Constraining the accretion flow density profile near Sgr A* using the L' -band emission of the S2 star

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ABSTRACT

Context. The density of the ambient medium around a supermassive black hole (SMBH) and the way it varies with distance plays an important role in our understanding of the inflow-outflow mechanisms in the Galactic centre (GC). This dependence is often fitted by spherical power-law profiles based on observations in the X-ray, infrared (IR), submillimetre (submm), and radio domains.

Aims. Nevertheless, the density profile is poorly constrained at the intermediate scales of 1000 Schwarzschild radii (R_s). Here we independently constrain the spherical density profile using the stellar bow shock of the star S2 which orbits the SMBH at the GC with the pericentre distance of 14.4 mas ($\sim 1500 R_s$).

Methods. Assuming an elliptical orbit, we apply celestial mechanics and the theory of bow shocks that are at ram pressure equilibrium. We analyse the measured IR flux density and magnitudes of S2 in the L' -band (3.8 micron) obtained over seven epochs in the years between 2004–2018. We put an upper limit on the emission from S2's associated putative bow shock and constrain the density profile of the ambient medium.

Results. We detect no significant change in S2 flux density until the recent periape in May 2018. The intrinsic flux variability of S2 is at the level of 2–3%. Based on the dust-extinction model, the upper limit on the number density at the S2 periape is $\sim 1.87 \times 10^9 \text{ cm}^{-3}$, which yields a density slope of at most 3.20. Using the synchrotron bow-shock emission, we obtain the ambient density of $\leq 1.01 \times 10^5 \text{ cm}^{-3}$ and a slope of ≤ 1.47 . These values are consistent with a wide variety of media from hot accretion flows to potentially colder and denser media comparable in properties to broad-line-region clouds. However, a standard thin disc can be excluded at the distance of S2's pericentre.

Conclusions. With the current photometry sensitivity of 0.01 mag, we are not able to make stringent constraints on the density of the ambient medium in the GC using S2-star observations. We can distinguish between hot accretion flows and thin, cold discs, where the latter can be excluded at the scale of the S2 periape. Future observations of stars in the S cluster using instruments such as Mid-IR Extremely Large Telescope Imager and Spectrograph at Extremely Large Telescope with the photometric sensitivity of as much as 10^{-3} mag will allow the GC medium to be probed at intermediate scales at densities as low as $\sim 700 \text{ cm}^{-3}$ in case of non-thermal bow-shock emission. The new instrumentation, in combination with discoveries of stars with smaller pericentre distances, will help to independently constrain the density profile around Sagittarius A* (Sgr A*).

Key words. infrared: stars – black hole physics – Galaxy: center

1. Introduction

The Galactic centre (GC) is a unique astrophysical setting where one can study the dynamical effects in one of the densest stellar clusters (Alexander 2005; Merritt 2013; Schödel et al. 2014) and the mutual interaction of stars, gaseous-dusty structures, the magnetic field, and the supermassive black hole (SMBH; Genzel et al. 2010; Eckart et al. 2017). The dynamical centre traced by orbiting stars coincides with the non-thermal compact and variable radio source Sagittarius A* (Sgr A*) within $0.03''$ (Menten et al. 1997). Because of its well-constrained mass of $M_{\text{BH}} = (4.15 \pm 0.13 \pm 0.57) \times 10^6 M_{\odot}$ (Parsa et al. 2017) based mainly on monitoring of fast-moving S stars (see also Schödel et al. 2002; Ghez et al. 2008; Gillessen et al. 2009, 2017; Boehle et al. 2016), Sgr A* is associated with the SMBH, leaving only a little space for alternative scenarios of its compact nature (Eckart et al. 2017). Given its distance of 8.1 kpc (Eisenhauer et al. 2003; Reid et al. 2003; Gravity Collaboration

2019), it is the nearest SMBH, which allows us to perform high-precision astrometric and interferometric observations of Sgr A* and its immediate surroundings. In recent years, this provided an opportunity to confirm general relativistic predictions by measuring the gravitational redshift (Gravity Collaboration 2018a; Do et al. 2019a) as well as the Schwarzschild precession (Gravity Collaboration 2020) by monitoring the bright S2 star (also referred to as S0-2) that orbits Sgr A* with a period of 16.2 years.

Sgr A* belongs to extremely low-luminous sources, with its bolometric luminosity that is eight to nine orders of magnitude below its Eddington limit (Narayan et al. 1998; Yuan & Narayan 2014). Its broadband spectral energy distribution is best described by a class of radiatively inefficient accretion flows (RIAFs; Yuan et al. 2003). In order to better comprehend its low-luminous, diluted accretion flow, it is of prime importance to constrain its density at large, intermediate, and eventually small spatial scales. This has been done by

analysing the radiative properties of its surroundings. At larger scales, the analysis of the X-ray bremsstrahlung emissivity profile of 1.3 keV plasma in the central hot bubble yielded a number density of $n_B \approx 26 \eta_f^{-1/2} \text{cm}^{-3}$ (with η_f being the filling factor), close to the Bondi radius (Baganoff et al. 2003; Wang et al. 2013), which lies at $r_B \approx 4'' (T_a/10^7 \text{K})^{-1} \approx 0.16 \text{pc}$ given the plasma temperature T_a (Wang et al. 2013). Density constraints at the scale of 10–100 Schwarzschild radii (hereafter denoted r_s) were obtained by analysing the polarised millimetre (mm) and submillimetre (submm) emission, which implies non-thermal self-absorbed synchrotron radiation in the mm domain that becomes optically thin at submm wavelengths (Aitken et al. 2000; Agol 2000; Bower et al. 2003). Marrone et al. (2006, 2007) detected the Faraday rotation of the polarised submm emission based on which they constrain the accretion-rate of Sgr A* to $\dot{M} \sim 2 \times 10^{-9} - 2 \times 10^{-7} M_\odot \text{yr}^{-1}$, which corresponds to the maximum number density of $n_a^{\text{max}} < 1.4 \times 10^7 \text{cm}^{-3}$ close to the event horizon. Generally, the hot accretion flow in the GC is described by a power-law density profile $n \propto r^{-\gamma}$, where the power-law slope is typically inferred to be $\gamma \sim 0.5 - 1.0$ (Wang et al. 2013) in agreement with the RIAF-type flows. Spherical Bondi-type flow with $\gamma \sim 3/2$ is also consistent with the X-ray surface brightness profile up to $3''$ from Sgr A* (Róžańska et al. 2015). Hence, the number densities of the ambient flow and the corresponding slopes are still quite uncertain, especially at intermediate spatial scales where the fast-moving S stars are located. This uncertainty was further enhanced by the possible detection of a cooler ($\sim 10^4 \text{K}$) and a denser disc with a number density of $\sim 10^5 - 10^6 \text{cm}^{-3}$ at the radius of $\sim 0.004 \text{pc}$, which is comparable to the semi-major axis of the bright S2 star (Murchikova et al. 2019). However, the detection by Murchikova et al. (2019) is questionable, especially the association of the recombination double-peak line of H30 α at 1.3 mm with the gaseous material at milliparsec spatial scales; it is especially controversial given that near-infrared (NIR) line maps, for example of the recombination Br γ line that can trace ionised material of 10^4K , do not seem to imply any existence of such a compact disc-like structure. On the other hand, there are indications of denser material traced by the Br γ line (Peißker et al. 2020a) and the blueshifted H30 α line (Royster et al. 2019) at larger spatial scales closer to the Bondi radius, which the study of Murchikova et al. (2019) could be associated with. This underlines the need for better and independent constraints on the accretion-flow density at the intermediate range of radii.

The fast-moving stars of the S cluster could in principle be used to constrain the accretion flow density at intermediate scales. As fast-moving probes, these stars drive shocks into the ambient medium and provide radiative energy that can temporarily affect the radiative properties of the accretion flow as well as form localised density and extinction enhancements. Any detected flares linked to an orbiting star would thus provide a way to infer density and other properties of the intercepted ambient medium. Quataert & Loeb (2005) analysed the possibility that the stellar wind shocks in the central arcsecond could contribute to the ambient TeV and non-thermal broad-band emission. It was predicted that the interaction of the fast-moving B-type S2 star with the ambient hot accretion flow is expected to form a bow shock, which should result in a month-long thermal X-ray bremsstrahlung flare with the peak luminosity of $L_X = 4 \times 10^{33} \text{erg s}^{-1}$ (Giannios & Sironi 2013; Christie et al. 2016). The contribution to the emerging X-ray flux is also expected to come from UV and optical photons of S2 that are Compton-upscattered by electrons of the inner hot RIAF (Nayakshin 2005). According to these models, the S2 bow-shock

X-ray luminosity is in principle comparable to the quiescent X-ray emission of Sgr A* and hence could be detected with the current X-ray instruments. The non-detection of any significant X-ray flare around the pericentre crossings of S2 star in 2002 and 2018 is consistent with a diluted hot RIAF flow in the central arcsecond (Yuan et al. 2003). The low X-ray flux from the bow shock is also in agreement with numerical 3D adaptive-mesh refinement simulations of the motion of S2 through the RIAF (Schartmann et al. 2018).

The detection and the monitoring of the dusty G2 source or the Dusty S-cluster Object (DSO; Gillessen et al. 2012; Eckart et al. 2013; Witzel et al. 2014; Valencia-S et al. 2015; Peißker et al. 2020b; Ciurlo et al. 2020) triggered an effort to predict and observe its potential interaction with the ambient environment. Narayan et al. (2012) predicted the radio synchrotron emission from the DSO bow shock to be comparable to the quiescent radio emission of Sgr A*. In theory, electrons in the ambient accretion flow are expected to be accelerated in the bow-shock region. The main sources of uncertainty are the bow-shock size, which depends on the stellar-wind characteristics and the ambient density, and the magnetic field strength that is enhanced in the bow shock. For the more realistic estimates of the DSO/G2 bow-shock size, assuming the low-mass star model, the radio synchrotron emission was calculated to be well below the radio emission of Sgr A*, and therefore no detectable flare was expected (Crumley & Kumar 2013; Zajaček et al. 2016). This was observationally confirmed by the non-detection of any enhanced radio or mm emission by Bower et al. (2015) and Borkar et al. (2016), who constrain the DSO/G2 cross-section to $< 2 \times 10^{29} \text{cm}^2$.

