

LETTER TO THE EDITOR

Formation of low-spinning $100 M_{\odot}$ black holes

K. Belczynski¹ and S. Banerjee^{2,3}

¹ Nicolaus Copernicus Astronomical Centre of the Polish Academy of Sciences, ul. Bartycka 18, 00-716 Warszawa, Poland
e-mail: chrisbelczynski@gmail.com

² Helmholtz-Institut für Strahlen- und Kernphysik, Nussallee 14-16, 53115 Bonn, Germany

³ Argelander-Institut für Astronomie, Auf dem Hügel 71, 53121 Bonn, Germany

Received 15 May 2020 / Accepted 2 August 2020

ABSTRACT

Aims. It is speculated that a merger of two massive stellar-origin black holes in a dense stellar environment may lead to the formation of a massive black hole in the pair-instability mass gap ($\sim 50\text{--}135 M_{\odot}$). Such a merger-formed black hole is expected to typically have a high spin ($a \sim 0.7$). If such a massive black hole acquires another black hole it may lead to another merger detectable by LIGO/Virgo in gravitational waves. Acquiring a companion may be hindered by gravitational-wave kick/recoil, which accompanies the first merger and may quickly remove the massive black hole from its parent globular or nuclear cluster. We test whether it is possible for a massive merger-formed black hole in the pair-instability gap to be retained in its parent cluster and have low spin. Such a black hole would be indistinguishable from a primordial black hole.

Methods. We employed results from numerical relativity calculations of black hole mergers to explore the range of gravitational-wave recoil velocities for various combinations of merging black hole masses and spins. We compared merger-formed massive black hole speeds with typical escape velocities from globular and nuclear clusters.

Results. We show that a globular cluster is highly unlikely to form and retain a $\sim 100 M_{\odot}$ black hole if the spin of the black hole is low ($a \lesssim 0.3$). Massive merger-formed black holes with low spins acquire high recoil speeds ($\gtrsim 200 \text{ km s}^{-1}$) from gravitational-wave kick during formation that exceed typical escape speeds from globular clusters ($\sim 50 \text{ km s}^{-1}$). However, a very low-spinning ($a \sim 0.1$) and massive ($\sim 100 M_{\odot}$) black hole could be formed and retained in a galactic nuclear star cluster. Even though such massive merger-formed black holes with such low spins acquire high speeds during formation ($\sim 400 \text{ km s}^{-1}$), they may avoid ejection since massive nuclear clusters have high escape velocities ($\sim 300\text{--}500 \text{ km s}^{-1}$). A future detection of a massive black hole in the pair-instability mass gap with low spin would therefore not be proof of the existence of primordial black holes, which are sometimes claimed to have low spins and arbitrarily high masses.

Key words. black hole physics – gravitational waves

1. Introduction

By the beginning of 2020 the LIGO/Virgo collaboration had published ten black hole–black hole (BH–BH) mergers from the first two (O1/O2) observational runs (Abbott et al. 2019a), and more than 50 BH–BH merger alerts from their recently concluded third observing run¹ (O3). The origin of these BH–BH mergers is still unknown.

Numerous formation scenarios and sites have been proposed in the literature to explain LIGO/Virgo BH–BH merger detections. The two most prominent proposals are the classical isolated binary evolution channel in galactic fields (Tutukov & Yungelson 1993; Lipunov et al. 1997; Voss & Tauris 2003) and dynamical formation in dense clusters (Miller & Hamilton 2002; Portegies Zwart et al. 2004; Gültekin et al. 2006). Triple stars are also proposed as BH–BH merger formation sites (Antonini et al. 2018), or isolated binaries with very rapid rotation leading to homogeneous evolution (Marchant et al. 2016). First generation of stars is yet another potential formation site Bond & Carr (1984).

Massive stars are subject to pair instabilities (Bond et al. 1984) and are not expected to form BHs with masses of $\sim 100 M_{\odot}$. We

note that typically this pair-instability mass gap is adopted to be broad, namely $\sim 50\text{--}135 M_{\odot}$, but recently it was claimed that the range may be narrower: $\sim 90\text{--}125 M_{\odot}$ (Farmer et al. 2020). The stellar-origin BHs are claimed to form with low spins $a \lesssim 0.2$ from single massive stars (Fuller et al. 2019; Belczynski et al. 2020). Mergers of two massive BHs in dense clusters can produce $\sim 100 M_{\odot}$ BHs, but such BHs will typically have large spin $a \sim 0.7$ (Gerosa & Berti 2017; Fishbach et al. 2017). Therefore, it seems that astrophysical channels that invoke stars, at least initially, to produce BHs cannot form BHs of $\sim 100 M_{\odot}$ with low spins. This inference could be potentially used to distinguish stellar-origin BHs from primordial BHs.

