

LETTER TO THE EDITOR

Gaia GraL: Gaia DR2 gravitational lens systems

I. New quadruply imaged quasar candidates around known quasars

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ABSTRACT

Context. Multiply imaged gravitationally lensed quasars are among the most interesting and useful observable extragalactic phenomena. Because their study constitutes a unique tool in various fields of astronomy, they are highly sought, but difficult to find. Even in this era of all-sky surveys, discovering them remains a great challenge, with barely a few hundred systems currently known.

Aims. We aim to discover new multiply imaged quasar candidates in the recently published *Gaia* Data Release 2 (DR2), which is the astrometric and photometric all-sky survey with the highest spatial resolution that achieves effective resolutions from 0.4'' to 2.2''.

Methods. We cross-matched a merged list of quasars and candidates with *Gaia* DR2 and found 1 839 143 counterparts within 0.5''. We then searched matches with more than two *Gaia* DR2 counterparts within 6''. We further narrowed the resulting list using astrometry and photometry compatibility criteria between the *Gaia* DR2 counterparts. A supervised machine-learning method, called extremely randomized trees, was finally adopted to assign a probability of being lensed to each remaining system.

Results. We report the discovery of two quadruply imaged quasar candidates that are fully detected in *Gaia* DR2. These are the most promising new quasar lens candidates from *Gaia* DR2 and a simple singular isothermal ellipsoid lens model is able to reproduce their image positions to within ~ 1 mas. This Letter demonstrates the discovery potential of *Gaia* for gravitational lenses.

Key words. gravitational lensing: strong – quasars: general – astrometry – methods: data analysis – catalogs – surveys

1. Introduction

Strong gravitational lenses (Einstein 1936; Zwicky 1937; hereafter GL) probe many key features in astronomy: e.g. dark matter halos of galaxies (e.g., Oguri & Keeton 2004), substructures in lensing galaxies (e.g., Metcalf & Zhao 2002), the determination of the Hubble constant independently of the cosmic distance ladder (e.g., Refsdal 1964; Suyu et al. 2017), properties of dark energy (e.g., Cao et al. 2015), and they might even reveal the shape of the central engine of quasars (e.g., Tomozeiu et al. 2018). However, since the first detection of a multiply imaged quasar (Walsh et al. 1979), GLs have been elusive. The discovery and characterization of these systems require exceptional imaging capabilities, which poses a challenge to ground-based surveys. Thus, the relatively small number of systems that have reliably been confirmed to be lensed has historically plagued

many of the potential studies that can be performed with these objects, mainly due to systematics at the level of individual lenses.

The data from the ESA/*Gaia* space mission (Gaia Collaboration 2016b) are expected to change this situation dramatically. *Gaia* is currently conducting the largest, most precise, and most accurate all-sky astrometric survey from space. Its main goal is to produce a three-dimensional dynamical map of the Milky Way based on the measurement of positions, parallaxes, and proper motions supplemented by spectrophotometric parameters for more than 10^9 stars. Moreover, the instruments also detect and transmit to the ground observations of millions of galaxies and quasars (Robin et al. 2012; Krone-Martins et al. 2013; de Souza et al. 2014; de Bruijne et al. 2015; Gaia Collaboration 2016b). Thus careful analyses of the *Gaia* data releases offer unique opportunities to perform the

first all-sky magnitude-limited census of strongly lensed quasars, down to image separations as small as $\sim 0.18''$.

Recently, [Finet & Surdej \(2016\)](#) have shown that, of the $\sim 6.6 \times 10^5$ quasars that would likely be observed by the satellite, nearly 2900 might be multiply imaged and resolved in the final *Gaia* data release. Of these, about 2650 are expected to be doubly imaged lenses and 250 systems are expected to consist of more than two lensed images. Hence, *Gaia* data may lead to an increase in the number of lensed quasars by more than an order of magnitude with respect to what is currently known. Thus, *Gaia* will provide invaluable data on new lensed quasars, which will be studied either individually or from a statistical point of view.

