelmIV-Alt* (Table 7).

2. Description of *Manuscript* and its epoch

Manuscript is organised by constellation and lists all 1028 entries from the star catalogue by Ptolemaios, plus four added stars, of which two are in the Pleiades. For all 1032 entries, column one of *Manuscript* gives the Latin name or description of the star, column seven provides the position in the catalogue of Ptolemaios, or for the added stars the statement 'in tabulis non extat' (not present in the table). Column eight gives the magnitude. The longitudes in *Manuscript* are larger by $21\frac{1}{4}$ degrees than those in the catalogue of Ptolemaios, to bring them to equinox 1586.

For a limited number of stars, the intermediate columns provide new measurements (angular distances between stars in columns two and three and meridional altitudes in column four) and the equatorial and ecliptic coordinates derived from them (in columns five and six, respectively). *Manuscript* gives newly determined ecliptic coordinates for 387 out of the 1032 entries. We collected these entries in *WilhelmIV*, and we numbered the stars in order of occurrence, with a W number (see Table 1). The three stars that are (knowingly) repeated in Ptolemaios also occur twice in the manuscript, so that *WilhelmIV* contains 384 independent entries. Three of the four added stars have ecliptic coordinates. We list these stars, together with the repeated entries and the entries with non-stellar identifications, in Table 2. For the fourth added star, in Aquila, two angular distances to other stars, but no equatorial or ecliptic position, are given; we did not include this star in *WilhelmIV*, and we refer to it as W 999. From the angular distances given, we derive that it corresponds to ϵ Aql.

For 384 entries (381 independent ones), the manuscript gives the angular distance between the entry and one or more stars named in column two. For all these entries the ecliptic coordinates are given (in Col. 6). We indicate the angular distance with ϕ. No ϕ is given for W 87, W 88, or W 224, but column six does give their ecliptic position. For five stars, the distance to a reference star is repeated, with the roles of star and reference star reversed; we refer to the second occurrence as reverse repeated angle. As we explain below, the position of W 224 (Aldebaran) is the anchor point from which absolute positions for other entries were determined.

For 346 entries (343 independent ones), *Manuscript* gives the altitude at meridian passage and the equatorial coordinates. For stars that are always above the horizon, both superior and inferior altitudes are listed, which are indicated by 'superne' or 'inferne'. Table 3 details the frequencies of angular distances and altitudes in *Manuscript*.

All angles in *Manuscript* are given in degrees and sexagesimal fractions; for latitudes or declinations, S or M was added for Septentrionalis (north) or Meridionalis (south), respectively. For the machine-readable tables, we converted the fraction into seconds:

$$q = G_q + \frac{M_q + F_q}{60}; \quad S_q = 60F_q; \quad q = \phi, h, \alpha, \delta, \beta, \qquad (1)$$

where ϕ, h, α, δ, and β refer to the angular distance, altitude in the meridian, right ascension, declination, and ecliptic latitude, respectively. The values of F_q in the catalogue are such that S_q is always an integer, one of the following 14: 0, 6, 10, 12, 15, 20, 24, 30, 36, 40, 45, 48, 50, or 54. Remarkably, 18 (=3/10) and 42 (=7/10) were not used. For ecliptic longitude, a zodiacal sign was used, which we converted into a number Z, ranging from 1 for Aries ♈ to 12 for Pisces ♓:

$$\lambda = (Z-1)30 + G_\lambda + \frac{M_\lambda + F_\lambda}{60}; \quad S_\lambda = 60F_\lambda. \tag{2}$$

In the preamble of the 121-star catalogue from 1586, the equatorial and ecliptic coordinates of Aldebaran are given and said to form the fundament of the coordinates of all other stars in the catalogue. The coordinates of Procyon are also given and said to be very accurate. A translation of this Latin preamble is provided by Schrimpf & Verbunt (2021). The coordinates of Aldebaran and Procyon in this preamble (Table 4) are identical to those given in *Manuscript*, with the exception of the longitude of Procyon, for which F_λ is larger by 15 in *Manuscript*.

For geophysical latitude ϕ_G, the declination of a star follows immediately from its altitude at meridional culmination with Eq. (9). Conversely, from the altitudes in *Manuscript* (54°17′ for Aldebaran and 44°54′ for Procyon) and their declinations, we can derive the value of ϕ_G used in constructing the catalogue. For both Aldebaran and Procyon, exactly the same value ϕ_G = 51°19′ follows. This agrees with the value given in Chapter 10 of Rothmann's Handbook.

Ecliptic coordinates were computed from equatorial coordinates with Eqs. (12) and (13), which require a value for the obliquity ϵ. From the equatorial and ecliptic coordinates of Aldebaran and Procyon in the preamble and *Manuscript*, we can compute the obliquity used in constructing the catalogue. The ecliptic coordinates derived from the equatorial coordinates match those of Aldebaran in the preamble and in the manuscript exactly and, due to rounding, those of Procyon approximately for obliquity ϵ = 23°31′ (Table 4). Our analysis in Sect. 4.3 of the full catalogue confirms this value for ϵ, somewhat surprisingly since values of ϵ = 23°31′24″ and ϵ = 23°31′30″ are given in Chapters 10 and 12 of Rothmann's Handbook, respectively.

Manuscript does not give the equinox. It is based on observations made in 1586 and 1587, and was presumably written in 1587, which suggests an equinox of 1586 (Wolf 1878). The manuscript of the catalogue for 121 stars gives an equinox of 1586. The stellar positions of the stars in this catalogue are, with few exceptions, identical to those of the manuscript. Therefore, we used 1 January 1586 (old style) = JD 2300345 as the epoch and the equinox of *Manuscript*.

The positions of the entries in *Manuscript* are anchored on the position of Aldebaran. Brahe determined the right ascension of this star to be 6 arcmin smaller than the value used in Kassel, and he discussed this discrepancy with Rothmann and Wilhelm IV (see Wolf 1878; Hamel 2002 for more detail). From the way the positions of the other stars were derived from the measurements (see Eqs. (7)–(9) and (11)), this would imply that all right ascensions in the Manuscript are too large by 6 arcmin if Brahe's value is correct. Comparison with the values computed from HIPPARCOS-2 data (van Leeuwen 2007) show that Brahe's criticism was justified (Table 4).

3. Identification and machine-readable tables

The method of identification of the stars largely follows the procedure outlined in Verbunt & van Gent (2010), with two modifications. For the computation of the star positions at the epoch of the catalogue, we used data from HIPPARCOS-2, the reanalysis of the HIPPARCOS Catalogue by van Leeuwen (2007). Furthermore, we took the limited spatial resolution of the naked human eye into account by merging stars within 2′ of one another. The visual magnitudes are from the original HIPPARCOS Catalogue (ESA 1997). On the basis of this, we prepared a version of the HIPPARCOS Catalogue converted to epoch 1 January 1586, as detailed in Appendix A. When referring to the HIPPARCOS Catalogue below, this converted version is meant.

Table 4. Equatorial and ecliptic positions of Aldebaran and Procyon.

Star	G_α	M_α	G_δ	M_δ	H	Z	G_λ	M_λ	F_λ	G_β	M_β	S_β	H
Aldebaran	63	10	15	36	S	♊	4	06	00	5	31	45	M
preamble	63	10	15	36	S	♊	4	06		5	31	45	M
computed from equatorial						♊	4	06	00	5	31	45	M
HIP	63	4	15	36	S	♊	4	00	21	5	29	49	M
Procyon	109	30	6	13	S	♋	20	11	15	15	56	20	M
preamble	109	30	6	13	S	♋	20	11		15	56	20	
computed from equatorial						♋	20	11	19	15	56	22	M
HIP	109	24	6	13	S	♋	20	04	41	15	56	06	M

Notes. For each star, the first line has the coordinates from *Manuscript*, the second line those of the preamble of the 121-star catalogue, the third line the ecliptic coordinates computed from the equatorial coordinates in *Manuscript* with ϵ = 23°31′, and the fourth line the coordinates in 1586 computed from HIPPARCOS data.

