Multiband nonthermal radiative properties of pulsar wind nebulae

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ABSTRACT

Aims. The nonthermal radiative properties of 18 pulsar wind nebulae (PWNe) are studied in the 1D leptonic model.

Methods. The dynamical and radiative evolution of a PWN in a nonradiative supernova remnant are self-consistently investigated in this model. The leptons (electrons/positrons) are injected with a broken power-law form, and nonthermal emission from a PWN is mainly produced by time-dependent relativistic leptons through synchrotron radiation and inverse Compton process.

Results. Observed spectral energy distributions (SEDs) of all 18 PWNe are reproduced well, where the indexes of low-energy electron components lie in the range of 1.0–1.8 and those of high-energy electron components in the range of 2.1–3.1. Our results show that $F_X/F_\gamma > 10$ for young PWNe; $1 < F_X/F_\gamma \leq 10$ for evolved PWNe, except for G292.0+1.8; and $F_X/F_\gamma \leq 1$ for mature/old PWNe, except for CTA 1. Moreover, most PWNe are particle-dominated. Statistical analysis for the sample of 14 PWNe further indicate that (1) not all pulsar parameters have correlations with electron injection parameters, but electron maximum energy and PWN magnetic field correlate with the magnetic field at the light cylinder, the potential difference at the polar cap, and the spin-down power; (2) the spin-down power positively correlates with radio, X-ray, bolometric, and synchrotron luminosities, but does not correlate with gamma-ray luminosity; (3) the spin-down power positively correlates with radio, X-ray, and γ -band surface brightness; and (4) the PWN radius and the PWN age negatively correlate with X-ray luminosity, the ratio of X-ray to gamma-ray luminosities, and the synchrotron luminosity.

Key words. pulsars: general – stars: winds, outflows – acceleration of particles – radiation mechanisms: non-thermal

1. Introduction

A pulsar wind nebula (PWN) is formed when the wind of a pulsar interacts with the ambient medium, either the supernova ejecta or the interstellar medium (e.g., Rees & Gunn 1974; Kennel & Coroniti 1984; Chevalier 2004; Gaensler & Slane 2006; Bucciantini 2008). It is generally believed that a PWN is mainly composed of a relativistic nonthermal lepton (electron/positron) plasma and magnetic field and can emit nonthermal photons from radio to γ -rays via synchrotron radiation and the inverse Compton (IC) process. Observations have shown that some PWNe, such as the Crab nebula, Kes 75, MSH 15-52, can emit nonthermal emission from radio to very high-energy (VHE) γ -ray bands (e.g., Bühler & Blandford 2014; Reynolds et al. 2017). Currently, about 37 TeV PWNe are firmly identified¹. More recently, the observations and relevant physical analysis of 19 TeV PWNe have been presented in Abdalla et al. (2017), where 14 TeV PWNe are firmly identified as PWNe by HESS observations. These observations provide an essential reason for studying PWNe.

Various models have been proposed to explain the nonthermal properties of PWNe and a brief review of current models and differences can be found in Torres et al. (2014). Here we focus on the one-dimensional (1D) model of the dynamical and radiative evolution of a PWN in a nonradiative SNR. Here we focus on the 1D model of the dynamical and radiative evolution of a PWN in a nonradiative supernova remnant (SNR) presented in Gelfand et al. (2009). In the frame of the model, the radiative properties during different phases of the PWN evolution

are investigated with a single power-law injection spectrum for the electrons/positrons (Gelfand et al. 2009), with the relativistic Maxwellian and a high-energy power-law tail injection spectrum for the electrons/positrons (Fang & Zhang 2010a), and with two possible forms of injected electron spectra: a broken power law and the sum of a power law at low energy, and a relativistic Maxwellian plus a high-energy power-law tail (Zhu et al. 2015). It has been shown that the broken power-law form for the electron injection is required when the model applies to the Crab nebula (Zhu et al. 2015). Recently, a model for describing PWN radiative properties during the dynamical evolutions of the PWN and the SNR is presented in Martin et al. (2016), the radius of the PWN during the free expansion phase and compression in this model is calculated through solving the equations given by Chevalier (2005), where the prescription is similar to that in Gelfand et al. (2009). The model is applied to a TeV PWN CTA 1 (Martin et al. 2016). We note that there is a kind of model in which a broken power-law injection spectrum for the electrons/positrons is assumed but the dynamics beyond reverberation is not included (e.g., Zhang et al. 2008; Bucciantini et al. 2011; Tanaka & Takahara 2011).

