The *Gaia* reference frame for bright sources examined using VLBI observations of radio stars (Corrigendum)

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This is a corrigendum to [Lindegren \(2020\)](#). Owing to a coding error in the implementation of the analysis method described in the paper, the results obtained from its application to *Gaia* DR2 data were significantly wrong. Although the main conclusions of the original paper remain unchanged, the results presented in Tables 2 and 3, Figs. 3–5, and Sects. 3.3, 3.4, 4, and 5 are substantially affected. The revised tables, figures, and portions of the text are given below.

3.3. Results

The direct application of the solution method in Sect. 2.3 to the full sample of 41 sources gives a very poor fit as measured by the loss function $Q \approx 2\,176\,000$ for $n \equiv \sum_i n_i = 224$ degrees of freedom, or a reduced chi-square of $Q/n \approx 9713$. This solution also gives unrealistically large values for the spin parameters, where $|\dot{\omega}| \approx 2.5 \text{ mas yr}^{-1}$. Inspection of the discrepancy measure Q_i/n_i of the individual sources shows that T Tau has by far the highest value at $Q_i/n_i \approx 247\,502$, followed by S Cr at $Q_i/n_i \approx 123\,397$, and so on. Removing T Tau from the sample and re-computing the solution and discrepancy measures gives $Q \approx 924\,169$ for $n = \sum_i n_i = 219$ degrees of freedom ($Q/n \approx 4220$). In this solution the source with the largest discrepancy measure is S Cr at $Q_i/n_i \approx 123\,714$. Removing this source as well and iterating the procedure until all sources but one have been removed gives a series of solutions with $k = 0, 1, \dots$ rejected sources. The evolution of $\max(Q_i/n_i)$ and Q/n as functions of k is shown in the left panels of Fig. 3; the corresponding orientation and spin parameters are shown in the right panels.

Errors in the rotation parameters caused by non-linear source motions and other model deficiencies are generally reduced as more outliers are removed, while the statistical (formal) uncertainties increase because fewer sources contribute to the solution. The optimum solution is a compromise between the opposite tendencies and may be found somewhere along the sequence of solutions described above. The rather smooth progression of the discrepancy measures in Figs. 3a and b gives no clear indication of the optimum k , except that it is probably in the range from 7 (removing the seven most obvious outliers) to ≈ 33

(after which $Q/n < 1$). The spin parameters in Fig. 3d show an abrupt change with the removal of HD 282630 at $k = 13$, after which the fluctuations, although not negligible, are roughly of a size that is compatible with the formal uncertainties. For the subsequent discussion we assume, somewhat arbitrarily, the solution at $k = 15$ as the baseline¹. This solution has $Q/n = 5.68$ with $n = 139$ degrees of freedom and the most discrepant source is PZ Cas with $Q_i/n_i \approx 28.604$.

The accepted and rejected sources and their individual discrepancy measures for the baseline solution are listed in Table 2. The rotation parameters are given in Table 3 under entry A, including the formal uncertainties calculated from the inverse normal matrix. The correlation matrix (for $T = \text{J2015.5}$) is

$$\text{corr}[\varepsilon_X(T), \varepsilon_Y(T), \varepsilon_Z(T), \omega_X, \omega_Y, \omega_Z] = \begin{bmatrix} +1.000 & +0.517 & +0.204 & +0.136 & +0.066 & +0.120 \\ +0.517 & +1.000 & +0.206 & +0.099 & +0.131 & +0.105 \\ +0.204 & +0.206 & +1.000 & +0.091 & +0.035 & +0.133 \\ +0.136 & +0.099 & +0.091 & +1.000 & -0.028 & +0.405 \\ +0.066 & +0.131 & +0.035 & -0.028 & +1.000 & -0.090 \\ +0.120 & +0.105 & +0.133 & +0.405 & -0.090 & +1.000 \end{bmatrix}. \quad (30)$$

Given the reduced chi-square of the solution, $Q/n \approx 5.68$, it is likely that the formal uncertainties underestimate the actual errors. More realistic estimates may be obtained by bootstrap resampling of the 26 accepted sources, yielding the uncertainties in the ‘Adopted’ entry of Table 3.

