Frequencies, chaos, and resonances: A study of orbital parameters of nearby thick-disc and halo stars

Helmer H. Koppelman\textsuperscript{1,2}, Jorrit H. J. Hagen\textsuperscript{1}, and Amina Helmi\textsuperscript{1}

\textsuperscript{1} Kapteyn Astronomical Institute, University of Groningen, Landleven 12, 9747 AD Groningen, The Netherlands
\textsuperscript{2} School of Natural Sciences, Institute for Advanced Study, Princeton, NJ 08540, USA
e-mail: koppelman@ias.edu

Received 10 September 2020 / Accepted 11 January 2021

ABSTRACT

Aims. We study the distribution of nearby thick-disc and halo stars in subspaces defined by their characteristic orbital parameters. Our aim is to establish the origin of the structure reported in particular in the $R_{\text{max}} - z_{\text{max}}$ space.

Methods. To this end, we computed the orbital parameters and frequencies of stars for a generic and for a Stäckel Milky Way potential.

Results. We find that for both the thick-disc and halo populations, very similar prominent substructures are apparent for the generic Galactic potential, while no substructure is seen for the Stäckel model. This indicates that the origin of these features is not merger-related, but due to the non-integrability of the generic potential. This conclusion is strengthened by our frequency analysis of the orbits of stars, which reveals the presence of prominent resonances, with $\sim 30\%$ of the halo stars associated with resonance families. In fact, the stars in resonances define the substructures seen in the spaces of characteristic orbital parameters. Intriguingly, we find that some stars in our sample and in debris streams are on the same resonance as the Sagittarius dwarf.

Conclusions. Our study constitutes a step towards disentangling the imprint of merger debris from substructures driven by internal dynamics. Given their prominence, these resonant-driven overdensities could potentially be useful in constraining the exact form of the Galactic potential.

Key words. Galaxy: structure – Galaxy: halo – Galaxy: kinematics and dynamics

1. Introduction

With the second data release of the Gaia space mission (Gaia Collaboration 2018b), a catalogue comprising the full range of 6D phase space information has become available (Katz et al. 2019). This subset has enabled detailed studies of interconnected dynamical processes taking place in the Milky Way. For example, substructures in integrals-of-motion spaces of nearby disc stars have been related to the Galactic bar or spiral arms (e.g., Dehnen 2000; Monari et al. 2017, 2019; Khoperskov et al. 2019; Trick et al. 2019; Hunt et al. 2019), or to interactions with satellite galaxies (Antoja et al. 2018; Laporte et al. 2018; Laporte & Minchev 2019).

Furthermore, the assembly history of the Milky Way can be studied by identifying and characterising substructures in integrals-of-motion space, as first put forward by Helmi & de Zeeuw (2000). Structures comprising tens to hundreds of stars have been linked to accreted dwarf galaxies whose debris can still be traced in the solar neighbourhood (Helmi et al. 1999, 2017; Chiba & Beers 2000; Klement et al. 2008, 2009; Williams et al. 2011; Myeong et al. 2018; Koppelman et al. 2018, 2019b; Yuan et al. 2020; Naidu et al. 2020, see also the reviews of Newberg & Carlin 2016; Klement 2010). Besides these small groups, recently the relics of a massive dwarf galaxy (now known as Gaia-Enceladus) have been identified in the same kind of spaces by Helmi et al. (2018), see also Belokurov et al. (2018).

With the rise of machine learning, it has become popular to deploy automated classification algorithms to search for substructures (Necib et al. 2020; Borsato et al. 2020; Du et al. 2019; Koppelman et al. 2019b; Ratzenböck et al. 2020; Yuan et al. 2020). Substructures are not necessarily always accreted (e.g., Gómez & Helmi 2010; Gómez et al. 2013; Jean-Baptiste et al. 2017), and therefore the interpretation of their origin is not always straightforward. In this work, we investigate structures reported in $R_{\text{max}} - z_{\text{max}}$ space (e.g., Haywood et al. 2018), supplemented by information obtained from the orbital frequencies of the stars.

Substructures identified in the space characterised by $R_{\text{max}} - z_{\text{max}}$ may be amenable to intuitive interpretation. Schuster et al. (2012) noted that the low-[$\alpha$/Fe] stars in the halo, identified in Nissen & Schuster (2010), have orbits that reach out to much larger $R_{\text{max}}$ and $z_{\text{max}}$ than the high-[$\alpha$/Fe] stars. A similar conclusion was reached by Haywood et al. (2018), who identified distinct ‘wedges’ in this space. These wedges are also present when using the chemically defined halo sample of Chiba & Beers (2000), with updated astrometry from Gaia DR2, ruling out that they are due to a kinematic bias. Haywood et al. (2018) relate the wedges to the merger with Gaia-Enceladus and link them to the impulsive heating of an ancient Milky Way disc. The authors suggest that the largest gap may hint at a phase transition in the assembly history of the Milky Way, from a significant to a quiescent accretion phase. A different view is presented by Amarante et al. (2020), who argue that these wedges are due to transitions between orbital families. Based on smoothly sampled halo orbits, these authors attribute the structures in this space to ‘a natural consequence of resonant effects’. This view is supported by Schuster & Allen (1997), who report on a gap in the distribution of $z_{\text{max}}$ that is reminiscent of the wedges in $z_{\text{max}}$ with...
$R_{\text{max}}$ space. These authors ascribe the gap in the $z_{\text{max}}$ distribution to chaotic diffusion, where the orbits ‘stick’ to one set of quasi-periodic orbits or another.

