Theoretical investigation of oscillator strengths and lifetimes in Ti II[★]

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ABSTRACT

Aims. Accurate atomic data for Ti II are essential for abundance analyses in astronomical objects. The aim of this work is to provide accurate and extensive results of oscillator strengths and lifetimes for Ti II.

Methods. The multiconfiguration Dirac–Hartree–Fock and relativistic configuration interaction (RCI) methods, which are implemented in the general-purpose relativistic atomic structure package GRASP2018, were used in the present work. In the final RCI calculations, the transverse-photon (Breit) interaction, the vacuum polarisation, and the self-energy corrections were included.

Results. Energy levels and transition data were calculated for the 99 lowest states in Ti II. Calculated excitation energies are found to be in good agreement with experimental data from the Atomic Spectra Database of the National Institute of Standards and Technology based on the study by Huldt et al. Lifetimes and transition data, for example, line strengths, weighted oscillator strengths, and transition probabilities for radiative electric dipole (E1), magnetic dipole (M1), and electric quadrupole (E2) transitions, are given and extensively compared with the results from previous calculations and measurements, when available. The present theoretical results of the oscillator strengths are, overall, in better agreement with values from the experiments than the other theoretical predictions. The computed lifetimes of the odd states are in excellent agreement with the measured lifetimes. Finally, we suggest a relabelling of the $3d^2(^1D)4p\,y^2D_{3/2}^o$ and $z^2P_{3/2}^o$ levels.

Key words. atomic data

1. Introduction

Iron-group elements have high cosmic abundances and display, due to their complex atomic structure, rich spectra in astronomical objects. Titanium is among the lighter elements in this group and is identified as an α -element that is primarily produced by captures of He nuclei. Studies of metal-poor stars, aimed to probe early galactic chemical compositions, show correlations with abundances of scandium and vanadium and thus a common production route with the iron-group elements (Snedden et al. 2016).

Spectroscopic analyses of astronomical objects rely on accurate transition data for elemental abundances, which in turn are crucial input to stellar and galactic evolution modelling. Data that are needed include wavelengths and oscillator strengths as well as hyperfine structure. The current paper addresses this important need.

We note that Ti II is one of the most abundant ions and its lines are found in the ultraviolet, visible, and near-infrared spectra of B, A, and F stars, for example. Furthermore, its parity forbidden nebular lines are found in stellar winds (Ryde 2009; Cowan et al. 2020; Hartman et al. 2004).

The energy levels of Ti II presented in the Atomic Spectra Database (ASD) of the National Institute of Standards and Technology (NIST; see Kramida et al. 2019) were compiled by Sugar

& Corliss (1985) and Saloman (2012) based on the results from Huldt et al. (1982). The observed spectral lines of Ti II were compiled from sources of Huldt et al. (1982), Allende Prieto & García López (1998), Pickering et al. (2001), and Aldenius (2009) for electric dipole (E1) transitions and of Hartman et al. (2004, 2005) and Bautista et al. (2006) for magnetic dipole (M1) and electric quadrupole (E2) transitions.

There are a number of experimental and theoretical studies of lifetimes and oscillator strengths of the Ti II ion. The sources adopted in the compilation by the NIST ASD are summarised in Table 1, and they include those of Roberts et al. (1973, 1975), Wolnik & Berthel (1973), Danzmann & Kock (1980), Kostyk & Orlova (1983), Pickering et al. (2001), Wiese et al. (2001), Morton (2003), Hartman et al. (2005), and Deb et al. (2008).

On the experimental side, a number of studies of lifetimes have been presented in the literature. Within the FERRUM project, Hartman et al. (2003) reported measurements of the long lifetime of the $3d^2(^3P)4s\,^4P_{5/2}$ level in Ti II, using the laser probing technique (LPT) (Lidberg et al. 1999) at the CRYRING storage ring (Abrahamsson et al. 1993). This measurement, however, resulted in a lifetime (28 ± 10 s) more than a factor of 2 longer than the theoretical estimate. The large uncertainty was due to the systematic effect of repopulation. Later on, within the same project, Hartman et al. (2005) extended the work on the measurement of lifetimes of other four metastable levels and determined the absolute transition probabilities for eight forbidden transitions originating from the levels $3d4s^2\,^2D_{3/2,5/2}$, using a combination of laboratory and astrophysical measurements.

[★] Full Table 3 is only available at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via <http://cdsarc.u-strasbg.fr/viz-bin/cat/J/A+A/643/A156>

Table 1. Summary of previous works on oscillator strengths and lifetimes in Ti II.

NIST database collection on oscillator strengths		
Roberts et al. (1973, 1975)	exp.	Beam-foil + stabilised arcs
Wolnik & Berthel (1973)	exp.	Shock tubes
Hartman et al. (2005)	exp.	LPT + astrophysical BF
Danzmann & Kock (1980)	exp.	Combined hook
Kostyk & Orlova (1983)	exp.	Solar spectra
Pickering et al. (2001, 2002)	exp.	LIF + FTS
Wiese et al. (2001)	exp.	HSAS
Morton (2003, 2004)	Compilation	
Deb et al. (2008)	theo.	CIV3
Other works on oscillator strengths		
Blackwell et al. (1982)	exp.	Oxford furnace
Thevenin (1989)	exp.	Solar spectra
Savanov et al. (1990)	Compilation	
Bizzarri et al. (1993)	exp.	LIF + FTS
Luke (1999)	theo.	CI
Kurucz (2017)	theo.	Semi-empirical
Wood et al. (2013)	exp.	LIF + FTS
Ruczkowski et al. (2014)	theo.	Semi-empirical
Lundberg et al. (2016)	exp. & theo.	LIF + FTS + HFR
Other works on lifetimes		
Roberts et al. (1973)	exp.	Beam-foil
Kwiatkowski et al. (1985)	exp.	LIF
Gosselin et al. (1987)	exp.	Beam-laser method
Bizzarri et al. (1993)	exp.	LIF
Langhans et al. (1995)	exp.	LIF
Hartman et al. (2003, 2005)	exp.	LPT
Bautista et al. (2006)	theo.	AUTOSTRUCTURE
Palmeri et al. (2008)	exp. & theo.	LPT + HFR
Deb et al. (2008)	theo.	CIV3
Lundberg et al. (2016)	exp. & theo.	LIF + HFR
Ruczkowski et al. (2016)	theo.	Semi-empirical
Kurucz (2017)	theo.	Semi-empirical

Notes. BF: Branching fraction; LPT: Laser probing technique; LIF: Laser induced fluorescence; FTS: Fourier transform spectroscopy; HSAS: High sensitivity absorption spectroscopy; CI: Configuration Interaction; and HFR: Relativistic Hartree–Fock (HFR).

Lifetimes for metastable levels were measured using the LPT. Branching fractions from some of these levels were derived from *Hubble* Space Telescope/STIS spectra of Eta Carinae. With the development of methods to eliminate the repopulation effects, Palmeri et al. (2008) re-measured the lifetime of $3d^2(^3P)4s^4P_{5/2}$ using the same technique at CRYRING and they determined a shorter lifetime (16 ± 2 s).

A number of other authors have measured the lifetimes of short-lived Ti II levels. Kwiatkowski et al. (1985), Bizzarri et al. (1993), and Langhans et al. (1995) performed the measurements using a laser-induced fluorescence (LIF) technique. Roberts et al. (1973) determined some lifetimes of Ti II levels using the beam-foil technique. These sources are also given in Table 1.

A frequently used method to determine transition probabilities or oscillator strengths for E1 transitions is to combine the lifetime for the upper level with branching fractions (BF). The lifetime can be measured using the LIF technique and the BF are determined from, for example, Fourier transform spectroscopy (FTS) of a laboratory light source. This approach has been adopted by Bizzarri et al. (1993), Pickering et al. (2001), Wood et al. (2013), and Lundberg et al. (2016) to derive absolute transitions probabilities. Wiese et al. (2001) measured the oscillator strengths for the Ti II vacuum ultraviolet (VUV) resonance

transitions $3d4s4p^4F_{3/2}^o - 4d^24s^4F_{3/2}$ at 191.0938 nm and $3d4s4p^4D_{1/2}^o - 4d^24s^4F_{3/2}$ at 191.0609 nm, using the high sensitivity absorption spectroscopy (HSAS) experiment.

There are a few theoretical works presenting lifetimes or radiative transition rates for the complex system of Ti II. Luke (1999) calculated transition rates for E1 transitions between the fine structure levels of the terms $3d^24p\{^4G^o, ^4F^o, ^2F^o, ^2D^o, ^4D^o, ^2G^o\}$ to the levels of the ground term $3d^24s^4F$ by using a configuration interaction (CI) method. Bautista et al. (2006) carried out calculations of radiative transition rates and lifetimes for states belonging to the $\{3d^24s, 3d^3, 3d^24p, 3d4s^2\}$ configurations using the AUTOSTRUCTURE package (Badnell 1986). However, some of the theoretical metastable lifetimes obtained significantly disagree with the earlier measurements (Hartman et al. 2003, 2005). Palmeri et al. (2008) presented theoretical lifetimes of 13 metastable levels, along with the transition probabilities for several decay channels from these metastable levels obtained from three pseudo-relativistic Hartree–Fock (HFR) calculations, that is, HFR(A), HFR(B), and HFR(C), using the Cowan code (Cowan 1981). Deb et al. (2008) performed calculations of E2 and M1 transition probabilities among all 37 even parity levels, using the CIV3 code of

Hibbert (1975), together with a semi-empirical fine-tuning technique to reproduce the experimental energy separations.