This Letter is the first published report from a group especially set up to unravel these possibilities of *Gaia*. The group is called *Gaia* gravitational lenses, or *Gaia* GraL. We aim to perform a systematic investigation of gravitational lenses in the various *Gaia* data releases, through lens candidate detection from *Gaia* data, ground-based follow-up observations and modeling of the lenses. We report here our analysis of the environment of 1 839 143 quasar candidates compiled from the literature with a positional counterpart in *Gaia* Data Release 2 (DR2, [Gaia Collaboration 2018](#)). Based on these data coupled with statistical cuts and machine-learning methods, we discover new quadruply imaged lensed quasar candidates.

In Sect. 2 we present the *Gaia* DR2 data adopted in this Letter and the compilation of our quasar list. The selection criteria for lensed quasar candidates and the probability assignment method are detailed in Sect. 3. We comment on a few candidates and discuss our results in Sect. 4. Finally, we summarize our findings and briefly present our plans in Sect. 5.

2. Data

2.1. *Gaia* data

Using the *Gaia* archive facility at ESAC ([Salgado et al. 2017](#)), we extracted the positions (α, δ), astrometric pseudo-colors ([Lindegren et al. 2018](#)), parallaxes (ϖ), proper-motions (μ_α, μ_δ) and fluxes in the G , G_{BP} and G_{RP} passbands ([Evans et al. 2018](#)), together with their associated uncertainties, from the table `gaiadr2.gaia_source` via an ADQL positional cross-match with the compiled quasar list (Sect. 2.2).

Although the *Gaia* onboard instrument can attain an angular resolution of $\sim 0.18''$, *Gaia* Data Release 1 (DR1, [Gaia Collaboration 2016a](#)) reached an effective angular resolution that was limited to the range $2''$ – $4''$ ([Arenou et al. 2017](#)). The raw data processing and the final filtering based on the astrometric quality excluded a large fraction of objects with small separations from *Gaia* DR1, because these objects are prone to larger errors. However, because the astrometric quality improves at each release, the expected angular resolution improves with the updated processing at each new data release.

In *Gaia* DR2, the effective resolution now reaches $\sim 0.4''$, with completeness for separations larger than $\sim 2.2''$ ([Gaia Collaboration 2018](#)). Moreover, as a result of the instrument design, the evolution of the data processing, and data calibration, the effective angular resolution is not the same for different types of *Gaia* DR2 data. The resolution described above applies strictly only for astrometry and G band photometry. The angular resolution of *Gaia* spectrophotometry in *Gaia* DR2 reaches $\sim 2''$, while its completeness reaches $\sim 3.5''$ ([Gaia Collaboration 2018](#)).

Table 1. Number of candidate clusters that passed our tests.

<i>Gaia</i> DR2 sources	Astrometry	Astrometry and color	Astrometry and color, $\Delta\theta \leq 1''$
2	16 500	320	28
3	1874	46	0
4	269	8	0
≥ 5	66	0	0

Notes. Many sources cannot be tested for color in the *Gaia* DR2.

2.2. Compiled quasar list

We first set up a long list of quasars by compiling and merging several catalogs of quasars or quasar candidates. Since these catalogs were constructed using different selection criteria based on spectroscopy, photometry and cuts in visible or near-infrared bands, the resulting list is inhomogeneous in reliability and quality of prior information.

The main contribution to our list comes from the largest currently available catalog, the Million Quasars Catalog (MILLIQUAS; [Flesch 2015, 2017](#)), which comprises 1 998 464 quasars or high-confidence quasar candidates over the entire sky. Furthermore, we considered the ALLWISE catalog of quasars (1 354 775 objects, [Secrest et al. 2015](#)). To avoid creating duplicates, for each ALLWISE entry we searched for MILLIQUAS sources within $1''$; when no counterpart was found, we added this ALLWISE source to our compiled quasar list. The same procedure was applied to the LQAC3 (321 945 sources, [Souchay et al. 2015](#)) and the SDSS Quasar catalog DR12Q (297 301 sources, [Pâris et al. 2017](#)). Our final list contains 3 112 975 objects, of which 1 839 143 have a counterpart in the *Gaia* DR2 within an angular separation smaller than $0.5''$.

As this list is intended to be exhaustive, it might contain contaminants. To discard the most obvious ones, we first filtered out the Galactic plane (we only kept $|b| > 15^\circ$) and the area of the Magellanic clouds. We then applied a soft astrometric test to keep objects with $|\varpi - 3\sigma_\varpi| < 4 \text{ mas}$ and $|\mu| - 3\sigma_\mu < 4 \text{ mas yr}^{-1}$ (μ stands for $\mu_\alpha \cos(\delta)$ and μ_δ). The reasons for the choice of these thresholds are based on the *Gaia* DR2 properties of known lenses; more details regarding these thresholds are given in Paper II ([Ducourant et al. 2018](#)). This step eliminates $\sim 1\%$ of the candidates.