In this work, we further explored the link between the $R_{\text{max}} - z_{\text{max}}$ space and orbital resonances, as mapped by frequency space. The space of orbital frequencies is particularly interesting for several reasons. Firstly, as shown by Gómez & Helmi (2010) and Gómez et al. (2010), individual streams associated with an accreted galaxy can be easily identified here. The time of accretion is quite precisely encoded in this space and can be determined from the separation between two adjacent streams in frequency space. Secondly, Valluri et al. (2012) argued that the strengths and locations of resonances for halo stars in frequency space are both dependent on the stellar distribution function, as well as on the global shape of the halo. In principle, the orientation of the halo with respect to the disc can be determined from the distribution of stars in frequency space. Moreover, Valluri et al. argued that the diffusion rates of the orbital frequencies can help in distinguishing between the true and an incorrect potential. Finally, the mapping of frequency space is a powerful tool to establish the different orbital families present in an ensemble. For separable and non-degenerate potentials the frequencies are directly related to the actions. Frequencies have the advantage that they can be easily and precisely calculated numerically. Also, in near-integrable potentials, for which the actions do not exist, the frequencies trace the different orbital families accurately (e.g., Laskar 1993).

We investigated the distribution of stars in the solar vicinity in spaces associated with different orbital parameters – such as pericentre, apocentre, eccentricity, $z_{\text{max}}$ and orbital frequencies – and explored the kinematics of the substructures identified in those subspaces. The paper is organised as follows. In Sect. 2, we introduce the data and present the selection criteria applied. Then, in Sect. 3, we identify substructures in subspaces of different orbital parameters. In Sect. 4, we perform an orbital frequency analysis and show how the structures in the various subspaces, in particular $R_{\text{max}} - z_{\text{max}}$, are linked to resonances in frequency space. We reflect on these results in Sect. 5 and present our conclusions in Sect. 6.

2. Data

We used the subset of Gaia DR2 with line-of-sight velocities (Gaia Collaboration 2018d). This subset – also referred to as the Gaia radial-velocity spectrometer (RVS) subset – of more than 7 million stars thus contains accurate measurements of the 3D positions and 3D velocities of the stars. Gaia DR2 parallaxes are known to suffer from systematic parallax errors that vary with position, $G$-band magnitude, and stellar type. Therefore, we used Bayesian distances $d$ derived by McMillan (2018). The author derived distances by assuming an overall parallax offset of $-29$ mas with an RMS error of 43 mas for the Gaia RVS subset as found for the full Gaia DR2 release (Lindgren et al. 2018). We focus here on a local sample with $d < 2.5$ kpc of high-quality parallaxes (parallax $\text{ax}_{\text{over} \text{error}} > 5$). These quality criteria leave us with a sample of 5015 006 stars.

For these stars, we computed positions and velocities in a galactocentric cylindrical coordinate system ($R$, $z$, $\phi$, $V_R$, $V_z$, $V_\phi$). We placed the solar position at $R_\odot = 8.2$ kpc (McMillan 2017)\(^1\) and $z_\odot = 0.014$ kpc (Binney et al. 1997). The Local Standard of Rest (LSR) velocity (i.e. $V_R(R_\odot) = 232.8$ km s$^{-1}$)

\(^1\) This value is consistent with the Gravity Collaboration (2019), who recently determined $R_\odot = 8178 \pm 13\_\text{stat} \pm 22\_\text{sys}$ pc.

\section{Results: Phase-space and integrals of motion}

For most of the analysis presented in this paper, we adopt the Milky Way potential determined by McMillan (2017), unless stated otherwise. This axisymmetric potential consists of a stellar thin and thick-disc, H\(_2\) gas disc, molecular gas disc, a flattened bulge, and a spherical halo component (NFW). This system has a virial mass of $1.3 \times 10^{12} M_\odot$, with $5.43 \times 10^{10} M_\odot$ in stars.

We integrated the orbits of halo stars in this potential for 80 Gyr, and 40 Gyr for the thick-disc stars. These long timescales were chosen because of our interest in computing the orbital frequencies (see Sect. 4). Numerically estimating the frequencies requires densely sampled orbits with many periods, typically

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Velocity distribution of the stars in our sample, highlighting the various kinematic selections. A logarithmic map of star counts is shown in the background (the smooth blue map). Stars within the cyan semi-circles are considered to belong to the thick-disc. The stars outside the outermost cyan curve are considered halo stars, they are split into a pure halo and a TDtail sample.}
\end{figure}
3.1. Velocity space

The local stellar distribution is enriched by several moving groups that are apparent as clumps in velocity space \((v_R, v_z, v_\phi)\). These structures might be related to stars born together or may be of dynamical nature (e.g., see Antoja et al. 2018, on the outer Lindblad resonance (OLR) mode due to the Galactic bar).