From the original sample of $m = 41$ sources, $k = 15$ were thus removed to obtain the baseline subset of $m - k = 26$ sources. It is not obvious that the process of successively removing the most discrepant source leads to the optimum subset in the sense that no other subset of the same size has a smaller Q/n . It is conceivable that a different procedure, for instance, starting from

¹ This choice of k is the same as made in the original version of this paper. Although that choice, based on an incorrect computation, is in principle irrelevant for the present analysis, the corrected results do not present a clear case for adopting a different k . As it happens, $k = 15$ results in exactly the same subset of rejected radio sources as in the original paper.

Fig. 3. Evolution of discrepancy measures and rotation parameters in a series of solutions for which the k most discrepant sources have been removed. (a) Discrepancy measure Q_i/n_i for the most discrepant source in the solution. Points are labelled with the name of the source. (b) Total discrepancy measure Q/n as a function of the number of rejected sources. (c) Estimated orientation parameters and (d) spin parameters as functions of the number of rejected sources. Error bars are formal $\pm 1\sigma$ uncertainties from the inverse normal matrix, i.e. not adjusted depending on the goodness-of-fit.

a smaller subset and adding the best-fitting sources (outward selection; Ben-Gal et al. 2010), would lead to a different result. While it is impractical to test all $\binom{41}{26} \simeq 6 \times 10^{10}$ possible combinations, an exhaustive search of the $\binom{33}{26} = 4\,272\,048$ different subsets of size 26 drawn from the 33 sources with the smallest discrepancy measure did not uncover a more favourable subset. Figure 4 shows Q/n versus the spin components for these solutions. Many of these solutions are very different from the baseline solution in terms of the spin components, but invariably their Q/n is then also significantly higher. This makes it credible that the spin parameters of the adopted solution are not the chance result of a particular combination of data for a few sources.

The solution gives improved estimates of the astrometric parameters of the sources, obtained by solving (21) for each i . These results are not tabulated, as they are practically the same as a weighted average of the data obtained by very long baseline interferometry (VLBI) and the *Gaia* data, after correcting the latter for the frame rotation and parallax zero-point. For example, the joint estimate of the parallax of Cyg X-1 is $\hat{\varpi} = 0.504 \pm 0.025$ mas, which is very close to the weighted mean of the VLBI value, 0.547 ± 0.041 mas, and the *Gaia* DR2 value after correction for the zero-point, 0.472 ± 0.032 mas.

3.4. Alternative solutions

The classical way to determine ω is to solve the over-determined system of Eqs. (8) and (9) using only the proper motion differences. In this process it is natural to assign a weight to each equation that is inversely proportional to the combined variances of the VLBI and *Gaia* proper motions. An equivalently weighted least-squares solution is obtained with the formalism of Sect. 2.3 simply by deleting in f_i all the VLBI data items that are not proper motions. The resulting normal equations, Eq. (22), are of course singular for the orientation parameters, but the lower right 3×3 part of the equations gives the desired solution for ω . Applying this procedure to the baseline subset of 26 sources gives the result shown as solution B in Table 3. The spin parameters are reasonably close to those of the baseline solution (A) for the X and Z components, while the Y component shows a more negative value. It is more interesting, however, that the formal uncertainties are significantly higher in solution B than in A: the uncertainty is a factor two higher for the Y component.

It thus appears that the positional VLBI data are at least as valuable as the proper motion data when estimating the spin, at least for the spread of VLBI epochs considered in this work. A direct test of this hypothesis is to make the complementary

Table 2. *Gaia* DR2 matches and solution statistics for the radio sources in Table 1.