Primordial BHs were proposed to form from density fluctuations in the very early Universe by Hawking (1971). These primordial BHs are sometimes employed to explain LIGO/Virgo BH–BH mergers, that is if they catch a companion BH (Clesse & García-Bellido 2017). Although in principle primordial BHs could have an arbitrary mass, various observational constraints exclude most possibilities, nevertheless allowing masses in the range $\sim 10\text{--}100 M_{\odot}$ (Carr & Silk 2018). Primordial BHs are usually argued to have very low spins $a \sim 0.01$ (De Luca et al. 2019).

It may appear that the detection of a $100 M_{\odot}$ BH with low spin (e.g., by LIGO/Virgo) would point to the primordial

¹ See <https://gracedb.ligo.org/superevents/public/03/>

formation scenario. However, there are several alternative astrophysical scenarios. It has been proposed in the past that numerous consecutive mergers of a seed BH with lighter companions (either other BHs or stars) with randomly oriented spins (so they cancel out in the final merger product) would not increase the spin of the seed BH (Miller 2002; Hughes & Blandford 2003; Antonini et al. 2019). Consecutive (runaway) mergers of stars in young stellar clusters were claimed to lead to the formation of quasi-stellar objects massive enough to form $\sim 100 M_{\odot}$ BHs (Portegies Zwart & van den Heuvel 2007; Fujii & Portegies Zwart 2013; Di Carlo et al. 2019). However, it is not clear how such a quasi-stellar object would avoid pair-instability explosion (no BH formation) or what the spin would be of a BH if pair instability was avoided somehow.

Here we investigate the validity of a different astrophysical scenario. We test the possibility that a merger of two massive stellar-origin BHs can produce a $\sim 100 M_{\odot}$ BH with low spin and with such spatial velocity that it could be retained in its parent globular or nuclear cluster. Such a BH could catch another BH and produce a merger detectable by LIGO/Virgo.

2. Calculations

We explore a range of mass ratios of BH–BH mergers, leading to a BH of $\approx 100 M_{\odot}$, by considering three combinations of component masses, namely $(M_1, M_2) = (50.0 M_{\odot}, 50.0 M_{\odot})$, $(70.0 M_{\odot}, 35.0 M_{\odot})$, and $(70.0 M_{\odot}, 20.0 M_{\odot})$. These choices are motivated by the range of mass ratios in BH–BH mergers observed to date ($\approx 0.3 - \approx 1.0$; Abbott et al. 2019b). For each combination, we consider two extreme cases, namely both component BHs are maximally spinning, i.e., their dimensionless spin magnitudes are $(a_1, a_2) = (1.0, 1.0)$ or both components are not spinning, i.e., $(a_1, a_2) = (0.0, 0.0)$. For each $(M_1, M_2), (a_1, a_2)$ combination, we perform 10^5 evaluations of the final merged BH’s dimensionless spin vector, \mathbf{a}_f , and the recoil kick velocity, \mathbf{v}_k , due to the anisotropic radiation of GW during the merger assuming that the component BH spins are uncorrelated and oriented isotropically with respect to the orbital angular momentum. We note that the energy carried away by the GW emission will result in the final BHs’ mass, M_f , being smaller than $M_1 + M_2$. Since this discrepancy is at most a few percent (but depends, in a complex manner, on the BH–BH binary mass ratios and spin vectors as numerical-relativity calculations suggest (Hughes 2009; Sperhake 2015), we ignore it for the present purpose and assign $M_f \approx M_1 + M_2$.

The vectors \mathbf{v}_k and \mathbf{a}_f of the merged BH were obtained using numerical relativity (NR) calculations for a wide range of mass ratio and spin configurations of the merging BHs (Pretorius 2005; Campanelli et al. 2007; Baker et al. 2008; van Meter et al. 2010) and fitting formulae exist that well reproduce the NR outcomes. In general, if the spins of the merging BHs are zero, \mathbf{v}_k will lie on the orbital plane and be aligned along the line joining the BHs just before the merger. Its magnitude, v_k , is zero (small) for equal-mass (extreme-mass-ratio) components and maximizes to $\approx 170 \text{ km s}^{-1}$ at the mass ratio of $\approx 1/2.9$. If the merging BHs have spins, then depending on the magnitudes and orientations of the spins, \mathbf{v}_k will also have an in-plane component perpendicular to the mass axis and a component perpendicular to the orbital plane. For (near-) maximally spinning BHs, this off-plane component of the merger recoil typically dominates and can well exceed 500 km s^{-1} (Baker et al. 2008; van Meter et al. 2010); for certain configurations, it can reach $\approx 3000 \text{ km s}^{-1}$ (Campanelli et al. 2007). In this work, we use the NR-based fitting formulae of van Meter et al. (2010) for the components of \mathbf{v}_k , which

incorporate cases where the spins of the BHs are inclined with respect to the orbital angular momentum and would therefore undergo spin-orbit precession during the in-spiral and merger phases of the BH–BH. These formulae agree with NR outcomes within 5%.