Figure 2 shows the velocity distribution of the three subsets that we analysed, as indicated by the insets. As is clearly apparent from this figure, these distributions are affected by the kinematic selection criteria used. We also highlight how the top (thick-disc) and middle rows (TDtail) look similar, whereas the bottom row (pureHalo) shows a different distribution, mostly in the middle and right columns corresponding to \(v_\phi \) vs. \(v_R\) and \(v_\phi\) vs. \(v_z\), respectively. The TDtail component (middle row) consists of stars with ‘hotter’ thick-disc-like kinematics, and it is likely the result of the proto-disc being puffed up during the

---

**Fig. 2.** Different projections of velocity space for the various subsets considered in this chapter: thick-disc, TDtail, and pure halo from top to bottom. The lower density of stars present in these top panels for small values of \(v_R\) and \(v_z\) is due to the exclusion of thin-disc stars in our selection. We note that even our pureHalo subset contains some stars with ‘hot’ thick-disc-like kinematics (as evidenced by the arch-like structure in the bottom central panel). Red dashed curves in the bottom panel outline the structures that are mentioned in the text.
Fig. 3. Orbital parameters spaces for the different subsets defined in Sect. 2. From top to bottom: thick-disc, TDecale, and pureHalo. The left panels correspond to the most often used space to identify merger debris (i.e. $L_z$, $E$). The substructure seen near $L_z = 250$ kpc km s$^{-1}$ for highly bound energies in the middle-left panel is the globular cluster M4 (also known as NGC 6221), whose extent is largely the result of limitations in the Gaia DR2 astrometry for very dense stellar fields. The subspaces shown in the middle and right panels depict a large amount of structure, which appears to be independent of the subset or population considered. We note that the top row shows a smaller dynamical range than the other two rows.

merger with Gaia-Enceladus (Helmi et al. 2018; Di Matteo et al. 2019).

The strong, radially elongated feature at $v_\phi = 0$ in the pureHalo sample (middle panel) is indicative of the debris of Gaia-Enceladus (cf. ‘the sausage’, Belokurov et al. 2018). The top and bottom parts of the crescent shape at $v_\phi \approx 100$ km s$^{-1}$ imply that this sample still contains a small fraction of heated thick-disc stars. It can be seen as an extension of the distribution shown in the middle panel of the TDecale (middle row). The elongated structure in $v_R$ and the crescent shape have been outlined with thin, red dashed lines.

In the bottom-left panel, we also see a clear overdensity of stars in an arch near $v_z \sim \pm 250$ km s$^{-1}$ for a range of values of $v_R$. These stars define the two clumps in the bottom-right panel seen at $(v_\phi \sim 120$ km s$^{-1}$, $v_z \sim \pm 250$ km s$^{-1})$, and they correspond to the Helmi streams (Helmi et al. 1999). The Helmi streams are marked with thin, red dashed lines.

3.2. Characteristic orbital parameters

In a static axisymmetric potential, such as the one considered here, the integrals of motion energy ($E$) and angular momentum in the z-direction ($L_z$) are conserved. However, stars that share a common origin phase-mix as they evolve in the Galactic potential (Helmi & White 1999), which results in complex configurations in phase space (Tremaine 1999), their distributions in, for example, $E - L_z$ space remain clumped.

We show the distribution of the stars in the three samples in $E - L_z$ space in Fig. 3 (see the left column). The middle and right columns show a combination of other orbital parameters that are sometimes used to identify substructures. The pureHalo sample (bottom row) shows by far the largest spread in the orbital parameters (we note, in particular, the larger dynamical range with regard to the top row). Its distribution is dominated by stars near $L_z \sim 0$ kpc km s$^{-1}$, which is the debris
Fig. 4. $R_{\text{max}}$ vs. $z_{\text{max}}$ for the thick-disc (left), TDtail (middle), and pureHalo (right) populations, computed for an integrable two-component Milky Way Stäckel potential. In all subsets, the amount of structure has drastically decreased. The clear overdensity of stars in the TDtail sample and at $R_{\text{max}} \approx 6.5$ kpc and $z_{\text{max}} \approx 0.5$ kpc corresponds to the globular cluster M4 and is thus of no specific interest.

identified as Gaia-Enceladus (Koppelman et al. 2018; Helmi et al. 2018). Besides this prominent structure, we see contamination from the hot thick-disc and several small overdensities such as that tentatively linked to an accretion event labelled Sequoia ($L_\perp \sim -2000$ kpc km s$^{-1}$, $E \sim -120$ 000 km$^2$ s$^{-2}$), and ($L_\perp \sim 1200$ kpc km s$^{-1}$, $E \sim -170$ 000 km$^2$ s$^{-2}$) for Thamnos (see Myeong et al. 2019 and Koppelman et al. 2019b, for more details).