The bow-shock synchrotron emission can be generalised to the S stars, predicting their broad-band synchrotron spectrum with the peak close to 1 GHz and associated monochromatic light-curves (Ginsburg et al. 2016). Ginsburg et al. (2016) estimated that the synchrotron emission from S-star bow shocks can be comparable to the Sgr A* radio (10 GHz) and IR flux densities (10^{14}GHz) for rather extreme combinations of the wind mass-loss rate and its terminal velocity – $(\dot{m}_w, v_w) = (10^{-5} M_\odot \text{yr}^{-1}, 10^3 \text{km s}^{-1})$ and $(\dot{m}_w, v_w) = (10^{-6} M_\odot \text{yr}^{-1}, 4 \times 10^3 \text{km s}^{-1})$. These values deviate from those inferred for S2 (Martins et al. 2008; Habibi et al. 2017): $(\dot{m}_w, v_w) = (< 3 \times 10^{-7} M_\odot \text{yr}^{-1}, 10^3 \text{km s}^{-1})$. For these observationally inferred values, the S star non-thermal flux densities are below those of Sgr A*. Another, independent way to constrain the density of the ambient medium is the detection of deviation from the Keplerian ellipse due to hydrodynamic drag force. Gillessen et al. (2019) claimed the detection of a significant deviation for the DSO/G2 object due to the drag force, from which they inferred a density of $4 \times 10^3 \text{cm}^{-3}$ at the DSO/G2 pericentre close to $1000 r_s$.

Here we report NIR L' -band emission from one of the brightest members of the S cluster, star S2. Close to its first observed pericentre passage in 2002, Clénet et al. (2004) reported a brightening of S2 in L' -band, which could be associated with its interaction with the ambient medium at the scale of 1500 Schwarzschild radii¹. Based on our reanalysis of the 2002 epoch and an additional seven epochs, including close to the subsequent pericentre passage in 2018, we can exclude any significant brightening related to the pericentre passage of S2 in L' -band. The intrinsic fractional variability of the S2 light curve is only $\sim 2.5\%$. Using the model of the thermal dust emission in the shocked ambient medium, we place an upper limit on

¹ The S2 pericentre distance according to Gravity Collaboration (2018a) is 1513 Schwarzschild radii.

Fig. 1. S2 star and known dusty objects close to its orbit. *Left panel:* position of S2 and the dusty source G1 close to the pericentre of S2 around 2002. We also show the orbit of the DSO (Dusty S-cluster Object, also known as G2). The points of the sources represent colour-coded radial velocities according to the axis on the right. The grey circle in the bottom left corner represents the diffraction limited FWHM of 98 mas corresponding to NACO L' -band. *Right panel:* position of S2 and dusty source G1 at the pericentre passage of S2 at 2018.38. The orbital parameters were adopted from [Gravity Collaboration \(2018a\)](#) (S2 star), [Valencia-S et al. \(2015\)](#) (DSO source), and [Witzel et al. \(2017\)](#) (G1 source).

the ambient density of $\lesssim 10^9 \text{ cm}^{-3}$, which can accommodate a wide variety of ambient media at the S2 pericentre distance of 1500 Schwarzschild radii, except for a standard thin cold disc. The bow-shock synchrotron model gives a tighter upper limit of $\sim 10^5 \text{ cm}^{-3}$. An improved photometric sensitivity of the next-generation of instruments, in particular Mid-IR Extremely Large Telescope Imager and Spectrograph (METIS) at Extremely Large Telescope (ELT), will help to further constrain the density at intermediate scales.

The current paper is structured as follows. Section 2 describes the observations and data-reduction methods used for this work. We then briefly discuss the photometry and its results for S2 in Sects. 2.1 and 2.2, respectively. In Sect. 3 we constrain the slope and density of the ambient accretion flow using S2 star observations in L' band. Section 3.1 is dedicated to the exploration of the extinction caused by the ambient gas and dust inside the Bondi radius. In Sect. 3.2, the upper limit of the ambient density is inferred based on the thermal bow-shock emission. In Sect. 3.3, we then constrain the ambient density based on the non-thermal bow-shock emission. We discuss the obtained density values in the broader context of the accretion flows and the GC gaseous-dusty structures in Sect. 4. Finally, in Sect. 5 we summarise the main results.

2. Observations and data reduction

Observations of the GC in the NIR were carried out with the European Southern Observatory (ESO) Very Large Telescope (VLT) Unit 1 (UT1)-YEPUN on Paranal, Chile. The data were obtained from the IR camera Coude NIR Camera (CONICA) and the Nasmyth Adaptive Optics System (NAOS) adaptive optics (AO) module, known as NACO. The central parsec of the GC has been frequently observed with the VLT in the L' -band ($3.8 \mu\text{m}$, $0.0271'' \text{ pixel}^{-1}$).

In Table 1, we summarise the obtained data that are analysed in this study. The total time-coverage of the data set is from 2004.317 until 2018.307. The only applied criterion on the epoch selection is the imaging quality, because a large number of the available data sets in the archive were observed during poor atmospheric conditions, in particular bad and fast seeing. We

used the standard data-reduction process that consists of the sky subtraction, flat-fielding, and bad pixel correction for all imaging data and combined them via a shift-add algorithm to obtain a final array size of $28'' \times 28''$ for all epochs.

The first observed periairpse of S2 occurred in 2002. The spatial resolution of NACO at $3.8 \mu\text{m}$ is $0.096''$, while the distance between Sgr A* and S2 in 2002 is approximately $0.040''$. Therefore, it is impossible to obtain the emission of S2 from the images completely separated from Sgr A* emission. Furthermore, we know that Sgr A* is flaring in the observation which is done in 2002 ([Genzel et al. 2003](#)), and in 2002 and 2003 the NIR-excess extended source denoted G1 is sufficiently close to the S2-Sgr A* system that it cannot be disentangled from the Sgr A*-S2 system ([Clénet et al. 2004](#); [Witzel et al. 2017](#)).

We show the superposition of S2 and G1 in Fig. 1, where we plot the projected orbits of S2, G1, and DSO/G2. Immediately before the pericentre passage around 2002, the mutual separation between S2 and G1 was less than 0.1 arcsec ; see Fig. 1 (left panel). The G1 cloud is currently going to larger distances from Sgr A* (more than 0.2 arcsec , see Fig. 1, right panel), and therefore potential changes of the brightness of S2 after 2002 pericentre passage would be due to other effects, most likely due to bow-shock emission, which we analyse in this contribution. Hence, to prevent further uncertainties in our analysis, we excluded the first observed periairpse of S2.

2.1. Photometry

Flux densities of S2 and calibrators were determined via aperture photometry using a circular aperture of 82 mas in L' -band. Dereddened fluxes were computed with the extinction of $A_L = 1.09 \pm 0.13$ ([Fritz et al. 2011](#)) and a zero magnitude value of $F_0(L) = 249 \text{ Jy}$.

The magnitude calibration is done by considering S26, S30, and S35 as calibrators using their known brightness from [Gillessen et al. \(2009\)](#). These S-stars are chosen because they are sufficiently isolated over the epochs of our study. In Fig. 2, we show the surroundings of S2 within about $\pm 1''$ on 9 May 2013.

In general, the grainy background at the centre of our Galaxy is variable due to the proper motion of the stars ([Sabha et al. 2012](#)). Therefore, to correct for background brightness, we used

Fig. 2. NACO L' -band mosaic of the GC in 2013.356. North is up and east is to the left. The identification of three stars (S26, S30 and S35) as photometric calibrators is depicted in particular. The black cross shows the IR counterpart of Sgr A*. In order to check any variability in the flux of S2, we used S65 as a reference star.

five apertures with the same size as our photometric apertures close enough to the calibrators and S2 to get an appropriate representative of the existing background. Because of the high-velocity S-stars, the background is highly variable, and therefore background apertures are not located at the same position for all the epochs considered in this study.

2.2. Photometry results

S2 is an early B-type dwarf of spectral type B0–B2.5 V with a stellar wind that has an estimated velocity of $v_w \sim 1000 \text{ km s}^{-1}$ and a mass-loss rate of $\dot{m}_w \lesssim 3 \times 10^{-7} M_\odot \text{ yr}^{-1}$ (Martins et al. 2008). We did the aperture photometry with local background subtraction for seven selected epochs from 2004 to 2018. S65 flux and magnitude were also studied in order to use this object as a test star to reveal any meaningful changes in S2 flux or magnitude.

The photometry results are presented in Table 1. As shown in Fig. 3, there is essentially no change detected in the measured flux density of S2 within the uncertainties over the seven epochs. The total coverage is 15 years, which is close to the S2 orbital period around Sgr A*, including epochs close to the pericentre passages of S2. The light curve has a mean value of the flux density of $\bar{F} = 8.88 \pm 0.33 \text{ mJy}$, with a difference between maximum and minimum values of 0.90 mJy. In terms of the L' -band magnitude, the mean magnitude of S2 in the L' -band is 11.12 ± 0.04 . The intrinsic variability of S2 is characterised by a normalised excess variance (Nandra et al. 1997),

$$\sigma_{\text{rms}}^2 = \frac{1}{N\bar{f}^2} \sum_{i=1}^N [(f_i - \bar{f})^2 - \sigma_i^2], \quad (1)$$

where f_i are individual flux measurements, $\bar{f} = 1/N \sum_{i=1}^N f_i$ is the mean flux density, and σ_i are individual measurement errors. We obtain the value of the fractional variability $F_{\text{var}} = \sqrt{\sigma_{\text{rms}}^2} = 2.52\%$ in terms of the flux density and $M_{\text{var}} = 0.22\%$ in terms of the magnitude. For the control star S65, we obtain $F_{\text{var}} = 4.53\%$, which is almost a factor of two larger than S2. These values are

fully consistent with no intrinsic changes in the L' -band flux density of S2 within measurement uncertainties. For completeness, the mean, the maximum difference, and the fractional variability F_{var} are listed in Table 1 for both S2 and S65.

In order to study the S2 flux and magnitude closely during its periape in May 2018 at around $1500 R_s$ distance from Sgr A*, we performed the aperture photometry on all single exposures from the observation on 2018.307, on the position of S2 and the IR counterpart of radio source Sgr A*. The physical separation of S2 and Sgr A* was only 1.38 mpc, corresponding to 34.6 mas, while the angular resolution is 99.5 mas in L' -band. The photometry results are shown in Fig. 4. As can be seen, there is no sudden increase in the received flux from the position, and therefore Sgr A* is in its quiescent state in L' -band over the observation 2018.307. In addition to the non-detection of any flare from the IR counterpart of Sgr A*, no apparent increase in the S2 flux was observed within the measurement uncertainties.