More recently, calculations of oscillator strengths of radiative E1 transitions and lifetimes for odd-parity states have been performed using a semi-empirical method (Ruczkowski et al. 2014, 2016). Lundberg et al. (2016) report oscillator strengths as well as lifetimes for highly excited states from laboratory measurements and calculations. The calculations were carried out using the HFR method of Cowan (1981), taking core-polarisation effects into account.

Extensive calculations of energy levels, oscillator strengths, and lifetimes in Ti II were also carried out by Kurucz using the semi-empirical approach, and these data are available in Kurucz (2017). The oscillator strengths from Kurucz (2017) for strong lines are well known to agree rather well with experimental values, but for weak lines they are inevitably more uncertain.

In the present work, ab initio calculations were performed for the 99 (37 even and 62 odd) lowest states belonging to the even $\{3d^24s, 3d^3, 3d4s^2\}$ configurations and to the odd $\{3d^24p, 3d4s4p\}$ configurations in Ti II. The calculations were performed based on the fully relativistic multiconfiguration Dirac–Hartree–Fock (MCDHF) and relativistic configuration interaction (RCI) methods, as implemented in the newest version of the general-purpose relativistic atomic structure package GRASP2018 (Froese Fischer et al. 2019). Energy levels as well as E1, M1, and E2 transition data were computed along with the corresponding lifetimes of these states.

2. Multiconfiguration Dirac–Hartree–Fock approach

In the MCDHF method (Grant 2007; Froese Fischer et al. 2016), the atomic state functions (ASFs) used to describe the states of the atom are approximate eigenfunctions of the Dirac–Coulomb Hamiltonian given by

$$H_{\text{DC}} = \sum_{i=1}^N [c\alpha_i \cdot \mathbf{p}_i + (\beta_i - 1)c^2 + V_i^N] + \sum_{i>j}^N \frac{1}{r_{ij}}, \quad (1)$$

where α and β are the 4×4 Dirac matrices, V_i^N is the monopole part of the electron–nucleus Coulomb interaction, c is the speed of light in atomic units, $\mathbf{p}_i \equiv -i\nabla$ is the electron momentum operator, and r_{ij} is the distance between electrons i and j . The ASFs (Ψ) are expanded over configuration state functions (CSFs, Φ):

$$\Psi(\gamma P J M_J) = \sum_{i=1}^{N_{\text{CSF}}} c_i \Phi(\gamma_i P J M_J). \quad (2)$$

The CSFs are jj -coupled many-electron functions built from products of one-electron Dirac orbitals. As for the notation, J and M are the angular quantum numbers, P is parity, and γ_i specifies the occupied subshells of the CSF with their complete angular coupling tree information, including for instance the orbital occupancy, coupling scheme, and other quantum numbers necessary to uniquely describe the CSFs.

The radial parts of the Dirac orbitals together with the expansion coefficients c_i of the CSFs are obtained in a relativistic self-consistent field (SCF) procedure by solving the Dirac–Hartree–Fock radial equations and the configuration interaction eigenvalue problem, resulting from applying the variational principle on the statistically weighted energy functional of the targeted states with terms added in order to preserve the orthonormality of the one-electron orbitals. The angular integrations needed for the construction of the energy functional are

based on the second quantisation method in the coupled tensorial form (Gaigalas et al. 1997, 2001).

Higher order interactions, such as the transverse photon interaction and quantum electrodynamic effects (vacuum polarisation and self-energy), are added to the Dirac–Coulomb Hamiltonian in subsequent RCI calculations (McKenzie et al. 1980). Keeping the radial components from the previous step fixed, the expansion coefficients c_i of the CSFs for the targeted states are obtained by solving the configuration interaction eigenvalue problem.

The transition data (e.g. transition probabilities and weighted oscillator strengths) between two states $\gamma' P' J'$ and $\gamma P J$ are expressed in terms of reduced matrix elements of the transition operator $\mathbf{T}$ (Grant 1974):

$$\langle \Psi(\gamma P J) \| \mathbf{T} \| \Psi(\gamma' P' J') \rangle = \sum_{j,k} c_j c'_k \langle \Phi(\gamma_j P J) \| \mathbf{T} \| \Phi(\gamma'_k P' J') \rangle. \quad (3)$$

For E1 and E2 transitions, there are the following two forms of the transition operator: the Babushkin and Coulomb forms, which are equivalent to the length and the velocity form in the non-relativistic quantum mechanics. Babushkin and Coulomb gauges for the exact solutions of the Dirac-equation give the same value of the transition moment (Grant 1974).

3. Computational schemes

Calculations were performed in the extended optimal level (EOL) scheme (Dyall et al. 1989) for the weighted average of the even and odd parity states. These states belong to the $\{3d^24s, 3d^3, 3d4s^2\}$ even configurations and the $\{3d^24p, 3d4s4p\}$ odd configurations. From the initial calculations and analysis of the eigenvector compositions, we identified a number of configurations giving considerable contributions to the total wave functions. Particularly, the two-electron excitation configurations $\{3p^43d^44s, 3p^43d^34s^2, \text{ and } 3p^43d^5\}$ are of crucial importance to predict the correct order of the energy levels for even states. These important configurations, together with the target configurations, define what is known as the multireference (MR). The MR sets for the even and odd parities are presented in Table 2, which also displays the number of CSFs in the final even and odd state expansions distributed over the different J symmetries.

The CSF expansions were determined using the multireference-single-double (MR-SD) method, allowing single and double (SD) substitutions from configurations in the MR to orbitals in an active set (AS; Olsen et al. 1988; Sturesson et al. 2007; Froese Fischer et al. 2016). In the MCDHF calculations, no substitutions were allowed from the Ar-like core ($1s^22s^22p^63s^23p^6$), which defines an inactive closed core. This means that only valence–valence (VV) electron correlation effects were taken into account in the MCDHF calculations. These calculations were followed by RCI calculations for an extended expansion, which was achieved by opening the $3s$ and $3p$ core shells for substitutions to include core–valence (CV) and core–core (CC) effects. In the RCI calculations, $1s^22s^22p^6$ was kept as an inactive closed core. The CV expansion was obtained by allowing SD substitutions from the valence orbitals and the $3s^23p^6$ core, with the restriction that there was one substitution at most from $3s^23p^6$ to the active set $\{8s, 8p, 8d, 8f, 8g, 8h, 7i\}$. The inclusion of CC correlations in the calculations decreases the excitation energies significantly and they also have large effects on the energy separations. Considering the fact that the inclusion of CC correlations increases the number of

Table 2. Summary of the active space construction.

Parity	MR	Extended MR in RCI	N_{CSFs}
Even	$3p^63d^3$, $3p^63d^24s$, $3p^63d4s^2$, $3p^63d4p^2$, $3p^64s^24d$, $3p^63d4s4d$, $3p^63d4s5s$, $3p^63d^24d$, $3p^63d4d^2$, $3p^64s4d^2$, $3p^64s4p^2$	$3p^43d^44s$ $3p^43d^34s^2$ $3p^43d^5$	14 089 101
Odd	$3p^63d^24p$, $3p^63d4s4p$, $3p^64s4p4d$, $3p^63d4s5p$, $3p^63d4p4d$, $3p^64s^24p$, $3p^64p4d^2$	$3p^43d^34s4p$ $3p^43d^44p$	15 573 967

Notes. MR: multireference, where the targeted configurations are labelled in boldface; and N_{CSFs} : number of CSFs.

Table 3. Transition data of E1, M1, and E2 transitions for Ti II from the present calculations.

Upper	Lower	EM	ΔE (cm ⁻¹)	λ (Å)	S	$\log(gf)$	A (s ⁻¹)
$3d^2({}_0^1S)4p^2P_{3/2}^o$	$3d^2({}_2^3F)4s^4F_{3/2}$	E1	63375	1577.901	3.245E-07	-7.201	4.295E+01
$3d^2({}_0^1S)4p^2P_{3/2}^o$	$3d^2({}_2^3F)4s^4F_{5/2}$	E1	63281	1580.248	1.274E-06	-6.607	1.679E+02
$3d^2({}_0^1S)4p^2P_{1/2}^o$	$3d^2({}_2^3F)4s^4F_{3/2}$	E1	63276	1580.356	8.055E-07	-6.807	2.120E+02
$3d^2({}_0^1S)4p^2P_{3/2}^o$	$3d^3({}_3^4F)4F_{3/2}$	E1	62467	1600.836	7.410E-08	-7.849	9.371E+00
$3d^2({}_0^1S)4p^2P_{3/2}^o$	$3d^3({}_3^4F)4F_{5/2}$	E1	62391	1602.785	3.301E-05	-5.200	4.161E+03
—	—	—	—	—	—	—	—

Notes. Upper and lower states, transition type, wavenumber, ΔE , wavelength, λ , line strength, S , weighted oscillator strength, $\log(gf)$, transition probability, A , provided by the present calculations are shown in the table. Wavelength and wavenumber values are adjusted to match the level energy values in the NIST ASD (Kramida et al. 2019). Only the first five rows are shown; the full table is available at the CDS.

CSFs very rapidly, which is beyond the reach of the available computational resources, the CC expansion was obtained by allowing SD substitutions from both $3s$ and $3p$ shells to the active set $\{8s, 8p, 8d\}$. The MR set in the RCI calculations was extended to include additional important configurations. For these configurations, single substitutions from valence and core orbitals were allowed to the active orbital set $\{8s, 8p, 8d\}$ and double substitutions to $\{4s, 4p, 4d\}$. The number of CSFs in the final even and odd state expansions are 14 089 101 and 15 573 967, respectively, and they are distributed over the different J symmetries.