In the middle panels of Fig. 3 we plot $R_{\text{max}}$ vs. $z_{\text{max}}$, while in the right panels we show the eccentricity vs. $z_{\text{max}}$ (i.e. a proxy for orbital inclination). The eccentricity is defined as $e = [r_p + r_a]/r_a$, where $r_p$ and $r_a$ denote the orbital pericentre and apocentre (measured as the minimum and maximum galactocentric distances over the whole integration interval). Particularly in these orbital parameters spaces, we see a large amount of structure. These structures look similar to the ‘wedges’ reported by Haywood et al. (2018) and computed for a different Milky Way potential than used here. Figure 3 shows that the structures are present in all the star samples explored, although with a different prominence. The fact that they are present independently of the population suggests that the phenomenon that causes them must be of a global nature and is unlikely to be due to individual (ancient) accretion events.

The shape and exact location of the structures shift slightly when changing the Galactic potential, but they appear to be generic. For example, we also integrated our samples using the potentials provided by Pfiff et al. (2014) and the MWPotential2014 (Bovy 2015) and found similar features (see Appendix A and also Haywood et al. 2018).

To gain further insight, we also decided to explore a less generic but integrable potential, namely a Milky Way-like potential of Stäckel form (derived from Batsleer & Dejonghe 1994; Famaey & Dejonghe 2003). Potentials of this form are separable and contain only regular orbits. Figure 4 shows the distributions of the subspace $R_{\text{max}}$ vs. $z_{\text{max}}$ for the stars in our various subsets now determined using a two-component Stäckel Milky Way potential of Famaey & Dejonghe (2003). This potential has a total mass of $M_{\text{tot}} = 4 \times 10^{11} M_\odot$, a disc mass with a fraction of $k = 0.11$ of $M_{\text{tot}}$, scale length $r_s = 7$ kpc, and a flattening of the disc and halo of $e_d = 0.75$ and $e_h = 1.02$, and it provides a good representation of, for example, the Galactic rotation curve. Figure 4 reveals rather smooth and featureless distributions, especially in comparison to the previous figure. This implies that the structures seen in Fig. 3 must be induced by the non-integrability of the generic Galactic potentials used. The structures could thus be due to resonances and the result of chaotic diffusion, a hypothesis that we explore further in Sect. 4.

3.3. Comparison to known substructures

To further strengthen this preliminary conclusion, we investigated the distribution of stars associated with known accreted substructures in the orbital parameter spaces just explored. Many candidate substructures have been identified since Gaia DR2, such as Gaia-Enceladus (Helmi et al. 2018), Sequoia (Myeong et al. 2019), and Thamnos (Koppelman et al. 2019b), and some have also been confirmed, such as the Helmi streams (Helmi et al. 1999; Koppelman et al. 2019a).

We took the stars that belong to these substructures according to Koppelman et al. (2019b) and noted their intersection with our pureHalo sample. In this way, we find 1186 Gaia-Enceladus stars, 54 members of the Helmi streams, 92 in Sequoia, and 1098 Thamnos stars.

The velocity distribution of the stars in these various substructures is shown in the top panels of Fig. 5. The bottom-left panel directly reflects the selection criteria used to identify them (where a lower limit in the energy $E$ has been somewhat artificially imposed on the Gaia-Enceladus stars to reduce overlap with the low-energy halo and the tail of the thick-disc (e.g., Feuillet & Feltzing 2020). This constraint depletes the number of stars with small $v_R$ and eccentricities $e < 0.8$. On the other hand, the middle and right-hand-side panels of the bottom row show that these accreted substructures are also themselves split into substructures or wedges, just like the stars in the various subsets considered in the previous section. For example, the nearly horizontally aligned gaps in ($R_{\text{max}}, z_{\text{max}}$)-space continue to be very prominent. Given the (tentative) range of accretion times for the various objects (~6 to 10 Gyr ago), and that their debris seems to be influenced in the same way, this analysis further supports the idea that the origin of the features seen in Fig. 3 must lie in a global phenomenon affecting the whole Galaxy.

4. Analysis: Orbital frequencies

4.1. Methods

In this section, we describe a frequency analysis we performed to explore whether resonances might play a role in
the substructures seen in the orbital parameter spaces. We estimated the frequencies numerically using the SuperFreq package\(^2\) (Price-Whelan 2015, see also Price-Whelan et al. 2016), which is a Python implementation very similar to the Numerical Analysis of Fundamental Frequencies (or NAFF) code written by Valluri & Merritt (1998) and Valluri et al. (2010, 2012), which in itself is an implementation of the NAFF technique of Laskar (1990; 1993, see also Wang et al. 2016, who provide a comparison between the NAFF algorithm and another commonly used orbital frequency and classification code by Carpenter & Aguilar 1998). The software finds the fundamental frequencies by computing the Fourier spectra for the phase space coordinates (more precisely: of a complex time series) used to describe the orbit. Since the orbits are computed in an axisymmetric coordinates (more precisely: of a complex time series) used to describe the orbit. Since the orbits are computed in an axisymmetric

In SuperFreq, the first fundamental frequency is the non-zero frequency with the highest amplitude. The algorithm then continues by moving down along the remaining frequency lines in the Fourier spectrum corresponding to the other coordinates, in order of decreasing amplitude. The next fundamental frequency must be different from the first one found, and so on. These fundamental frequencies \(\Omega = (\Omega_r, \Omega_z, \Omega_\phi)\) are not necessarily equal to the dominant frequencies \(\omega\) in the Fourier spectra of each coordinate. The fundamental frequencies found are also not necessarily equal to the frequencies in which the angle variables in action-angle coordinates vary with time. For regular orbits, the recovered fundamental frequencies will, however, be a linear combination of these ‘more fundamental’ (action-angle) frequencies (Valluri et al. 2012).