Name	<i>Gaia</i> DR2 identifier	<i>G</i> (mag)	RUWE	E_i (mas ⁻²)	Ω_i (mas ⁻² yr ²)	n_i	Q_i/n_i	Accepted (0/1)
SY Scl	2335529621301024128	10.09	1.28	0.0	20.2	5	2.556	1
S Per	459101393719884800	7.80	1.27	0.0	16.6	5	4.246	1
LS I +61 303	465645515129855872	10.39	0.92	1.4	755.1	5	9.823	1
UX Ari	118986060277836160	6.33	6.33	0.7	12.1	10	766.906	0
HD 22468	3263936692671872384	5.60	1.01	0.3	143.7	5	11.515	1
V1271 Tau	69876712724339456	11.44	1.15	54.6	197.4	5	0.467	1
V811 Tau	65190048708380160	12.09	1.08	42.1	240.4	5	4.684	1
HD 283447	163184366130809984	9.98	3.99	21.8	23.1	10	19983.091	0
V410 Tau	164518589131083136	10.32	0.98	1156.3	339.1	5	0.915	1
V1023 Tau	164513538249595136	11.65	3.27	3.9	27.1	5	1704.823	0
HD 283572	164536250037820160	8.80	1.02	10.2	273.7	10	1.273	1
T Tau	48192969034959232	9.63	1.69	12.0	165.6	5	255047.336	0
HD 283641	152104381299305600	10.80	1.62	210.6	110.1	5	7.636	1
V1110 Tau	147940663906992512	9.96	0.90	147.6	203.3	5	88.639	0
HD 282630	156900622818205312	10.33	1.02	255.1	260.1	5	73.700	0
T Lep	2962625495403737600	6.44	1.39	0.0	5.3	5	73.666	0
V1961 Ori	3209424108758593408	11.61	1.09	605.3	201.2	5	5.042	1
Brun 334	3017270879709003520	10.82	1.10	699.3	169.8	5	4.894	1
V1321 Ori	3209531650444835840	10.13	0.96	176.8	72.1	5	2.995	1
MT Ori	3017364127743299328	11.32	1.16	479.0	97.3	5	71.869	0
V1046 Ori	3209634905754969856	6.53	2.21	100.1	14.8	5	127.846	0
HD 37150	3017294622299833216	6.51	0.98	294.4	95.9	5	62.206	0
TYC 5346-538-1	3015742318025842944	10.65	0.93	205.8	59.6	5	8.258	1
HD 290862	3219148872492984192	10.18	1.65	6.4	13.6	5	19.693	1
[SSC75] M 78 11	3219148185299243776	12.51	1.10	371.8	167.7	5	702.849	0
VY CMa	5616197646838713728	7.17	17.20	0.0	0.7	5	43.772	0
S CrI	3595101382979440256	6.42	1.40	0.2	27.5	5	124262.639	0
BH CVn	1475118788534734592	4.73	0.85	0.3	77.4	5	2.213	1
S CrB	1277100833181122816	7.40	1.70	0.0	8.4	3	5.828	1
σ^2 CrB	1328866562170960512	5.41	1.14	1.9	158.2	5	65.332	0
U Her	1200834239913483392	7.04	1.35	0.0	14.3	3	5.294	1
Haro 1-6	6049142032584969088	12.24	0.96	2.6	91.7	5	1.725	1
DoAr 51	6047570826172040960	12.52	6.43	10.2	3.2	5	7.127	1
W 40 IRS 5	4270236599432306560	12.13	2.98	46.5	15.4	5	17316.589	0
RR Aql	4234448531043979520	8.18	1.55	0.0	4.0	3	12.770	1
Cyg X-1	2059383668236814720	8.52	0.98	6.3	736.5	10	1.122	1
HD 199178	2162964329341318656	7.01	0.88	1.0	463.1	5	0.573	1
SS Cyg	1972957892448494592	11.69	1.42	32.9	181.1	5	11.217	1
AR Lac	1962909425622345728	5.89	0.82	1.9	933.1	5	2.119	1
IM Peg	2829193299742131328	5.66	1.43	4.0	115.3	10	1.233	1
PZ Cas	2012859963999694848	6.64	1.06	0.2	40.9	5	28.604	1

Notes. The quantity G is the mean magnitude in the *Gaia* photometric band; RUWE is the re-normalised unit weight error (Lindegren 2018) of the astrometric solution for the source in *Gaia* DR2. The quantities E_i and Ω_i are the formal weights potentially contributed by the source to the estimation of $\epsilon(T)$ and ω , computed using Eq. (29), and n_i the number of VLBI data items on the source. The second to last column is the discrepancy measure from Eq. (28), and the last column tells whether the source was accepted (1) or not (0) in the baseline solution.

solution to what is described above, that is, deleting all proper motion items in f_i . The result is shown as solution C in Table 3. In this case, the spin is more precise (in terms of formal uncertainties) than in solution B where (only) the proper motions are used, although not as precise as in A. It can be noted that only 23 of the 26 sources contribute to solution C because no positional VLBI data are provided for S CrB, U Her, and RR Aql, although these sources belong to the baseline subset.