To investigate nonthermal radiative properties of each PWN and statistical features of PWNe, observed multiwaveband data for each PWN are required. However, not all PWNe are observed at different bands, due to observation limits, so 18 PWNe from known PWNe are selected here according to the following criteria: (1) the period and period derivative of central pulsar in each PWN are known, and (2) nonthermal emission at radio, X-ray, and TeV bands are detected. In this selection, N 158A is not detected at TeV band, G310.6-1.6 is detected with an upper limit at TeV band, G292.0+1.8 is detected with upper limits at GeV

¹ <http://tevcat.uchicago.edu/>

band. These three PWNe are included in the sample because of their energetic pulsars. These PWNe are divided into three groups based on the ages of PWNe that are adopted in our calculations (see Sect. 3). The nonthermal radiative property for each PWN is studied in the leptonic model with a broken power-law injection for the electrons/positrons, and the correlation features of all 18 PWNe are presented.

The structure of this article is as follows. In Sect. 2 we briefly review the model. In Sect. 3 we apply the model to 18 PWNe, and calculate the spectral energy distribution (SED) of each PWN. In Sect. 4 we study correlation properties of the PWNe, and we give our conclusions and discussions in Sect. 5.

2. Model description

For completeness, we briefly review the model for dynamical and radiative evolution of a PWN inside a SNR given in [Zhu et al. \(2015\)](#), which is closely based on that developed by [Gelfand et al. \(2009\)](#); see also [Fang & Zhang 2010a](#)). In this model, the large-scale evolution of a composite SNR depends on the mechanical energy E_{sn} of the explosion, the density n_{H} of the ambient medium, the mass M_{ej} of the supernova ejecta, and the spin-down power $L(t)$ of the pulsar. For a given pulsar with a period P , a period derivative $\dot{P}$, a braking index n , and an initial spin-down power L_0 , the spin-down power $L(t)$ at time t is given by ([Gaensler & Slane 2006](#))

$$L(t) = L_0 \left(1 + \frac{t}{\tau_0} \right)^{-(n+1)/(n-1)}, \quad (1)$$

where $\tau_0 = (2\tau_c)/(n-1) - T_{\text{age}}$ is the initial spin-down timescale of the pulsar, $\tau_c = P/2\dot{P}$ is the characteristic age of the pulsar, and T_{age} is the age of the PWN. Since $L(t)$ can be estimated by $L(t) = 4\pi^2 I \dot{P}/P^3$, where I is the pulsar moment of inertia (here $I = 10^{45} \text{ g cm}^2$ is used), the evolution of the spin-down power can be determined (i.e., L_0 and τ_0 can be estimated) if T_{age} is known. Therefore, the main parameters of a given pulsar are P , $\dot{P}$, and n . At present, the values of the braking index n for some pulsars have been measured, and we use the measured values if available, otherwise we assume $n = 3$. We call these parameters the pulsar and ejecta parameters.

The evolution of isotropic electron distribution $N(E, t)$ in the PWN is calculated by using

$$\frac{\partial N(E, t)}{\partial t} = \frac{\partial}{\partial E} [\dot{E}N(E, t)] + \frac{N(E, t)}{\tau(E, t)} + Q(E, t), \quad (2)$$

where $\dot{E}$ is the energy-loss rate of the particles with an energy E which includes the contributions of synchrotron radiation, inverse Compton scattering, and adiabatic losses and $\tau(E, t)$ is the escape time (for the details of their processes, see [Zhang et al. 2008](#)). The last term on the right-hand side in Eq. (2), $Q(E, t)$, is a source term, i.e., the injected electron number per unit energy per unit time, and is assumed to be a broken power-law form

$$Q(E, t) = Q_0(t) \begin{cases} \left(\frac{E}{E_b} \right)^{-\alpha_1} & \text{for } E \leq E_b, \\ \left(\frac{E}{E_b} \right)^{-\alpha_2} & \text{for } E_b \leq E < E_{\text{max}}, \end{cases} \quad (3)$$

where $Q_0(t)$ is a time-dependent parameter, E_b is the break energy, $\alpha_1 < 2$ and $\alpha_2 > 2$ are respectively the spectral indexes of the injection rate at $E \leq E_b$ and $E > E_b$, and E_{max} is the maximum energy of the electrons.

Here the spin-down power of the pulsar is assumed to be distributed between electrons ($\dot{E}_e(t) = \eta_e L(t)$) and magnetic fields

($\dot{E}_B = \eta_B L(t)$) [$\eta_e + \eta_B = 1$] ([Gelfand et al. 2009](#)), then $Q_0(t)$ can be estimated from $(1 - \eta_B)L(t) = \int_0^{E_{\text{max}}} Q(E, t) E dE$, which gives

$$Q_0(t) = \begin{cases} \frac{(1 - \eta_B)L(t)}{E_b^2} \frac{(2 - \alpha_1)(2 - \alpha_2)}{\alpha_1 - \alpha_2} & \alpha_2 > 2 \\ \frac{(1 - \eta_B)L(t)}{E_b^2} \left[\frac{1}{2 - \alpha_1} + \log \left(\frac{E_{\text{max}}}{E_b} \right) \right] & \alpha_2 = 2. \end{cases} \quad (4)$$