3. Method

3.1. Neighbor extraction and parameter cuts: clusters

We matched the *Gaia* DR2 for detections within a $6''$ radius around each object of our quasar list, and kept a record of quasars with one or more neighbors. The quasar neighbors were also astrometrically filtered as described in Sect. 2.2, eliminating $\sim 10\%$ of them. We call the remaining systems clusters. We then applied a series of tests between the *Gaia* DR2 detections to verify a 3σ statistical compatibility between their astrometric ($\varpi, \mu_\alpha, \mu_\delta$ and astrometric pseudo-color) and photometric parameters (colors computed from G, G_{BP} and G_{RP} fluxes when available in *Gaia* DR2). These tests aimed to discard stellar contaminants, and might also discard the lensing galaxy (if detected). The tests, for instance $|\varpi_i - \varpi_j| \leq 3\sigma_{|\varpi_i - \varpi_j|}$ for the parallax, were applied between each possible pair (i, j) of the *Gaia* DR2 sources in each cluster. Clusters in which at least one

Table 2. Relative astrometry and flux ratios of our newly discovered gravitationally lensed quasar candidates (GRAL113100–441959 and GRAL122629–454209), and of a rediscovered system (GRAL203802–400815), from *Gaia* DR2 alone.

Image	GRAL113100–441959			GRAL122629–454209			GRAL203802–400815		
	$\Delta\alpha \cos(\delta)$ ($''$)	$\Delta\delta$ ($''$)	<i>G</i> -band flux ratio	$\Delta\alpha \cos(\delta)$ ($''$)	$\Delta\delta$ ($''$)	<i>G</i> -band flux ratio	$\Delta\alpha \cos(\delta)$ ($''$)	$\Delta\delta$ ($''$)	<i>G</i> -band flux ratio
A	0.000(2)	0.000(1)	1.000(17)	0.000(1)	0.000(1)	1.000(6)	0.000(1)	0.000(1)	1.000(9)
B	0.345(1)	−0.325(1)	0.947(10)	−3.066(1)	−1.705(1)	0.688(3)	1.515(1)	−0.029(1)	0.963(5)
C	−1.282(1)	−0.425(1)	0.475(6)	0.065(1)	−5.453(1)	0.659(3)	−0.793(1)	1.676(1)	0.763(4)
D	−0.343(1)	−1.511(1)	0.402(5)	4.986(2)	−2.092(2)	0.434(4)	1.379(2)	2.057(2)	0.550(8)

Notes. The astrometric errors are typically smaller than ~ 1 mas for all sources. The larger flux ratio errors in the A and B images of the GRAL113100–441959 candidate might originate from their smaller separation ($\sim 0.4''$), which is at the limit of the effective resolution of *Gaia* DR2.

pair of *Gaia* DR2 sources passed all tests were considered for further analysis, as described in Sect. 3.2.

Based on the tests applied on astrometric data, 16 500 clusters have two *Gaia* DR2 sources, 1874 clusters have three sources, 269 clusters have four sources and 66 clusters have five or more *Gaia* DR2 sources. This test was adopted to define the clusters for which the lensing probability was computed (Sec. 3.2). However, for a subset of these sources, the *Gaia* DR2 colors can be computed. The number of clusters that passed all the tests – defining more reliable candidates – is 372. They are split into 8, 46, and 320 clusters that are characterized by four, three, and two *Gaia* DR2 sources, respectively. These results are summarized in Table 1. We note that of the doubly imaged systems that passed all the tests, 121 clusters have separations smaller than $3''$, while 28 clusters attain $1''$. Only one of these 28 stringently selected clusters is a previously known gravitational lens, namely SDSS1332+0347.

Although several catalogs used in our compiled quasar list explicitly exclude known GLs, our list contains counterparts for 136 known GLs. This set contains 21 quadruply imaged systems of which 19 have two or more *Gaia* DR2 counterparts, and 9 have four counterparts. Of 11 previously known triply imaged systems, 10 were detected with two or more counterparts, while of the 104 doubly imaged systems, 79 show two counterparts. Thus from the 136 known GLs in our quasar list, we recovered 108 GLs with two or more *Gaia* DR2 counterparts.