For the 2002.663 epoch, we measure $17.41 \pm 1.18 \text{ mJy}$ at the –at that time– combined position of S2, G1, and L' -band counterpart of Sgr A*. Taking into account S2's mean stellar flux density of $8.88 \pm 0.33 \text{ mJy}$ (reddened with $A_L = 1.09$) and the G1 L' -band flux density of $\sim 2.58 \pm 1.40 \text{ mJy}$ close to the 2002 epoch (Witzel et al. 2017), the flux of Sgr A* ends up being around $6.03 \pm 0.52 \text{ mJy}$. Schödel et al. (2011) obtains the mean flux of Sgr A* in the L' -band as $4.33 \pm 0.18 \text{ mJy}$, which leads us to 1.70 mJy surplus in the measured flux density. However, due to the lack of a longer (at least days) light-curve time coverage during the S2 periape passage of the variable source black hole counterpart Sgr A* in 2002, it is not possible to determine whether this flux density excess is due to a Sgr A* flare or the S2 stellar bow-shock effect. However, since during the subsequent pericentre passage in 2018 the flux density of S2 is within the uncertainty comparable to its mean value (see Table 1), the excess can be attributed to the Sgr A* flare, unless the accretion flow density changes dramatically by several orders of magnitude from one periape to another. The flux excess of 1.70 mJy that corresponds to the magnitude change of $\sim 0.19 \text{ mag}$ would require the pericentre number density of $\sim 4.2 \times 10^{10} \text{ cm}^{-3}$ according to the dust-extinction model we present in Sect. 3.2. Considering the synchrotron bow-shock emission (see Sect 3.3), the required density at the pericentre is smaller, $n_a \sim 2.1 \times 10^5 \text{ cm}^{-3}$, but still larger by two orders of magnitude than expected for the RIAF density profile with $\gamma \approx 1$, $n_a \sim 7.2 \times 10^3 \text{ cm}^{-3}$. Such a large density present only for epoch 2002 is unlikely. The excess flux of 1.70 mJy can therefore be attributed to the L' -band flare of Sgr A*. In the L' -band, Sgr A* is permanently variable (Schödel et al. 2011) with the flux densities in the range between 1 mJy and 10 mJy (Ghez et al. 2005; Eckart et al. 2008; Dodds-Eden et al. 2009).

3. Constraining the slope and density of the ambient accretion flow using S2's L' -band emission

In this section, we develop a simple method for inferring the density slope and the maximum density at the pericentre of a wind-blowing star using the observed light curve and an analytical dust extinction model. This method is suitable for stars with an unresolved bow shock. For a fully resolved bow shock at least two positions of a star along an elliptical orbit, one can infer the density slope from the bow-shock size ratio and an orbital eccentricity; see Appendix B for more details. Furthermore, we also consider the non-thermal bow-shock

Table 1. L' -band flux and magnitude of S2 and S65 for seven epochs over a 15-year interval from 2004.317 to 2018.307.

UT date	Decimal date	S2 flux (mJy)	S65 flux (mJy)	S2 mag	S65 mag	Observation ID
2004 April 25	2004.317	9.02 ± 0.30	14.56 ± 0.94	11.10 ± 0.04	10.59 ± 0.07	60.A-9026(A)
2005 May 14	2005.367	9.30 ± 0.30	15.07 ± 0.91	11.07 ± 0.04	10.55 ± 0.07	073.B-0085(D)
2006 May 29	2006.408	9.30 ± 0.30	15.07 ± 0.91	11.07 ± 0.04	10.55 ± 0.07	077.B-0552(A)
2007 April 01	2007.249	8.88 ± 0.19	14.49 ± 0.63	11.12 ± 0.02	10.59 ± 0.05	179.B-0261(A)
2008 May 26	2008.402	8.54 ± 0.15	14.19 ± 0.59	11.16 ± 0.02	10.61 ± 0.05	081.B-0648(A)
2013 May 09	2013.356	8.40 ± 0.25	14.82 ± 1.12	11.18 ± 0.03	10.57 ± 0.08	091.C-0159(A)
2018 April 22	2018.307	8.70 ± 0.10	13.62 ± 0.33	11.13 ± 0.01	10.64 ± 0.03	0101.B-0052(B)
Mean		8.88 ± 0.33	14.55 ± 0.48	11.12 ± 0.04	10.59 ± 0.03	
difference (max – min)		0.90	1.45	0.11	0.09	
$F_{\text{var}} = \sqrt{\sigma_{\text{rms}}^2} (\%)$		2.52	4.53	0.22	0.51	

Notes. The DIT for all cases is 0.200 s. Light-curve characteristics, namely the mean, the difference between the maximum and the minimum, and the fractional variability are also included. With the fractional variability of 2.52%, no significant difference is detected for S2 flux within the uncertainties. The reported fluxes are dereddened.

Fig. 3. Light curve of S2 and S65 in L' -band over seven observational epochs. **Fig. 4.** Light curve at the position of S2 and the IR counterpart of Sgr A* and our reference star, S65, in 2018.307 in L' -band. The light curve shows that the IR counterpart of Sgr A* is in quiescence during our observation, and therefore the light curve belongs to S2.

emission as an independent way of constraining the ambient density. The S2 star is currently intensively monitored by the Very-Large-Telescope Interferometer GRAVITY, as well as the NIR imager NACO and SINFONI at VLT, which is capable of performing integral field spectroscopy. Using the

data over the past 25 years, post-Newtonian effects were measured, including the gravitational redshift (Gravity Collaboration 2018a) and the Schwarzschild precession (Parsa et al. 2017; Gravity Collaboration 2020), which are consistent with general relativity so far. Its highly eccentric orbit ($e = 0.885$) implies that the ambient gas density is expected to change along its orbit; it is expected to be the lowest close to its apocentre and the highest close to the pericentre due to the radial power-law profile of the hot X-ray atmosphere (Wang et al. 2013). The dust component is expected to coexist to a certain extent in this hot environment, which is also manifested by the presence of L' -band dusty sources in the central ~ 0.04 pc, namely DSO/G2, G1, and several other dusty and bow-shock sources in the central arcsecond (Gillessen et al. 2012; Valencia-S et al. 2015; Witzel et al. 2017). In particular, for the orbital solutions and radiative properties of the dust-enshrouded objects, see Peißker et al. (2020b) and Ciurlo et al. (2020). An increase in the ambient gas-and-dust density ratio along with the change in the orbital velocity are expected to lead to a larger local extinction around S2 due to the formation of a bow shock, and hence to a change in the NIR magnitudes and corresponding colour indices. We considered here in particular the NIR L' -band emission of S2, which can trace such a change as S2 moves through the accretion flow of different gas and dust density; see Fig. 5 for an illustration. A potential formation of a stellar bow shock associated with S2 can enhance the colour change despite the insufficient capability of current NIR instruments to resolve the bow-shock structure; see Fig. 5. Both the detection as well as the non-detection of the L' -band magnitude can be used to constrain the density as well as the slope of the gas-and-dust density distribution. The density changes along the S2 orbit depends on the density slope γ . Assuming the radial gas distribution, $n_a \approx n_0(r/r_0)^{-\gamma}$, where $\gamma > 0$, the density ratio between the pericentre and the apocentre can be estimated as,

$$\frac{n_{a,P}}{n_{a,A}} = \left(\frac{r_A}{r_P}\right)^\gamma = \left(\frac{1+e}{1-e}\right)^\gamma. \quad (2)$$

As the orbital eccentricity is well constrained for S2, $e = 0.885$, the ratio can be estimated to be $n_{a,P}/n_{a,A} \sim 4.05, 16.39$, and 66.36 for the plausible density slopes of $0.5, 1.0$, and 1.5 , where the first two values represent the radiatively inefficient accretion flow (see e.g. Xu et al. 2006; Wang et al. 2013) and the last value corresponds to the spherical steady Bondi flow (Różańska et al. 2015). As the variation in density is large (by a factor of 16.4) for a rather small variation in the density slope (by a factor of

Fig. 5. Illustrative figure of the motion of S2 through the hot flow around Sgr A* of variable density. The density may change by as much as an order of magnitude from the apocentre to the pericentre, from several hundred particles per cubic centimetre to several thousand particles per cubic centimetre, respectively, depending on the power-law slope of the radial number density distribution. For the exemplary calculation, we used $n_a = n_B(r/r_B)^{-1}$, where $n_B = 20.3 \text{ cm}^{-3}$ is the particle density at the Bondi radius. The white circle in the bottom left corner depicts the L' -band point spread function corresponding to 8m-class telescopes, from which it is apparent that the S2 bow shock remains unresolved, especially close to the pericentre.

Fig. 6. Temporal evolution of the number density of the ambient medium for the three different density slopes of the radial density profile, $n_a = n_0(r/r_0)^{-\gamma}$, where γ is equal to 0.5, 1.0, and 1.5; see the plot legend. The dashed red lines indicate the number densities that were observationally inferred by Baganoff et al. (2003) (26 cm^{-3} at the Bondi radius) and Gillessen et al. (2019) (4000 cm^{-3} at $\sim 1000 r_s$).

three), the L' -band observations of S2 can be used to constrain the density distribution of the hot accretion flow close to Sgr A* at the length scales of the order of 1500 Schwarzschild radii, where there are essentially no constraints for the ambient density (see, however, Gillessen et al. 2019).

In the further model, we approximate the orbit of S2 with an ellipse with a semimajor axis of $0.12538'' \approx 5 \text{ mpc}$ and an eccentricity of 0.88473 according to recent GRAVITY measurements (Gravity Collaboration 2018a). These orbital elements imply that S2 covers a large distance range, with a pericentre distance of $r_p = 119.2 \text{ AU} = 1512 r_s$ and an apoc-

Fig. 7. Total SED of the S2 star (black solid line) around L' band.

entre distance of $r_a = (1 + e)/(1 - e)r_p = 16.35r_p = 24724 r_s$. The power-law density distribution is adopted according to the Chandra X-ray studies of the extended emission of Sgr A* in the X-ray domain (Baganoff et al. 2003; Wang et al. 2013). The density distribution can be scaled with respect to the Bondi capture radius, $n_a \approx n_{a,B}(r/r_B)^{-\gamma}$, where $n_{a,B} \approx 26\eta_f^{-1/2} \text{ cm}^{-3}$ and $r_B \approx 4''(T_a/10^7 \text{ K})^{-1} = 0.16 \text{ pc}$ (Baganoff et al. 2003; Wang et al. 2013). Figure 6 shows the number gas density per cubic centimetre (cm) that is expected to be intercepted by S2 star during its entire orbit (zero time stands for the orbit periape). Different lines stand for different density slopes (0.5; 1.0; 1.5) according to the key. The bottom dashed red line depicts the inferred number density close to the Bondi radius according to X-ray observations of Baganoff et al. (2003), while the upper dashed red line stands for the number density of $n_a^{\text{DSO/G2}} \sim 4 \times 10^3 \text{ cm}^{-3}$ inferred for the periape of the DSO/G2 object (Gillessen et al. 2019).

We see that due to an eccentric orbit, S2 can in principle serve as a good probe of the ambient gaseous-dusty medium. For the S2 stellar blackbody emission, we adopt the stellar parameters that can reproduce the L' flux density of $\sim 9 \text{ mJy}$. For the model with $\log(L_{S2}/L_\odot) = 4.54$, $T_{S2} = 26800 \text{ K}$, $R_{S2} = 6 R_\odot$, we obtain the L' flux density of $F_L = 8.9 \text{ mJy}$; see also Fig. 7 for the spectral energy distribution (SED) of S2 around L' -band.