Resonant orbits are those for which the fundamental frequencies are commensurable. That is, a resonance is defined to exist if \(n \cdot \Omega = 0\), for any vector \(n = (n_r, n_z, n_\phi)\) comprising integer numbers (where at most one element is zero). In theory, there are an infinite number of resonances for combinations of arbitrarily large vector elements \(n_i\); here, we only consider those with \(|n_i| \leq 5\). In a spherical potential, the true ‘fundamental’ frequencies corresponding to the nature of the potential satisfy \(\Omega_r / 2 \leq |\Omega_\phi| \leq \Omega_\phi\) (Binney & Tremaine 2008), where the lower limit corresponds to any orbit in a homogeneous sphere (harmonic oscillator), and the upper limit corresponds to orbits in a Kepler potential (point mass). In the epicyclic approximation, \(\Omega_c / \Omega_\phi = 1 / \sqrt{2} \approx 0.707\) for a flat rotation curve \(v_{\text{circ}}(r) = \text{const.}\). In an axisymmetric disc potential, vertical oscillations for most disc stars are expected to have the shortest periods (\(\Omega_c\) is the largest in \(\Omega\)).

The NAFF methodology provides reliable frequencies for regular orbits that have been traced for 20–30 periods, so we limited our analysis to stars with a minimum of 20 orbital periods. All thick-disc stars and almost all halo stars (99.76%) are

---

\(^2\) https://superfreq.readthedocs.io/en/latest/

\(^3\) By definition, the fundamental frequencies are non-zero. However, a zero frequency line in the spectrum of a coordinate can indicate that the orbit’s centre is not on the origin. For example, a zero-frequency in \(z\) indicates an asymmetric orbit such as a ‘banana’ orbit.
H. H. Koppelman et al.: Chaos and resonances

Fig. 6. Frequency maps for the different populations. From top to bottom: thick-disc, TDtail, and pureHalo population. From left to right: cycling through the combinations of frequencies. The panels, especially those in the left column, show a clear presence of resonances in the form of straight lines and of depleted regions around them.

integrated long enough to satisfy this criterion. Most halo stars are integrated for 100–600 orbital periods, while most thick-disc stars for 150–300 orbital periods.

4.2. Results

We commence the frequency analysis by showing the distribution of fundamental orbital frequencies for each of the populations or subsets considered in Fig. 6. The top row shows a density map, and because of the size of the sample, the colours indicate the logarithm of the number of stars, with bright orange being the densest and blue being the least dense. Resonances are clearly discernible in this space as well-populated thin straight lines. Most of the resonances are present in the $\Omega_R$ vs. $\Omega_z$ space (left column). The regions around these resonances are depleted of stars, which is common for non-integrable potentials and indicates the presence of chaotic zones (e.g., see Fig. 1 of Price-Whelan et al. 2016, for an illustration of this phenomenon). There are no very obvious resonances in $\Omega_{\phi}$ vs. $\Omega_R$ (middle panels, although these might be expected for thin-disc stars, particularly in relation to the Galactic bar, see Dehnen 2000).

We find that roughly 10% of the stars in the thick-disc sample are on resonant orbits in $\Omega_z/\Omega_R$. For the TDtail and for the pureHalo samples, these fractions are much higher: namely 25% and 28%, respectively. Stars are said to belong to a certain resonance if the orbital frequency ratio for the pair of coordinates deviates at most 0.001 from the rational number of the resonance under consideration. To establish if the resonances are related to the structures we identified in the previous section, we now discuss the most dominant resonances in each of the subsets.
A selection of the most dominant resonances in the thick-disc sample is shown in Fig. 7 (top row). These are the \( \Omega_z : \Omega_R = 1:1 \) (blue), 2:1 (yellow), 1:2 (green), 3:2 (red), 3:4 (purple), and 5:4 (brown) resonances. The percentages of stars in each of these resonances are listed in Table 1. The middle row of Fig. 7 shows that the stars on these resonances occupy specific regions in the orbital parameter spaces. There is a strong connection between the frequencies and features seen in these spaces. The overdensities first shown in Fig. 3 correspond directly to some of the highlighted resonances. On the other hand, depleted regions and gaps in the orbital parameter spaces indicate regions of unstable orbits. Such orbits are likely trapped by the resonances explored here, leaving empty regions around the corresponding frequencies. This is, for example, clearly seen for the \( \Omega_z : \Omega_R = 1:2 \) (in green) and \( \Omega_z : \Omega_R = 3:2 \) (in red) resonances. The bottom panels of Fig. 7 show that stars on a given resonance have a broad range of velocities. However, some resonances (e.g., the 1:2) occupy only a small region in certain projections of velocity space.