Figure 4 of Lindegren et al. (2018) suggests that the transition from the faint to the bright reference frame in *Gaia* DR2 does not occur abruptly at $G = 13$, but happens gradually from $G \simeq 13$ to $\simeq 11$ mag. Several of the sources in Table 1 have

magnitudes in the transition interval and may therefore not contribute fully to the determination of the rotation parameters. In solution D the model of the *Gaia* data in (11) is modified so that the applied rotation is $\mathbf{x}$ multiplied by the magnitude-dependent function

$$\phi(G) = \begin{cases} 1 & \text{if } G \leq 11, \\ (13 - G)/2 & \text{if } 11 < G \leq 13, \\ 0 & \text{if } G > 13. \end{cases} \quad (31)$$

This improves the overall fit significantly ($Q = 747.6$ in D against 789.5 in A), although the discrepancy measure increases

Table 3. Summary of the different solutions for the orientation and spin parameters.

Solution	Orientation at $T = \text{J2015.5}$ (mas)			Spin (mas yr^{-1})		
	$\varepsilon_X(T)$	$\varepsilon_Y(T)$	$\varepsilon_Z(T)$	ω_X	ω_Y	ω_Z
A	-0.019 ± 0.032	$+1.304 \pm 0.074$	$+0.553 \pm 0.026$	-0.068 ± 0.023	-0.051 ± 0.027	-0.014 ± 0.028
B	–	–	–	-0.050 ± 0.036	-0.139 ± 0.055	$+0.002 \pm 0.038$
C	-0.001 ± 0.036	$+1.413 \pm 0.085$	$+0.559 \pm 0.033$	-0.056 ± 0.028	-0.057 ± 0.028	-0.019 ± 0.035
D	$+0.026 \pm 0.034$	$+1.465 \pm 0.079$	$+0.629 \pm 0.028$	-0.051 ± 0.025	-0.066 ± 0.029	-0.015 ± 0.030
Adopted	-0.019 ± 0.158	$+1.304 \pm 0.349$	$+0.553 \pm 0.135$	-0.068 ± 0.052	-0.051 ± 0.045	-0.014 ± 0.066

Notes. The table gives the estimated orientation and spin components for different solutions with their uncertainties (after $\pm$). All solutions use the same subset of 26 sources, denoted as accepted in Table 2. Solution A is the baseline solution described in Sect. 3.3. Solutions B–D are the alternative solutions described in Sect. 3.4: B using only proper motions, C using only positions, and D using the same data as A and the magnitude dependent model in Eq. (31). The ‘Adopted’ solution is identical to A, but the uncertainties are estimated by bootstrap resampling of the 26 sources (while for A–D the uncertainties are formal values computed from the inverse normal matrix).

for two of the six accepted sources in the magnitude range 11–13 (V1961 Ori and DoAr 51). The VLBI data therefore support the magnitude dependent model, although not unambiguously. The resulting rotation parameters (for $G \lesssim 11$) are not significantly different from the baseline solution.

4. Discussion

4.1. Estimated rotation parameters

The results in Table 3 yield a spin for the *Gaia* DR2 bright reference frame of $\omega = (-0.068, -0.051, -0.014) \text{ mas yr}^{-1}$, which is not statistically significant in view of the uncertainties obtained by the bootstrap method. Nevertheless, the consistently negative values of ω_X and ω_Y obtained for a wide range of k in Fig. 3d do suggest a marginally significant spin of the order of 0.1 mas yr^{-1} . This is supported by the qualitative agreement between solutions B and C, which use different subsets of the VLBI data.

With regard to the orientation parameters $\varepsilon(2015.5)$, the Y and Z components are statistically significant in view of the uncertainties obtained by the bootstrap method and indicate a total orientation error of about 1.4 mas . The large uncertainty of the Y component reflects the unfavourable celestial distribution of the more recent VLBI observations included in the analysis (Sect. 3.5).