3.1. Extinction estimate inside the Bondi radius

We assume that the ambient gas inside the Bondi radius has a power-law profile, $n_a \approx n_{a,B}(r/r_B)^{-\gamma}$, which we scaled to the number density at the Bondi radius r_B . To estimate the local extinction due to the gas and dust inside the Bondi radius, we calculate the local column density of hydrogen,

$$N_h = \int_{r_{S2}}^{r_B} n_a(r) dr = \frac{n_{a,B} r_B^\gamma}{1 - \gamma} (r_B^{1-\gamma} - r_{S2}^{1-\gamma}), \quad (3)$$

where we integrated from a certain inner radius associated with S2, in this case its pericentre distance of $r_p = a_{S2}(1 - e_{S2}) \approx 0.577 \text{ mpc}$, up to the Bondi radius. The column density estimate for the number density of $n_B \approx 26 \text{ cm}^{-3}$ (Baganoff et al. 2003) at the Bondi radius and the maximum density power-law slope of $\gamma \approx 1.5$ for the Bondi-type flow is $N_h(\gamma = 1.5) = 4.02 \times 10^{20} \text{ cm}^{-2}$, whereas for the flat slope of $\gamma = 0.5$, we get $N_h(\gamma = 0.5) = 2.41 \times 10^{19} \text{ cm}^{-2}$. The visual extinction due to the ambient gas inside the Bondi radius for the steep density

Fig. 8. Number density and thickness of the two shock layers in the two-shock approximation model as a function of the distance from Sgr A* (in Schwarzschild radii). *Left:* number density (expressed in cm^{-3}) of the shock driven into the ambient medium (red lines) and of the shocked stellar wind (blue lines) calculated for three specific values of the ambient density slope: $\gamma = 0.5, 1.0$, and 1.5 . *Right:* thickness (expressed in cm) of the stellar-wind (blue lines) and ambient shock layers (red lines) calculated for the three different density slopes as in the *left panel*.

profile ($\gamma = 1.5$) is given by $A_V \approx 5.6 \times 10^{-22} N_h (\text{cm}^{-2}) = 0.23 \text{ mag}$ (see e.g. [Reina & Tarengi 1973](#); [Gorenstein 1975](#); [Predehl & Schmitt 1995](#)), which corresponds to the extinction $A_K \sim 0.1 A_V = 0.023 \text{ mag}$ in the NIR K -band and $A_L \approx 0.5 A_K = 0.012 \text{ mag}$ in the NIR L' -band (see [Schödel et al. 2010](#), for the extinction analysis towards the GC), in which our observations were carried out. For the flat density profile of $\gamma = 0.5$, we get $A_L \approx 0.0007 \text{ mag}$. Therefore, no increase in the L' -band magnitude within uncertainties is expected from the S2 plunging in deeper towards higher ambient densities. In summary, the upper limit on the ambient extinction inside the Bondi radius is $A_L \lesssim 0.012 \text{ mag}$. The only substantial increase at longer NIR wavelengths can be produced locally due to the bow-shock formation characterised by its density, radial scale, and thickness, which allows us to estimate the extinction.

3.2. Constraining the ambient density based on the thermal bow-shock emission

The S2 bow shock is modelled using a two-shock scenario ([Dyson 1975](#)), where one shock is driven into the ambient medium (ambient shock) and the other is formed from the shocked stellar wind (stellar-wind shock). These two layers are separated by a contact discontinuity at a distance approximately given by the stagnation radius; see Eq. (B.1). This treatment is based on the analytical theory of [Dyson \(1975\)](#), who modelled an interaction of a fast stellar wind with an expanding gas globule. In the first approximation, two layers (warm and cold) mix to form one shocked layer. In the second approximation, two layers are separated by a contact discontinuity. We apply the second approximation in a similar way to [Scoville & Burkert \(2013\)](#) for the GC NIR-excess source DSO/G2 interacting with the ambient diluted medium (see also their Fig. 3 for the illustration). The extinction estimate is based on the number density of gas and dust inside the shock and its characteristic length-scale, or ‘thickness’. At the stagnation point, the ambient shock is characterised by the number density $n_{\text{ams}} = 4n_a$ and a thickness of $t_{\text{ams}} = 0.65R_0$. The stellar wind shock has a characteristic number density of $n_{\text{sws}} = n_w v_w^2 / c_s^2$, where $n_w = \dot{m}_w / 4\pi R_0^2 v_w \mu m_H$ is the stellar wind number density with the mean molecular weight of $\mu = 0.5$ (ionised gas) and the hydrogen mass of m_H and the sound speed of the stellar wind gas $c_s = (k_B T_{\text{sw}} / \mu m_H)^{1/2}$ is eval-

uated for the temperature of $T_{\text{sw}} = 10^4 \text{ K}$. The thickness of the stellar wind is given by $t_{\text{sws}} = 2.3(c_s / v_w)^2 R_0$. We show both the number density and the thickness of both shocks in Fig. 8 as a function of the distance of S2 from Sgr A* in Schwarzschild radii. We see that the ambient shock is thicker and less dense than the stellar wind shock for all ambient density profiles considered in the legend.

In our model, we further consider that the dust is more likely to exist in the ambient medium through which S2 moves along its orbit, and therefore the ambient shock is expected to be more relevant in terms of the localised extinction and its contribution in the L' -band. The B-type stars typically do not contain dust in their stellar winds, and therefore the stellar wind shock is not likely to contribute to L' -band emission. However, for completeness, we consider both shock layers in assessing the constraints on the ambient number density. This is also due to the complex dynamics of dust grains and their potential of being dragged by a denser material in the stellar-wind shock; especially smaller grains may be affected (see e.g. [van Marle et al. 2011](#)). During the motion of S2, the two shock layers are affected by hydrodynamic instabilities that lead to the mixing of both layers ([Schartmann et al. 2018](#)), which is likely to lead to the exchange of material and the actual density of the gaseous-dusty mixed layer may be closer to the denser stellar-wind shock.

We calculate the limits on the ambient number density at the S2 periape as well as the limits for the density slope γ . The upper limit is obtained if we associate the upper limit for the extinction with the standard deviation, $A_{\text{shock}} \lesssim 0.04 \text{ mag}$. Another estimate is based on the fractional variability with $A_{\text{shock}} \sim 0.02 \text{ mag}$. The values of the density and the slope are inferred from the extinction-column density relation $A_V \approx 5.6 \times 10^{-22} N_h (\text{cm}^{-2})$, where $N_h \sim n_{\text{shock}} t_{\text{shock}}$ with the shock number density n_{shock} and a shock thickness t_{shock} for each shock layer as explained above. The extinction in L' -band is approximately given by $A_L = 0.5 \times 0.1 \times A_V$ ([Schödel et al. 2010](#)). This model allows us to compute the whole light curve along the S2 trajectory. In Table 2, we list the density slope and the number density constraints for each extinction value, with the shock layer taken into account. For the ambient shock, which is more likely to lead to an increased dust emission, the upper limits on the number density are in the range $n_a^{\text{ams}} \lesssim 7.15 \times 10^8 - 1.87 \times 10^9 \text{ cm}^{-3}$ with the density slope $\gamma \lesssim 3.03 - 3.20$. For the stellar-wind

Table 2. Summary of constraints for the ambient density close to the GC at the S2 periaapse based on the two extinction limits, 1σ and fractional variability, and analytical bow-shock models for both the ambient shock and the stellar-wind shock thermal emission.

Thermal emission – upper limit	Power-law slope	Ambient number density (periaapse)
Ambient shock ($A = 0.04$)	3.20	$1.87 \times 10^9 \text{ cm}^{-3}$
Stellar-wind shock ($A = 0.04$)	1.57	$1.86 \times 10^5 \text{ cm}^{-3}$
Thermal emission – upper limit	Power-law slope	Ambient number density (periaapse)
Ambient shock ($A = 0.02$)	3.03	$7.15 \times 10^8 \text{ cm}^{-3}$
Stellar-wind shock ($A = 0.02$)	1.41	$7.53 \times 10^4 \text{ cm}^{-3}$
Non-thermal emission – upper limit	Power-law slope	Ambient number density (periaapse)
Peak flux density of 0.91 mJy (maximum flux difference)	1.47	$1.01 \times 10^5 \text{ cm}^{-3}$
Peak flux density of 0.22 mJy (fractional variability)	1.17	$1.88 \times 10^4 \text{ cm}^{-3}$

Notes. In addition, we include the ambient density upper limit for the non-thermal synchrotron emission based on the maximum flux difference of 0.90 mJy and the fractional variability of 0.2 mJy.

shock, the number densities are smaller by four orders of magnitude and the density slope is smaller by a factor of two, $n_a^{\text{sws}} \lesssim 7.53 \times 10^4 - 1.86 \times 10^5 \text{ cm}^{-3}$ and $\gamma \lesssim 1.41 - 1.57$.

Figure 9 shows a comparison of model light curves with the actual L' -band data, with the upper panels representing the larger extinction value of $A = 0.04$ based on the standard deviation of the light-curve points and the lower panels showing results with the smaller extinction of $A = 0.02$ based on the fractional variability. The peak of the observed light curve has an apparent offset from the peak of the modelled light curves (at the periaapse), which is most likely a systematic effect and not an intrinsic brightening.

3.3. Constraining the ambient density based on the non-thermal bow-shock emission

The thermal bow-shock emission in L' -band is clearly affected by the presence of dust along the S2 orbit. While on one hand the dust is clearly present in dusty objects whose orbits lie in the S cluster, the hot ambient medium likely destroys the dust particles continually. In cases where the presence of dust is diminished along the S2 orbit, the thermal emission of the bow shock in the L' -band is even smaller than we predicted in Sect. 3.2.

Another way to constrain the ambient density at the S2 pericentre is to take into consideration the broadband non-thermal synchrotron emission of the bow shock, which does not depend on the presence of dust. When the bow shock develops ahead of the star, it provides a volume where electrons can be accelerated by the enhanced magnetic field. Subsequently, they cool off by emitting synchrotron emission. Ginsburg et al. (2016) used the same mechanism to claim that the non-thermal emission of stellar bow shocks in the S cluster may be comparable to the radio and NIR emission of Sgr A*, and therefore could be detected.

One can reverse the task and instead constrain the upper limit of the ambient density based on the statistical variations we detect in the L' -band light curve of S2. According to Table 1, the maximum flux difference in our light curve is 0.90 mJy, while the rms fractional variability is at the level of 0.2 mJy. Therefore, the maximum contribution of the bow-shock synchrotron emission in L' -band can only be at this flux density level, which allows us to place an upper limit on the ambient density.