If the spins of the merging BH components are zero, then the only source of angular momentum of the merged BH is the BH–BH orbital angular momentum. Accordingly, the dimensionless spin of the merged BH, \mathbf{a}_f , will be aligned with the orbital angular momentum, \mathbf{L} , and, for equal-mass components, will have the magnitude $a_f \approx 0.7$ (Pretorius 2005). When the merging BHs have finite, misaligned spins, \mathbf{a}_f is generally misaligned relative to \mathbf{L} and its magnitude is augmented (suppressed) with respect to the nonspinning-merger value if the spins are pro-aligned (anti-aligned). For most configurations, \mathbf{L} dominates the angular momentum budget of the BH–BH system (Rezzolla et al. 2008) and hence the angle between \mathbf{a}_f and \mathbf{L} is typically $< 10^\circ$. In this work, we use the NR-based fitting functions of Rezzolla et al. (2008) to compute \mathbf{a}_f for each of the randomly chosen component-spin orientations. The NR-based fitting formulae used in this work are described in Appendix A.

For each combination, $(M_1, M_2), (a_1, a_2)$, of the merging BH masses and dimensionless spin magnitudes (see above), we perform 10^5 evaluations of \mathbf{a}_f and \mathbf{v}_k . Each evaluation proceeds as follows: the inclinations and the azimuths of the component BH spins, $(\theta_1, \theta_2), (\phi_1, \phi_2)$, with respect to the orbital angular momentum, \mathbf{L} , are chosen randomly and independently of each other from a uniform distribution within the range $(0, 2\pi)$. Next, the NR fitting formulae, as described in Appendix A, are applied to evaluate the in-plane and off-plane components of the vector, \mathbf{v}_k , of the merger recoil kick and the magnitude of the final spin, \mathbf{a}_f . We used dedicated numerical routines for this purpose, in which the formulae in Appendix A are implemented to evaluate the \mathbf{v}_k -components and \mathbf{a}_f (and θ_f) simultaneously. Given the pure analytical nature and medium complexity of these expressions, each evaluation sequence takes less than a second; 10^5 iterations typically take $< 3 \text{ h}$ on a single $\sim 3 \text{ Ghz}$ processor. That way, the distributions of \mathbf{v}_k and \mathbf{a}_f for each $(M_1, M_2), (a_1, a_2)$ combination are obtained for isotropic and uncorrelated orientations of the spins of the merging BHs. As explained in Appendix A, the effect of the undetermined angles on the fitting formulae is automatically cancelled out by the choice of a large number of isotropically distributed spin orientations. Therefore, the distributions of \mathbf{v}_k and \mathbf{a}_f depend only on the $(M_1, M_2), (a_1, a_2)$ combination.

Figure 1 shows the probability, P , that $a_f < a_{f,\text{crit}}$ as a function of $a_{f,\text{crit}}$, with $a_{f,\text{crit}}$ being a chosen threshold value of a_f , for the three component-mass combinations and for $a_1 = a_2 = 1.0$. The fractions plotted in this figure are out of 10^5 trials (see above). Because of the dominance of \mathbf{L} , the final angular momentum of the BH–BH system can be significantly suppressed only for strongly anti-aligned spin orientations of the merging BHs. Accordingly, the fraction of configurations that suppress the system’s angular momentum to produce \mathbf{a}_f below a certain $a_{f,\text{crit}}$ decreases strongly with $a_{f,\text{crit}}$, as Fig. 1 demonstrates. Only mergers with the most dissimilar BH masses ($70.0 M_{\odot}, 20.0 M_{\odot}$) can yield $a_f < 0.1$, and that too with a very small probability ($P \approx 0.05$). Equal-mass, maximally spinning BHs cannot yield a_f below ≈ 0.35 and, with the mass ratio of 0.5, the lower limit of a_f is ≈ 0.2 .

On the other hand, when $a_1 = a_2 = 0.0$ the orbital angular momentum, which is necessarily perpendicular to the orbital plane, is the only resource of angular momentum of the merging BH–BH system. In that case, both \mathbf{a}_f and \mathbf{v}_k are unique and

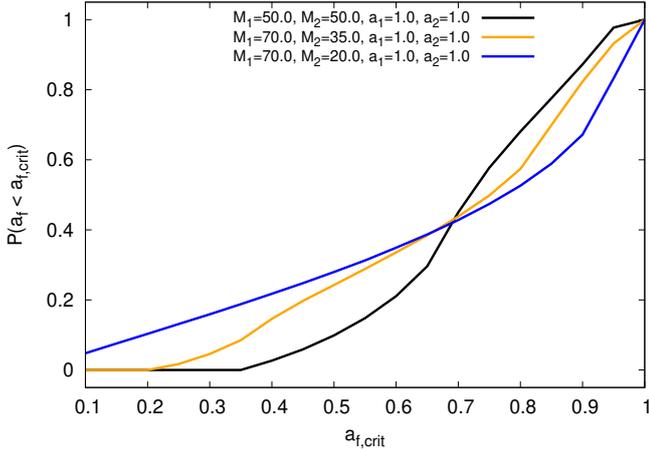


Fig. 1. Probability, $P(a_f < a_{f,crit})$, that the dimensionless spin magnitude, a_f , of a merged BH is less than the value $a_{f,crit}$, as a function of $a_{f,crit}$. Each curve corresponds to a particular mass combination of the maximally spinning components of the merging BH–BH, as indicated in the legend. The probabilities are obtained out of 10^5 trials of random and uncorrelated orientations of the spins of the BH components.