Figure 7 shows that most of the resonances can be associated with a contiguous region in the orbital parameters spaces. However, this figure also reveals a higher complexity for the \( \Omega_z : \Omega_R = 1:1 \) and the \( \Omega_z : \Omega_R = 2:1 \) resonances. The stars of these families are found in two regions that surround the most prominent gap of Fig. 3, located near \( R_{\text{max}} \sim 8 \text{ kpc} \) and \( z_{\text{max}} \sim 2 \text{ kpc} \). This potentially means that these resonances are not solely responsible for this depleted region. We return to this point in the next section.

Following the same procedure for the halo samples, we show the TDtail and the pureHalo samples separately in Figs. 8 and 9, respectively. In addition to the \( \Omega_z : \Omega_R \) resonances identified in the thick-disc sample, we also find the 3:4 and 5:2 being populated. Just as we saw for the thick-disc sample, the stars associated with the resonances also occupy specific regions in the orbital parameter spaces.

### Table 1. Percentage of stars in the various sub-samples associated with the listed \( \Omega_z : \Omega_R \) resonances.

<table>
<thead>
<tr>
<th>sample</th>
<th>1:1</th>
<th>2:1</th>
<th>1:2</th>
<th>3:2</th>
<th>3:4</th>
<th>5:4</th>
<th>5:2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thick-disc [%]</td>
<td>5</td>
<td>0.3</td>
<td>0.2</td>
<td>3.6</td>
<td>0.03</td>
<td>0.7</td>
<td>0</td>
</tr>
<tr>
<td>TDtail [%]</td>
<td>10.9</td>
<td>4.2</td>
<td>0.6</td>
<td>4.8</td>
<td>3.1</td>
<td>1.9</td>
<td>0</td>
</tr>
<tr>
<td>pureHalo [%]</td>
<td>10.1</td>
<td>1.7</td>
<td>6.2</td>
<td>3.9</td>
<td>4.3</td>
<td>1.8</td>
<td>0.4</td>
</tr>
</tbody>
</table>

**Notes.** The percentages are calculated with respect to the entire sample – not just relatively to the stars on resonances. We note that the quoted percentages here are specific to the McMillan (2017) potential.

A 4.3. Commonalities between sub-samples

In all three sub-samples, we find similar families of resonances to be dominant. The \( \Omega_z : \Omega_R = 1:1 \) is the most prominent resonance in all samples. In the thick-disc sample, the other dominant resonance is 3:2. For the TDtail sample, this resonance is also the second most dominant frequency, but the 2:1 and 3:4 resonances are also prominent. In the pureHalo sample, the second most dominant resonance is 2:1, followed by 3:4 and 3:2.

We now return to the most prominent gap seen in Fig. 3 and present in all sub-samples, which is the gap that goes through \( (R_{\text{max}}, z_{\text{max}}) \sim (8, 2) \text{ kpc} \). For practical purposes, we focused on the thick-disc subset. We have previously noted from the middle row panels of Fig. 7 that the gap also divides the stars associated with the resonances \( \Omega_z : \Omega_R = 1:1, 2:1, \) and 1:2 in two separate zones. This implies that these resonances on their own can not fully explain the presence of a gap in the \( R_{\text{max}} \text{ vs. } z_{\text{max}} \) space.

To gain further insight, we selected stars around the gap for the \( \Omega_z : \Omega_R = 1:1 \) resonance, as shown in the leftmost panel of...
Fig. 10. The rightmost panel of this figure shows that the stars in this region cluster around $\Omega_z : \Omega_r = 2:3$, but that those above the gap have $\Omega_z/\Omega_r < 2/3$, while those below have values greater than 2/3. Similar results are found for the other resonant families, with the gap seen in the characteristic orbital parameters of stars associated with the $\Omega_z : \Omega_r = 2:1$ resonance being related to $\Omega_z : \Omega_r = 1:3$. Overlapping chaotic zones, which are related to resonances, enhance chaotic diffusion (e.g., Laskar 1993). We surmise that the most prominent gap in $(R_{\text{max}}, z_{\text{max}})$ shown here is due to the 3D $(\Omega_b : \Omega_r : \Omega_z) = (2:2:3)$ resonance.

Fig. 8. Similar to Fig. 7, but for the T Dtype sample.

Fig. 9. Similar to Figs. 7 and 8, but for the pureHalo sample.
Interestingly, a similar gap in \( z_{\text{max}} \) space was already discussed by Schuster & Allen (1997). Our analysis agrees well with the assessment of these authors, who ascribed the gap to the diffusion of chaotic orbits between two quasi-periodic orbit families. That is, the chaotic orbits ‘stick’ to the quasi-periodic regions – one on each side of the gap – and quickly diffuse in the regions between them. Schuster & Allen suggested that such a structure appears because >40% of the halo stars are on chaotic orbits in their Milky Way model (which is different to the one used in this work).