As mentioned in the Introduction, the non-zero spin of the bright reference frame of *Gaia* DR2 is also seen in a comparison with proper motions of HIPPARCOS stars calculated from the position differences between *Gaia* DR2 and the HIPPARCOS catalogue, divided by the $\sim 24 \text{ yr}$ epoch difference. This comparison was made by Brandt (2018) in the course of constructing The HIPPARCOS–*Gaia* Catalog of Accelerations (HGCA). For 115 663 HIPPARCOS stars the HGCA gives three essentially independent sets of proper motions, namely (i) as measured by HIPPARCOS around epoch 1991.25, (ii) as measured by *Gaia* (in DR2) around epoch 2015.5, and (iii) as calculated from the HIPPARCOS–*Gaia* position differences divided by the epoch difference. The HGCA is intended for orbit fitting and for identifying candidate stars with substellar or dark companions. To this end, Brandt (2018) made a careful cross-calibration of the three sets of proper motions in order to eliminate any systematic offsets, in particular the global rotations. For the rotation between sets (ii) and (iii) the result as given in Brandt’s Table 1 is $\omega = (-0.081, -0.113, -0.038) \text{ mas yr}^{-1}$. Brandt used the same sign convention for this vector as in this paper, so that his result can be directly compared with our Table 3.

The comparison with Brandt (2018) rests on the assumption that the HIPPARCOS–*Gaia* position differences yield proper

motions that are absolute; that is, they are expressed in a reference frame that is non-rotating with respect to the International Celestial Reference System (ICRS).² The validity of this assumption depends on the quality of the positional reference frame of HIPPARCOS at epoch 1991.25 and of the positional reference frame for the same stars in *Gaia* DR2 at epoch 2015.5. If both sets of positions are aligned with the ICRS at their respective epochs, the proper motions calculated from the position differences must clearly be non-rotating with respect to the ICRS.

According to the present analysis, the bright reference frame of *Gaia* DR2 was however offset from ICRS by more than 1 mas at epoch 2015.5, as given by $\varepsilon(2015.5)$ in Table 3. To take this into account in the spin comparison, Brandt’s estimate quoted above needs to be increased by $\varepsilon(2015.5)/(24.25 \text{ yr})$. This yields the corrected estimate of $\omega = (-0.082, -0.059, -0.015) \text{ mas yr}^{-1}$, which is in good agreement with the adopted spin solution in Table 3. The uncertainty of this estimate is $\sqrt{0.6^2 + 0.4^2}/24.25 \approx 0.03 \text{ mas yr}^{-1}$ per axis, which was obtained by combining the RMS orientation error of the HIPPARCOS reference frame at epoch 1991.25 (in Vol. 3, Sect. 18.7 of The HIPPARCOS and *Tycho* Catalogues, ESA 1997, given as 0.6 mas per axis) and the uncertainty of $\varepsilon(2015.5)$, taken to be 0.4 mas .

The old HIPPARCOS positions thus provide an independent estimate of the spin, which agrees with the results from the VLBI observations and is of comparable or even better accuracy. However, because its uncertainty is dominated by the unknown orientation errors of the HIPPARCOS reference frame, which will not improve, the method will be of limited value for the validation of future *Gaia* data releases. In contrast, the VLBI method, as discussed in Sect. 4.4, has great potential for future improvement.

The origin of the non-zero rotation parameters for the bright reference frame of *Gaia* DR2 is a deficiency in the specific instrument calibration model used for the DR2 astrometric solution. The relevance of this deficiency for the reference frame was not recognised at the time when the DR2 data were prepared and validated, and although its effect on the bright sources was noted (e.g. Fig. 4 in Lindegren et al. 2018), no explanation of its origin was offered. This is now understood, and the calibration model is being improved with a view towards avoiding a similar error in future data releases (see Appendix B).

² Brandt (2018) adopted the (bright) reference frame of *Gaia* DR2 for the HGCA, therefore the published proper motions of type (iii), called `pmra_hg` and `pmdec_hg` in his Table 5, are not absolute in our sense. To place these values on the ICRS, the applied cross-calibration corrections must be subtracted, i.e. use `pmra_hg-crosscal_pmra_hg` and `pmdec_hg-crosscal_pmdec_hg`.