In view of the systematic offset of 6′ in right ascension noted by Brahe, which is confirmed in Table 4 and further discussed in Sect. 6, we searched for counterparts with

$$\Delta\alpha = \alpha - \alpha_{\rm HIP}; \quad \Delta\alpha_c = \Delta\alpha - 6'; \quad \Delta\delta = \delta - \delta_{\rm HIP} \tag{3}$$

by looking for the entry with the smallest value of

$$\Delta_c = 2\arcsin\sqrt{\sin^2\frac{\Delta\delta}{2} + \cos\delta\cos\delta_{\rm HIP}\sin^2\frac{\Delta\alpha_c}{2}}. \tag{4}$$

This equation, the haversine function, is also accurate for very small angles. For stars for which *Manuscript* gives ecliptic coordinates only, we converted the ecliptic coordinates to equatorial coordinates, using the value for the obliquity that was used in Kassel, ϵ = 23°31′. In virtually all cases, the nearest star is the counterpart, and in some cases a brighter star at a slightly larger distance is the counterpart. For W 112 and W 247, the counterpart is a star cluster (see Table 2).

The machine-readable version of *Manuscript* was divided intro three tables to minimise the number of empty slots. *WilhelmIV* contains the equatorial and ecliptic coordinates and the magnitudes resulting from the observations in Kassel, corresponding to columns one, five, six, and eight of *Manuscript* (Table 5), and results of our analysis with the HIPPARCOS Catalogue. In addition to the positional errors Δ_c derived from the corrected right ascensions with Eq. (1), we also list positional errors when no correction for the offset in right ascension is applied. For this, we converted the equatorial coordinates $\alpha_{\rm HIP}$ and $\delta_{\rm HIP}$ to ecliptic coordinates $\lambda_{\rm HIP}$ and $\beta_{\rm HIP}$ with the value of the obliquity in 1586 according to modern theory $\epsilon(1586)$ = 23.°4931 (Seidelman 1992, Eq. 3.222-1). For each HIPPARCOS entry with $V \leq 5.3$, we computed the angular distance Δ between the position in the old catalogue and the position computed from HIPPARCOS data with

$$\Delta\lambda = \lambda - \lambda_{\rm HIP}; \quad \Delta\beta = \beta - \beta_{\rm HIP} \tag{5}$$

and

$$\Delta = 2\arcsin\sqrt{\sin^2\frac{\Delta\beta}{2} + \cos\beta\cos\beta_{\rm HIP}\sin^2\frac{\Delta\lambda}{2}}. \tag{6}$$

Table 5. First and eigth line of the machine-readable table *WilhelmIV*.

W	C	Con	i	G_α	M_α	S_α	G_δ	M_δ	S_δ	H_δ	Z	G_λ	M_λ	S_λ	G_β	M_β	S_β	H	V	HIP	I	V_H	$\Delta\alpha$	$\Delta\delta$	Δ_c	$\Delta\lambda$	$\Delta\beta$	Δ	Description
1	1	UMi	1	005	46	00	87	04	00	S	3	22	48	10	66	01	15	S	2	11767	1	2.0	11.2	0.3	0.4	1.4	−1.9	2.0	Stella Polaris
...																													
8	2	UMa	1								4	17	15	40	40	12	00	S	4	41704	1	3.3	5.6	0.0	0.2	3.0	−0.7	2.4	In rostro.

Notes. Column 1 sequence number W of the entry, Cols. 2 and 3 sequence number C and name of the constellation, Col. 4 sequence number i of the entry within the constellation, Cols. 5–11 equatorial coordinates G_α, M_α, S_α and G_δ, M_δ, S_δ and H_δ (see Eq. (1)), Cols. 12–19 ecliptic coordinates Z, G_λ, M_λ, S_λ and G_β, M_β, S_β and H (see Eqs. (1) and (2)), Col. 20 magnitude V in *Manuscript*. Columns 21–26 Hipparcos counterpart, identification flag I, magnitude V_H of the Hipparcos counterpart, positional errors $\Delta\alpha$, $\Delta\delta$, and Δ_c in arcminutes (see Eqs. (3) and (4)), Cols. 27–29 positional errors $\Delta\lambda$, $\Delta\beta$ and Δ in arcminutes (see Eqs. (5) and (6)), Col. 30 name or description of the star as given in column one of *Manuscript*. The identification flag indicates an identification with 1 = nearest star (with $V < 5.3$), 2 = not nearest star, 3 = merged Hipparcos star (see Appendix A), 9 = star cluster, and 6 = repeated entry. The full table is available at the CDS.

Table 6. First lines of the machine-readable table *WilhelmIV-Dist*, with examples of a repeat entry ($F = 1$) and of a reverse repeat entry ($F = 2$).

W	W_r	F	ϕ			$\phi - \phi_{HIP}$	Reference star
			G_ϕ	M_ϕ	S_ϕ	(′)	
1	92	0	44	40	12	−1.1	Cauda Cygni
1	104	0	28	35	45	−0.8	Ad coxas Cassiopeae
1	132	0	43	23	30	−0.9	Capella
2	132	0	47	21	00	−0.9	Capella
...							
60	25	0	20	04	00	−0.8	Ulti: Caudae Urs: ma:
...							
80	25	1	20	04	00	−0.8	Ult: Caudae Urs: ma:
...							
121	132	0	23	39	20	−1.2	Capella
...							
132	121	2	23	39	20	−1.2	Cap: Medusae

Notes. Column 1 sequence number W for the entry, Col. 2 sequence number W_r for the reference star, Col. 3 flag F for repeated entries (1 for a repeated entry and 2 for a reversed repeated angle), Cols. 4–6 angular distance ϕ between entry and reference star in terms of G_ϕ, M_ϕ, and S_ϕ (Eq. (1)), Col. 7 difference in arcminutes between this angle and the value ϕ_{HIP} computed from Hipparcos-2 data, and Col. 8 name or description of the reference star. Columns 9 and 10 of the machine-readable file (not shown in Table 6) give the Hipparcos numbers of star and reference star. Two angles are given in *Manuscript* for a star which is not itself an entry; this star is listed in *WilhelmIV-Dist* with $W = 999$. The full table is available at the CDS.

Table 7. First lines of the machine-readable table *WilhelmIV-Alt*.

W	F	h				$h - h_{HIP}$	HIP
		G_h	M_h	S_h		(′)	
1	0	54	15	00	S	−0.1	11767
1	0	48	23	00	I	0.5	11767
4	0	65	29	00	S	0.8	72607
4	0	37	09	00	I	−0.4	72607
...							
80	1	81	01	00	M	−1.1	76041

Notes. Column 1 sequence number W, Col. 2 the repeat flag (1 for a repeated entry), Cols. 3–6 altitude h in terms of G_h, M_h, and S_h and indicator S for superior, I for inferior, or M for southern (meridional) culmination. Column 7 difference between tabulated altitude h and true altitude h_{HIP} computed from Hipparcos-2 data with Eqs. (7)–(9) and the modern value $\phi_G = 51°31367$. Column 8 Hipparcos number of the star. The full table is available at the CDS.

Table 8. Refraction as a function of altitude according to Rothmann.

h	R_R	h	R_R	h	R_R	h	R_R
2°	13′40″	9°	4′40″	16°	1′20″	23°	30″
2 3°	12′20″	10°	3′50″	17°	1′10″	24°	25″
4°	11′00″	11°	3′10″	18°	1′00″	25°	20″
5°	9′35″	12°	2′40″	19°	50″	26°	15″
6°	8′10″	13°	2′10″	20°	45″	27°	10″
7°	6′50″	14°	1′50″	21°	40″	28°	5″
8°	5′40″	15°	1′35″	22°	35″	≥29°	0

We consider all our identifications to be secure.

WilhelmIV-Dist gives the angles measured between entries and reference stars, corresponding to columns one to three of *Manuscript* (Table 6). All reference stars listed in the second column of *Manuscript* also occur in the first column, and thus can be identified with a W number. *WilhelmIV-Alt* contains the altitude(s) for each star, Col. 4 of *Manuscript*.