For the calculation of the bow-shock synchrotron light curve and broad-band spectrum, we follow the model of Sądowski et al. (2013), originally developed for the estimate of the bow-shock emission of the DSO/G2 object; see also Zajaček et al. (2016). As in Sądowski et al. (2013) and Zajaček et al. (2016), for

calculating synchrotron light curves and spectra, we use the temperature profile of the ADAF, $T_a = 9.5 \times 10^{10}(r/r_s)^{-1}$ (Yuan et al. 2003), with the ratio between the magnetic and the gas pressure of $\chi = P_{\text{mag}}/P_{\text{gas}} = 0.3$, and the Mach number of the star at the pericentre distance of S2, $\mathcal{M} = 2.6$. Other relevant parameters were adopted from particle-in-cell simulations performed by Sądowski et al. (2013); specifically, the slope of the high-energy power-law electron distribution, $p = 2.4$, the minimum Lorentz factor, $\zeta = 7.5$, and the fraction of accelerated electrons to high energies, $\eta = 0.05$. Specifically for S2, we used its mass-loss rate $\dot{m}_w \approx 3 \times 10^{-7} M_\odot \text{ yr}^{-1}$ and the terminal wind velocity $v_w \sim 10^3 \text{ km s}^{-1}$ inferred from spectroscopy (Martins et al. 2008; Habibi et al. 2017).

In contrast to Sądowski et al. (2013), we perform only the plowing synchrotron model, in which the accelerated electrons are kept in the shocked region and radiate in the shocked magnetic field. The plowing model is more likely to contribute in L' -band than the local model, in which accelerated electrons leave the shock and radiate in the unshocked magnetic field. Due to the short cooling time for the NIR L' -band synchrotron emission, $t_{\text{cool}} \approx 0.25(B/0.08 \text{ G})^{-3/2}(v_c/8 \times 10^{13} \text{ Hz})^{-1/2} \text{ yr}$, the local model would have a negligible contribution.

In Fig. 10 (left panel), we plot the L' -band synchrotron light curve for the peak flux density of 0.91 mJy, which corresponds to the maximum flux difference, and the light curve with the peak flux density of 0.22 mJy, which corresponds to the fractional variability. The non-thermal bow-shock emission is also compared with the Sgr A* mean flux density in L' -band (Schödel et al. 2011), the S2 total flux, and the actual S2 measured flux density from 2018.307, which are all larger by at least a factor of a few than the bow-shock non-thermal emission. In the right panel of Fig. 10, we plot the SED of the S2 bow-shock synchrotron emission calculated for the pericentre for both L' -band flux density values (maximum difference and fractional variability). We also compare the synchrotron bow-shock SED with the SED of Sgr A* calculated using the advection-dominated accretion flow model (ADAF; Yuan et al. 2003; Yuan & Narayan 2014). Taking into account the maximum flux difference as an upper limit for the L' -band synchrotron contribution, we get the pericentre number density of $n_a = 1.01 \times 10^5 \text{ cm}^{-3}$ (the slope of 1.47). The fractional variability flux limit yields the number density of $1.88 \times 10^4 \text{ cm}^{-3}$ (the density slope of 1.17). Both limiting cases are summarised in Table 2. For both density limits, the bow-shock synchrotron flux density would be comparable to that of Sgr A* at frequencies of a few GHz, where the peak of its SED occurs.

Fig. 9. Comparison of model light curves based on the mean S2 emission and the modelled thermal emission of its bow shock (solid green and blue lines) with the observed L' -band emission of S2 (red points with errorbars). *Upper panels:* modelled thermal emission of the bow shock is associated with the extinction of $A = 0.04$ mag, which was inferred from the standard deviation of observed light curve points. We model the thermal dust emission of the ambient bow shock (solid green line in *left panel*) and the thermal emission of the stellar-wind shock (solid blue line in *right panel*) separately. The horizontal solid black line represents the light curve mean value, while the dashed black lines stand for the standard deviation. *Lower panels:* as in the upper panels, but for the extinction of $A = 0.02$ mag based on the excess variance of the observed light-curve points.

To separate Sgr A* and the predicted S2 bow-shock emission is challenging as it requires highly sensitive measurements at cm to decimetre (dm) wavelengths with an angular resolution of a tenth of an arcsecond or better. However, around the time of the periape passage, long-term (weeks to a few months) light curves should reveal an overall increase of the spatially unresolved emission of both sources. Such a data set is not available at present. Very-long-baseline interferometry (VLBI) measurements at GHz frequencies (or cm wavelengths) could in principle be possible due to the high brightness temperature of the S2 bow-shock emission at its pericentre. We outline here a few basic estimates. For the assumed maximum non-thermal excess in L' -band of ~ 1 mJy, the calculated peak of the bow-shock synchrotron spectrum is at 4.61 GHz. The peak flux density is expected to be 0.86 Jy at the S2 pericentre, which is comparable to or even larger than the flux density of Sgr A* at cm wavelengths. The characteristic size of the bow shock given by the stagnation distance would be $R_0 \sim 5.6 \times 10^{13}$ cm at the pericentre, which gives the angular scale of ~ 0.5 mas, which is below the resolution capabilities of the cm VLBI, namely ~ 1 mas at best. The angular distance between the S2 radio source and

Sgr A* at the pericentre is ~ 14.4 mas, which is smaller than the intrinsic size of Sgr A* at 6 cm (~ 20 mas), because of λ^2 scatter-broadening of the size of Sgr A* (Bower et al. 2006; Doeleman et al. 2008). The brightness temperature² of the S2 bow shock at its pericentre would be large, $T_B \sim 8.5 \times 10^{10}$ K, comparable to that of Sgr A*, $\gtrsim 2 \times 10^{10}$ K (Krichbaum et al. 1998; Doeleman et al. 2008). However, before and after the S2 pericentre, its brightness temperature is expected to drop rather fast, remaining above 10^6 K from the epoch 2017.08 up to 2020.54, with the peak at the pericentre in 2018.38. Taking into account the smaller contribution of S2 bow-shock non-thermal emission at L' -band at the level of ~ 0.2 mJy, which results from a smaller ambient density, mainly the characteristic size is affected; it increases to $R_0 = 1.29 \times 10^{14}$ cm, which corresponds to the angular scale of 1.1 mas, which is at the limit of the

² We estimate the brightness temperature using the standard relation derived from the Rayleigh-Jeans approximation in the radio domain, $T_B = \frac{c^2}{2k_B} \frac{F_\nu}{\Omega \nu^2}$, where the solid angle $\Omega \approx \pi R_0^2 / d_{GC}^2$ with R_0 being the stagnation radius of the bow shock.

Fig. 10. Non-thermal contribution of the S2 bow shock to its L' -band emission. In the *left panel*, we plot the L' -band light curve as calculated for the S2 bow-shock synchrotron emission for the peak flux of 0.91 mJy (solid black line) and for the peak flux of 0.22 mJy (dashed black line). For comparison, we also show the fractional variability of S2 (dotted green line) and its maximum flux difference (dot-dashed line). The measured mean flux of Sgr A* in L' -band, 4.33 ± 0.18 mJy, is depicted by a dashed blue line. The sum of the mean flux density of S2 with the maximum potential contribution of the non-thermal bow-shock emission is represented by a solid red line. The measured flux density of S2 from 2018.307 is marked by an orange cross. In the *right panel*, we show the SED of S2 bow-shock synchrotron emission as calculated for the S2 pericentre for the L' -band flux of 0.91 mJy (solid line) and for the L' -band flux of 0.22 mJy (dashed line). The orange dot-dashed line marks the L' -band frequency range and the red star symbol represents the mean flux density of S2 in L' -band. The blue solid line represents the SED of Sgr A* based on the advection dominated accretion flows (ADAFs; Yuan et al. 2003; Yuan & Narayan 2014).

resolving power of the cm VLBI. However, as for a lower density the resulting spectrum of the S2 bow shock is shifted towards the lower frequency at 1.5 GHz (see Fig. 10), the bow-shock size is actually smaller than the typical VLBI beam FWHM of $\sim 3''$. With the peak flux of 435 mJy, the brightness temperature of $T_B \sim 7 \times 10^{10}$ K is again comparable to that of Sgr A* and remains above 10^6 K in the time window between 2016.98 and 2020.90. In general, the cm VLBI observations could reveal a deviation from the Gaussian core of Sgr A* in the epochs of ~ 1.5 years before and after the pericentre when the brightness temperature is still above 10^6 K. At the pericentre, it is not possible to resolve the S2 bow-shock emission because of the scatter-broadening of Sgr A* and the bow-shock source. Only the flux excess could be detected; however, in that case a corresponding light curve at cm wavelengths would need to be analysed to distinguish the long-term bow-shock excess from the short-term stochastic radio flares.

Based on the upper limit inferred from the maximum synchrotron contribution to the L' -band emission of S2, accretion flows exceeding the particle density of $\sim 10^5 \text{ cm}^{-3}$ at $\sim 1500 r_s$ are unlikely. On the other hand, the presence of the Bondi-type flow with $\gamma \sim 3/2$ cannot be excluded based on our L' -band light-curve analysis.

4. Discussion

Our observational analysis of the L' -band emission of the star S2 yields a light curve that is essentially flat within uncertainties and shows intrinsic flux variations due to the interaction of S2 with the ambient medium or due to intrinsic stellar variability at the level of $\sim 2.5\%$ only, as given by the fractional variability. Using the theory of a two-layer bow shock (Dyson 1975), we are therefore only able to place an upper limit on the ambient density at the S2 periape as well as an upper limit on the density slope. The upper limit considering the contribution of the ambient bow shock, which is more likely to

contain dust, is $n_a \lesssim 1.87 \times 10^9 \text{ cm}^{-3}$, and the upper limit for the slope is $\gamma \lesssim 3.20$. These values change only slightly when one considers the smaller extinction of ($A = 0.02$) as indicated by the excess variance, $n_a \sim 7.15 \times 10^8 \text{ cm}^{-3}$ and $\gamma = 3.03$. If the stellar-wind shock layer were found to contain dust, which would require the turbulence and the mixing of shocked layers, then the density and the slope constraints would decrease to $n_a = 7.53 \times 10^4 - 1.86 \times 10^5 \text{ cm}^{-3}$ and $\gamma = 1.41 - 1.57$. The non-thermal synchrotron emission of the bow-shock appears to put tighter constraints on the S2 pericentre density: $n_a \lesssim 1.01 \times 10^5 \text{ cm}^{-3}$ for the maximum flux difference and $n_a \lesssim 1.88 \times 10^4 \text{ cm}^{-3}$ for the fractional variability. For the slope, we obtain $\gamma \lesssim 1.47$ for the maximum flux difference and $\gamma \lesssim 1.17$ for the fractional variability.