depend only on the mass ratio of the merging BHs, as determined by the NR fitting formulae employed here. Therefore, $P(a_f < a_{f,crit})$ is a step function jumping from zero to unity at the value of $a_{f,crit}$ that is equal to the unique a_f for the chosen mass ratio. This is demonstrated in Fig. 2. With nonspinning BHs, the lowest a_f obtained here is ≈ 0.5 for mergers with the smallest mass ratio (≈ 0.3).

Since our focus here is the low-spinning BH–BH merger products, it would be worth considering the GW recoil kick magnitudes, v_k , such merged BHs would receive as a result of the merger. This is shown in Fig. 3. Here, the distributions of v_k are shown for those $a_1 = a_2 = 1.0$ mergers that lead to $a_f < 0.1$, 0.3 , and 0.4 for the mass ratios ≈ 0.3 , 0.5 , and 1.0 , respectively (for $a_1 = a_2 = 0.0$, we have $a_f > 0.5$; see above). As evident, the strongly anti-aligned mergers that yield $a_f < 0.1$ have a relatively narrow range of recoil kicks, $200 \text{ km s}^{-1} \lesssim v_k \lesssim 700 \text{ km s}^{-1}$, and their distribution is strongly peaked at $v_k \approx 400 \text{ km s}^{-1}$. For $a_{f,crit} = 0.3$ and 0.4 and mass ratios ≥ 0.5 , the v_k distributions are peaked at smaller values but have tails extending up to 1600 km s^{-1} . Qualitatively, any BH–BH configuration (out of spinning BH components) that leads to a small a_f would emit GW relatively symmetrically leading to a relatively low recoil. But due to the required low mass ratio (see Fig. 1), the configuration just before the merger is still asymmetrical leading to recoils of typically 400 km s^{-1} .

3. Discussion and conclusions

We have shown that a merger of two rapidly spinning ($a_1 \approx a_2 \approx 1$) and unequal mass ($q \lesssim 0.3$) BHs can produce a $\sim 100 M_{\odot}$ BH with low spin and sufficiently low spatial velocity (from GW kick) for it to be retained in a nuclear cluster. Such a BH, under the assumption that it catches another BH, can manifest itself in high-frequency GW observations (LIGO/Virgo).

So far, (O1/O2) LIGO/Virgo observations seem to indicate that the detected BH–BH mergers have component BHs with low spins (Abbott et al. 2019b; see also Belczynski et al. 2020; Banerjee 2020), with the exception of GW170729 and GW190412 which are consistent with a moderately spinning BH (Abbott et al. 2019b; The LIGO Scientific Collaboration &

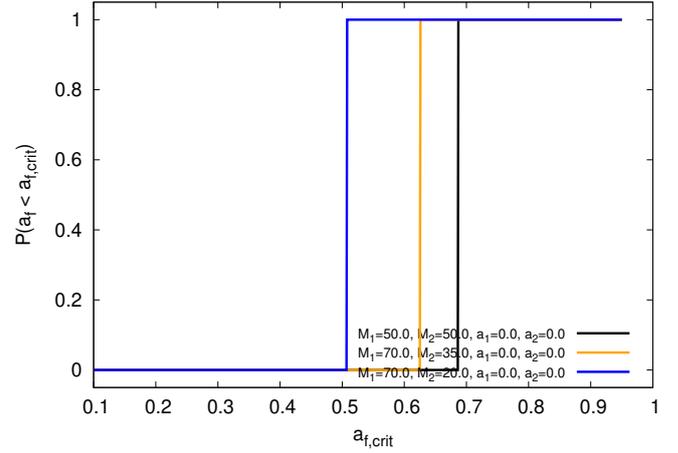


Fig. 2. Same plot as Fig. 1 but for nonspinning BH components.

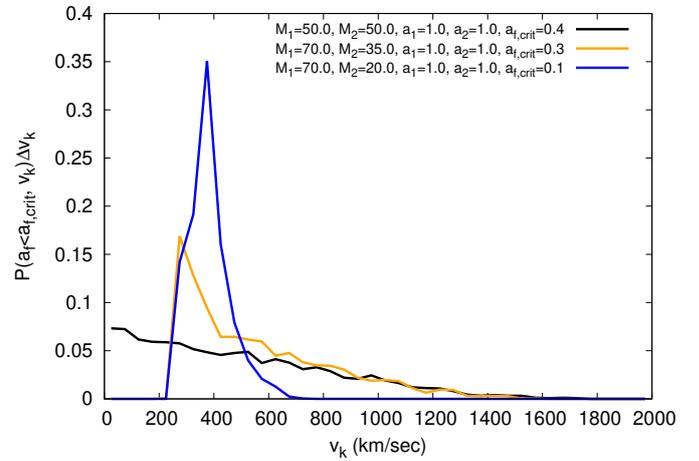


Fig. 3. Distributions of GW recoil kick magnitudes, v_k , for those mergers of maximally spinning BHs that yield a low-spinning merged BH with $a_f < a_{f,crit}$. The BH-mass combinations and the correspondingly chosen $a_{f,crit}$ are indicated in the legend.