4. Accuracy of the computations

The accuracy of computation can be determined from tabulated numbers in *Manuscript* that were derived from other tabulated numbers.

4.1. Altitudes and declinations

The relations between the geographical latitude ϕ_G of the observer, the declination δ of a star, and its altitude h at meridian passage for culminations above (superior) or below (inferior) the celestial north pole in the northern direction as well as culminations in the southern (meridional) direction are

$$\delta = \phi_G - h + \frac{\pi}{2} \quad \text{superior culmination,} \tag{7}$$

$$\delta = -\phi_G + h + \frac{\pi}{2} \quad \text{inferior culmination, and} \tag{8}$$

$$\delta = \phi_G + h - \frac{\pi}{2} \quad \text{meridional culmination,} \tag{9}$$

respectively. In these equations h is the true altitude. The observed or apparent altitude h_a is affected by atmospheric refraction R:

$$h_a = h + R. \tag{10}$$

The values for refraction used by Rothman, R_R, are listed in Table 8; according to Rothmann, no refraction occurs for $h \geq$

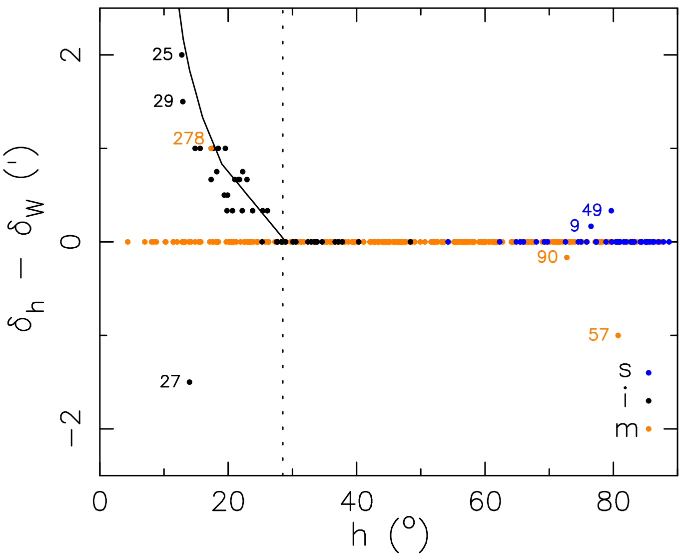

Fig. 1. Difference between the declination δ_h computed from the altitude with Eqs. (7)–(9) and the declination δ_W listed in *Manuscript* for $\phi_G = 51°19'$. With few excaptions, the computations are exact for superior and southern culminations. The values δ_h from inferior culmination are systematically too high for low altitudes. Values for which $\delta_h \neq \delta_W$ are labelled with *W* for those entries for which only one altitude is listed, and for W 9. The solid curve gives the refraction R_R according to Rothmann. Stars to the right of the vertical dashed line have $R = 0$, according to Rothmann.

29°. As noted above, all entries in *Manuscript* for which an altitude is listed have equatorial coordinates. A single altitude is listed for 311 entries, namely 293, ten, and eight entries from meridional, superior, and inferior culmination, respectively; for 32 entries, the altitudes are listed for both superior and inferior culmination. If we enter these altitudes and $\phi_G = 51°19'$ into Eqs. (7)–(9), we obtain declinations δ_h equal to the catalogued declinations δ_W listed in column five of *Manuscript* for all but three and two values obtained from meridional and superior culmination, respectively, as shown in Fig. 1. For meridional altitudes, $h < 29°$; this implies that the tabulated altitudes are true altitudes, corrected for refraction – or at least considered as such in the computation of δ. Twenty-four of the inferior altitudes lead to a different declination than given in the catalogue, that is $\delta_h \neq \delta_W$. The differences in declinations derived from altitudes of inferior culmination with the catalogued values roughly follow the atmospheric refraction, leading to the assumption that these are apparent altitudes. However, the correspondence is not exact, which we are inclined to ascribe to rounding and/or copying errors.

Thus, the 378 altitudes listed in the Manuscript, and collected by us in *WilhelmIV-Alt*, lead to 343 independent entries with declinations. The other altitudes are three repeated entries and 32 altitudes of inferior culminations, which are not used.

4.2. Angles between stars and right ascensions

The standard equation for the angular distance ϕ between two stars may be rewritten in equatorial coordinates as

$$\alpha = \alpha_r + \arccos\left(\frac{\cos\phi - \sin\delta\sin\delta_r}{\cos\delta\cos\delta_r}\right), \qquad (11)$$

showing that the right ascension α of a star can be computed from its declination δ if the angle ϕ is known to a reference star with known equatorial coordinates α_r and δ_r. *Manuscript* contains 500 values for ϕ between entries, of which 410 are independent and between stars with listed equatorial coordinates. Of these 410 entries, 273 have ϕ with one reference star, 67 with two reference stars (i.e. 134 values of ϕ), and one (W 1) with three reference stars.

To retrace the steps presumably taken in Kassel, we used the tabulated values of ϕ in subsequent iterations. In the first iteration, we have one reference star, Aldebaran, for which angles were given to 21 entries with declination known from an altitude measurement. The first iteration thus allows the computation of the right ascension for these 21 entries. Seven of these, which occur in the second column of *WilhelmIV-Dist*, can be used as a reference star in the second iteration. After eight iterations, a total of 64 stars have been used as a reference star, and the equatorial coordinates of 342 stars are known. Only one star with equatorial coordinates in *Manuscript*, W 288, is not found in these iterations; the angle ϕ in *WilhelmIV-Dist* between this star and W 291 does not help as no altitude h and thus no equatorial coordinates are listed for W 291.

To check the accuracy of the computation of the right ascension α_{eq} from the right-hand side of Eq. (11) with the catalogue value α_W, we must take into account that only 14 values for the number of seconds S_α are allowed as tabulated values (as explained with Eq. (1)). Thus, each computed value $S_{\alpha_{eq}}$ is bracketed by two allowed values. For the 273 entries with ϕ given to a single reference star, the catalogued value S_α corresponds to the nearest bracketing value $S_{\alpha_{eq}}$ in 181 cases, to the other bracketing value in 54 cases, and to neither in 38 cases.

For the 137 entries with ϕ given to two or, in the case of W 1, three reference stars, the catalogued value of α corresponds to the nearest bracketing value in 74 cases, to the other bracketing value in 28 cases, and to neither in 35 cases. In those cases where both values of α are the same after rounding, no choice is necessary. If the two (or three) computed values for α are different, the catalogue sometimes gives the value derived from the average (for example for W 29); sometimes it gives the value derived from just one computed value of α. We have not been able to determine a criterion on the basis of which this one value is chosen: It can be the one found in the lowest or in the highest iteration, the one found with the largest or smallest angle ϕ to the reference star, or the one found with the reference stars with the lowest or highest absolute value of the declination $|\delta_r|$.

To see whether in computing the right ascensions the tabulated values of δ or the values computed from altitudes were used, we compared the results for α_{eq} for the reference star W 25 for both options. This indicates that the tabulated values were used.

Figure 2 illustrates the difference distribution between the catalogued value α_W and the value α_{eq} computed with Eq. (11). The distributions are shown for the exact values α_{eq} computed with Eq. (11) and for the values α_{eq} rounded to the nearest allowed integers of S_α. For entries with more than one ϕ, we used the first ϕ only to avoid double counting one entry in the catalogue; this leaves 341 values. The difference between the two curves shows the effect of rounding: The rounding masks small computational errors when the catalogued value is the bracketing allowed value closest to the exactly computed one; it increases the error when the catalogued value corresponds to the other bracketing value. As a result, the median value of $|\alpha_{eq} - \alpha_W|$ is $1''.3$ for exact α_{eq}; however, after rounding, 217 entries have $|\alpha_{eq} - \alpha_W| = 0$. The average and rms of $\alpha_{eq} - \alpha_W$ are $0''.7$ and $6''.6$ for the exact α_{eq} values, and $0''.6$ and $6''.7$ for the rounded α_{eq} values. As we see below, these errors are negligible with respect to the measurement errors.