4. Results and discussions

4.1. Energy levels

The energies and wave function composition in LS -coupling for the 99 (37 even and 62 odd) lowest states for Ti II are given in Table A.1. In the calculations, the labelling of the eigenstates is determined by the LSJ -coupled CSF with the largest coefficient in the expansion resulting from the transformation from jj -coupling to LSJ -coupling using the method by Gaigalas et al. (2017).

The accuracy of the wave functions from the present calculation was evaluated by comparing the calculated energy levels with experimental data from Huldt et al. (1982) provided via the NIST ASD. In most cases, the differences of the computed energy levels compared to the experimental data are less than 1.0%. The largest disagreements are for the $4F$ states in the

Fig. 1. Comparison of $\log(gf)$ values of the current study with the values available in the NIST ASD.

ground configuration $3d^24s$ and the first excited configuration $3d^3$, and the differences are about 4%. The averaged uncertainty of computed energy spectra compared to NIST data is 1.06%.

In addition, the Landé g -factors for the 99 states in Ti II were derived from the wave function expansions. The results were collected and made available in a recent paper by Li et al. (2020), in which they provide a collection of theoretical Landé g -factors for ions of astrophysical interest.

Fig. 2. Comparison of the theoretical $\log(gf)$ values to those published by Blackwell et al. (1982). *Left panel:* this work (black square). *Right panel:* Ruczkowski et al. (2014) (red square); Lundberg et al. (2016) (blue circle); and Kurucz (2017) (magenta triangle).

Fig. 3. Comparison of the theoretical $\log(gf)$ values to those published by Bizzarri et al. (1993). *Left panel:* this work (black square). *Right panel:* Ruczkowski et al. (2014) (red square); Lundberg et al. (2016) (blue circle); and Kurucz (2017) (magenta triangle).

4.2. Transition rates and oscillator strengths

Transition data, for example, wavenumbers, wavelengths, line strengths, weighted oscillator strengths, as well as transition probabilities of E1, M1, and E2 transitions are given in Table 3; a full table is available at the CDS. We note that the wavenumbers and wavelengths are adjusted to match the level energy values in the NIST ASD.

The uncertainties of the computed LS -allowed E1 transition rates can be estimated by the relative differences between values in length and velocity forms, dT (Froese Fischer 2009; Ekman et al. 2014):

$$dT = \frac{|A_l - A_v|}{\max(A_l, A_v)}, \quad (4)$$

where A_l and A_v are the transition rates in length and velocity form. For all the strong E1 transitions with $A \geq 10^6 \text{ s}^{-1}$, which are dominated by the LS -allowed transitions, dT always remains below 20%; the mean dT of these strong transitions is 0.122. Contrary to the strong transitions, 891 out of 1302 E1 transitions are weaker transitions with $A < 10^6 \text{ s}^{-1}$, which are associated with larger dT values; the average dT for these transitions is 0.509. These weak transitions are mainly LS -forbidden E1 transitions, for example, the intercombination and two-electron one-photon transitions. For these types of transitions, as discussed in Ekman et al. (2014), dT can no longer be used as an uncertainty

estimate for each individual transition rate and it should only be used if averaging over a very large sample. On the other hand, the transitions are governed by the outer part of the wave functions; the length form is more sensitive to this part of the wave functions and it is generally considered to be the preferred form (Grant 1974; Hibbert 1974). Therefore, only transition data and lifetimes in length form are presented in this work.

The accuracy of calculated transition rates in length form can be estimated by extensive comparisons with other theoretical works and experimental results. In Fig. 1, $\log(gf)$ values from the present work are compared with available results from NIST ASD. The NIST data are compiled based on the results from Roberts et al. (1973, 1975), Wolnik & Berthel (1973), Danzmann & Kock (1980), Kostyk & Orlova (1983), Pickering et al. (2001), Wiese et al. (2001), and Morton (2003) for E1 transitions and from Hartman et al. (2005) and Deb et al. (2008) for M1 and E2 transitions. As shown in the figure, the agreement between the $\log(gf)$ values computed in the present work and the respective results from the NIST database are rather good for most of the transitions. Furthermore, the computed $\log(gf)$ values are compared with the results from various experimental sources (Blackwell et al. 1982; Bizzarri et al. 1993; Pickering et al. 2001, 2002; Wood et al. 2013), which were carried out by a combination of LIF and FTS methods, as seen in Figs. 2–5 in the left panels for the present work and the right panels for other recent theoretical results (Ruczkowski et al. 2014;

Fig. 4. Comparison of the theoretical $\log(gf)$ values to those published by Pickering et al. (2001, 2002). *Left panel:* this work (black square). *Right panel:* Ruczkowski et al. (2014) (red square); Lundberg et al. (2016) (blue circle); and Kurucz (2017) (magenta triangle).

Fig. 5. Comparison of the theoretical $\log(gf)$ values to those published by Wood et al. (2013). *Left panel:* this work (black square). *Right panel:* Ruczkowski et al. (2014) (red square); Lundberg et al. (2016) (blue circle); and Kurucz (2017) (magenta triangle).

Table 4. Comparison between experimental and calculated oscillator strengths ($\log(gf)$) for the VUV resonance transitions.

Upper	Lower	$\lambda(\text{\AA})$	$\log(gf)$					
			Present	HFR ^(a)	Semi-empirical ^(b)	Exp. ^(c)	Exp. ^(d)	Exp.-new ^(d)
$3d4s4p\ ^4F_{3/2}^o$	$3d^24s\ ^4F_{3/2}$	1910.612	-0.363	-0.38	-0.372	-0.381(16.7)	-0.41(9)	0.388
$3d4s4p\ ^4D_{1/2}^o$	$3d^24s\ ^4F_{3/2}$	1910.954	-0.398	-0.54	-0.470	-0.407(13.3)	-0.55(8)	0.400

References. ^(a)Lundberg et al. (2016); ^(b)Kurucz (2017); ^(c)Wiese et al. (2001); and ^(d)Pickering et al. (2001).

Lundberg et al. 2016; Kurucz 2017). From the figures, we note an overall better agreement between the present results and experimental values, especially for transitions with $\log(gf) \geq 1.0$. The comparisons between other theoretical values, that is, semi-empirical calculations (Ruczkowski et al. 2014; Kurucz 2017) and HFR calculations (Lundberg et al. 2016) as well as experimental results show a much wider scatter.

In Table 4, the computed and measured $\log(gf)$ values for the two VUV resonance transitions are given. Wiese et al. (2001) measured the $\log(gf)$ values using the HSAS experiment at the Synchrotron Radiation Center in Stoughton, Wisconsin. Pickering et al. (2001) determined the two values by a combi-

nation of FTS branching fractions and Kurucz's calculated lifetimes. The discrepancy between the values obtained from the two different methods are quite large for the 1910.954 Å transition, and the present computed value is in good agreement with the one from Wiese et al. (2001). We have revised the Pickering et al. (2001) data by combining their BF's with our lifetimes, which should be more accurate than previous calculations. The new data is included in the last column in Table 4. It can be seen that the new $\log(gf)$ values are in excellent agreement with those from Wiese et al. (2001).

In addition, the present work can be compared with other theoretical calculations. In Fig. 6, $\log(gf)$ values from the present

Fig. 6. Comparison of $\log(gf)$ values of the current study with the values from other theoretical work. *Top left:* Luke (1999). *Top right:* Ruczkowski et al. (2014). *Bottom left:* Lundberg et al. (2016). *Bottom right:* Kurucz (2017).

Table 5. Comparison between experimental and calculated forbidden transition probabilities.

Upper	Lower	Type	$\lambda(\text{\AA})$	$A \text{ (s}^{-1}\text{)}$				
				Present	CI ^(a)	HFR(B) ^(b)	Semi-empirical ^(c)	Exp. ^(d)
$3d4s^2\ ^2D_{3/2}$	$3d^2(^3F)4s\ ^2F_{5/2}$	M1,E2	4918.226	2.314	4.127	3.18	2.338	2.15(20)
	$3d^2(^3F)4s\ ^2F_{7/2}$	E2	4984.182	0.366	0.662	0.50	0.367	0.42(8)
	$3d^2(^1D)4s\ ^2D_{3/2}$	M1,E2	6153.610	0.311	0.349	0.34	0.343	0.53(16)
	$3d^2(^1D)4s\ ^2D_{5/2}$	M1,E2	6166.426	0.106	0.106	0.11	0.119	0.048(16)
	$3d^3\ ^2G_{7/2}$	E2	6264.328	0.182	0.143	0.16	0.267	0.20(9)
$3d4s^2\ ^2D_{5/2}$	$3d^2(^3F)4s\ ^2F_{7/2}$	M1,E2	4927.262	2.087	3.881	2.90	2.074	1.9(3)
	$3d^2(^1D)4s\ ^2D_{5/2}$	M1,E2	6079.536	0.364	0.410	0.40	0.399	0.41(9)
	$3d^3\ ^2G_{9/2}$	E2	6220.964	0.188	0.146	0.17	0.276	0.19(5)

References. ^(a)Deb et al. (2008); ^(b)Palmeri et al. (2008); ^(c)Kurucz (2017); and ^(d)Hartman et al. (2005).

work are compared with the results from CI (Luke 1999) as well as semi-empirical (Ruczkowski et al. 2014; Kurucz 2017) and HFR (Lundberg et al. 2016) calculations, when available. As shown in the figure, the deviations between the $\log(gf)$ values computed in the present work and respective results from other sources are small for values down to -1.5 , with a much wider scatter for the weaker lines. Given the overall good agreement between the present work and experimental results observed

in the above mentioned discussions, we suggest that the current results together with the experimental values are used as a reference.