Therefore, the L' -band data is consistent with the S2 star interacting with a hot, diluted ambient flow with a power-law density slope of $\gamma \sim 0.5$ (generally the advection-dominated flow (ADAF); Wang et al. 2013) and with a spherical Bondi-type solution with a density slope of $\gamma \sim 1.5$ (also consistent with the convection-dominated flow (CDAF); Różańska et al. 2015). Moreover, we cannot exclude the possibility that S2 interacts with an even denser and colder type of medium, such as the proposed cool disc with densities in the interval of $n_{\text{CD}} \sim 10^5 - 10^6 \text{ cm}^{-3}$ (Murchikova et al. 2019). Indeed, the uppermost limit for the density $n_a < 1.87 \times 10^9 \text{ cm}^{-3}$ is well in the range for the number density of broad-line region (BLR) clouds $n_{\text{BLR}} \sim 10^8 - 10^{11} \text{ cm}^{-3}$ (Gaskell 2009). Although we do not expect the presence of BLR clouds in such a low-luminosity nucleus as Sgr A* (see, however, Bianchi et al., 2019 for the discovery of the compact BLR in the low-luminosity Seyfert galaxy NGC3147 with $L/L_{\text{Edd}} \sim 10^{-4}$), we cannot exclude the presence of denser gaseous-dusty structures, with which stars can occasionally interact, such as dust-enshrouded objects monitored in the S-cluster (Peißker et al. 2020b; Ciurlo et al. 2020). This is also supported by the multiphase medium of Sgr A* on the scale of one parsec (Moser et al. 2017), which could also be

present on smaller scales due to for example thermal instability (Róžańska et al. 2017). In particular, Peiðker et al. (2020a) report a detection of the proper motion of a Bry filament, whose estimated distance is close to the Bondi radius at ~ 0.2 pc. At comparable scales, Royster et al. (2019) used the H30 α line to trace a blueshifted, ionised gas with a velocity of between -480 and -300 km s $^{-1}$, which appears to be outflowing. Hence, the environment close to the Bondi radius is complex in terms of both kinematics and density.

Based on the flat L' -band light curve and the upper limits on the density, we can exclude the presence of a cold, thin accretion disc (Shakura & Sunyaev 1973), whose number densities close to the periastron of S2 at $\sim 1500 r_s$ can be estimated using the relation for the number density as given by the standard accretion disc theory; see also Frank et al. (2002). Using the mean atomic weight of $\mu = 0.615$ for the fully ionised gas, we obtain

$$n_{\text{thin}} = 2.7 \times 10^{12} \left(\frac{\alpha}{0.1} \right)^{-7/10} \left(\frac{\dot{m}}{10^{-8} M_{\odot} \text{ yr}^{-1}} \right)^{11/20} \times \left(\frac{M_{\bullet}}{4 \times 10^6 M_{\odot}} \right)^{5/8} \left(\frac{R}{1500 r_s} \right)^{-15/8} f^{11/5} \text{ cm}^{-3}, \quad (4)$$

where the factor f stands for $f = [1 - (r_s/R)^{1/2}]^{1/4}$. We calculate a number density range of $n_{\text{disc}} = 7.6 \times 10^{11} - 9.6 \times 10^{12} \text{ cm}^{-3}$ for the accretion rate in the range $\dot{m} = 10^{-9} - 10^{-7} M_{\odot} \text{ yr}^{-1}$ and a viscous parameter of $\alpha = 0.1$ for the S2 pericentre distance. In Fig. 11, we plot the thin disc density profiles up to $100 r_s$, since the density profile below this radius is relatively uncertain and the accretion flow likely breaks up due to the accumulation of poloidal magnetic flux, which results in a magnetically arrested disc (MAD; Narayan et al. 2003). This transition occurs where the magnetic energy density and the gravitational potential energy are at balance, $B^2/8\pi = GM_{\bullet}\rho_{\text{flow}}/R_m$, from which we can easily derive the magnetospheric radius,

$$R_m \sim \frac{8\pi GM_{\bullet}\rho_{\text{flow}}}{B^2} \quad (5)$$

$$= 90.5 \left(\frac{M_{\bullet}}{4 \times 10^6 M_{\odot}} \right) \left(\frac{n_{\text{flow}}}{10^6 \text{ cm}^{-3}} \right) \left(\frac{B_{\text{pol}}}{10 \text{ G}} \right)^{-2} r_s, \quad (6)$$

where the poloidal magnetic field is scaled to 10 G and the number density of the flow to 10^6 cm^{-3} according to the synchrotron models of the Sgr A* flares (Yusef-Zadeh et al. 2006; Eckart et al. 2012). The putative MAD part of the flow is highlighted by the blue vertical line in Fig. 11.

The thin disc is expected to contain dust from the sublimation radius towards larger distances. If we associate the effective thin disc temperature, $T_{\text{eff}} = [3GM_{\bullet}\dot{m}/(8\pi r^3\sigma_B)]^{1/4}$, with the dust sublimation temperature of $T_{\text{sub}} = 1000$ K, we get a relation for the dust sublimation radius (see also Czerny & Hryniewicz 2011)

$$r_{\text{sub}} = 163 \left(\frac{M_{\bullet}}{4 \times 10^6 M_{\odot}} \right)^{1/3} \left(\frac{\dot{m}}{10^{-7} M_{\odot} \text{ yr}^{-1}} \right)^{1/3} \left(\frac{T_{\text{sub}}}{1000 \text{ K}} \right)^{-4/3} r_s. \quad (7)$$

We obtain the range of $r_{\text{sub}} = 35 - 163 r_s$ for the accretion rate range of $\dot{m} = 10^{-9} - 10^{-7} M_{\odot} \text{ yr}^{-1}$. Hence, the dusty bow-shock layer should be formed when the S2 plunges through the disc midplane close to its periastron at $\sim 1500 r_s$. The formation of the shock in the disc material would lead to the extinction increase in L' -band magnitude by 0.65 mag for the density of $4.9 \times 10^{11} \text{ cm}^{-3}$ and by as much as 2.30 mag for the density of

$6.13 \times 10^{12} \text{ cm}^{-3}$. We can therefore exclude the possibility that the S2 star crosses a thin, dense, and cold disc.

Although the comparison with the thin disc density in Fig. 11 may seem purely academic for the Sgr A* environment, it is still instructive to compare our upper limits with the thin-disc density for two reasons. Firstly, although the presence of a thin disc was implicitly excluded already based on X-ray and mm observations (Yuan et al. 2003; Baganoff et al. 2003; Dexter et al. 2010), the upper density limit inferred from the stellar bow shock is independent of these previous measurements. Secondly, the transition between the hot advection-dominated accretion flows (ADAFs) and thin discs at such low accretion-rate values, such as those for Sgr A*, is still not well understood. For α discs, the viscously stable branch at low $\dot{m}$ exists in the stability curve of surface density–accretion rate or Σ – $\dot{m}$ plot and these discs would be both geometrically and optically thin; see e.g. Frank et al. (2002). Furthermore, they would primarily cool via the free-free emission instead of black-body radiation (Tylenda 1981) and one of the main reasons why such a disc is likely not present for Sgr A* is the short time for plasma to cool down via bremsstrahlung due to the low-angular momentum of the flow. On the other hand, they are low accretion-rate systems where the thin disc was detected, such as for example the maser source NGC4258 (Gammie et al. 1999) and NGC3147 (Bianchi et al. 2019). Most likely, the boundary conditions of the flow, that is, either feeding by stellar winds or the inflow of gas from larger scales, determine the transition between the ADAF and the thin disc at low $\dot{m}$ and vice versa. In addition, hybrid models of discs have been suggested, with an outer thin disc part and an inner hot ADAF part, with the transition at 10–100 gravitational radii (Gammie et al. 1999).

In order to better constrain the density and lower the upper density limit and the slope, the photometry precision needs to improve, with values below 0.01 mag. Also, a more precise theory for the bow shock formation and its corresponding NIR emission needs to be taken into account. However, the current methodology that we provide here can be directly transferred to the more precise photometry.

In terms of future instruments, the METIS at ELT will be particularly well suited for high-precision photometry in near- and mid-IR domains. All observing modes of METIS will initially work with a single conjugate adaptive optics (SCAO) system at or close to the diffraction limit of the 39m ELT. At a wavelength of $3.5 \mu\text{m}$, the ELT will allow operation at an angular resolution of 23 milliarcseconds. Here, METIS will have a point source sensitivity ($10\text{-}\sigma$ in 1 hour) of 21.2 mag (or about $1 \mu\text{Jy}$) (Brandl et al. 2018a,b). In the N band at a wavelength around $10 \mu\text{m}$, the point-source sensitivity can be expected to be about 1.6 orders of magnitude lower. The field of view of about $10'' \times 10''$ in imaging and $1.0'' \times 0.5''$ in IFU (integral field unit) spectroscopy mode at L - and M -bands is ideally suited for the densely populated GC region. However, the limiting sensitivity for flux density measurements in this field will be determined by confusion due to source crowding. Hence, the effective sensitivity available to determine flux densities and to establish light curves can safely be assumed to be an order of magnitude lower than the nominal point-source sensitivity, but will still be about an order of magnitude better than the currently reached sensitivity. For specific estimates of the potentially probed density range, we consider the limiting flux density value for the bow-shock emission in the L' -band to be 0.01 mJy, which corresponds to an $\sim 10^{-3}$ mag difference in the S2 star L' -band magnitude. Taking into account the model of thermal ambient-shock emission, which is the least sensitive in terms of the

The next-best target to observe both the potential thermal and non-thermal bow-shock emission at the periaapse would be S62 in early 2023 which appears to be a suitable time to observe the closest S-star to the SMBH by the JWST. This will be an opportunity to shed further light on the density of the accretion flow around Sgr A* along the S62 orbital plane (PeiBker et al. 2020c), in particular at its pericentre distance of $r_p^{S62} \approx 225 r_s \sim 18$ AU, where the conditions of the accretion flow were only constrained via the rotation measure (Marrone et al. 2006, 2007); see Fig. 11. The detection of the bow-shock emission of S62 would provide an independent means of assessing the ambient number density

NIR bow-shock emission from the S stars, which is expected to be most prominent at the pericentre, could be detected via peculiar, longer K_s or L' -band flares. Moreover, the total bow-shock flare duration would be of the order of a few months up to one year and the peak flux is coincident with the pericentre of the star, see our model predictions in Fig. 9 for the thermal dust emission and in Fig. 10 for the non-thermal synchrotron emission associated with the bow shock. The start of the flux increase would be half a year before the pericentre. In terms of the shocked stellar wind, [Giannios & Sironi \(2013\)](#) and [Christie et al. \(2016\)](#) calculated the timescale of approximately one month for the X-ray bremsstrahlung flare produced by S2 bow-shock with the luminosity of 4×10^{33} erg s $^{-1}$. However, [Schartmann et al. \(2018\)](#) argued that for the expected accretion-flow density, the X-ray emission from the bow shock is beyond the detection limit. [Ginsburg et al. \(2016\)](#) calculated the non-thermal synchrotron emission, which for the typical parameters assumed for S2, the mass-loss rate of $\dot{m}_w \sim 10^{-7} M_\odot$ yr $^{-1}$

and the terminal wind velocity of $v_w \sim 10^3 \text{ km s}^{-1}$, is below the quiescent emission of Sgr A*. Therefore, the stochastic IR flares of Sgr A* (Witzel et al. 2012, 2018), including the very bright ones (Do et al. 2019b), are not associated with the stellar bow shocks in the S cluster as they evolve on a timescale of several hours only, which can be interpreted as an orbital timescale of plasmoids close to the innermost stable circular orbit (Gravity Collaboration 2018b). In other words, some short-term NIR flares could be associated with short-period stars orbiting Sgr A* on the scale of the innermost stable circular orbit (Leibowitz 2020), which could be disentangled from the stochastic background via their periodic or quasi-periodic signal. However, the bright IR flare analysed by Do et al. (2019b) could be associated with a change of accretion state due to the recent pericentre passage of S2 in 2018 or even the pericentre passage of DSO/G2 in 2014. The time delay between the pericentre and the average flare statistics may be interpreted via the viscous timescale of the hot accretion flow, which can be of the order of one to ten years (Czerny et al. 2013).