3.2. Identification of lens candidates from clusters

The next step consists of classifying the identified clusters with respect to their probability of being a GL candidate. A GL candidate is defined here as a configuration of images (i.e. relative positions and differences in magnitudes) that can be well reproduced with a non-singular isothermal ellipsoid in presence of an external shear model (NSIEg, Kormann et al. 1994; Kovner 1987). This model effectively describes the mass distribution of massive early-type galaxies in the region where multiple images occur (Gilman et al. 2017). The complete procedure we adopted to distinguish GL candidates from accidental groups of stars is detailed in Paper III (Delchambre et al. 2018), which is dedicated to a large blind search for gravitational lenses in the entire *Gaia* DR2 catalog. Here we provide a description of the basic principles.

During the process, we assigned to each candidate a probability based on extremely randomized trees (ERT, Geurts et al. 2006). This probability reflects the match between a candidate and the learning set used to build the ERT and thus does not constitute a probability in the mathematical sense. These can be

matched to an expected ratio of identification of the GL candidates (the true-positive rate, hereafter TPR), however, and to an expected ratio of misclassification of groups of stars as GL candidates (the false-positive rate, hereafter FPR) through the use of a cross-validation procedure. Our ERT models were trained using a set of 106 623 188 simulated NSIEg lensed-image configurations composed of more than three components, uniformly covering the most common ranges of parameters of the lens model. An equal number of contaminant observations were also produced to correspond to accidental groups of sources (i.e. stars and/or quasars) in our learning sample. However, as these simulations do not follow a realistic distribution of galaxies/stars parameters, the derived TPR and FPR are only estimates. Each of these simulated configurations was then altered by the addition of noise, in agreement with the uncertainties of the *Gaia* astrometric and photometric observations. Furthermore, as we expect some lensed images to be missing from the *Gaia* DR2, all combinations of the three and four images were considered for building the ERT. We note that the fifth image from the NSIEg lens model, being often out of reach of the *Gaia* photometric sensitivity, was accordingly not taken into account. These ERT models are referred to in the following as ABCD, ABC, ABD, ACD, and BCD where A, B, C, and D identify the images we used during the learning phase of the corresponding ERT, assuming they are sorted in ascending order of *G* magnitude (i.e., A is the brightest image).

All the three- and four-image configurations that passed the astrometric tests were then processed using these models. The TPR associated with a probability above 0.9 in the models ABCD, ABC, ABD, ACD, and BCD is given by 0.741, 0.398, 0.391, 0.244, and 0.274, respectively, while the corresponding FPR is given by 0.006, 0.025, 0.024, 0.022, and 0.024.

4. New lensed quasar candidates in *Gaia* DR2

We report the first results of this effort in searching GL among the current and upcoming *Gaia* data releases. We discovered two new candidates, namely GRAL113100–441959, and GRAL122629–454209, and we codiscovered a recent system GRAL203802–400815 (or WGD2038–4008, Agnello et al. 2018), each of which is composed of four images.

GRAL113100–441959 and GRAL203802–400815 are characterized by an ERT probability higher than 0.95. In addition to these candidates, five known systems were also rediscovered with similar probabilities: HE0435–1223, PG1115+080, PKS1830–211, RXJ1131–1231, and WFI2033–4723. Our other lens candidate, GRAL122629–454209, is characterized by a much lower ERT probability of 0.22, mainly due to large flux