For the M1 and E2 forbidden transitions, there are a number of measurements of transition rates A . In Table 5, the computed A values are compared with experimental results by Hartman et al. (2005), which were obtained using a combination of laboratory and astrophysical measurements. The computed

Fig. 7. Comparison of $\log(A)$ values for M1+E2 transitions between even states of the current study with the values from Deb et al. (2008).

A values from the present work and semi-empirical calculations from Kurucz's database (Kurucz 2017) are in much better agreement with those measurements than from the other theoretical results, that is, the HFR calculation by Palmeri et al. (2008) and the CI calculation by Deb et al. (2008), which were compiled in the NIST ASD. Computed A values from the present work are within the uncertainties of experimental measurements, except for transitions $3d4s^2\ ^2D_{3/2} - 3d^2(^1D)4s\ ^2D_{3/2,5/2}$, for which, on the contrary, the various theoretical works give much closer values. Finally, in Fig. 7, we compare the computed M1 and E2 transition rates with values from the CI calculations by Deb et al. (2008). As shown in the figure, the deviations are small for most of the strong transitions, but with a much wider scatter for the weaker lines.

4.3. Lifetimes

The calculated lifetimes are compared with other theoretical predictions and experimental values, when available, in Tables A.2 and A.3 for even and odd states, respectively. The agreement with Kurucz's semi-empirical results (Kurucz 2017) is overall good for both even and odd states.

In Fig. 8, the lifetimes from experiments (Palmeri et al. 2008; Hartman et al. 2003, 2005) using the LPT approach are compared with the results from the present calculations and from other theoretical works (Kurucz 2017; Deb et al. 2008; Palmeri et al. 2008; Grant 1974; Bautista et al. 2006). As can be seen from the figure, the theoretical values from this work fall into, or only slightly outside, the range of the estimated uncertainties of the experimental values. On the contrary, the differences between the measured and computed lifetimes by Deb et al. (2008), using the CIV3 code, are larger for all of these values.

For odd states, a number of measurements of lifetimes (Roberts et al. 1973; Kwiatkowski et al. 1985; Gosselin et al. 1987; Bizzarri et al. 1993; Langhans et al. 1995; Lundberg et al. 2016) are compared with the theoretical results from the present work, semi-empirical calculations (Ruczkowski et al. 2016; Kurucz 2017), and HFR calculations (Lundberg et al. 2016). As shown in the figure, the measured lifetimes from Roberts et al. (1973) are overall larger than the values from other measurements, and all the calculated values from the present work agree with those of the latter within the experimental errors. The Kurucz's values, which are frequently used

Fig. 8. Comparison of the experimental (with error bars) and calculated lifetimes (in s) for even states of Ti II. (a) Kurucz (2017); (b) Deb et al. (2008); (c) Palmeri et al. (2008); (d) Bautista et al. (2006); (e) Hartman et al. (2005); and (f) Hartman et al. (2003). We note that ilev, a label in the x -axis, corresponds to the ilev in Table A.1.

as a reference in many literature, are in agreement with experimental values in most cases, except for the $3d^2(^3F)^3F4p\ ^2G_{7/2}^o$, $3d^2(^1D)^1D4p\ ^2D_{5/2}^o$, and $3d^2(^1D)^1D4p\ ^2F_{5/2}^o$ levels.

4.4. Labelling of the $3d^2(^1D)4p\ y^2D_{3/2}^o$ and $z^2P_{3/2}^o$ levels

The NIST label classification for Ti II adheres to the analysis by Huldt et al. (1982), who assigned the $39\,603\text{ cm}^{-1}$ level as $3d^2(^1D)4p\ ^2D_{3/2}^o$ and the $39\,233\text{ cm}^{-1}$ level as $^2P_{3/2}^o$. Due to the strong mixing between the states, the assignment is uncertain and it was later questioned by Bizzarri et al. (1993). Based on lifetime measurements, they suggested that the assignments for the two levels should be interchanged. However, they did not update the identifications, according to their suggestion, and the lifetime values and oscillator strengths in later publications (Pickering et al. 2001, 2002; Wood et al. 2013) are presented using the original identification. The present calculations support the suggestion of Bizzarri. Our calculated level energies are $39\,357\text{ cm}^{-1}$ for $^2D_{3/2}^o$ and $39\,723\text{ cm}^{-1}$ for $^2P_{3/2}^o$, suggesting the experimental level $39\,233\text{ cm}^{-1}$ to be $^2D_{3/2}^o$ and the level $39\,603\text{ cm}^{-1}$ to be $^2P_{3/2}^o$. By reassigning the two levels, there is also a significant improvement in the agreement between our theoretical and the measured $\log(gf)$ values. As shown in Fig. 10, after correcting the labels, the agreement is very good for most of the transitions. Further support for the reassignment is offered by the lifetimes, which are 5.48 and 4.42 ns, respectively, for $^2D_{3/2}^o$ and $^2P_{3/2}^o$ from the present work, relative to 4.5(2) and 5.5(3) ns from Bizzarri et al. (1993) using the original identifications. Again, the reassignment of the two levels results in a perfect agreement between the theoretical and measured lifetimes (see Table A.3). Please note that all the labels in this paper have been corrected to the new labels.

5. Summary and conclusions

In the present work, extended transition data of E1, M1, and E2 transitions and lifetimes are made available for Ti II using MCDHF and RCI methods. The computations of transition properties in the systems of Ti II are challenging, mainly due to the strong interaction between the terms and configurations of

Fig. 9. Comparison of the experimental (with error bars) and calculated lifetimes (in ns) for odd states of Ti II. (a) Kurucz (2017); (b) Ruczkowski et al. (2016); (c) Bizzarri et al. (1993); (d) Kwiatkowski et al. (1985); (e) Roberts et al. (1973); (f) Langhans et al. (1995); (g) Gosselin et al. (1987); and (h) Lundberg et al. (2016). We note that ilev, a label in the x -axis, corresponds to the ilev in Table A.1.

Fig. 10. Comparison of the theoretical $\log(gf)$ values for the transitions involving $3d^2(^1D)4p^2D_{3/2}$ or $^2P_{3/2}$ levels to measured values by Pickering et al. (2001, 2002) and Wood et al. (2013), both before and after the interchange of the labelling of the two levels.

the even states. Thus, some of the states are strongly mixed, and highly correlated wave functions are needed to accurately predict their LS -composition. It is found that the inclusion of $\{3p^43d^44s, 3p^43d^34s^2 \text{ and } 3p^43d^5\}$ configurations in MR is very important to predict the correct order of the even energy levels.

We have performed an extensive comparison of the computed oscillator strengths and lifetimes with other theoretical and experimental results. There is a significant improvement in the accuracy in the computed oscillator strengths of E1 transitions compared with previous theoretical works.

The computed lifetimes of odd states are all in excellent agreement with the measured lifetimes. We suggest that the current results be used as a reference for Ti II for various astrophysical applications.

A careful comparison of the energies of the levels, the oscillator strengths, as well as the radiative lifetimes with the experimental values has made possible the identification of the mislabelling of the $3d^2(^1D)4p \ y^2D_{3/2}^o$ and $z^2P_{3/2}^o$ levels adopted in the NIST ASD. The assignments for the $^2D_{3/2}^o$ and $^2P_{3/2}^o$ levels should be interchanged.

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Appendix A: Atomic state function composition in LS-coupling, energy levels, and lifetimes for the Ti II ion.

Table A.1. Atomic state function composition (up to three *LS* components with a contribution >0.02 of the total wave function) in *LS*-coupling and energy levels (in cm⁻¹) for Ti II.