5. Conclusions

For the brightest star in the S cluster, S2, we obtained a light curve in the NIR L' -band, which is flat within the measurement uncertainties with the fractional flux variability of only 2.52% in terms of the L' -band flux density. When we associate the flux density standard deviation of 0.04 mag with the local dust extinction due to the bow shock, we can place an upper limit of $n_a < 1.87 \times 10^9 \text{ cm}^{-3}$ on the ambient number density at $\sim 1500 r_s$ for the warm bow shock driven into the ambient medium, which is expected to contain dust. For the colder and the denser stellar-wind shock, the number density limit is $n_a < 1.86 \times 10^5 \text{ cm}^{-3}$, which is nevertheless less firm because of the limited presence of dust in stellar winds of B-type stars. Considering the non-thermal bow-shock emission, which does not rely on the existence of dust, we can place an upper density limit of $1.01 \times 10^5 \text{ cm}^{-3}$.

These limits cannot yet constrain the type of the hot accretion flow and therefore both the radiatively inefficient accretion flow with the density profile of $\gamma = 0.5$ and the spherical Bondi-type flow with $\gamma = 1.5$ can accommodate these number densities. However, we can firmly exclude the standard, thin accretion disc extending all the way to $1500 r_s$, the density of which would be at least two orders of magnitude greater and the star-disc interactions would produce a shock with a local extinction of 0.65–2.30 mag in L' band, which is much larger than the inferred 1σ excess of 0.04 mag of the L' -band light curve, and a difference of 0.11 mag between the maximum and the minimum magnitude.

In terms of future prospects, a major challenge is to detect and directly resolve bow shocks associated with the S-stars in NIR bands because of the high confusion and stellar density close to Sgr A*. It may become possible when high-resolution mid-IR imaging becomes available with the next generation of instruments, namely METIS at the ELT, because bow shocks are generally more prominent because of the dust emission in mid-IR bands. High-precision photometry associated with METIS with the flux sensitivity of the $\sim 0.01 \text{ mJy}$ in L' -band is expected to improve the density constraints by two to three orders of magnitude in comparison with our current upper limits. For the non-thermal bow-shock emission, the direct detection of the flux excess is plausible in cases where the hot-flow density slope is larger than $\gamma \sim 0.5$ between the Bondi radius and the S2 pericentre distance.

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Appendix A: Flux table

Table A.1. Dereddened fluxes of S2 and S65 in L' -band in 2018.307.

UT time	S2 (mJy)	S65 (mJy)
05:57:34.4735	9.84 ± 0.11	14.48 ± 0.25
05:59:43.8697	9.00 ± 0.02	12.38 ± 0.06
06:00:25.8029	10.03 ± 0.27	15.61 ± 0.62
06:02:34.9990	1.63 ± 0.05	2.94 ± 0.23
06:03:17.5402	8.15 ± 0.01	12.43 ± 0.05
06:05:26.7352	8.68 ± 0.04	13.34 ± 0.17
06:06:08.8654	9.15 ± 0.13	13.81 ± 0.35
06:08:16.4643	10.21 ± 0.38	15.21 ± 0.83
06:09:01.9952	9.64 ± 0.26	14.70 ± 0.63
06:11:10.5931	8.72 ± 0.10	13.86 ± 0.36
06:45:59.1812	10.07 ± 0.13	13.38 ± 0.26
06:48:07.7927	9.90 ± 0.10	14.93 ± 0.24
06:48:49.5321	12.52 ± 0.63	15.67 ± 0.92
06:53:54.6491	10.21 ± 0.04	14.05 ± 0.08
06:56:44.5710	10.41 ± 0.18	14.11 ± 0.36
06:57:29.3005	8.13 ± 0.03	12.32 ± 0.20
06:59:38.1146	8.63 ± 0.03	13.97 ± 0.13
07:34:08.2162	8.03 ± 0.05	13.4 ± 0.26
07:37:02.3746	8.14 ± 0.03	12.95 ± 0.17
07:39:11.7988	8.26 ± 0.02	12.55 ± 0.14
07:39:54.9392	8.75 ± 0.06	12.57 ± 0.22
07:42:48.1058	7.82 ± 0.17	12.75 ± 0.87
07:44:58.3101	8.76 ± 0.06	14.46 ± 0.21
07:47:53.4375	8.27 ± 0.17	14.10 ± 0.69
08:26:49.3672	7.40 ± 0.03	12.60 ± 0.32
08:28:59.1627	7.93 ± 0.03	12.82 ± 0.02
08:29:42.3116	9.18 ± 0.06	14.00 ± 0.24
08:31:51.9218	8.37 ± 0.02	12.53 ± 0.11
08:32:36.0531	8.61 ± 0.08	12.88 ± 0.29
08:34:47.4474	10.03 ± 0.13	13.62 ± 0.26
08:35:30.1790	8.73 ± 0.11	14.57 ± 0.42
08:38:23.3029	8.79 ± 0.25	15.14 ± 0.88
08:40:33.6955	7.45 ± 0.03	13.72 ± 0.03
09:16:05.5965	8.28 ± 0.03	13.75 ± 0.13
09:18:58.3609	7.72 ± 0.02	14.01 ± 0.18
09:21:09.3821	7.09 ± 0.03	13.30 ± 0.34
09:21:52.5156	7.66 ± 0.02	13.09 ± 0.24
09:24:03.3459	7.49 ± 0.02	12.31 ± 0.23
09:24:46.0790	9.80 ± 0.25	16.09 ± 0.66
09:26:55.2871	8.83 ± 0.05	13.62 ± 0.17
09:27:38.2308	8.46 ± 0.10	13.89 ± 0.49
09:29:47.4548	8.89 ± 0.07	14.16 ± 0.22
10:05:26.0274	8.25 ± 0.02	14.11 ± 0.08
10:07:35.2335	8.32 ± 0.10	16.33 ± 0.41
10:08:18.1657	8.71 ± 0.10	13.24 ± 0.46
10:10:30.5604	9.35 ± 0.18	14.43 ± 0.49
10:11:11.8925	9.24 ± 0.03	15.21 ± 0.12
10:13:20.2894	8.34 ± 0.03	12.99 ± 0.19
10:14:03.2225	8.48 ± 0.02	13.07 ± 0.12
10:16:13.2227	8.44 ± 0.09	12.47 ± 0.47
10:19:10.3576	10.80 ± 0.47	16.81 ± 1.12

As described in Sect. 2.2, we performed photometry for the single exposures of the observation 0101.B-0052(B) in 2018.307. As shown in Fig. 4 there is no significant change in flux at the position of S2 and the IR counterpart of Sgr A*,

suggesting that Sgr A* is in its quiescent period in L' -band during the observation epoch of 2018.307.

Appendix B: The S2 star bow-shock evolution along the orbit: formula for the density slope

The size of a bow shock formed due to the supersonic motion of the star close the supermassive black hole is scaled by the stand-off distance R_0 , which depends on two stellar parameters, the mass-loss rate $\dot{m}_w$ and the terminal wind velocity v_w , as well as on the density of the ambient medium ρ_a and the relative velocity of the star with respect to the surrounding medium v_{rel} . In the most general form, it can be expressed in the following way (Wilkin 1996; Zhang & Zheng 1997; Christie et al. 2016),

$$R_0 = \left(\frac{\dot{m}_w v_w}{\Omega(1 + \alpha)\rho_a v_{\text{rel}}^2} \right)^{1/2} \simeq C_\star v_{\text{rel}}^{-1} \rho_a^{-1/2}, \quad (\text{B.1})$$

where Ω stands for the solid angle into which the stellar wind is blown (in case it is fully isotropic $\Omega = 4\pi$). The coefficient α is the ratio of thermal and the ram pressure of the ambient medium, $\alpha = P_{\text{th}}/P_{\text{ram}}$. The supersonic motion takes place when the Mach number is larger than one, $M = v_{\text{rel}}/c_s = 1/\sqrt{\kappa\alpha} > 1$, where c_s is the sound speed and κ is an adiabatic index. The last equality in Eq. (B.1) is valid for the supersonic limit when $\alpha \rightarrow 0$, that is, when the thermal pressure is low. The term $C_\star = (\dot{m}_w v_w / \Omega)^{1/2}$ then represents a stellar parameter that we assume to be constant during one orbital period, which for S-stars with a semi-major axis a is typically (in years),

$$\tau_{\text{orb}} = 11.85 \left(\frac{a}{0.1''} \right)^{3/2} \left(\frac{M_\bullet}{4 \times 10^6 M_\odot} \right)^{-1/2} \text{ yr}. \quad (\text{B.2})$$

The stellar parameter $C_\star$ may be evaluated for the mass-loss rate $\dot{m}_w$ and the terminal wind velocity v_w as inferred for S2 based on spectroscopic information (Martins et al. 2008),

$$C_\star = 7.1 \times 10^{12} \left(\frac{\dot{m}_w}{10^{-7} M_\odot \text{ yr}^{-1}} \right)^{1/2} \left(\frac{v_w}{10^3 \text{ km s}^{-1}} \right)^{1/2} \text{ g}^{1/2} \text{ cm}^{1/2} \text{ s}^{-1}. \quad (\text{B.3})$$

Now we consider the relation between the scale lengths of the bow shock R_0 at two different positions. We assume that the number density profile near the GC as a function of distance r from the black hole has a power-law form:

$$n_a = n_0 \left(\frac{r}{r_0} \right)^{-\gamma}, \quad (\text{B.4})$$

where the power-law index γ is positive, $\gamma \geq 0$. It follows directly from Eq. (B.1) that the ratio of the standoff radii between two positions of the bow shock along the orbit is as follows,

$$\frac{R_{01}}{R_{02}} = \frac{v_{\text{rel}2}}{v_{\text{rel}1}} \left(\frac{\rho_{a2}}{\rho_{a1}} \right)^{1/2} = \frac{v_{\text{rel}2}}{v_{\text{rel}1}} \left(\frac{n_{a2}}{n_{a1}} \right)^{1/2} = \frac{v_{\text{rel}2}}{v_{\text{rel}1}} \left(\frac{r_1}{r_2} \right)^{\gamma/2}, \quad (\text{B.5})$$

where the subscripts 1 and 2 stand for two positions along the elliptical orbit characterised by velocities $v_{\text{rel}1}$ and $v_{\text{rel}2}$ and distances from the focus (SMBH) r_1 and r_2 ($r_1 > r_2$), respectively. The last equality in Eq. (B.5) is derived using a general expression for the power-law density profile, Eq. (B.4). Unless otherwise indicated in the text, we consider the ambient medium to be stationary with respect to the star, and therefore the relative velocity is approximately given by the orbital velocity of the star

around the GC, $v_{\text{rel}} \simeq v_*$. By taking this into consideration, we obtain a slightly modified relation for the ratio,

$$\frac{R_{01}}{R_{02}} = \frac{v_{*2}}{v_{*1}} \left(\frac{n_{a2}}{n_{a1}} \right)^{1/2} = \left(\frac{r_1}{r_2} \right)^{(\gamma+1)/2} \left(\frac{2a-r_2}{2a-r_1} \right)^{1/2}. \quad (\text{B.6})$$