The Virgo Collaboration 2020). This was explained in terms of Tayler-Spruit magnetic dynamo (Spruit 2002; Fuller et al. 2019) which efficiently removes angular momentum from massive stars and leads to the formation of low-spinning BHs in BH–BH mergers (Belczynski et al. 2020; Bavera et al. 2020). If this applies to all first-generation BHs, then the only way to produce a merger of two highly spinning BHs is to have a merger of two second-generation BHs, as these will typically have large spin (Gerosa & Berti 2017; Fishbach et al. 2017). Alternatively, it seems that nature can produce rapidly spinning BHs, as these are observed in Galactic and extra-galactic high-mass X-ray binaries (HMXRBs): Cyg X-1: $a = 0.98$, LMC X-1: $a = 0.92$, and M33 X-7: $a = 0.84$ (Miller & Miller 2015). The apparent tension between LIGO/Virgo BHs and those in HMXRBs can be understood if the specific evolutionary scenarios are different for both populations (e.g., Qin et al. 2019). Whether or not the tension is fully understood (which is still debated), it is clear that nature can produce both slow- and rapidly spinning BHs, and if the rapidly spinning BHs merge they are the potential candidates for our scenario.

Stars can possibly form BHs massive enough to satisfy the total mass requirement and the uneven mass ratio limit. As noted in Sect. 1, stars can possibly form BHs (first generation) as

massive as $\sim 90 M_{\odot}$ and still avoid pair-instability pulsation mass loss. Such a massive BH would then need to dynamically pair with a $\sim 20\text{--}30 M_{\odot}$ BH in a dense stellar cluster, which is possible as Monte Carlo and direct N -body computations show (Rodríguez et al. 2019; Di Carlo et al. 2019; Banerjee 2020). As we demonstrate here, the formation of a low-spinning $\sim 100 M_{\odot}$ BH induces a strong GW recoil kick that accelerates the merged BH to a high speed of $\sim 400 \text{ km s}^{-1}$. This removes all such BHs from globular clusters (mass $\sim 10^5\text{--}10^6 M_{\odot}$, size $\sim \text{pc}$; Harris 1996; Baumgardt & Hilker 2018), which have typical central escape velocities of $\lesssim 70 \text{ km s}^{-1}$ (e.g., Georgiev et al. 2009). However, such BHs can potentially be retained in nuclear clusters (mass $\sim 10^6\text{--}10^8 M_{\odot}$, size $\sim \text{pc}$; Schödel et al. 2014) that typically have much higher central escape velocities of $\sim 300\text{--}500 \text{ km s}^{-1}$ (Georgiev et al. 2016, 2009; Schödel et al. 2014; Miller & Lauburg 2009).

We note that current LIGO/Virgo observations do not provide spin magnitude measurements of merging BHs. These observations provide measurements of the BH–BH effective spin parameter, which is a projection of both BH spins weighted by BH mass on the binary’s orbital angular momentum vector. This means that if the effective spin is low then the BH spins are low; the spin vectors are facing in opposite directions causing them to cancel out; or the spin vectors are in the orbital plane. However, future observations may provide large numbers of massive BHs and this may allow for statistical assessment of BH spin magnitudes. Alternatively, there is hope for measuring spin precession in the inspiral waveform in future. This would also allow the spin magnitudes of merging BHs to be estimated.

In future work, we intend to explore the effect of more recent NR results (e.g., Lousto & Zlochower 2013; Hofmann et al. 2016) and the mass–spin dependence of BHs as obtained from stellar-evolutionary calculations (Belczynski et al. 2020). Also, the formation rate estimate of the proposed scenario would require careful analysis of nuclear cluster evolution folded with detailed stellar and binary evolution. Such an estimate is beyond the scope of this paper.

Here, we simply point out (proof-of-principle) the possibility of the formation of a $100 M_{\odot}$, low-spinning BH of astrophysical origin. This adds to existing scenarios of a seed BH growing in mass by accretion of lower mass stars or light BHs (Miller 2002; Hughes & Blandford 2003) and BH formation from a runaway stellar merger product (Portegies Zwart & van den Heuvel 2007; Fujii & Portegies Zwart 2013; Di Carlo et al. 2019). These astrophysical scenarios are to be contrasted with any claims that such BHs (if they exist and are detected) must be of primordial origin. Despite the fact that the channel pointed out here is potentially of lower probability than the other channels, it follows from basic physics (numerical relativity) and requires only a specific astrophysical site (nuclear cluster), of which there are many in the Universe.