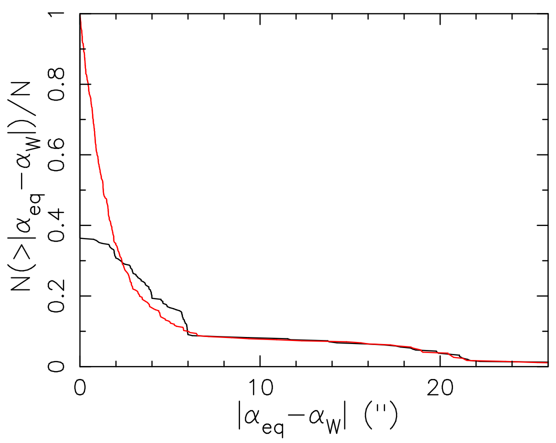

Fig. 2. Complement of the normalised empirical cumulative distribution function ECDF, i.e. 1−ECDF for the absolute values of the differences between the right ascension $\alpha_{\rm eq}$ computed with Eq. (11) and the catalogued right ascension $\alpha_{\rm W}$. Red: vertical axis for the exact values $\alpha_{\rm eq}$. Black: for the values $\alpha_{\rm eq}$ rounded to the nearest allowed integer of S_α.

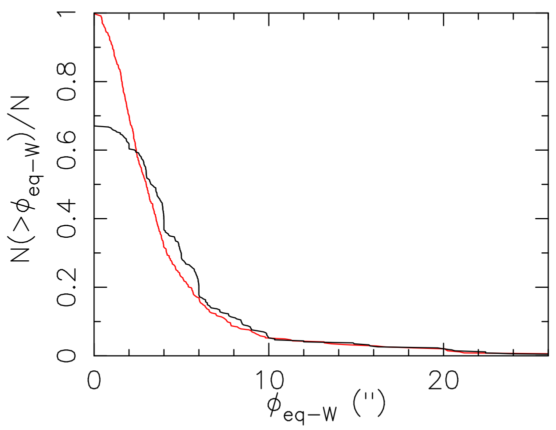

Fig. 3. Complement to the normalised empirical cumulative distribution function, i.e. 1-ECDF, of the differences $\phi_{\rm eq-W}$ between the catalogued position and the position computed with Eqs. (12) and (13). Red: vertical axis for the exact computed values $\lambda_{\rm eq}$ and $\beta_{\rm eq}$. Black: for the values rounded to the nearest allowed integer of S_λ and S_β.

4.3. Conversion of equatorial to ecliptic coordinates

The equations which convert equatorial into ecliptic coordinates are

$$\tan\lambda = \frac{\sin\alpha\cos\epsilon + \tan\delta\sin\epsilon}{\cos\alpha} \tag{12}$$

and

$$\sin\beta = \sin\delta\cos\epsilon - \cos\delta\sin\alpha\sin\epsilon. \tag{13}$$

We compare the values for λ and β obtained with these equations from the equatorial coordinates in column five of *Manuscript* with the values listed in Col. 6. In accordance with our finding in Sect. 2 and Table 4, we set the obliquity to $\epsilon = 23°31'$.

The angular difference $\phi_{\rm eq-W}$ between the computed and catalogued positions in ecliptic coordinates is found with

$$\phi_{\rm eq-W} = 2\arcsin\sqrt{\sin^2\frac{\beta_{\rm eq}-\beta_{\rm W}}{2} + \cos\beta_{\rm eq}\cos\beta_{\rm W}\sin^2\frac{\lambda_{\rm eq}-\lambda_{\rm W}}{2}}. \tag{14}$$

Figure 3 shows the complement to the normalised empirical cumulative distribution function ECDF, that is 1−ECDF of the differences $\phi_{\rm eq-W}$ found with Eqs. (12)–(14) for the exact computed values of $\lambda_{\rm eq}$ and $\beta_{\rm eq}$ and for their values rounded to the nearest allowed values. The average and rms values in our first calculation were very high because of three outliers, W 55, W 214, and W 374. Comparison of the calculated and catalogue positions indicates errors in *Manuscript* for these entries, and we emended the ecliptic coordinates accordingly (see Appendix B.3). For 113 entries, the rounding leads to $\phi_{\rm eq-W} = 0$. The median value of $\phi_{\rm eq-W}$ is 3.″0 for exact calculation; however, after rounding to allowed values, it becomes 3.″3. The average and rms values are 4.″1 and 4.″2 for the exact values, and 3.″9 and 4.″7 for the rounded values.

If we adopt a value of the obliquity 1′ higher or lower than 23°31′, the average and rms values of $\phi_{\rm eq-W}$ increase strongly. This confirms that the value actually used is indeed 23°31′.

4.4. Position from angles with two other stars

For 41 catalogue entries, ecliptic coordinates are given, but no altitudes or equatorial coordinates. For 37 of these, angles ϕ_i are listed to two reference stars with known coordinates λ_i, β_i, $i = 1, 2$. Each angle ϕ_i defines a circle around its reference star, and one of the intersections between the circles gives the previously unknown position λ, β for one of the 37 stars. To calculate these, we rewrote the equivalent of Eq. (11) in ecliptic coordinates in the form of two non-linear equations for two unknowns λ and β:

$$F_i(\lambda,\beta) = \cos\beta\cos\beta_i\cos(\lambda-\lambda_i) + \sin\beta\sin\beta_i - \cos\phi_i = 0. \tag{15}$$

These can be solved iteratively. Whether this is the method by which the ecliptic coordinates of the 37 stars were determined is not clear. The catalogued coordinates of the four stars without altitude and with no or just one angle to another star indicate that not all measured altitudes and/or angles are catalogued.

On the assumption that the coordinates of 37 entries were determined from Eq. (15), presumably by trial and error, we checked its accuracy in two ways. First, we computed $|F_1(\lambda,\beta)| + |F_2(\lambda,\beta)|$ from the values λ, β, λ_i, β_i, and ϕ_i given in the catalogue. Twenty-two values are less than 10^{-5}, and fifteen lie between 10^{-4} and 10^{-5}, indicating an excellent accuracy of the computations. Second, we used a modern code to solve Eq. (15) by numerically minimising $|F_1(\lambda,\beta)| + |F_2(\lambda,\beta)|$ as a function of λ and β, and we computed the angle $\phi_{\rm eq12-W}$ between the optimal computed position $\lambda_{\rm eq12}$, $\beta_{\rm eq12}$ and the catalogued position λ_W, β_W. As initial guess for the iterative solution of Eq. (15), we used the catalogued position, and we ended the iteration when $|F_1| + |F_2| < 10^{-6}$. The results are shown in Fig. 4, $\phi_{\rm eq12-W}$ for the exactly computed position, and for the computed position rounded to the nearest allowed values of S_λ and S_β. The median is 3.″5 for the exact and 5.″0 for the rounded solutions.

5. Accuracy of the measurements

By comparing the catalogued altitudes at culmination and the angles between stars with values computed from modern HIPPARCOS data and modern refraction values, we can determine the accuracy of the measurements.

5.1. Measured altitudes

As we have seen in Sect. 4.1, the derivation of the declinations of the entries in *Manuscript* indicates that the catalogued altitudes

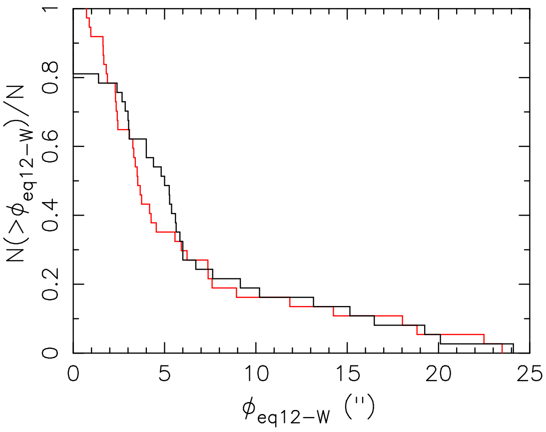

Fig. 4. Complement to the normalised empirical cumulative distribution function, i.e. 1-ECDF of the differences $\phi_{\rm eq12-W}$ between the position computed with Eq. (15) and the catalogued position. Red: vertical axis for the exact computed values $\lambda_{\rm eq12}$ and $\beta_{\rm eq12}$. Black: for the values rounded to the nearest allowed integer of S_λ and S_β.