ilev	State	<i>LS</i> -composition	<i>E_{RCI}</i>	<i>E_{NIST}</i>
1	$3d^2(^3F) ^3F 4s ^4F_{3/2}$	0.93	0	0
2	$3d^2(^3F) ^3F 4s ^4F_{5/2}$	0.93	90	94
3	$3d^2(^3F) ^3F 4s ^4F_{7/2}$	0.93	215	226
4	$3d^2(^3F) ^3F 4s ^4F_{9/2}$	0.93	375	393
5	$3d^3(^4F) ^4F_{3/2}$	0.94	963	908
6	$3d^3(^4F) ^4F_{5/2}$	0.94	1 034	984
7	$3d^3(^4F) ^4F_{7/2}$	0.94	1 132	1 087
8	$3d^3(^4F) ^4F_{9/2}$	0.94	1 253	1 216
9	$3d^2(^3F) ^3F 4s ^2F_{5/2}$	0.90	4 775	4 629
10	$3d^2(^3F) ^3F 4s ^2F_{7/2}$	0.90	5 032	4 898
11	$3d^2(^1D) ^1D 4s ^2D_{3/2}$	$0.71 + 0.14 3p^6 3d^3(^2D) ^2D + 0.05 3p^6 3d^3(^1D) ^2D$	8 835	8 711
12	$3d^2(^1D) ^1D 4s ^2D_{5/2}$	$0.73 + 0.13 3p^6 3d^3(^2D) ^2D + 0.05 3p^6 3d^3(^1D) ^2D$	8 867	8 744
13	$3d^3(^2G) ^2G_{7/2}$	$0.89 + 0.04 3p^6 3d^2(^1G) ^1G 4s ^2G$	9 256	8 998
14	$3d^3(^2G) ^2G_{9/2}$	$0.89 + 0.04 3p^6 3d^2(^1G) ^1G 4s ^2G$	9 370	9 118
15	$3d^3(^4P) ^4P_{1/2}$	$0.85 + 0.04 3p^6 3d^2(^3P) ^3P 4s ^4P + 0.03 3p^6 3d^3(^3P) ^2P$	9 506	9 364
16	$3d^3(^4P) ^4P_{3/2}$	$0.82 + 0.05 3p^6 3d^2(^3P) ^2P + 0.04 3p^6 3d^2(^3P) ^3P 4s ^4P$	9 537	9 396
17	$3d^3(^4P) ^4P_{5/2}$	$0.89 + 0.04 3p^6 3d^2(^3P) ^3P 4s ^4P$	9 655	9 518
18	$3d^3(^2P) ^2P_{1/2}$	$0.61 + 0.26 3p^6 3d^2(^3P) ^3P 4s ^2P + 0.03 3p^6 3d^2(^3P) ^3P 4s ^4P$	9 938	9 851
19	$3d^2(^3P) ^3P 4s ^4P_{1/2}$	$0.86 + 0.06 3p^6 3d^3(^4P) ^4P$	9 969	9 873
20	$3d^2(^3P) ^3P 4s ^4P_{3/2}$	$0.89 + 0.04 3p^6 3d^3(^4P) ^4P$	10 022	9 931
21	$3d^3(^3P) ^2P_{3/2}$	$0.59 + 0.25 3p^6 3d^2(^3P) ^3P 4s ^2P + 0.07 3p^6 3d^3(^4P) ^4P$	10 057	9 976
22	$3d^2(^3P) ^3P 4s ^4P_{5/2}$	$0.88 + 0.04 3p^6 3d^3(^4P) ^4P$	10 111	10 025
23	$3d^3(^2D) ^2D_{3/2}$	$0.51 + 0.20 3p^6 3d^3(^2D) ^2D + 0.20 3p^6 3d^2(^1D) ^1D 4s ^2D$	12 920	12 629
24	$3d^3(^2D) ^2D_{5/2}$	$0.52 + 0.20 3p^6 3d^3(^2D) ^2D + 0.19 3p^6 3d^2(^1D) ^1D 4s ^2D$	13 043	12 758
25	$3d^3(^2H) ^2H_{9/2}$	0.93	13 061	12 677
26	$3d^3(^2H) ^2H_{11/2}$	0.93	13 154	12 775
27	$3d^2(^1G) ^1G 4s ^2G_{9/2}$	$0.89 + 0.04 3p^6 3d^3(^2G) ^2G$	15 598	15 258
28	$3d^2(^1G) ^1G 4s ^2G_{7/2}$	$0.89 + 0.04 3p^6 3d^3(^2G) ^2G$	15 607	15 266
29	$3d^2(^3P) ^3P 4s ^2P_{1/2}$	$0.64 + 0.27 3p^6 3d^3(^3P) ^2P$	16 889	16 516
30	$3d^2(^3P) ^3P 4s ^2P_{3/2}$	$0.64 + 0.27 3p^6 3d^3(^3P) ^2P$	16 992	16 625
31	$3d^3(^2F) ^2F_{7/2}$	0.91	21 358	20 892
32	$3d^3(^2F) ^2F_{5/2}$	0.91	21 415	20 952
33	$3d ^2D 4s ^2D_{3/2}$	$0.64 + 0.12 3p^6 3d^3(^2D) ^2D + 0.11 3p^6 3d^3(^2D) ^2D$	25 080	24 961
34	$3d ^2D 4s ^2D_{5/2}$	$0.62 + 0.12 3p^6 3d^3(^2D) ^2D + 0.12 3p^6 3d^3(^2D) ^2D$	25 316	25 193
35	$3d^2(^3F) ^3F 4p ^4G_{5/2}^o$	0.93	29 561	29 544
36	$3d^2(^3F) ^3F 4p ^4G_{7/2}^o$	0.94	29 744	29 735
37	$3d^2(^3F) ^3F 4p ^4G_{9/2}^o$	0.94	29 969	29 968
38	$3d^2(^3F) ^3F 4p ^4G_{11/2}^o$	0.94	30 233	30 241
39	$3d^2(^3F) ^3F 4p ^4F_{3/2}^o$	$0.92 + 0.02 3p^6 3d^2(^3F) ^3F 4p ^2D^o$	30 818	30 836
40	$3d^2(^3F) ^3F 4p ^4F_{5/2}^o$	0.93	30 935	30 959
41	$3d^2(^3F) ^3F 4p ^4F_{7/2}^o$	0.93	31 084	31 114
42	$3d^2(^3F) ^3F 4p ^2F_{5/2}^o$	$0.78 + 0.08 3p^6 3d^2(^1D) ^1D 4p ^2F^o + 0.03 3p^6 3d^2(^3F) ^3F 4p ^2D^o$	31 258	31 208
43	$3d^2(^3F) ^3F 4p ^4F_{9/2}^o$	0.94	31 263	31 301
44	$3d^2(^3F) ^3F 4p ^2F_{7/2}^o$	$0.82 + 0.08 3p^6 3d^2(^1D) ^1D 4p ^2F^o$	31 530	31 491
45	$3d^2(^3F) ^3F 4p ^2D_{3/2}^o$	$0.77 + 0.08 3p^6 3d^2(^3P) ^3P 4p ^2D^o + 0.02 3p^6 3d^2(^3F) ^3F 4p ^4D^o$	31 779	31 757
46	$3d^2(^3F) ^3F 4p ^2D_{5/2}^o$	$0.72 + 0.08 3p^6 3d^2(^3P) ^3P 4p ^2D^o + 0.05 3p^6 3d^2(^3F) ^3F 4p ^4D^o$	32 038	32 026
47	$3d^2(^1S) ^1S 4s ^2S_{1/2}$	0.92	32 129	31 788
48	$3d^2(^3F) ^3F 4p ^4D_{1/2}^o$	0.92	32 527	32 532
49	$3d^2(^3F) ^3F 4p ^4D_{3/2}^o$	$0.89 + 0.02 3p^6 3d^2(^3F) ^3F 4p ^2D^o$	32 593	32 603
50	$3d^2(^3F) ^3F 4p ^4D_{5/2}^o$	$0.86 + 0.05 3p^6 3d^2(^3F) ^3F 4p ^2D^o$	32 683	32 698
51	$3d^2(^3F) ^3F 4p ^4D_{7/2}^o$	0.91	32 746	32 767
52	$3d^3(^2D) ^2D_{3/2}$	$0.52 + 0.19 3p^6 3d ^2D 4s ^2D + 0.15 3p^6 3d^3(^2D) ^2D$	32 754	32 275
53	$3d^3(^2D) ^2D_{5/2}$	$0.52 + 0.20 3p^6 3d ^2D 4s ^2D + 0.14 3p^6 3d^3(^2D) ^2D$	32 798	32 333

Notes. Energy levels are given relative to the ground state.

Table A.1. continued.