Figures 7–9 show that the velocity distributions of stars associated with resonances reveal the presence of clumps, which could potentially be confused with merger debris (e.g., the Arcurus stream, see Navarro et al. 2004 or Kushniruk & Bensby 2019). A more recent example is the reported prograde stellar stream, Nyx, with azimuthal velocities around 140 km s\(^{-1}\) (Necib et al. 2020), which could also be due to the Galactic bar acting on thick-disc-like stars, but which we did not consider here (e.g., see Monari et al. 2013 or Antoja et al. 2015). We note that, in most cases, the clumps associated with resonances do not correspond to overdensities in velocity space for the full data set, as can be seen from a comparison of Figs. 7–9 to Fig. 2.

4.4. Known accreted substructures in frequency space

We now explore the regions of frequency space occupied by the accreted substructures introduced in Sect. 3.3. Their kinematics and orbital parameters are shown in Fig. 5, while Fig. 11 shows their distribution in frequency space.

Figure 11 reveals that Gaia-Enceladus and Thamnos are seen to span rather large regions in frequency space. They are affected by resonances just like the whole halo subset. The Sequoia stars occupy a small region close to and around the \( \Omega_R : \Omega_z = 1:2 \) resonance. With the other hand, the Helmi streams (in red) occupy two relatively narrow regions in \( \Omega_R \) vs. \( \Omega_z \), and they also appear to be split in \( \Omega_R \) vs. \( \Omega_z \), with one of the two groups being on the \( \Omega_z : \Omega_R = 1:2 \) resonance. There are stars both from the positive and negative \( z \)-velocity clumps on the 1:2 resonance, with the associated stars being slightly colder kinematically. The splitting into clumps in frequency space could potentially be due to different wraps of the debris streams (Gómez & Helmi 2010), but clearly a much higher number of stars would be needed to confirm this interpretation.

5. Discussion

The analysis carried out in this work shows that substructures present in spaces associated with the orbital parameters of stars can be driven by properties of the gravitational potential in which the stars move. In particular, we see that a realistic (but non-integrable) Milky Way potential – with a disc and halo components – leads to the presence of well-populated orbital resonances. The trapping of orbits at resonances and chaotic zones leads to the depletion of stars in the regions around these resonances.
These results do not strongly depend on the choice for the gravitational potential assumed for the Milky Way, provided this is generic enough (and not integrable, e.g., of Stäckel form). We also find that in two other commonly used Galactic potentials (Piffel et al. 2014; Bovy 2015), very similar substructures can be observed in $K_{\text{max}}$ vs. $z_{\text{max}}$, as is shown in Appendix A, and that they are associated with resonant families, and hence with the presence of non-integrability in the system.

Our analysis (as also concluded by Amarante et al. 2020) therefore directly shows that not all substructures are related to accretion, as is often considered in the literature, nor to the settling of the gravitational potential after major merger activity as had been suggested by Haywood et al. (2018). Nonetheless, the characteristics of substructures from merger events may sometimes be related to the presence of resonances. For example, we see that some of the stars in the Helmi streams appear to be close to a resonance, $\Omega_z : \Omega_R \approx 1:2$. This perhaps explains why its stars are distributed asymmetrically in velocity space (the stream with $v_z < 0$ has more stars than that with $v_z > 0$). This asymmetry was used to constrain its time of accretion to approximately 5-8 Gyr ago, which is a puzzlingly low value given that these stars are on relatively bound orbits. Since stars near a resonance take longer to spread out in space (Vogelsberger & White 2008), this can lead to an underestimation of their time of accretion. This means that chaotic dynamics may have to be considered when modelling the evolution of tidal debris, as already hinted by Price-Whelan et al. (2015) in their modelling of cold thin streams farther away in the halo.

Probably the best way to tell whether substructures are related to accretion events is via a chemical tagging analysis. Stars that originate from a satellite will follow characteristic tracks in chemical abundance space. On the other hand, stars that group together because of dynamical resonances have no reason to be chemically distinct from other stars.

How stars populate different resonant families depends on their distribution function and the gravitational potential in which they move. In this work, we only considered that of the Milky Way, but a recent analysis of nearby thin-disc stars has shown that the Sagittarius dwarf also plays a role in their dynamics (see Antoja et al. 2018). Interestingly, if we integrate the orbit of Sagittarius in the McMillan (2017) potential for 40 Gyr with the initial conditions derived by the Gaia Collaboration (2018c) and apply the SuperFreq code to obtain its frequencies (i.e. we repeat the analysis done for the stars in our sample), we find that Sagittarius falls on the $\Omega_z : \Omega_R = 1:2$ and $\Omega_R : \Omega_\phi = 1:1$ resonance. The stars in our sample on the $\Omega_z : \Omega_\phi = 1:2$ resonance (green points in Figs. 7–9) define a rather distinct branch in frequency space. They also have rather different velocities, with high $v_z$ values. This intriguing coincidence probably deserves further investigations, as this is also the resonance where some of the Helmi stream stars lie.