$h_{\rm W}$ are the true altitudes h. The altitude $h_{\rm a}$ measured in Kassel is found by adding the refraction $R_{\rm R}$ according to Rothmann (see Eq. (10)). From the modern HIPPARCOS data, we determined the declination in 1586, and from this the actual true altitude $h_{\rm HIP}$ with Eqs. (7)–(9) to which the modern value R for refraction was added. Repeated entries were ignored, as were the two entries representing a cluster: W 112 and W 247. Atmospheric refraction, especially at low altitudes, depends on meteorological conditions, and in the absence of detailed information can be given only for average conditions. For this, we used an equation by Sæmundsson (1986, cited in Meeus 1998):

$$R(') = 1.02/\tan\left(h(^\circ) + \frac{10\overset{\circ}{.}3}{h(^\circ)/1^\circ + 5.11}\right). \quad (16)$$

The division by 1° in the denominator is explicitly shown to indicate that the denominator is dimensionless. The difference between the altitudes observed in Kassel and those predicted from HIPPARCOS data is

$$\Delta h = h_{\rm W} + R_{\rm R} - (h_{\rm HIP} + R). \quad (17)$$

In the computation of $h_{\rm HIP}$ from the declination, we used the modern value $\phi_{\rm G} = 51°18'49\overset{''}{.}212$; this is $10\overset{''}{.}788$ less than the value determined in Kassel – or about 330 m on the ground. If the altitudes tabulated in *Manuscript* are the apparent altitudes h_a (the rationale for this possibility is given below), comparison with modern data gives

$$dh = h_{\rm W} - (h_{\rm HIP} + R) = \Delta h - R_{\rm R}. \quad (18)$$

For $h > 29°$, where $R_{\rm R} = 0$, $dh = \Delta h$. In Fig. 5, we show Δh and dh as a function of the altitude, together with linear fits. To eliminate undue influence of outliers, we iteratively determined the rms of the distances to the best fit and removed points at distances of more than 3 rms. For three entries, the catalogue value is clearly wrong: W 108, W 283, and W 367 with dh = $17\overset{'}{.}4$, $-49\overset{'}{.}1$, and $-61\overset{'}{.}8$, respectively. The rms values show that rounding S_h to allowed values has a negligible effect and may be ignored in the analysis.

Figure 5 shows two remarkable features. First, the points from superior culmination on average give a lower difference Δh than the points from meridional culminations at the same altitude. In addition to results for fits to all points, we therefore show in Table 9 the results for fits to data from southern and northern culminations separately. For fits to all h and for fits to $h > 29°$, the dependence of Δh and dh on altitude h is significantly shallower for the northern culminations than for the southern culminations.

The second striking feature of Fig. 5 is the non-zero slope of the fit to Δh that ill describes the points at $h < 29°$, which curve upwards towards lower h. The results for the fits of Δh to points at $h > 29°$ are incompatible with the fits to all h, when data combined from northern and southern culminations are considered, and also when data from northern or southern culminations are considered separately. In contrast the fits of dh to data at $h > 29°$ are compatible with the corresponding fits for all h, especially when southern and northern culminations are fitted separately. A striking illustration of this is provided by the three points with the lowest altitude, which are well outside the 3-rms range in the fit for Δh, but well within the 3-rms range in the fits for dh (Fig. 5).

If only points more than 4 rms are iteratively removed in the fitting procedure, all values for the intercept a and slope b of the best fits are the same within the error as those listed in Table 9. Hence our conclusions are independent of the exact criterion for inclusion and rejection of points in the fit.

We are thus faced with the conundrum that the comparison between the altitudes and declinations in the catalogue indicates that the altitudes are the true altitudes, whereas the comparison of the altitudes with those predicted from modern HIPPARCOS data suggests that they are the apparent altitudes. For the fits to all points and to those from southern culminations only Δh and dh are close to zero at altitude 90°. This indicates that the measurement apparatus was aligned to the zenith rather than to the horizon, understandably in hilly country. Most points from northern culminations have $\Delta h < 0$ and $dh < 0$, which may indicate a misalignment with the zenith for these measurements. The dependence of Δh and dh on altitude h could be due to an inaccurate scale division. The problem with this explanation is the near absence of a dependence of Δh on h for the northern culminations.

5.2. Angles between stars

Manuscript lists the angular distances ϕ between stars in column two. From the modern HIPPARCOS data, we computed the angle $\phi_{\rm HIP}$ at the time of the observation and subtracted this from ϕ to obtain the measurement error. The results are shown in Fig. 6. There is no dependence on ϕ and the average difference is compatible with zero. A straight-line fit, iteratively removing outliers at more than 3 rms, gives

$$\phi - \phi_{\rm HIP}\,(') = 0.04(9) - 0.001(3)\phi(^\circ).$$

Assuming that the average value is zero, the rms of $\phi - \phi_{\rm HIP}$ is $0\overset{'}{.}73$, which is sufficiently large to ensure that rounding S_ϕ to allowed values has a negligible effect. The furthest outliers are the angles between W 108-W 105 and between W 339-W 331, for which $\phi - \phi_{\rm HIP} = 26\overset{'}{.}1$ and $-20\overset{'}{.}6$, respectively.

6. Accuracy of the catalogue

Figure 7 shows the distributions of the magnitudes of the stars as given in *WilhelmIV*, and of the modern magnitudes of their counterparts. The magnitudes in the catalogue correlate well with the modern magnitudes. As expected in a catalogue of a limited size, most entries are bright. For the geographical latitude of Kassel,

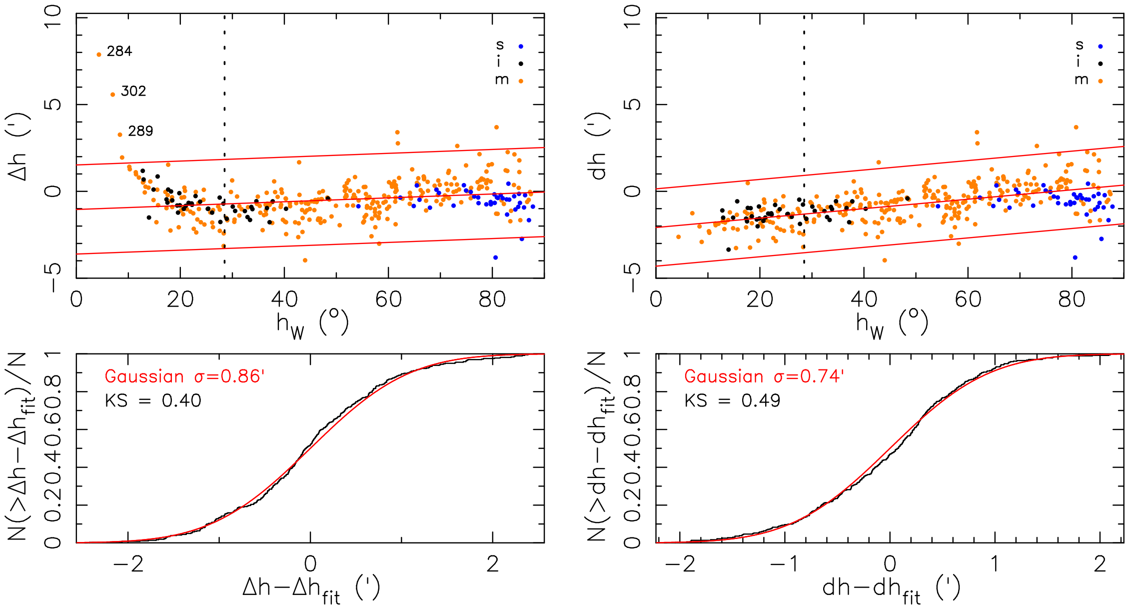

Fig. 5. *Top*: differences Δh (*left*) and dh (*right*) between the altitudes measured in Kassel and the altitudes that should have been measured according to modern data (Eqs. (17) and (18)). The solid lines show the best linear fit and its 3-rms range. Points outside this range were ignored in the fit. To the right of the dashed vertical line $R_R = 0$ and thus $\Delta h = dh$. *Bottom*: cumulative distribution of the difference between the measured Δh and dh and their best fit, compared with the distribution expected for a Gaussian with σ equal to the rms of the differences with the best fit. KS indicates the p-value from a two-sided Kolmogorov-Smirnov test for which the observed points are drawn from the Gaussian.