ilev	State	LS-composition	E_{RCI}	E_{NIST}
54	$3d^2(\frac{3}{2}F)^3F4p^2G_{7/2}^\circ$	$0.89 + 0.05\ 3p^6\ 3d^2(\frac{1}{2}G)^1G4p^2G^\circ$	34 742	34 543
55	$3d^2(\frac{3}{2}F)^3F4p^2G_{9/2}^\circ$	$0.89 + 0.05\ 3p^6\ 3d^2(\frac{1}{2}G)^1G4p^2G^\circ$	34 936	34 749
56	$3d^2(\frac{3}{2}P)^3P4p^2S_{1/2}^\circ$	0.94	37 499	37 431
57	$3d^2(\frac{1}{2}D)^1D4p^2D_{3/2}^\circ$	$0.51 + 0.29\ 3p^6\ 3d^2(\frac{1}{2}D)^1D4p^2P^\circ + 0.05\ 3p^6\ 3d^2(\frac{3}{2}P)^3P4p^2D^\circ$	39 357	39 233
58	$3d^2(\frac{1}{2}D)^1D4p^2D_{5/2}^\circ$	$0.70 + 0.06\ 3p^6\ 3d^2(\frac{3}{2}P)^3P4p^2D^\circ + 0.05\ 3p^6\ 3d^2D4s^3D4p^2D^\circ$	39 599	39 477
59	$3d^2(\frac{1}{2}D)^1D4p^2P_{3/2}^\circ$	$0.58 + 0.24\ 3p^6\ 3d^2(\frac{1}{2}D)^1D4p^2D^\circ + 0.05\ 3p^6\ 3d^2(\frac{3}{2}P)^3P4p^4S^\circ$	39 723	39 603
60	$3d^2(\frac{1}{2}D)^1D4p^2P_{1/2}^\circ$	0.92	39 789	39 675
61	$3d^2(\frac{1}{2}D)^1D4p^2F_{5/2}^\circ$	$0.77 + 0.09\ 3p^6\ 3d^2(\frac{3}{2}F)^3F4p^2F^\circ + 0.05\ 3p^6\ 3d^2(\frac{1}{2}D)^1D4p^2D^\circ$	40 143	39 927
62	$3d^2(\frac{3}{2}P)^3P4p^4S_{3/2}^\circ$	$0.89 + 0.05\ 3p^6\ 3d^2(\frac{1}{2}D)^1D4p^2P^\circ$	40 200	40 027
63	$3d^2(\frac{1}{2}D)^1D4p^2F_{7/2}^\circ$	$0.77 + 0.09\ 3p^6\ 3d^2(\frac{3}{2}F)^3F4p^2F^\circ + 0.06\ 3p^6\ 3d^2(\frac{3}{2}P)^3P4p^4D^\circ$	40 292	40 075
64	$3d^2(\frac{3}{2}P)^3P4p^4D_{1/2}^\circ$	0.92	40 423	40 330
65	$3d^2(\frac{3}{2}P)^3P4p^4D_{3/2}^\circ$	0.91	40 515	40 426
66	$3d^2(\frac{3}{2}P)^3P4p^4D_{5/2}^\circ$	0.90	40 668	40 582
67	$3d^2(\frac{3}{2}P)^3P4p^4D_{7/2}^\circ$	$0.86 + 0.05\ 3p^6\ 3d^2(\frac{1}{2}D)^1D4p^2F^\circ$	40 883	40 798
68	$3d^2(\frac{3}{2}P)^3P4p^4P_{1/2}^\circ$	$0.91 + 0.03\ 3p^6\ 3d^2D4s^3D4p^4P^\circ$	42 084	41 997
69	$3d^2(\frac{3}{2}P)^3P4p^4P_{3/2}^\circ$	$0.91 + 0.03\ 3p^6\ 3d^2D4s^3D4p^4P^\circ$	42 154	42 069
70	$3d^2(\frac{3}{2}P)^3P4p^4P_{5/2}^\circ$	$0.90 + 0.03\ 3p^6\ 3d^2D4s^3D4p^4P^\circ$	42 289	42 209
71	$3d^2(\frac{1}{2}G)^1G4p^2G_{7/2}^\circ$	$0.89 + 0.05\ 3p^6\ 3d^2(\frac{3}{2}F)^3F4p^2G^\circ$	44 088	43 741
72	$3d^2(\frac{1}{2}G)^1G4p^2G_{9/2}^\circ$	$0.88 + 0.05\ 3p^6\ 3d^2(\frac{3}{2}F)^3F4p^2G^\circ$	44 127	43 781
73	$3d^2(\frac{3}{2}P)^3P4p^2D_{5/2}^\circ$	$0.76 + 0.09\ 3p^6\ 3d^2(\frac{1}{2}D)^1D4p^2D^\circ + 0.05\ 3p^6\ 3d^2(\frac{3}{2}F)^3F4p^2D^\circ$	45 286	44 902
74	$3d^2(\frac{3}{2}P)^3P4p^2D_{3/2}^\circ$	$0.74 + 0.09\ 3p^6\ 3d^2(\frac{1}{2}D)^1D4p^2D^\circ + 0.04\ 3p^6\ 3d^2(\frac{3}{2}F)^3F4p^2D^\circ$	45 296	44 915
75	$3d^2(\frac{3}{2}P)^3P4p^2P_{1/2}^\circ$	$0.87 + 0.03\ 3p^6\ 3d^2D4s^3D4p^2P^\circ + 0.02\ 3p^6\ 3d^2D4s^1D4p^2P^\circ$	45 761	45 472
76	$3d^2(\frac{3}{2}P)^3P4p^2P_{3/2}^\circ$	$0.84 + 0.03\ 3p^6\ 3d^2D4s^3D4p^2P^\circ + 0.02\ 3p^6\ 3d^2(\frac{3}{2}P)^3P4p^2D^\circ$	45 839	45 549
77	$3d^2(\frac{1}{2}G)^1G4p^2H_{9/2}^\circ$	0.93	46 047	45 674
78	$3d^2(\frac{1}{2}G)^1G4p^2H_{11/2}^\circ$	0.94	46 277	45 909
79	$3d^2(\frac{1}{2}G)^1G4p^2F_{7/2}^\circ$	$0.84 + 0.07\ 3p^6\ 3d^2D4s^3D4p^2F^\circ + 0.02\ 3p^6\ 3d^2(\frac{1}{2}D)^1D4p^2F^\circ$	47 737	47 467
80	$3d^2(\frac{1}{2}G)^1G4p^2F_{5/2}^\circ$	$0.84 + 0.07\ 3p^6\ 3d^2D4s^3D4p^2F^\circ + 0.02\ 3p^6\ 3d^2(\frac{1}{2}D)^1D4p^2F^\circ$	47 893	47 625
81	$3d^2D4s^3D4p^4F_{3/2}^\circ$	$0.73 + 0.15\ 3p^6\ 3d^2D4s^3D4p^4D^\circ$	52 134	52 330
82	$3d^2D4s^3D4p^4D_{1/2}^\circ$	0.88	52 184	52 339
83	$3d^2D4s^3D4p^4F_{5/2}^\circ$	$0.61 + 0.27\ 3p^6\ 3d^2D4s^3D4p^4D^\circ$	52 280	52 472
84	$3d^2D4s^3D4p^4D_{3/2}^\circ$	$0.73 + 0.16\ 3p^6\ 3d^2D4s^3D4p^4F^\circ$	52 288	52 459
85	$3d^2D4s^3D4p^4D_{5/2}^\circ$	$0.60 + 0.29\ 3p^6\ 3d^2D4s^3D4p^4F^\circ$	52 447	52 631
86	$3d^2D4s^3D4p^4F_{7/2}^\circ$	$0.51 + 0.39\ 3p^6\ 3d^2D4s^3D4p^4D^\circ + 0.02\ 3p^6\ 3d^2D4s^3D5p^2F^\circ$	52 510	52 705
87	$3d^2D4s^3D4p^4D_{7/2}^\circ$	$0.49 + 0.40\ 3p^6\ 3d^2D4s^3D4p^4F^\circ$	52 650	52 847
88	$3d^2D4s^3D4p^4F_{9/2}^\circ$	$0.90 + 0.02\ 3p^6\ 3d^2D4s^3D5p^4F^\circ$	52 866	53 097
89	$3d^2D4s^1D4p^2D_{5/2}^\circ$	$0.53 + 0.28\ 3p^6\ 3d^2D4s^3D4p^2D^\circ + 0.06\ 3p^6\ 3d^2(\frac{1}{2}D)^1D4p^2D^\circ$	53 507	53 555
90	$3d^2D4s^1D4p^2D_{3/2}^\circ$	$0.53 + 0.28\ 3p^6\ 3d^2D4s^3D4p^2D^\circ + 0.06\ 3p^6\ 3d^2(\frac{1}{2}D)^1D4p^2D^\circ$	53 546	53 597
91	$3d^2D4s^3D4p^4P_{1/2}^\circ$	$0.89 + 0.03\ 3p^6\ 3d^2(\frac{3}{2}P)^3P4p^4P^\circ$	56 030	56 223
92	$3d^2D4s^3D4p^4P_{3/2}^\circ$	$0.88 + 0.03\ 3p^6\ 3d^2(\frac{3}{2}P)^3P4p^4P^\circ$	56 054	56 249
93	$3d^2D4s^3D4p^4P_{5/2}^\circ$	$0.88 + 0.03\ 3p^6\ 3d^2(\frac{3}{2}P)^3P4p^4P^\circ$	56 124	56 326
94	$3d^2D4s^1D4p^2F_{5/2}^\circ$	$0.73 + 0.14\ 3p^6\ 3d^2D4s^3D4p^2F^\circ + 0.04\ 3p^6\ 3d^2(\frac{1}{2}G)^1G4p^2F^\circ$	59 328	59 322
95	$3d^2D4s^1D4p^2P_{3/2}^\circ$	$0.46 + 0.27\ 3p^6\ 3d^2D4s^3D4p^2P^\circ + 0.17\ 3p^6\ 3d^2(\frac{1}{2}S)^1S4p^2P^\circ$	59 411	59 388
96	$3d^2D4s^1D4p^2F_{7/2}^\circ$	$0.72 + 0.15\ 3p^6\ 3d^2D4s^3D4p^2F^\circ + 0.04\ 3p^6\ 3d^2(\frac{1}{2}G)^1G4p^2F^\circ$	59 476	59 468
97	$3d^2D4s^1D4p^2P_{1/2}^\circ$	$0.42 + 0.27\ 3p^6\ 3d^2D4s^3D4p^2P^\circ + 0.21\ 3p^6\ 3d^2(\frac{1}{2}S)^1S4p^2P^\circ$	59 485	59 440
98	$3d^2(\frac{1}{2}S)^1S4p^2P_{1/2}^\circ$	$0.62 + 0.25\ 3p^6\ 3d^2D4s^1D4p^2P^\circ + 0.03\ 3p^6\ 3d^2(\frac{3}{2}P)^3P4p^2P^\circ$	63 812	63 277
99	$3d^2(\frac{1}{2}S)^1S4p^2P_{3/2}^\circ$	$0.65 + 0.22\ 3p^6\ 3d^2D4s^1D4p^2P^\circ + 0.03\ 3p^6\ 3d^2(\frac{3}{2}P)^3P4p^2P^\circ$	63 930	63 375

Table A.2. Comparison of the experimental (with the uncertainty in the last digit given in parentheses) and calculated lifetimes (in s) for even states of Ti II.