External influences such as Sagittarius or the Large Magellanic Cloud are known to have an effect on the dynamics of the Galactic disc and of streams, both of which are so cold kinematically, easily revealing their presence (Garavito-Camargo et al. 2019; Petersen & Peñarrubia 2020a,b; Cunningham & Garavito-Camargo 2020; Erkal et al. 2020; Vasiliev et al. 2021). The long orbital periods of these satellites, in comparison to those of halo stars in the solar vicinity, would suggest that the perturbations will act adiabatically on their orbits. More precise conclusions would need orbital integrations including those systems as well as the ‘live’ response of the Galaxy. Of course, all of this does not take away the main conclusions of this work, no matter the exact equilibrium state of the Milky Way halo: structures that arise in the space of orbital parameters can be purely due to the gravitational potential model considered and are not necessarily related to merger events.

6. Conclusions

We studied the dynamical properties of nearby stars in the thick-disc and stellar halo using data from the second data release of the Gaia mission. We explored spaces of characteristic orbital parameters for a generic Galactic potential and found that these host a large amount of structures. Further analysis using orbital frequencies shows that these structures are due to the presence of resonant families (as claimed also by Amarante et al. 2020, see also Schuster & Allen 1997), and to the depletion of orbits around them due to non-integrability. Therefore, these structures reflect intrinsic properties of generic gravitational potentials of the Milky Way.

These findings are interesting on their own and highlight that a large number of stars in these components (nearly 30% for the halo sample and specifically for the McMillan (2017) potential) are on resonances. This gives us hope to use them to more precisely pinpoint the gravitational potential of the Milky Way. Valluri et al. (2012) argued that most stars are launched on regular tori, and that therefore chaotic diffusion should occur. The expectation would thus be that the fraction of stars on such irregular orbits would be the lowest for the true potential (as this would indicate self-consistency between the distribution function and the gravitational potential).

Frequency analysis is also interesting, because it can reveal the individual streams originating in an accretion event (Gómez & Helmi 2010). Our analysis shows hints of such imprints, but the number of stars is too low to reach meaningful conclusions. Furthermore, it may be particularly challenging to identify the individual streams or wraps, as these may be disguised and trapped by the resonances present in the phase space of halo stars.

Acknowledgements. We gratefully acknowledge financial support from a VICI grant and a Spinoza Prize from the Netherlands Organisation for Scientific Research (NWO) and HHK is grateful for the support from the Martin A. and Helen Choollian Membership at the Institute for Advanced Study. This work has made use of data from the European Space Agency (ESA) mission Gaia (http://www.cosmos.esa.int/gaia), processed by the Gaia Data Processing and Analysis Consortium (DPAC, http://www.cosmos.esa.int/web/gaia/dpac/consortium). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the Gaia Multilateral Agreement. For the analysis, the following software packages have been used: vaxx (Breddels & Veljanoski 2018), numpy (Van Der Walt 2011), matplotlib (Hunter 2007), jupyter notebooks (Kluyver et al. 2016).

References


A37, page 11 of 13
Appendix A: Variations between Milky Way models

We explore here the level of the model dependency of the structures in $R_{\text{max}} - z_{\text{max}}$ space analysed in the main text by comparing two key figures for a set of often-used Milky Way models. These are the MWpotential2014 model (Bovy 2015) and the Pfiff et al. (2014) model. For simplicity, we only show our findings for the pureHalo sample, which naturally has the largest range in $R_{\text{max}} - z_{\text{max}}$ space and therefore populates the full range of structures seen in this space. Figures A.1 and A.2 summarise our findings, and they clearly show that the variations between the models are small.

Figure A.1 shows the same space as the one shown in the middle panel in the bottom row of Fig. 3. For simplicity, we also show the version using the McMillan17 potential here, this panel is the same as the one in the main text. The wedges in $R_{\text{max}} - z_{\text{max}}$ space are present in all models, although they are clearly not equivalent. The large-scale structures (i.e. the three large wedges) are in approximately the same location. However, the fine substructure on top of these large wedges appears to be more sensitive to the model. We do note that some of this is due to the visualisation method and the sample size. For a larger sample that populates a larger fraction of the $R_{\text{max}} - z_{\text{max}}$ space, we expect to find more and finer substructures.

Similarly, Fig. A.2 shows the variation of the bottom-left panel of Fig. 6 as a function of the three Milky Way models (the McMillan17 panel is also equivalent to the one in the main text and is shown to facilitate comparison). Also in this space, the similarities between the three models are apparent.

---

**Fig. A.1.** Comparison of the $R_{\text{max}} - z_{\text{max}}$ space for three often-used Milky Way models. The left panel is equivalent to the bottom middle panel of Fig. 3 in the main text and is repeated here for convenience.

**Fig. A.2.** Comparison of the $\Omega_{\text{R}} - \Omega_{\text{z}}$ space for three often-used Milky Way models. The left panel is equivalent to the bottom-left panel of Fig. 6 in the main text and is repeated here for convenience.