Table 9. Results of straight line fits $a + bh$ to Δh (Eq. (17)) and dh (Eq. (18)) as a function of h.

Fit to:		Range	$a(')$	$b('/°)$	rms	n	n_{rej}
Δh	msi	all h	−1.0(1)	0.011(2)	0.′86	359	15
Δh	m	all h	−1.3(1)	0.016(3)	0.′93	282	10
Δh	si	all h	−0.8(1)	0.003(2)	0.′54	78	4
$\Delta h, dh$	msi	$h \geq 29°$	−1.8(2)	0.023(3)	0.′76	275	11
$\Delta h, dh$	m	$h \geq 29°$	−2.1(2)	0.030(3)	0.′78	225	8
$\Delta h, dh$	si	$h \geq 29°$	−1.3(2)	0.011(3)	0.′46	50	3
dh	msi	all h	−2.1(1)	0.027(2)	0.′74	362	12
dh	m	all h	−2.4(1)	0.034(2)	0.′75	282	10
dh	si	all h	−1.5(1)	0.013(2)	0.′43	78	4

Notes. The columns indicate which data points (meridional, superior, and inferior culmination) are fitted, which range of h was included in the fit, the intercept a and slope b of the fit, the rms of the differences between the data points and fit, the number of points n used in the fit, and n_{rej} the number of points rejected because they lie more than 3 rms from the fitted line.

$\phi_G = 51°19'$, stars with $\delta < -38°41'$ are always below the geometric horizon, and the practical observation limit is closer to the celestial equator. The southernmost star in *WilhelmIV* is ϵ Sgr, at $\delta = -34°20'$, followed by Fomalhout at $\delta = -31°41'$. *WilhelmIV* is virtually complete at modern magnitude $V < 3.0$ and $\delta > -31°$: Only γ^2Sgr, γ Hya, γ Eri, and α^2CVn are missing. Extending the magnitude limit to $V < 3.5$ shows that 24 more stars with $\delta > -31°$ are missing.

The excellent overall accuracy of *WilhelmIV* is indicated by the fact that all entries can be securely identified with HIPPARCOS counterparts, or in two cases clusters of stars. In Sect. 4.3, we have shown that equatorial coordinates were accurately converted into ecliptic coordinates. We therefore include in our analysis equatorial coordinates which are not in *WilhelmIV* but were computed by us from the ecliptic coordinates. For small angles Δ between the catalogue and HIPPARCOS positions, we have approximately $\cos\delta \simeq \cos\delta_{HIP}$ and

$$\Delta \simeq \sqrt{(\Delta\alpha\cos\delta)^2 + (\Delta\delta)^2}. \quad (19)$$

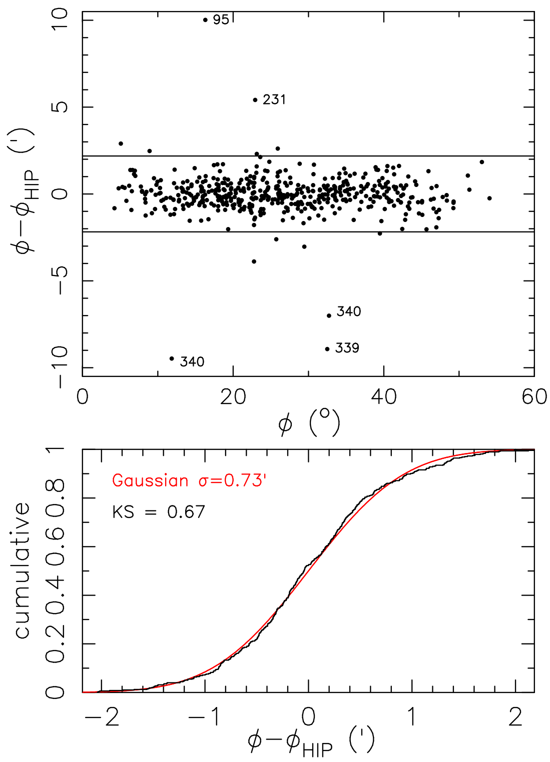

Fig. 6. *Top*: differences between the angles ϕ between two stars as given in *Manuscript* and the angles ϕ_{HIP} derived from modern HIPPARCOS data, as a function of ϕ. For some outliers, W is indicated (Table 6). *Bottom*: cumulative distribution of the differences compared with a Gaussian centred on zero.

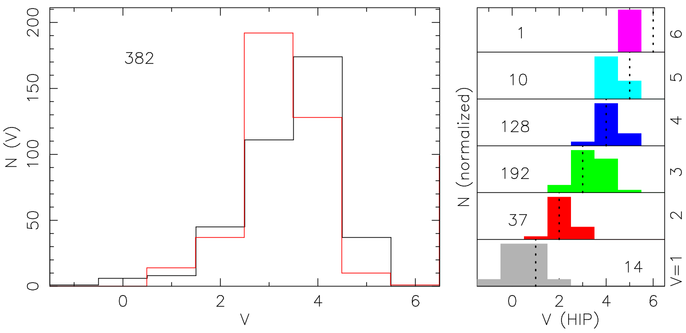

Fig. 7. *Left*: distribution of the magnitudes given in *WilhelmIV* (black) and of the magnitudes of their counterparts in the HIPPARCOS catalogue (red) for all securely identified stars. *Right*: distribution of the magnitudes of the HIPPARCOS counterparts, separately for each magnitude in *WilhelmIV*. The numbers show the number of stars in each frame.

Table 10. Average and rms of various distributions for values within frame limits of Fig. 8.

	Ave	rms		Ave	rms	Source
Δ	5.6	1.7	Δ_c	1.4	1.0	This article
			Δ	2.8	2.0	Catalogue Brahe
$\Delta\lambda$	5.1	2.1	$\Delta\beta$	0.3	2.6	This article
$\Delta\alpha_c$	0.3	1.1	$\Delta\delta$	0.7	1.0	This article
$\Delta\alpha_c$	0.2	1.2	$\Delta\delta$	0.8	1.5	Rothenberg [(a)]

Notes. [(a)]In Hamel (2002), for entries with $|\Delta\alpha_c| < 10'$ and $|\Delta\delta| < 10'$. Rothenberg also applied a correction of 6′ to the right ascensions; his results for $\Delta\alpha_c$ and $\Delta\delta$ are very similar to ours. Uncharacteristically large errors are due to computing or copying an error, and their contribution to the average and rms is unrealistic. Hence we only used values within the frame limits in the computation of average values and rms values.

A measurement error d in the direction of right ascension leads to an error of $\Delta\alpha \simeq d/\cos\delta$, hence to a large value for $\Delta\alpha$ at high declinations. This masks the effect of a systematic offset. For this reason, we show in Fig. 8 (top left) the distribution of $\Delta\alpha$ (Eq. (3)) for angles $|\delta| < 20°$ only. The average is $\simeq 6\overset{'}{.}5$, which is close to the offset of 6′ of the vernal equinox in *WilhelmIV* found by Brahe (cf. Table 4), and this justifies our correction to the catalogued right ascensions before we identified the entry. Astronomers in the 17th century knew about the offset found by Brahe, and thus they could correct for it. Curtz (1666) approximated the correction by moving the equinox of the catalogue to 1593, seven years later. Figure 8 (top right) shows the distributions of $\Delta\alpha_c \cos\delta$, and $\Delta\delta$, which reflect the measurement errors.