Acknowledgements. We thank the anonymous referee for constructive comments and useful suggestions that have helped to improve the manuscript. KB acknowledges support from the Polish National Science Center (NCN) grant Maestro (2018/30/A/ST9/00050). SB acknowledges the support from the Deutsche Forschungsgemeinschaft (DFG; German Research Foundation) through the individual research grant “The dynamics of stellar-mass black holes in dense stellar systems and their role in gravitational-wave generation” (BA 4281/6-1; PI: S. Banerjee). The authors thank Cole Miller and Mirek Giersz for useful comments and criticisms that have helped to improve the manuscript.

They acknowledge the hospitality of the National Astronomical Observatories of China (NAOC), Beijing where a good part of this work has been done.

References

- Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2019a, *ApJ*, **882**, L24
 Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2019b, *Phys. Rev. X*, **9**, 031040
 Antonini, F., Rodríguez, C. L., Petrovich, C., & Fischer, C. L. 2018, *MNRAS*, **480**, L58
 Antonini, F., Gieles, M., & Gualandris, A. 2019, *MNRAS*, **486**, 5008
 Baker, J. G., Boggs, W. D., Centrella, J., et al. 2007, *ApJ*, **668**, 1140
 Baker, J. G., Boggs, W. D., Centrella, J., et al. 2008, *ApJ*, **682**, L29
 Banerjee, S. 2020, ArXiv e-prints [arXiv:2004.07382]
 Baumgardt, H., & Hilker, M. 2018, *MNRAS*, **478**, 1520
 Bavera, S. S., Fragos, T., Qin, Y., et al. 2020, *A&A*, **635**, A97
 Belczynski, K., Klencek, J., Fields, C. E., et al. 2020, *A&A*, **636**, A104
 Blanchet, L., Qusailah, M. S. S., & Will, C. M. 2005, *ApJ*, **635**, 508
 Bond, J. R., & Carr, B. J. 1984, *MNRAS*, **207**, 585
 Bond, J. R., Arnett, W. D., & Carr, B. J. 1984, *ApJ*, **280**, 825
 Campanelli, M., Lousto, C., Zlochower, Y., & Merritt, D. 2007, *ApJ*, **659**, L5
 Carr, B., & Silk, J. 2018, *MNRAS*, **478**, 3756
 Clesse, S., & García-Bellido, J. 2017, *Phys. Dark Univ.*, **15**, 142
 De Luca, V., Desjacques, V., Franciolini, G., Malhotra, A., & Riotto, A. 2019, *J. Cosmol. Astropart. Phys.*, **2019**, 018
 Di Carlo, U. N., Giacobbo, N., Mapelli, M., et al. 2019, *MNRAS*, **487**, 2947
 Farmer, R., Renzo, M., de Mink, S., Fishbach, M., & Justham, S. 2020, *ApJ*, submitted [arXiv:2006.06678]
 Favata, M., Hughes, S. A., & Holz, D. E. 2004, *ApJ*, **607**, L5
 Fishbach, M., Holz, D. E., & Farr, B. 2017, *ApJ*, **840**, L24
 Fitchett, M. J. 1983, *MNRAS*, **203**, 1049
 Fujii, M. S., & Portegies Zwart, S. 2013, *MNRAS*, **430**, 1018
 Fuller, J., Piro, A. L., & Jermyn, A. S. 2019, *MNRAS*, **485**, 3661
 Georgiev, I. Y., Hilker, M., Puzia, T. H., Goudfrooij, P., & Baumgardt, H. 2009, *MNRAS*, **396**, 1075
 Georgiev, I. Y., Böker, T., Leigh, N., Lützgendorf, N., & Neumayer, N. 2016, *MNRAS*, **457**, 2122
 Gerosa, D., & Berti, E. 2017, *Phys. Rev. D*, **95**, 124046
 González, J. A., Spherhake, U., Brüggemann, B., Hannam, M., & Husa, S. 2007, *Phys. Rev. Lett.*, **98**, 091101
 Gültekin, K., Miller, M. C., & Hamilton, D. P. 2006, *ApJ*, **640**, 156
 Harris, W. E. 1996, *AJ*, **112**, 1487
 Hawking, S. 1971, *MNRAS*, **152**, 75
 Hofmann, F., Barausse, E., & Rezzolla, L. 2016, *ApJ*, **825**, L19
 Hughes, S. A. 2009, *ARA&A*, **47**, 107
 Hughes, S. A., & Blandford, R. D. 2003, *ApJ*, **585**, L101
 Kidder, L. E. 1995, *Phys. Rev. D*, **52**, 821
 Lipunov, V. M., Postnov, K. A., & Prokhorov, M. E. 1997, *Astron. Lett.*, **23**, 492
 Lousto, C. O., & Zlochower, Y. 2013, *Phys. Rev. D*, **87**, 084027
 Marchant, P., Langer, N., Podsiadlowski, P., Tauris, T. M., & Moriya, T. J. 2016, *A&A*, **588**, A50
 Miller, M. C. 2002, *ApJ*, **581**, 438
 Miller, M. C., & Hamilton, D. P. 2002, *MNRAS*, **330**, 232
 Miller, M. C., & Lauburg, V. M. 2009, *ApJ*, **692**, 917
 Miller, M. C., & Miller, J. M. 2015, *Phys. Rep.*, **548**, 1
 Morawski, J., Giersz, M., Askar, A., & Belczynski, K. 2018, *MNRAS*, **481**, 2168
 Portegies Zwart, S. F., Baumgardt, H., Hut, P., Makino, J., & McMillan, S. L. W. 2004, *Nature*, **428**, 724
 Portegies Zwart, S. F., & van den Heuvel, E. P. J. 2007, *Nature*, **450**, 388
 Pretorius, F. 2005, *Phys. Rev. Lett.*, **95**, 121101
 Qin, Y., Marchant, P., Fragos, T., Meynet, G., & Kalogera, V. 2019, *ApJ*, **870**, L18
 Rezzolla, L., Barausse, E., Dorband, E. N., et al. 2008, *Phys. Rev. D*, **78**, 044002
 Rodríguez, C. L., Zevin, M., Amaro-Seoane, P., et al. 2019, *Phys. Rev. D*, **100**, 043027
 Schödel, R., Feldmeier, A., Kunneriath, D., et al. 2014, *A&A*, **566**, A47
 Spherhake, U. 2015, *Classical Quantum Gravity*, **32**, 124011
 Spruit, H. C. 2002, *A&A*, **381**, 923
 The LIGO Scientific Collaboration, & The Virgo Collaboration 2020, ArXiv e-prints [arXiv:2004.08342]
 Tutukov, A. V., & Yungelson, L. R. 1993, *MNRAS*, **260**, 675
 van Meter, J. R., Miller, M. C., Baker, J. G., Boggs, W. D., & Kelly, B. J. 2010, *ApJ*, **719**, 1427
 Voss, R., & Tauris, T. M. 2003, *MNRAS*, **342**, 1169