State	This work	Other calculations	Experiments
$3d^2(^3F)4s^4F_{5/2}$	3.192e+4	2.770e+4 ^(a) ; 2.77e+4 ^(b)	
$3d^2(^3F)4s^4F_{7/2}$	1.167e+4	1.008e+4 ^(a) ; 1.01e+4 ^(b)	
$3d^2(^3F)4s^4F_{9/2}$	9.070e+3	7.813e+3 ^(a) ; 7.83e+3 ^(b)	
$3d^3(^4F)^4F_{3/2}$	9.994e+5	1.189e+6 ^(a) ; 1.47e+6 ^(b)	
$3d^3(^4F)^4F_{5/2}$	5.940e+4	5.025e+4 ^(a) ; 5.08e+4 ^(b)	
$3d^3(^4F)^4F_{7/2}$	2.428e+4	2.033e+4 ^(a) ; 2.04e+4 ^(b)	
$3d^3(^4F)^4F_{9/2}$	2.041e+4	1.712e+4 ^(a) ; 1.72e+4 ^(b)	
$3d^2(^3F)4s^2F_{5/2}$	4.533e+2	5.000e+2 ^(a) ; 3.20e+2 ^(b)	
$3d^2(^3F)4s^2F_{7/2}$	4.336e+2	4.525e+2 ^(a) ; 3.19e+2 ^(b)	
$3d^2(^3D)4s^2D_{3/2}$	8.452e+1	8.130e+1 ^(a) ; 6.01e+1 ^(b)	
$3d^2(^3D)4s^2D_{5/2}$	9.988e+1	9.804e+1 ^(a) ; 7.08e+1 ^(b)	
$3d^3(^3G)^2G_{7/2}$	1.694e+2	1.647e+2 ^(a) ; 1.44e+2 ^(b)	
$3d^3(^3G)^2G_{9/2}$	1.166e+2	1.103e+2 ^(a) ; 9.30e+1 ^(b)	
$3d^3(^3P)^4P_{1/2}$	1.072e+1	9.434e+0 ^(a) ; 1.26e+1 ^(b)	
$3d^3(^3P)^4P_{3/2}$	1.144e+1	1.010e+1 ^(a) ; 1.34e+1 ^(b)	
$3d^3(^3P)^4P_{5/2}$	1.096e+1	9.709e+0 ^(a) ; 1.30e+1 ^(b)	
$3d^3(^3P)^2P_{1/2}$	1.604e+2	8.197e+1 ^(a) ; 2.02e+2 ^(b)	
$3d^3(^3P)^2P_{3/2}$	9.192e+1	7.299e+1 ^(a) ; 1.05e+2 ^(b)	
$3d^2(^3P)4s^4P_{1/2}$	1.297e+1	1.248e+1 ^(a) ; 1.23e+1 ^(b) ; 1.36e+1 ^(c) ; 1.30e+1 ^(d) ; 1.27e+1 ^(e)	
$3d^2(^3P)4s^4P_{3/2}$	1.332e+1	1.222e+1 ^(a) ; 1.26e+1 ^(b) ; 2.28e+1 ^(c) ; 1.43e+1 ^(d) ; 1.31e+1 ^(e) ; 3.11e+1 ^(f)	1.8(4)e+1 ^(g)
$3d^2(^3P)4s^4P_{5/2}$	1.379e+1	1.245e+1 ^(a) ; 1.31e+1 ^(b) ; 1.43e+1 ^(c) ; 1.36e+1 ^(d) ; 1.35e+1 ^(e) ; 1.64e+1 ^(f)	2.8(10)e+1 ^(h) ; 1.6(2)e+1 ⁽ⁱ⁾
$3d^3(^3D)^2D_{3/2}$	2.312e+1	2.193e+1 ^(a) ; 2.73e+1 ^(b) ; 1.82e+1 ^(c) ; 1.90e+1 ^(d) ; 1.81e+1 ^(e)	
$3d^3(^3D)^2D_{5/2}$	2.767e+1	2.564e+1 ^(a) ; 3.37e+1 ^(b) ; 2.15e+1 ^(c) ; 2.25e+1 ^(d) ; 2.15e+1 ^(e)	2.4(3)e+1 ⁽ⁱ⁾
$3d^3(^3H)^2H_{9/2}$	2.359e+1	2.494e+1 ^(a) ; 3.33e+1 ^(b)	
$3d^3(^3H)^2H_{11/2}$	2.689e+1	2.849e+1 ^(a) ; 3.97e+1 ^(b)	
$3d^2(^3G)4s^2G_{9/2}$	6.523e+1	6.803e+1 ^(a) ; 5.13e+1 ^(b) ; 5.62e+1 ^(c) ; 5.68e+1 ^(d) ; 5.21e+1 ^(e)	
$3d^2(^3G)4s^2G_{7/2}$	7.080e+1	7.299e+1 ^(a) ; 5.69e+1 ^(b) ; 5.97e+1 ^(c) ; 6.05e+1 ^(d) ; 5.71e+1 ^(e)	
$3d^2(^3P)4s^2P_{1/2}$	8.243e+0	8.333e+0 ^(a) ; 8.72e+0 ^(b) ; 7.21e+0 ^(c) ; 7.08e+0 ^(d) ; 7.00e+0 ^(e) ; 12.7e+0 ^(f)	14(3)e+0 ^(g) ; 7.7(7)e+0 ⁽ⁱ⁾
$3d^2(^3P)4s^2P_{3/2}$	8.082e+0	8.197e+0 ^(a) ; 8.48e+0 ^(b) ; 7.12e+0 ^(c) ; 7.01e+0 ^(d) ; 6.87e+0 ^(e)	7.0(6)e+0 ⁽ⁱ⁾
$3d^3(^3F)^2F_{7/2}$	2.222e+0	2.188e+0 ^(a) ; 2.97e+0 ^(b) ; 2.04e+0 ^(c) ; 2.14e+0 ^(d) ; 2.34e+0 ^(e)	
$3d^3(^3F)^2F_{5/2}$	2.184e+0	2.137e+0 ^(a) ; 2.95e+0 ^(b) ; 2.00e+0 ^(c) ; 2.11e+0 ^(d) ; 2.31e+0 ^(e)	
$3d^2D4s^2D_{3/2}$	3.006e-1	2.865e-1 ^(a) ; 1.8e-1 ^(b) ; 2.18e-1 ^(c) ; 2.29e-1 ^(d) ; 2.17e-1 ^(e) ; 1.92e-1 ^(f)	2.9(1)e-1 ^(g)
$3d^2D4s^2D_{5/2}$	3.179e-1	3.049e-1 ^(a) ; 1.9e-1 ^(b) ; 2.28e-1 ^(c) ; 2.42e-1 ^(d) ; 2.29e-1 ^(e) ; 1.96e-1 ^(f)	3.3(2)e-1 ^(g)
$3d^2(^3S)4s^2S_{1/2}$	2.366e-1	2.096e-1 ^(a)	
$3d^3(^3D)^2D_{3/2}$	1.074e-2	2.242e-2 ^(a)	
$3d^3(^3D)^2D_{5/2}$	1.532e-2	2.740e-2 ^(a)	

References. Semi-empirical: ^(a)Kurucz (2017); CI: ^(b)Deb et al. (2008); HFR(A): ^(c)Palmeri et al. (2008); HFR(B): ^(d)Palmeri et al. (2008); HFR(C): ^(e)Palmeri et al. (2008); AUTOSTRUCTURE: ^(f)Bautista et al. (2006); LPT: ^(g)Hartman et al. (2005); LPT: ^(h)Hartman et al. (2003); and LPT: ⁽ⁱ⁾Palmeri et al. (2008).

Table A.3. Comparison of the experimental (with the uncertainty in the last digit given in parentheses) and calculated lifetimes (in ns) for odd states of Ti II.

State	This work	Other calculations	Experiments
$3d^2(^3F)4p^4G_{5/2}^o$	5.659	5.988 ^(a) ; 6.1 ^(b)	5.7(3) ^(c) ; 5.9(6) ^(d) ; 6.6(7) ^(e)
$3d^2(^3F)4p^4G_{7/2}^o$	5.598	5.917 ^(a) ; 6.0 ^(b)	5.6(3) ^(c) ; 5.8(5) ^(d) ; 7.8(8) ^(e)
$3d^2(^3F)4p^4G_{9/2}^o$	5.520	5.848 ^(a) ; 6.0 ^(b)	5.6(3) ^(c) ; 5.7(6) ^(d) ; 7.3(7) ^(e)
$3d^2(^3F)4p^4G_{11/2}^o$	5.434	5.780 ^(a) ; 5.9 ^(b)	5.6(3) ^(c) ; 5.7(7) ^(d) ; 7.2(7) ^(e)
$3d^2(^3F)4p^4F_{3/2}^o$	4.098	4.202 ^(a) ; 3.9 ^(b)	4.1(2) ^(c) ; 4.2(4) ^(d) ; 4.5(3) ^(f)
$3d^2(^3F)4p^4F_{5/2}^o$	4.057	4.149 ^(a) ; 3.8 ^(b) ; 3.76 ^(g)	4.1(2) ^(c) ; 4.1(3) ^(d) ; 4.3(3) ^(f) ; 3.87(20) ^(h)
$3d^2(^3F)4p^4F_{7/2}^o$	4.033	4.132 ^(a) ; 3.8 ^(b)	4.1(2) ^(c) ; 4.4(6) ^(d) ; 4.5(2) ^(f)
$3d^2(^3F)4p^4F_{9/2}^o$	4.015	4.098 ^(a) ; 3.8 ^(b)	4.1(2) ^(c) ; 4.3(4) ^(d) ; 5.4(8) ^(e) ; 4.2(2) ^(f)
$3d^2(^3F)4p^2F_{5/2}^o$	7.028	7.092 ^(a) ; 7.2 ^(b)	6.8(3) ^(c)
$3d^2(^3F)4p^2F_{7/2}^o$	6.896	6.993 ^(a) ; 7.1 ^(b)	6.8(3) ^(c)
$3d^2(^3F)4p^2D_{3/2}^o$	6.874	6.849 ^(a) ; 6.8 ^(b) ; 5.87 ^(g)	6.6(3) ^(c) ; 6.3(10) ^(d) ; 6.10(20) ^(h)
$3d^2(^3F)4p^2D_{5/2}^o$	6.782	6.757 ^(a) ; 6.6 ^(b)	6.6(3) ^(c) ; 6.5(9) ^(d)
$3d^2(^3F)4p^4D_{1/2}^o$	3.783	3.788 ^(a) ; 3.5 ^(b)	3.9(2) ^(c) ; 4.0(4) ^(d) ; 4.0(3) ^(f)
$3d^2(^3F)4p^4D_{3/2}^o$	3.842	3.846 ^(a) ; 3.6 ^(b)	4.0(2) ^(c) ; 4.1(5) ^(d) ; 4.0(3) ^(f)
$3d^2(^3F)4p^4D_{5/2}^o$	3.911	3.922 ^(a) ; 3.7 ^(b) ; 3.47 ^(g)	4.0(2) ^(c) ; 3.9(4) ^(d) ; 4.01(6) ⁽ⁱ⁾ ; 5.2(8) ^(e) ; 4.2(3) ^(f) ; 3.86(20) ^(h)
$3d^2(^3F)4p^4D_{7/2}^o$	3.846	3.846 ^(a) ; 3.6 ^(b) ; 3.40 ^(g)	4.0(2) ^(c) ; 4.1(5) ^(d) ; 4.2(3) ^(f) ; 3.75(20) ^(h)