The shift of 6′ in the right ascensions leads to an erroneous location of the ecliptic, to the right of the correct location, in the computation of ecliptic coordinates from equatorial coordinates (Eqs. (12) and (13)). This causes an error of 6′ in λ at $\lambda = 90°$ or 270°, but to an error smaller by a factor of $\cos\epsilon$ at the equinoxes. It has no effect on β at $\lambda = 90°$ or 270°, but at the vernal and autumnal equinoxes it leads to errors of about $-6' \sin\epsilon \simeq -2\overset{'}{.}6$ and $+2\overset{'}{.}6$, respectively. Thus the offset in right ascension is reflected in an offset in the longitudes, visible in the distribution of $\Delta\lambda\cos\beta$, and to a double peak in the distribution of $\Delta\beta$ as shown in Fig. 8 (bottom left). The error also increases the angle Δ (Eq. (6)), which peaks slightly above 6′, as shown by the black curve in Fig. 8 (bottom right). The pronounced tails of the distributions of $\Delta\lambda\cos\beta$ and of Δ derived from the (uncorrected) ecliptic coordinates arise because the effect of the offset of the vernal equinox decreases with declination with $\cos\delta$.

Correcting the right ascensions by subtracting 6′ much improves the accuracy of the catalogue (Table 10), as shown for right ascension and declination in Fig. 8 (top right) and for the total error by the blue curve in Fig. 8 (bottom right). To enable a fair comparison with the positional accuracy of the star catalogue of Brahe – as edited by Kepler (1627) – we show the errors Δ only for those stars that are present in both catalogues.

7. Discussion and comparison with Brahe

The observations of stars made at the castle of Kassel, under the patronage of Wilhelm IV, Landgraf von Hessen-Kassel, with instruments built by Jost Bürgi, reached an accuracy an order of magnitude better than anything achieved before. Meridional altitudes and angles between stars were measured with an accuracy of 44″. The accuracy of instruments and measurements profited from the exchanges on these topics between Wilhelm IV and Tycho Brahe (Hamel 2002). The computations with which the measurements were converted to celestial coordinates were very accurate as well. An error of about 6′ in the position of the vernal equinox, however, corrupted the celestial coordinates, which were tabulated for 382 stars and two open star clusters in ecliptic coordinates and for 342 stars and one open cluster in equatorial coordinates. The offset of the vernal equinox was discovered by Brahe; if a correction is made for this offset, the positional accuracy Δ_c of *WilhelmIV* is better by a factor of two than that of the star catalogue of Brahe himself (Table 10).

Wesley (1978) determined the accuracy of the measurements with various instruments by Brahe. He determined the measurement accuracy for eight fundamental stars, used by Brahe as reference stars for measurements on other stars, by comparing the meridian altitudes given by Brahe with modern computation. This is what we did in Sect. 5.1 for *WilhelmIV-Alt*. Our results are compared with those by Wesley for the mural quadrant of Brahe in Table 11. The measurement errors in *WilhelmIV-Alt* are on the whole smaller, averaging about half the errors of the mural quadrant of Brahe.

The star catalogue *WilhelmIV* is corrupted by the offset of the vernal equinox. Thus, the positional errors in *WilhelmIV* are larger than those for most of the stars in Brahe's catalogue. The catalogue by Brahe, however, is corrupted for a significant fraction of its entries by errors in converting measurements to celestial positions, for example by confusing measurements of different stars and by scribal errors, as shown by Rawlins (1993). Of the 366 stars that are common to *WilhelmIV* and the star catalogue of Brahe (Kepler 1627), no less than 36 have a positional error larger than 10′ in Brahe's catalogue, but only eight in even the uncorrected version of *WilhelmIV*. It is therefore a moot question as to which of the two catalogues is more accurate. However, when we consider the positional errors in *WilhelmIV* after correction for the offset in right ascensions, the combination of more accurate measurements and more diligent computing causes *WilhelmIV* to be more accurate (Fig. 8, bottom right).

The value of the obliquity used in *WilhelmIV* for the conversion of ecliptic coordinates into equatorial ones, or vice versa, is more accurate than the value $\epsilon(1601) = 23°21'30''$ used by Brahe. The geographical latitudes used in *WilhelmIV* and by Brahe are both remarkably accurate, and they have no consequence for the conversion of altitudes into declinations. This conversion is affected, however, by the erroneous values used for atmospheric refraction.

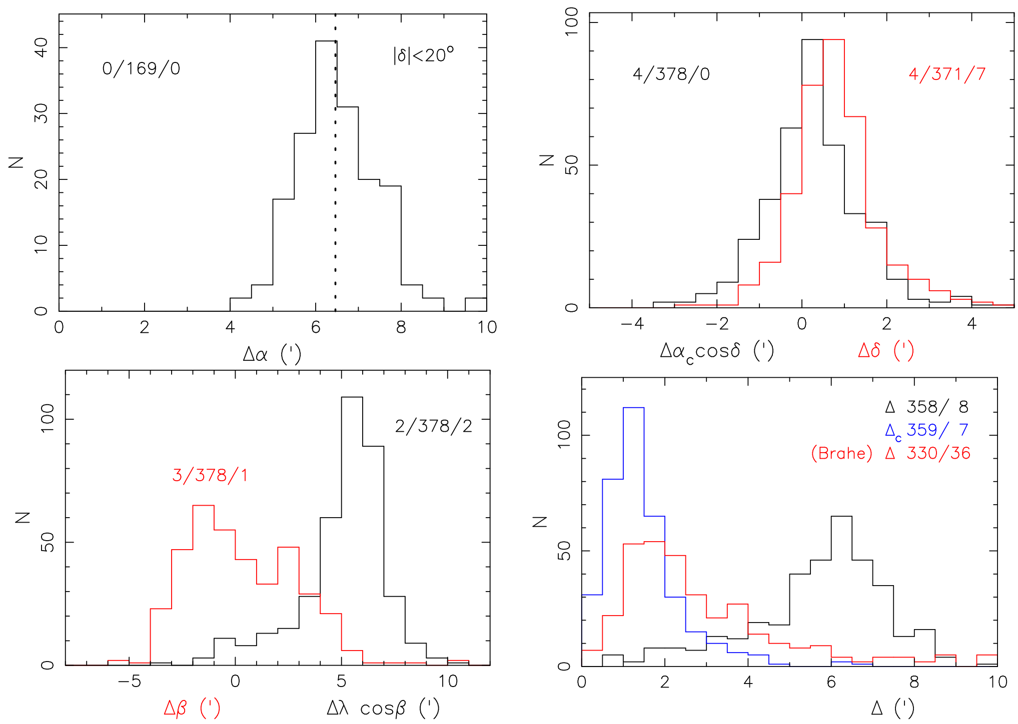

Fig. 8. Distributions of various positional errors. Entries identified with clusters and duplicate entries are not included. The histograms include entries for which equatorial coordinates are not given in *WilhelmIV*, but computed by us from the ecliptic coordinates. The numbers in the frame indicate the numbers of values to the left of, within, and to the right of the frame limits. *Top left*: distribution of $\Delta\alpha$ (Eq. (3)) for entries with $|\delta| < 20°$. The average shift, indicated with a vertical dashed line, is 6.′5. *Top right*: distributions of $\Delta\alpha_c \cos\delta$ (black) and of $\Delta\delta$ (red) – see Eq. (3). *Bottom left*: distributions of $\Delta\lambda \cos\beta$ (black) and $\Delta\beta$ (red) – see Eq. (5). *Bottom right*: distributions of the position errors Δ (black, Eq. (6)) and Δ_c (blue, Eq. (4)). For a fair comparison with the star catalogue of Brahe (edition by Kepler 1627, position errors shown in red), we selected only the 366 entries that are both in *WilhelmIV* and in Brahe's catalogue.

