Temperature-averaged and total free-free Gaunt factors for κ and Maxwellian distributions of electrons[★] (Corrigendum)

Miguel A. de Aveliz^{1,2} and Dieter Breitschwerdt²

¹ Department of Mathematics, University of Évora, R. Romão Ramalho 59, 7000 Évora, Portugal
 e-mail: maveliz@galaxy.lca.uevora.pt

² Zentrum für Astronomie und Astrophysik, Technische Universität Berlin, Hardenbergstrasse 36, 10623 Berlin, Germany

A&A, 580, A124 (2015), <https://doi.org/10.1051/0004-6361/201526104>

Key words. atomic processes – radiation mechanisms: general – ISM: general – galaxies: ISM – errata, agenda

1. Introduction

A missing normalization coefficient in Eqs. (11) and (12) of [de Aveliz & Breitschwerdt \(2015\)](#) affected the results shown in Figs. 3 and 4 and discussed in Sect. 3 of that earlier publication. Here, we present the corrected version of the equations and the corresponding results discussed in Sect. 3 and shown in Figs. 3 and 4 of that paper. In addition we provide supplementary material in the form of tables covering a larger parameter space than the one previously used. These tables are available at the CDS.

2. Temperature-averaged and total Gaunt factor

During the interaction of an electron with the Coulomb field of an ion of ionic state z and atomic number Z the amount of free-free energy emitted per unit time and unit volume is given by

$$\frac{dP_{\text{ff}}}{du} = \frac{8\pi^2}{hc^3} \left(\frac{2}{3m_e} \right)^{3/2} e^6 z^2 n_e n_{z,z} k_B T N_e \langle g_{ff}(\gamma^2, u) \rangle, \quad (1)$$

with z denoting the ionic state of an ion with atomic number Z and number density $n_{z,z}$, n_e is the electron number density, h is the Planck constant, c is the speed of light, e is the electron charge in Coulombs, m_e is the electron mass, k_B is the Boltzmann constant, T is the temperature, and $\langle g_{ff}(\gamma^2, u) \rangle$ is the temperature averaged Gaunt factor given by (see, e.g., [de Aveliz & Breitschwerdt 2017](#))

$$\langle g_{ff}(\gamma^2, u) \rangle = \frac{(k_B T)^{1/2}}{N_e} \times \int_0^{+\infty} \frac{1}{(y+u)^{1/2}} f[(y+u)k_B T] g_{ff} \left(\epsilon_i = \frac{y+u}{\gamma^2}, \epsilon_f = \frac{y}{\gamma^2} \right) dy \quad (2)$$

where f is the electron distribution function, $u = h\nu/k_B T$ (ν is the frequency of the emitted photon), $\gamma^2 = z^2 \text{Ry}/k_B T$ (Ry is the Rydberg constant), and N_e is a normalization coefficient

defined such $\langle g_{ff}(\gamma^2, u) \rangle = 1$ when $g_{ff}(\gamma^2, u) = 1$. For the κ and Maxwell-Boltzmann (MB) electron distribution functions N_e is given by

$$N_e = \begin{cases} \frac{2}{\sqrt{\pi k_B T}} \int_0^{+\infty} G(y+u; \kappa) dy & \text{if } \kappa > 3/2 \\ \frac{2e^{-u}}{\sqrt{\pi k_B T}} & \text{if MB,} \end{cases} \quad (3)$$

with

$$G(x; \kappa) = \frac{A_\kappa}{\left[1 + \frac{x}{\kappa - 3/2} \right]^{\kappa+1}}, \quad (4)$$

and

$$\int_0^{+\infty} G(y+u; \kappa) dy = A_\kappa \frac{\kappa - 3/2}{\kappa} \left(1 + \frac{u}{\kappa - 3/2} \right)^{-\kappa}, \quad (5)$$

which becomes e^{-u} when $\kappa \rightarrow \infty$ (Fig. 1). In these expressions

$$A_\kappa = \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)(\kappa - 3/2)^{3/2}} \text{ and } \kappa > 3/2. \quad (6)$$

Therefore, Eq. (11) in [de Aveliz & Breitschwerdt \(2015\)](#) must be rewritten as

$$\frac{dP_{\text{ff}}}{du} = C_{\text{ff}} z^2 n_e n_{z,z} T^{1/2} \times \begin{cases} \frac{1}{\int_0^{+\infty} G(y+u; \kappa) dy} \int_0^{+\infty} g_{\text{ff}}(\gamma^2, u) G(y+u; \kappa) dy & \text{if } \kappa > 3/2 \\ e^{-u} \int_0^{+\infty} g_{\text{ff}}(\gamma^2, u) e^{-y} dy & \text{if MB.} \end{cases} \quad (7)$$

where

$$C_{\text{ff}} = 16 \left(\frac{2\pi}{3m_e} \right)^{3/2} \frac{e^6 k_B^{1/2}}{hc^3} = 1.4256 \times 10^{-27} \text{ erg cm}^3 \text{ s}^{-1} \text{ K}^{-1/2}.$$

[★] The set of tables is only available at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via <http://cdsarc.u-strasbg.fr/viz-bin/qcat?J/A+A/620/C1>

Fig. 1. Variation of Integral (5) with κ as function of u . The solid black line represents e^{-u} which is overlapped by the integral as $\kappa \rightarrow \infty$.