References. Semi-empirical: ^(a)Kurucz (2017); Semi-empirical: ^(b)Ruczowski et al. (2016); LIF: ^(c)Bizzarri et al. (1993); LIF: ^(d)Kwiatkowski et al. (1985); Beam-foil: ^(e)Roberts et al. (1973); LIF: ^(f)Langhans et al. (1995); HFR: ^(g)Lundberg et al. (2016); LIF: ^(h)Lundberg et al. (2016); and Beam-laser: ⁽ⁱ⁾Gosselin et al. (1987).

Table A.3. continued.

State	This work	Other calculations	Experiments
$3d^2(^3F)4p\ ^2G_{7/2}^o$	4.633	4.854 ^(a) ; 5.5 ^(b)	4.6(2) ^(c) ; 6.7(16.7) ^(e)
$3d^2(^3F)4p\ ^2G_{9/2}^o$	4.642	4.854 ^(a) ; 5.5 ^(b)	4.6(2) ^(c) ; 4.8(4) ^(d) ; 6.2(15.5) ^(e)
$3d^2(^3P)4p\ ^2S_{1/2}^o$	5.901	5.917 ^(a) ; 5.0 ^(b)	10.0(>25) ^(e)
$3d^2(^1D)4p\ ^2D_{3/2}^o$	5.484	5.495 ^(a) ; 5.1 ^(b)	5.5(3) ^(c) ; 6.7(>16.7) ^(e)
$3d^2(^1D)4p\ ^2D_{5/2}^o$	6.385	5.435 ^(a) ; 6.1 ^(b)	6.1(3) ^(c) ; 8.0(12) ^(e)
$3d^2(^1D)4p\ ^2P_{3/2}^o$	4.422	4.464 ^(a) ; 4.2 ^(b)	4.5(2) ^(c)
$3d^2(^1D)4p\ ^2P_{1/2}^o$	3.864	3.831 ^(a) ; 3.6 ^(b)	4.0(2) ^(c)
$3d^2(^1D)4p\ ^2F_{5/2}^o$	3.729	4.425 ^(a) ; 3.7 ^(b)	3.9(2) ^(c) ; 5.9(8.8) ^(e)
$3d^2(^1D)4p\ ^2F_{7/2}^o$	3.644	3.759 ^(a) ; 3.5 ^(b)	3.8(2) ^(c) ; 4.1(4) ^(d) ; 6.7(1) ^(e)
$3d^2(^3P)4p\ ^4S_{3/2}^o$	3.483	3.521 ^(a) ; 3.1 ^(b)	3.6(2) ^(c) ; 5.5(8) ^(e)
$3d^2(^3P)4p\ ^4D_{1/2}^o$	4.311	4.484 ^(a) ; 3.3 ^(b)	4.4(2) ^(c) ; 4.6(5) ^(d)
$3d^2(^3P)4p\ ^4D_{3/2}^o$	4.302	4.464 ^(a) ; 3.3 ^(b)	4.5(2) ^(c)
$3d^2(^3P)4p\ ^4D_{5/2}^o$	4.260	4.425 ^(a) ; 3.3 ^(b)	4.3(2) ^(c) ; 4.2(6) ^(d) ; 6.1(9.1) ^(e)
$3d^2(^3P)4p\ ^4D_{7/2}^o$	4.199	4.310 ^(a) ; 3.3 ^(b)	4.2(2) ^(c)
$3d^2(^3P)4p\ ^4P_{1/2}^o$	4.886	4.808 ^(a) ; 4.6 ^(b)	4.8(2) ^(c)
$3d^2(^3P)4p\ ^4P_{3/2}^o$	4.845	4.785 ^(a) ; 4.6 ^(b)	4.8(2) ^(c)
$3d^2(^3P)4p\ ^4P_{5/2}^o$	4.828	4.762 ^(a) ; 4.6 ^(b)	4.8(2) ^(c) ; 5.7(8.6) ^(e)
$3d^2(^1G)4p\ ^2G_{7/2}^o$	3.521	3.571 ^(a) ; 3.1 ^(b)	3.6(2) ^(c)
$3d^2(^1G)4p\ ^2G_{9/2}^o$	3.512	3.571 ^(a) ; 3.1 ^(b)	3.7(2) ^(c) ; 5.5(8.2) ^(e)
$3d^2(^3P)4p\ ^2D_{5/2}^o$	4.706	4.808 ^(a) ; 5.5 ^(b)	4.7(2) ^(c) ; 6.5(9.7) ^(e)
$3d^2(^3P)4p\ ^2D_{3/2}^o$	4.745	4.831 ^(a) ; 5.5 ^(b)	4.7(2) ^(c)
$3d^2(^3P)4p\ ^2P_{1/2}^o$	5.548	5.525 ^(a) ; 5.2 ^(b)	5.5(3) ^(c)
$3d^2(^3P)4p\ ^2P_{3/2}^o$	5.523	5.525 ^(a) ; 5.3 ^(b)	5.5(3) ^(c) ; 7.2(10.8) ^(e)
$3d^2(^1G)4p\ ^2H_{9/2}^o$	4.709	4.902 ^(a) ; 4.8 ^(b)	4.7(2) ^(c) ; 6.4(9.6) ^(e)
$3d^2(^1G)4p\ ^2H_{11/2}^o$	4.606	4.808 ^(a) ; 4.7 ^(b)	4.6(2) ^(c) ; 6.0(9) ^(e)
$3d^2(^1G)4p\ ^2F_{7/2}^o$	5.643	5.208 ^(a) ; 5.0 ^(b)	5.5(3) ^(c) ; 6.1(15.2) ^(e)
$3d^2(^1G)4p\ ^2F_{5/2}^o$	5.616	5.155 ^(a) ; 5.0 ^(b)	5.4(3) ^(c)
$3d\ ^2D\ 4s\ ^3D\ 4p\ ^4F_{3/2}^o$	5.007	6.173 ^(a) ; 4.5 ^(b)	
$3d\ ^2D\ 4s\ ^3D\ 4p\ ^4F_{5/2}^o$	4.162	5.525 ^(a)	
$3d\ ^2D\ 4s\ ^3D\ 4p\ ^4D_{1/2}^o$	2.282	2.315 ^(a) ; 2.3 ^(b)	
$3d\ ^2D\ 4s\ ^3D\ 4p\ ^4D_{3/2}^o$	2.587	2.439 ^(a)	
$3d\ ^2D\ 4s\ ^3D\ 4p\ ^4D_{5/2}^o$	2.912	2.564 ^(a)	
$3d\ ^2D\ 4s\ ^3D\ 4p\ ^4D_{7/2}^o$	3.289	2.545 ^(a)	
$3d\ ^2D\ 4s\ ^3D\ 4p\ ^4F_{7/2}^o$	3.665	5.525 ^(a)	
$3d\ ^2D\ 4s\ ^3D\ 4p\ ^4F_{9/2}^o$	6.903	7.042 ^(a)	
$3d\ ^2D\ 4s\ ^1D\ 4p\ ^2D_{5/2}^o$	3.631	3.846 ^(a)	4.4(11) ^(e)
$3d\ ^2D\ 4s\ ^1D\ 4p\ ^2D_{3/2}^o$	3.651	3.876 ^(a)	
$3d\ ^2D\ 4s\ ^3D\ 4p\ ^4P_{1/2}^o$	3.712	3.831 ^(a)	
$3d\ ^2D\ 4s\ ^3D\ 4p\ ^4P_{3/2}^o$	3.735	3.831 ^(a)	
$3d\ ^2D\ 4s\ ^3D\ 4p\ ^4P_{5/2}^o$	3.727	3.817 ^(a)	
$3d\ ^2D\ 4s\ ^1D\ 4p\ ^2F_{5/2}^o$	2.958	3.058 ^(a)	
$3d\ ^2D\ 4s\ ^1D\ 4p\ ^2F_{7/2}^o$	2.921	3.012 ^(a)	3.0(7.5) ^(e)
$3d\ ^2D\ 4s\ ^1D\ 4p\ ^2P_{3/2}^o$	7.631	6.993 ^(a)	
$3d\ ^2D\ 4s\ ^1D\ 4p\ ^2P_{1/2}^o$	8.064	7.353 ^(a)	3.0(>7.5) ^(e)
$3d^2(^1S)4p\ ^2P_{1/2}^o$	2.753	2.809 ^(a)	
$3d^2(^1S)4p\ ^2P_{3/2}^o$	2.735	2.841 ^(a)	