Table 11. Measurement accuracies determined from meridian altitudes for the mural quadrant of Brahe, determined by Wesley (1978), and for *WilhelmIV-Alt*, determined in Sect. 5.1, for eight of the fundamental stars of Brahe.

	α Ari	α Tau	μ Gem	β Gem	α Leo	α Vir	α Aql	α Peg	$< \lvert dh \rvert >$	$d\epsilon$	$d\phi_G$
WilhelmIV-Alt	−53″	−32″	−36″	2″	2″	−37″	−28″	−21″	26″	1′25″	11″
Brahe	116″	60″	24″	26″	−26″	−22″	−100″	20″	49″	1′54″	2″

Notes. The table also lists the errors $d\epsilon$ in the obliquity and $d\phi_G$ in the geographical latitude.

Wilhelm IV and his collaborators did not publish their work, as we mentioned in the Introduction, but it is interesting to know that the accuracy of Brahe was not unique. Of course, Brahe published not only his star catalogue, but many other works on the renovation of astronomy. The accuracy of his planetary positions enabled Kepler to show that Mars moves in an ellipse with the Sun in a focal point, and by extension that all planets do. Thus Brahe and Kepler together revolutionised astronomy.

To our knowledge, *Manuscript* is the first star catalogue to give both equatorial coordinates and ecliptic coordinates. Earlier star catalogues by Ptolemaios (±150), al-Sufi (±964), and Ulugh Beg (1437), as well as the later star catalogue of Brahe (1627) only provided ecliptic coordinates. Riccioli (1651) and Hevelius (1690) also provided both ecliptic and equatorial coordinates. de Houtman (1603) gave the first printed star catalogue with equatorial coordinates, but no ecliptic coordinates.

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Appendix A: The HIPPARCOS catalogue for epoch 1586.

Table A.1. Stars added by us to the HIPPARCOS-2 Catalogue.

HIP	V	$\alpha(2000)$	$\delta(2000)$	μ_α	μ_δ	
		(h)	(°)	(mas yr^{-1})		
31067	6.2	6.519444	16.938748	−13.30	−48.23	a
52800	6.9	10.794485	−15.243544	16.47	−43.81	a
55203	3.8	11.3031	31.5308	−453.70	−591.40	b
59273	6.7	12.1581	−11.8542	0.70	−69.30	c
78727	4.2	16.0727	−11.3736	−63.2	−27.0	d
80579	6.7	16.4528	−47.5491	−6.1	−20.4	e
115125	5.2	23.3185	−13.4548	302.	−92.	d

Notes. a. Notes in printed version of HIPPARCOS (ESA 1997); b. UCAC4 (Zacharias et al. 2012) c. *Tycho*-2 (Høg et al. 2010) d. PPM (Röser & Bastian 1988; Bastian & Röser 1993), e. *Gaia*-DR1 (Gaia Collaboration 2016).

For a number of stars, the original HIPPARCOS Catalogue did not find a solution for the proper motion. These stars are not present in the revised HIPPARCOS-2 Catalogue by van Leeuwen (2007). We used the position of these stars as given in the HIPPARCOS Catalogue (ESA 1997) and collected their proper motions from various sources, as listed in Table A.1.

To take into account that some HIPPARCOS stars cannot be separated by the human eye, we proceeded as follows. We selected all stars in the original HIPPARCOS Catalogue with $V < 7.0$ and read the positions of these stars in the revised HIPPARCOS-2 Catalogue. We corrected these positions for proper motion over the interval between the HIPPARCOS epoch 1991.25 and the *Manuscript* epoch 1586 and for precession from HIPPARCOS equinox 2000.0 to the *Manuscript* equinox 1586, with Eqs. (3.211) and (3.212-2) of Seidelman (1992). For each star, we found all stars within 2′ at that epoch and merged them into one entry. Thus 80 pairs and two triples were merged into 82 entries. To determine the position of the merged entry, the positions of its components were weighted with their fluxes, separately for the right ascension and for the declination. The magnitude of the merged entry was found by adding the fluxes of its components. Thus with f_i the flux normalised to the flux of a star with $V = 0$:

$$f_i = 10^{-0.4V_i};\ \alpha = \sum_i (f_i\alpha_i) \Big/ \sum_i f_i\ ;\ \delta = \sum_i (f_i\delta_i) \Big/ \sum_i f_i \qquad \text{(A.1)}$$

and

$$V = -2.5 \log \sum_i f_i. \qquad \text{(A.2)}$$

Appendix B: Emendations to *Manuscript*

B.1. Angular distances between stars

For W 36, the name of the reference star is not given. The angular distance indicates W 24, which we entered in *WilhelmIV-Dist*.

Following W 165, 'In Cauda' (in the tail) sc. of Aquila, is an entry 'Extrema Caudae' (the end of the tail), which is indicated in Col. 7 as 'in tabulis non extat' (not present in the tables sc. of Ptolemaios). Two angles to reference stars are given for this entry, towards Aquila (W 163) and towards Rostrum Cygni (the beak of the Swan), which we identify with W 89. These angles imply that 'Extrema Caudae' corresponds to HIP 93244 (ϵ Aql). That star indeed is not in the star catalogue of Ptolemaios.

For W 340 and W 341, Canis maior (W 353) is indicated as the reference star, but it is excluded by the angular distance; we emended this to Canis minor (W 352).

B.2. Altitudes

For W 64, we emended 18°14′to 58°14′.

B.3. Ecliptic coordinates

On the basis of Sect. 4.3, we emended the following.
W 55. β 55°41′10″ to 54°41′10″.
W 214. β 02°49′54″ to 02°47′54″.
W 374. λ ♍ 2^{d}42^{m}50^s to ♍ 2^{d}41^{m}50^s.

Appendix C: Notes on individual identifications

Merged with is abbreviated with m.w.
W 26. HIP 85829 (V = 4.84) m.w. HIP 85819 (V = 4.89).
W 32. HIP 86614 (V = 4.57) m.w. HIP 86620 (V = 5.81).
W 53. HIP 110991 (V = 4.07) m.w. HIP 110988 (V = 6.31).
W 59. HIP 75411 (V = 4.31) m.w. HIP 75415 (V = 6.51).
W 73. HIP 79043 (V = 5.00) m.w. HIP 79045 (V = 6.25).
W 83. HIP 91971 (V = 4.34) m.w. HIP 91973 (V = 5.73).
W 89. HIP 95947 (V = 3.05) m.w. HIP 95951 (V = 5.12).
W 99. We chose HIP 99675 (31 Cyg, V = 3.8, Δ = 3.′4) as the counterpart above the closer, but fainter HIP 99639 (30 Cyg, V = 4.8, Δ = 2.′4).
W 108. We chose HIP 5542 (θ Cas, V = 4.3, Δ = 31.′7)) as the counterpart above the closer, but fainter HIP 5536 (ρ Cas, V = 5.2, Δ = 24.′0).
W 156. HIP 92946 (V = 4.62) m.w. HIP 92951 (V = 4.98).
W 174. HIP 102532 (V = 4.27) m.w. HIP 102531 (V = 5.15).
W 251. HIP 43103 (V = 4.03) m.w. HIP 43100 (V = 6.58).
W 277. HIP 78820 (V = 2.56) m.w. HIP 78821 (V = 4.90).
W 283. Notwithstanding the large positional error δ = 42.′8, we consider the identification secure.
W 312. HIP 5737 (V = 5.21) m.w. HIP 5743 (V = 6.44).
W 317. HIP 5131 (V = 5.33) m.w. HIP 5132 (V = 5.55).
W 340. HIP 26220 (V = 4.98) m.w. HIP 26221 (V = 5.13) and HIP 26224 (V = 6.71. The original HIPPARCOS catalogue (ESA 1997) lists V=4.98 for HIP 26220, but error 0.00 for $B - V$; this may indicate that the photometry is suspect. Ducati (2002) gave V = 6.73. Since HIP 26220 is a Herbig Ae/Be star, and thus variable, its magnitude is probably variable.
W 367. The positional error Δ = 60.′7 strongly suggests an error of 1° in writing or copying the correct result.