The total free-free power associated with an ion (Z, z) given by

$$P_{ff}(T) = C_{ff} z^2 n_e n_{z,z} T^{1/2} \int_0^{+\infty} \langle g_{ff}(\gamma^2, u) \rangle f(u) du \quad (8)$$

with

$$f(u) = \begin{cases} \int_0^{+\infty} G(y+u; \kappa) dy & \text{if } \kappa > 3/2 \\ e^{-u} & \text{if MB.} \end{cases} \quad (9)$$

The integral in the RHS of (8) is the total free-free Gaunt factor. When $\kappa \rightarrow \infty$ the Integral in (9) tends to e^{-u} and total Gaunt factor for the MB distribution of electrons as defined by [Karzas & Latter \(1961\)](#) is recovered

$$g_{ff}(T) = \int_0^{+\infty} \langle g_{ff}(\gamma^2, u) \rangle e^{-u} du. \quad (10)$$

The top panel of Fig. 2 and the panels in Fig. 3 display the variation with γ^2 of the temperature-averaged Gaunt factor, $\langle g_{ff}(\gamma^2, u) \rangle$, for different values of $u \in [10^{-4}, 10^4]$ for the MB and $\kappa = 2, \dots, 1200$ distributions. The bottom panel of Fig. 2 highlights in a magnified image of the region $\gamma^2 \in [10^{-2}, 10^6]$ the distribution with γ^2 of $\langle g_{ff}(\gamma^2, u) \rangle$ for different values of u .

It is clear that as κ increases the temperature-averaged κ distributed Gaunt factors approach those calculated with the MB distribution. However, the speed (with κ variation) of this approach depends on the values of u . For $u < 10$ this approach is faster than for $u \geq 10$ (compare top panel of Fig. 2 and the two panels in Fig. 3). For values as high as $\kappa = 1200$ there is still no overlap between the κ and Maxwellian values of $\langle g_{ff}(\gamma^2, u) \rangle$ for $u > 10^2$ and $\gamma^2 < 10$ (Fig. 3). This is a consequence of the slow approach to e^{-u} of the integral $\int_0^{+\infty} G(y+u; \kappa) dy$ for $u \geq 10$ in comparison to $u < 10$ (Fig. 1).

Fig. 2. Temperature-averaged Gaunt factors calculated for $\kappa = 2, 3, 5$ and MB (black line in both panels) distributions of electrons for the range $10^{-5} \leq \gamma^2 \leq 10^{10}$ (top panel) and a magnification of the region $\gamma^2 \in [10^{-2}, 10^6]$ and $\langle g_{ff}(\gamma^2, u) \rangle \in [0.9, 1.5]$ (bottom panel).

Figure 4 displays the total free-free Gaunt factor, $\langle g_{ff}(\gamma^2) \rangle$, calculated for the MB and $\kappa = 2, 3, 5, 10, 15$, and 25 distributions. For larger κ the $\langle g_{ff}(\gamma^2) \rangle$ almost overlap with the Maxwellian value as shown in the magnification of the regions $\langle g_{ff}(\gamma^2) \rangle \in [-1.0, 0.8]$ and $\langle g_{ff}(\gamma^2) \rangle \in [1.38, 1.45]$ (top panel

Fig. 3. Temperature-averaged Gaunt factors calculated for $\kappa = 10, 15, 25$, and 50 (left panel), $100, 500, 1000$, and 1200 (right panel) and MB (black line in both panels) distributions of electrons for the range $10^{-5} \leq \gamma^2 \leq 10^{10}$. We note the slow approach $\langle g_{ff}(\gamma^2, u) \rangle$ to the Maxwellian values for $u > 10$ and $\gamma^2 < 10$ for large values of the κ parameter.

Fig. 4. Total free-free Gaunt factor calculated for $\kappa(2, 3, 5, 10, 15$, and $25)$ and MB distributions.

Fig. 5. Magnification of the total free-free Gaunt factor in two regions in γ^2 and in the $\langle g_{ff}(\gamma^2) \rangle$ profile shown in Fig. 4 but for $\kappa = 50, 100, 500$, and 1000 . The solid black line refers to the Maxwellian value.

of Fig. 5), and $\langle g_{ff}(\gamma^2) \rangle \in [6, 10]$ and $\langle g_{ff}(\gamma^2) \rangle \in [0.995, 1.005]$ (bottom panel of same Figure). It turns out that even for κ param-

eters as high as 1000 there is still a slight difference between $\langle g_{ff}(\gamma^2) \rangle$ calculated for the κ and MB distributions.

3. Tables

Supplementary material is available at the CDS with a set of tables referring to the temperature-averaged Gaunt factor versus γ^2 and different u , and a table for the total Gaunt factor versus γ^2 for results obtained with the $\kappa = 2, 3, 5, 10, 15, 25, 50, 100$, and 500 and MB electron distributions. The parameter space comprises $\gamma^2 \in [10^{-5}, 10^{10}]$ and $u \in [10^{-12}, 10^{11}]$.

Acknowledgements. This research was supported by the project Enabling Green E-science for the SKA Research Infrastructure (ENGAGE SKA), reference POCI-01-0145-FEDER-022217, funded by COMPETE 2020 & FCT, Portugal.

References

- de Avillez, M. A. & Breitschwerdt, D. 2015, [A&A](#), **580**, A124
 de Avillez, M. A. & Breitschwerdt, D. 2017, [ApJS](#), **232**, 12
 Karzas, W. J. & Latter, R. 1961, [ApJS](#), **6**, 167