Appendix A: Numerical relativity fitting formulae

Fitting formulae based on NR are used to calculate the GR merger recoil kick and final spin of the merged BH, as discussed in Sect. 2. The recoil kick, \mathbf{v}_k , is computed using the fitting formulae as given in van Meter et al. (2010). These formulae are based on NR calculations of BH–BH mergers (Baker et al. 2008; van Meter et al. 2010) where the component BH spins are generally inclined with respect to the orbital axis, resulting in spin-orbit precession (i.e., precession of the BHs’ spin vectors and the orbital axis around the total angular momentum vector of the BH–BH system) of the merging BH–BH.

The GR merger recoil kick is given by

$$\mathbf{v}_k = (v_{\perp m} + v_{\perp s} \cos \xi) \mathbf{e}_1 + v_{\perp s} \sin \xi \mathbf{e}_2 + v_{\parallel} \mathbf{e}_3. \quad (\text{A.1})$$

Here, $v_{\perp m}$ is the mass-ratio-determined recoil, $v_{\perp s}$ is the in-orbital-plane spin-determined recoil, v_{\parallel} is the off-plane (parallel to the orbital axis) spin-determined recoil, \mathbf{e}_1 and \mathbf{e}_3 are the unit vectors along the directions of the BHs’ separation and the orbital axis just before the merger, respectively, and $\mathbf{e}_2 \equiv \mathbf{e}_1 \times \mathbf{e}_3$. $\xi = 215^\circ$ is a constant as obtained in the NR fitting by Baker et al. (2008).

The expressions for the in-plane and off-plane recoils are to some extent motivated by post-Newtonian treatments (Fitchett 1983; Kidder 1995; Favata et al. 2004; Blanchet et al. 2005; Baker et al. 2007; González et al. 2007; Campanelli et al. 2007) and are given by

$$v_{\perp m} = A\eta^2 \sqrt{1 - 4\eta(1 + B\eta)}, \quad (\text{A.2})$$

$$v_{\perp s} = H \frac{\eta^2}{(1+q)} (a_2 \cos \theta_2 - qa_1 \cos \theta_1), \quad (\text{A.3})$$

$$\begin{aligned} v_{\parallel} &= \frac{K_2 \eta^2 + K_3 \eta^3}{q+1} \\ &\times [qa_1 \sin \theta_1 \cos(\phi_1 - \Phi_1) - a_2 \sin \theta_2 \cos(\phi_2 - \Phi_2)] \\ &+ \frac{K_s(q-1)\eta^2}{(q+1)^3} \\ &\times [q^2 a_1 \sin \theta_1 \cos(\phi_1 - \Phi_1) + a_2 \sin \theta_2 \cos(\phi_2 - \Phi_2)]. \end{aligned} \quad (\text{A.4})$$