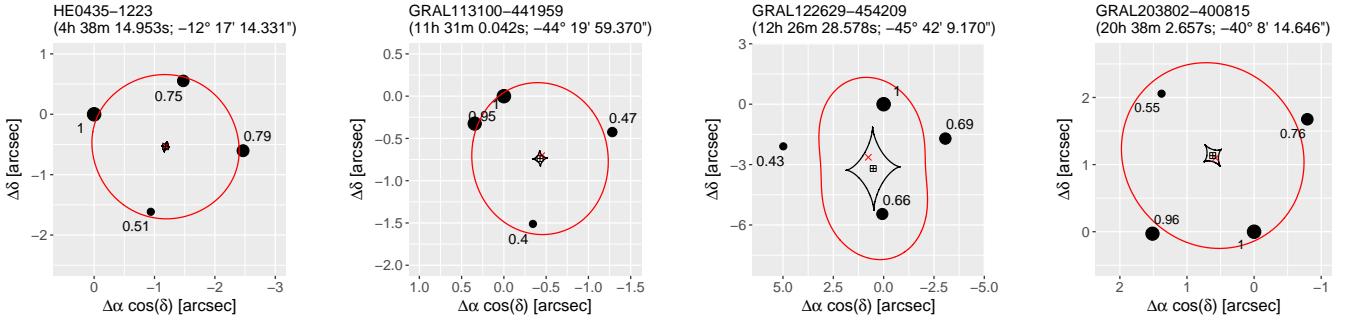


Fig. 1. Multiply-imaged gravitationally lensed quasars and candidates detected from *Gaia* DR2. Black dots locate the astrometric positions from *Gaia* DR2. The numbers associated with each position indicate the *G* band flux ratios with respect to the brightest image. The leftmost chart corresponds to the well-known lens HE0435–1223. The following three systems are our newly discovered quadruply-imaged candidates, and an independent rediscovery of a recently reported system, from *Gaia* DR2 data alone. Several products obtained from our simple lens modeling are represented: the critical curves (red), the caustics (black), the source positions (red crosses), and the predicted deflector centroids (black squares).

uncertainties. This is expected because the sources composing this system are near *Gaia* detection limit. In this regime, the *Gaia* DR2 photometry might show biases. Moreover, this faint candidate also lies at a nearby distance ($\sim 2''$) to a brighter source (with $G \sim 16.3$), and some of their observations might even share a single assigned window (e.g., Evans et al. 2018).

As a first step in the validation process, our systems were modeled using the public code `lensmodel` (v1.99, Keeton 2001). We adopted a simple SIE model plus external shear, using the lensed-image positions and *G*-band flux ratios, except for GRAL122629–454209, as observational constraints. The image positions are extremely well reproduced (to within 1 mas), whereas the flux ratios agree reasonably well with those measured. A more detailed Bayesian modeling of the *Gaia* GrAL quasar lens candidates is in preparation (Paper IV; Wertz et al. in prep.). The *Gaia* relative astrometry and photometry of the new quasar lens candidates are reported in Table 2 and are illustrated in Fig. 1, which also shows some modeling results.

In addition to the candidates reported here, we found 65 systems composed of three detections that are characterized by an ERT probability of being a quadruply imaged system higher than 0.95. These candidates, along with systems characterized with lower probabilities and doubly imaged candidates, will be subject of dedicated studies.

5. Conclusions

We reported the discovery of two new multi-imaged quasar candidates directly extracted from *Gaia* Data Release 2, and an independent rediscovery of a recent system. This result demonstrates the great potential of the ESA/*Gaia* mission in this field and that the *Gaia* data releases will provide an invaluable dataset that is well suited for extragalactic and cosmological studies.

This study is the first *Gaia* data only detection of lensed quasar candidates. We have shown that in an up-to-date list of quasars and quasar candidates comprising more than 3.1 million objects, 1.8 million have a counterpart in the *Gaia* DR2. By applying astrometric and photometric selection criteria, we have derived a list of *Gaia* DR2 source clusters. Employing a machine-learning method trained with an NSIEg lens model, we derived lensing probabilities for all our candidates that are composed of three and four components. Finally, we modeled a selection of promising lens candidates, demonstrating that they are sufficiently reliable for further investigations. Of course, in order to prove the lensing nature of these candidates, spectro-

scopic and high-angular resolution multi-band imaging observations are required.

Gaia was not designed for extragalactic studies. However, it is a transversal all-sky survey that will touch many expected and unexpected areas of astronomy, and this Letter is a vivid illustration of this capability. *Gaia* promises substantial advances in the study of strongly lensed, multiply imaged quasars because by its final data release, it will provide the highest spatial resolution of any all-sky survey. Moreover, it also provides exquisite astrometric and photometric data of the observed sources, including time-series in the final data releases. *Gaia* will thus enable us to set strong constraints in the modeling of gravitationally lensed quasar systems, and it will constrain cosmological parameters derived from one of the most interesting phenomena in nature.

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