Fig. 4. Sensitivity of the solution to the selection of sources. The plots show the spin parameters and associated Q/n values for about four million solutions using different subsets of the sources as described in the text. The colour shows the smallest Q/n in each bin. The dashed lines denote the spin parameters for the baseline solution A with $Q/n \approx 5.68$.

4.3. Binarity and source structure

In Fig. 5 the source DoAr 51 stands out because it has a large RUWE = 6.43, while obtaining a reasonable fit in the solution ($Q_i/n_i = 7.127$). This object is a triple system consisting of an equal-mass pair with a period of about eight years, a separation of approximately 56 mas at 2015.5, and a fainter tertiary

Fig. 5. Discrepancy measures of the sources in solution A against the RUWE in the *Gaia* DR2 astrometry. The case of DoAr 51 is discussed in Sect. 4.3.

component at a separation of about 790 mas (Schaefer et al. 2018). This triple configuration could explain the high RUWE obtained with *Gaia*. The VLBI observations detect both components of the close pair, and the data in Table 1 refer to their centre of mass (Ortiz-León et al. 2017). The near-coincidence of the VLBI and *Gaia* epochs, together with the near-coincidence of the optical photocentre with the centre of mass of the close pair and the rather large uncertainty of the *Gaia* proper motion (~ 1 mas yr $^{-1}$), could explain why the discrepancy measure is not higher. With $\Omega_i = 3.2$ mas $^{-2}$ yr 2 , this object contributes less to the determination of ω than any of the other accepted sources in Table 2. DoAr 51 is a good example of an object for which an extended model along the lines in Sect. 2.4 could drastically increase the usefulness of the data.

5. Conclusions

This paper provides a rigorous mathematical framework for estimating the orientation and spin of the *Gaia* reference frame, in which the *Gaia* data are optimally combined with VLBI measurements of bright radio sources. The simultaneous estimation of the orientation (ϵ) and spin (ω) is essential for achieving the best accuracy. The method takes full advantage of past and future single-epoch VLBI measurements of *Gaia* sources for the determination of the spin. Independent estimates of their proper motions from VLBI can be incorporated into the solution, but are not required by the method.

Applied to published VLBI data for a sample of 41 bright ($G \leq 13$ mag) radio sources, the method gives the rotation parameters summarised in Table 3. The solution retains 26 of the investigated radio sources, while 15 are rejected based on a statistical discrepancy measure sensitive to binarity and source structure. The results indicate that the bright reference frame of *Gaia* DR2 at the reference epoch 2015.5 is offset from the ICRS by about 1.3 mas in Y and 0.6 mas in Z . The solution for the spin indicates a rotation rate of the order of 0.1 mas yr $^{-1}$. The components of the spin solution are not statistically significant in comparison with their uncertainties from the bootstrap method. Nevertheless, they agree well with independent estimates obtained from a comparison of the extrapolated *Gaia* DR2 positions with the HIPPARCOS catalogue at epoch J1991.25 (Lindegren et al. 2018; Brandt 2018) if the latter estimates are

corrected for the alignment error of *Gaia* DR2 at the *Gaia* epoch. The accuracy of the present study is limited by the relatively small number of radio stars included, by the uncertainties of the *Gaia* DR2 proper motions, and by issues related to the astrophysical nature of the sources.

The origin of the spin of the bright reference frame of *Gaia* DR2 is understood and measures have been taken to avoid this problem in future data releases (see Appendix B). Nevertheless, it is important that the consistency of future reference frames can be validated across the full range of magnitudes and the present method offers such a possibility for the bright stars. As many as possible of the already existing VLBI measurements of radio stars should be used for this purpose, but it is very desirable to complement this by re-observing many of these sources in the coming years, and if possible, add new targets to the list for improved robustness. The use and re-calibration of old, possibly unpublished data should be pursued. In this context, the unique historical value of positional VLBI measurements needs to be stressed. As argued in Sect. 4.5, observers should ensure that

relevant intermediate data and meta-information are preserved for optimal future uses of their data.

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