The last equality is obtained using the expression for the orbital velocity of a star on an elliptic orbit, or the opposite equation, $v_* = \sqrt{\mu(2/r - 1/a)}$, where $\mu = GM_*$. In cases where the orbital elements for a given S star are constrained, one can invert Eq. (B.6) to get the power-law index γ as function of the bow-shock standoff ratio at two positions $R_{01}(r_1)/R_{02}(r_2)$, orbital distances r_1 and r_2 , and the semi-major axis a ,

$$\gamma_{12} = \frac{2 \log [R_{01}(r_1)/R_{02}(r_2)] + \log [(2a-r_1)/(2a-r_2)]}{\log (r_1/r_2)} - 1. \quad (\text{B.7})$$

The number of parameters in relation B.7 may be reduced when evaluated at special points on the orbit, especially the pericentre (true anomaly $\nu = 0^\circ$), the apocentre ($\nu = 180^\circ$), and the semi-latus rectum ($\nu = 90^\circ$). From classical celestial mechanics, we easily obtain the ratios for corresponding distances and velocities at the apocentre and the pericentre for a general elliptical orbit with an eccentricity e . The ratio of the distances from the supermassive black hole is $(r_P/r_A) = (1-e)/(1+e)^3$ and the ratio of velocities is $(v_{\text{orbP}}/v_{\text{orbA}}) = (1+e)/(1-e)$. Thus, the ratio between the apocentre and the pericentre bow-shock size is simply a function of two parameters: the orbital eccentricity e and the power-law index for the ambient density distribution γ ,

$$\frac{R_{0A}}{R_{0P}} = \left(\frac{1+e}{1-e} \right)^{1+\gamma/2}. \quad (\text{B.8})$$

In the same way as in Eq. (B.7), one can express the power-law index in the following way,

$$\gamma_{AP} = 2[\log (R_{0A}/R_{0P}) / \log [(1+e)/(1-e)] - 1]. \quad (\text{B.9})$$

A similar simple expression is obtained when evaluating the ratio between the points at the latus-rectum intersection with the ellipse and the pericentre. We denote the standoff distance at that point $R_{0,90}$ (the true anomaly is 90 degrees). The following relation then holds:

$$\frac{R_{0,90}}{R_{0P}} = \frac{(1+e)^{(\gamma+2)/2}}{(1+e^2)^{1/2}}, \quad (\text{B.10})$$

from which the density power-law index trivially follows (or from Eq. (B.7)),

$$\gamma_{LP} = \frac{2 \log [R_{0,90}/R_{0P}] + \log (1+e^2)}{\log (1+e)} - 2. \quad (\text{B.11})$$

As the star passes through the medium with a variable density n_a , the bow-shock emission is also expected to change. An upper limit for the thermal bow-shock emission is given by the rate of thermalised kinetic terms of the stellar and ambient winds, $L_{\text{th}} = 1/2 \dot{m}_w (v_{\text{rel}}^2 + v_w^2) \simeq 1/2 \dot{m}_w (v_*^2 + v_w^2)$ (Wilkin et al. 1997). The ratio of thermal bow-shock luminosities between two points is then given by,

$$L_{\text{th1}}/L_{\text{th2}} = (\beta_1^2 + 1)/(\beta_2^2 + 1), \quad (\text{B.12})$$

³ We use the indices P, A, and 90° to denote the quantities evaluated at the pericentre, apocentre, and the semi-latus rectum, respectively.

Table B.1. Power-law index γ and the corresponding ratio between the apocentre and the pericentre bow-shock sizes and the thermal emission luminosities calculated for different eccentricities.

e	γ	$(R_{0A}/R_{0P})_0$ (Eq. (B.8))	$L_{\text{thA}}/L_{\text{thP}}$ (S2)
0	≥ 0	1	1
0.5	0.0	3.0	0.19
0.5	0.5	3.9	0.19
0.5	1.0	5.2	0.19
0.5	2.0	9.0	0.19
0.9	0.0	19.0	1.8×10^{-2}
0.9	0.5	39.7	1.8×10^{-2}
0.9	1.0	82.8	1.8×10^{-2}
0.9	2.0	361.0	1.8×10^{-2}

Fig. B.1. Colour-coded ratio R_{0A}/R_{0P} of stellar bow-shock sizes between the apocentre and the pericentre of the corresponding orbit in the $e - \gamma$ plane (see also Eq. (B.8)). The white and black lines correspond to the contours of the constant ratio with numbers indicating the value of R_{0A}/R_{0P} for a given contour.

where $\beta = v_*/v_w$. Table B.1 summarises different exemplary values of the index γ and the corresponding values of the ratio of bow-shock sizes between the apocentre and the pericentre – Eq. (B.8) – as well as the ratio of luminosities of thermal emission, Eq. (B.12), for different eccentricities. As a standard prototype of S stars, we take S2 with inferred $a \simeq 0.123''$, $e \simeq 0.88$, and $v_w = 1000 \text{ km s}^{-1}$ (Martins et al. 2008; Habibi et al. 2017; Gravity Collaboration 2018a).

In Fig. B.1, we plot the colour-coded ratio R_{0A}/R_{0P} in the $e - \gamma$ plane (eccentricity–power-law index). We see that in general the ratio R_{0A}/R_{0P} has a broad range between 1 and 10^4 for plausible values of the density power-law slope, $\gamma \in (0, 2)$, and the orbital eccentricity, $e \in (0, 0.99)$. For the mean eccentricity of $e = 2/3$ in the thermal (isotropic) distribution of eccentricities, $n(e)de = 2ede$, and the flat density profile of $\gamma = 0.5 - 1.0$ (Wang et al. 2013), the ratio R_{0A}/R_{0P} is between 7.5 and 11.2 according to Eq. (B.8). *Effect of thermal pressure.* In the analysis above, we considered the thermal pressure of the ambient medium to be negligible with respect to the ram pressure, that is, $\alpha = P_{\text{th}}/P_{\text{ram}} \rightarrow 0$. Yet, at different distances from the SMBH, the factor $\alpha(r) = P_{\text{th}}/P_{\text{ram}} = c_s(r)^2/v_*(r)^2$ may in principle change and become non-negligible. The ratio of standoff radii between the apocentre and the pericentre of the orbit then becomes

$$\frac{R_{0A}}{R_{0P}} = \left(\frac{R_{0A}}{R_{0P}} \right)_0 \left(\frac{1 + \alpha_P}{1 + \alpha_A} \right)^{1/2}, \quad (\text{B.13})$$

where $(R_{0A}/R_{0P})_0$ is the ratio with a neglected thermal pressure, Eq. (B.8). The thermal term at the pericentre may be evaluated as follows,

$$\alpha_P = c_s(r_P)^2 / v_\star(r_P)^2 = \frac{\kappa k_B T_P}{\bar{\mu} m_p} \frac{a}{\mu} \frac{1-e}{1+e}, \quad (\text{B.14})$$

where T_P is the gas temperature at the pericentre, $\bar{\mu}$ is the mean molecular weight, and m_p is a proton mass. The ratio of the thermal terms at the apocentre and the pericentre is then

$$\frac{\alpha_A}{\alpha_P} = \frac{T_A}{T_P} \left(\frac{1+e}{1-e} \right)^2 = \left(\frac{r_P}{r_A} \right)^\delta \left(\frac{1+e}{1-e} \right)^2 = \left(\frac{1+e}{1-e} \right)^{2-\delta}, \quad (\text{B.15})$$

where $\delta \geq 0$ is a power-law index of the spherical temperature profile of the ambient medium, $T_a(r) = T_0(r_0/r)^\delta$, where $T_0 = T_a(r_0)$. The extra term in Eq. (B.13) would vanish if $\alpha_P = \alpha_A$, which would occur if $e = 0$. Using Eq. (B.15) we may rewrite Eq. (B.13) in the following form,

$$\frac{R_{0A}}{R_{0P}} = \left(\frac{R_{0A}}{R_{0P}} \right)_0 \underbrace{\left[\frac{1 + \alpha_P(T_P, a, e)}{1 + \alpha_P(T_P, a, e) \left(\frac{1+e}{1-e} \right)^{2-\delta}} \right]^{1/2}}_{F_{AP}(T_P, a, e, \delta)}. \quad (\text{B.16})$$

For the estimation of the factor F_{AP} in brackets, which arises because of thermal pressure, we consider the orbital elements of

S2 and the following temperature profile of the ambient medium (Psaltis 2012; Broderick et al. 2011),

$$T_P = T_0 \left(\frac{GM_\bullet}{rc^2} \right)^\delta, \quad (\text{B.17})$$

where $\delta = 0.84$ and $T_0 = 9.5 \times 10^{10}$ K. After calculation, the correction term is ≈ 0.8 for S2 star, and so every ratio in Table B.1 should be slightly adjusted, $R_{0A}/R_{0P} \approx 0.8(R_{0A}/R_{0P})_0$, if the temperature profile, Eq. (B.17), holds. Including the thermal pressure term, the formula for the density power-law index γ based on the ratio of the apocentre and the pericentre standoff radii changes accordingly,

$$\gamma'_{AP} = 2[(\log(R_{0A}/R_{0P}) - \log F_{AP}) / \log[(1+e)/(1-e)] - 1]. \quad (\text{B.18})$$

Similarly, using the semi-latus rectum–pericentre ratio $R_{0,90}/R_{0P}$, one can derive the slope γ'_{LP} modified by the thermal term F_{LP} in the following way,

$$\gamma'_{LP} = \frac{2[\log(R_{0,90}/R_{0P}) - \log F_{LP}] + \log(1+e^2)}{\log(1+e)} - 2, \quad (\text{B.19})$$

where the thermal term is expressed by

$$F_{LP}(T_P, a, e) = \left[\frac{1 + \alpha_P(T_P, a, e)}{1 + \alpha_P(T_P, a, e) \frac{(1+e)^{2-\delta}}{(1-e)^2}} \right]^{1/2}. \quad (\text{B.20})$$