In Eqs. (A.2)–(A.4), A , B , H , K_2 , K_3 , and K_s are constants as obtained by NR fitting, $q \equiv m_1/m_2$ ($m_1 \leq m_2$) is the mass ratio of the merging BHs with dimensionless spin magnitudes (a_1, a_2), $\eta \equiv q/(1+q)^2$, (θ_1, θ_2) are the inclination angles that the BHs’ spins project to orbital axis, (ϕ_1, ϕ_2) are the azimuth angles that the spins project to the separation axis, and (Φ_1, Φ_2) are phase angles depending on q and the initial separation of the merging BHs’ (Baker et al. 2008; van Meter et al. 2010). Since, in this work, the BHs’ spins are assumed to be uncorrelated and randomly oriented (i.e., $\theta_1, \theta_2, \phi_1, \phi_2$ are random in the range 0° – 360°) over a large number of trials (see Sect. 2), the terms $\cos(\phi_1 - \Phi_1)$ and $\cos(\phi_2 - \Phi_2)$ will average out in Eq. (A.4). Hence, $\Phi_1 = \Phi_2 = 0$ is assumed, as also done in Morawski et al. (2018). The fitting constants are $A = 1.35 \times 10^4 \text{ km s}^{-1}$, $B = -1.48$, $H = 7540 \text{ km s}^{-1}$ (Baker et al. 2008) and

$K_2 = 32\,092 \text{ km s}^{-1}$, $K_3 = 108\,897 \text{ km s}^{-1}$, $K_s = 15\,375 \text{ km s}^{-1}$ (van Meter et al. 2010).

The final spin magnitude of the merged BH, \mathbf{a}_f , and its tilt angle with respect to the orbital axis (orbital angular momentum) at a large separation of the binary, θ_f , are obtained using the NR-fitting formulae of Rezzolla et al. (2008). The final spin is given by

$$\begin{aligned} \mathbf{a}_f &= \frac{1}{(1+q)^2} \left[a_2^2 + a_1^2 q^4 + 2a_1 a_2 q^2 \cos \alpha \right. \\ &\quad \left. + 2(a_2 \cos \theta_2 + a_1 q^2 \cos \theta_1) l q + l^2 q^2 \right]^{1/2}, \end{aligned} \quad (\text{A.5})$$

with (see also, Morawski et al. 2018)

$$\cos \alpha \equiv \hat{a}_1 \cdot \hat{a}_2 = \cos \theta_2 \cos \theta_1 + \sin \theta_2 \sin \theta_1 \cos(\phi_1 - \phi_2), \quad (\text{A.6})$$

where \mathbf{l} is a vector which is parallel to the orbital angular momentum at a large binary separation (Rezzolla et al. 2008) and whose magnitude is given by

$$\begin{aligned} l &= \frac{s_4}{(1+q^2)^2} (a_2^2 + a_1^2 q^4 + 2a_2 a_1 q^2 \cos \alpha) \\ &\quad + \left(\frac{s_5 \eta + t_0 + 2}{1+q^2} \right) (a_2 \cos \theta_2 + a_1 q^2 \cos \theta_1) \\ &\quad + 2\sqrt{3} + t_2 \eta + t_3 \eta^2. \end{aligned} \quad (\text{A.7})$$

Here, the fitting constants are $s_4 = -0.129$, $s_5 = -0.384$, $t_0 = -2.686$, $t_2 = -3.454$, and $t_3 = 2.353$.

The component of the final spin vector parallel to the direction of the (large-separation) orbital angular momentum is

$$a_f^{\parallel} \equiv \mathbf{a}_f \cdot \hat{l} = \frac{a_2 \cos \theta_2 + a_1 q^2 \cos \theta_1 + l q}{(1+q)^2}, \quad (\text{A.8})$$

so that the tilt angle of the final spin is given by $\cos \theta_f = a_f^{\parallel} / a_f$.

We note that in Eqs. (A.5)–(A.8), the inclination and azimuthal angles of the BH spins are, $\theta_1, \theta_2, \phi_1$, and ϕ_2 , are with respect to the binary’s orbit and its axis (i.e., the direction of the orbital angular momentum) at a sufficiently large binary separation, on which the ansatz for the final spin is based (Rezzolla et al. 2008). On the other hand, the angles in the GR recoil kick formulae, as given by Eqs. (A.2)–(A.4), are with respect to the instantaneous orbital plane just before the merger which would be oriented differently from the large-separation orbital plane if the merging BH–BH system undergoes spin-orbit precession (see above). However, since, in this work, the spins of the BHs are assumed to be uncorrelated and randomly oriented over a large number of trials (Sect. 2), the angles can in effect be used synonymously in both sets of equations.

Also, all of the above equations are expressed following the convention $m_1 \leq m_2$ as often employed in NR and as they are implemented in the subroutines used in this work. The usual precautionary measures are taken to ensure this convention in these subroutines. However, in the main text of this paper, the astrophysical convention of $M_1 \geq M_2$ is followed.