On the other hand, the maximum energy of the electrons can be estimated by the condition that the Larmor radius of the electrons inside the PWN is smaller than the termination shock radius of the PWNe by the containment factor $\epsilon < 1$, which is given by

$$E_{\text{max}} \approx 3e\epsilon \sqrt{\frac{\eta_B L(t)}{c}}, \quad (5)$$

where e is the electron charge (e.g., [Zhu et al. 2015](#)). We note that E_{max} can also be estimated by the balance between the synchrotron losses and acceleration, which gives $E_{\text{max}}^{\text{syn}} \approx [3(m_e c^2)^2 / 4e] \sqrt{\pi / e B_{\text{pwn}}}$. For the parameters of PWNe used here (see Tables 1–3), $E_{\text{max}} < E_{\text{max}}^{\text{syn}}$ is always satisfied. Hence, the injection parameters involving electron injection are E_b , α_1 , α_2 , η_B , and ϵ .

In such a model, the dynamical and radiative properties for a given PWN can be self-consistently studied [Gelfand et al. \(2009\)](#); see also [Fang & Zhang 2010a](#); and [Zhu et al. 2015](#)). On the one hand, the time evolutions of the SNR radius $R_{\text{snr}}(t)$, reverse shock radius $R_{\text{rs}}(t)$, PWN radius $R_{\text{pwn}}(t)$, the position of the neutron star $R_{\text{psr}}(t)$, the termination shock radius $R_{\text{ts}}(t)$, and the magnetic field $B_{\text{pwn}}(t)$ of the PWN can be calculated. On the other hand, the time evolutions of electron spectrum and the spectral energy distribution (SED) of nonthermal photons can be obtained. It should be pointed out that different electron injection forms and electron energy losses will lead to the change in dynamical features of the PWN. In this paper, the broken power-law injection of the electrons (see Eq. (3)) and escape term of the electrons (see the second term on the left-hand side of Eq. (2)) are considered, so the dynamical features for a given PWN will be different from those given in [Gelfand et al. \(2009\)](#), who assumed a single power-law injection without electron escape (e.g., Crab nebulae, see [Zhu et al. 2015](#)). We note that both $R_{\text{pwn}}(t)$ and $B_{\text{pwn}}(t)$ play important roles in the evolution of the energy losses and the SED for a given PWN. Therefore, in our calculations the allowed ranges of $R_{\text{pwn}}(t)$ are limited by the values given in [Abdalla et al. \(2017\)](#); see their Tables 1 and 3).

In our calculations, nonthermal photons are produced through synchrotron radiation and inverse Compton (IC) scattering of relativistic electrons (e.g., [Zhu et al. 2015](#)). For the synchrotron radiation, the emissivity given in [Zhang et al. \(2008\)](#) is used, which includes the effect of electron pitch angle. The magnetic field B_{pwn} of a PWN evolves with time and can be estimated by Eq. (A.6) in Appendix A. We note that [Torres et al. \(2014\)](#) did not consider the effect of electron pitch angle (see also [Tanaka & Takahara 2010](#); [Martin et al. 2012](#)), which will result in parameter differences between our model and the model in [Torres et al. \(2014\)](#) for a given PWN.

For IC scattering, soft photon fields consist of four components: the cosmic microwave background (CMB) in our calculations, the galactic far-infrared (FIR) background, the near-infrared (NIR) and optical photon field due to the stars, and synchrotron radiation. The energy density and temperature of the CMB photons are $U_{\text{CMB}} = 0.25 \text{ eV cm}^{-3}$ and $T_{\text{CMB}} = 2.7 \text{ K}$, and the energy densities and temperatures of FIR (U_{FIR} and T_{FIR})

Table 1. Parameters for young PWNe.

PWN	N 158A	G21.5-0.9	Crab nebula	Kes 75	G310.6-1.6	3C 58
Pulsar and ejecta parameters						
P (ms)	50.5	61.8	33.4	324.0	31.18	65.7
$\dot{P}$ (s s^{-1})	4.79×10^{-13}	2.02×10^{-13}	4.23×10^{-13}	7.08×10^{-12}	3.89×10^{-14}	1.93×10^{-13}
n	2.129	3.0	2.509	2.19	3.0	3.0
τ_c (yr)	1671	4850	1296	726	12 708	5397
$L(t)$ (erg s^{-1})	1.47×10^{38}	3.38×10^{37}	4.53×10^{38}	8.21×10^{36}	5.06×10^{37}	2.68×10^{37}
T_{age} (yr)	760	900	940	1000	1500	2400
L_0 (erg s^{-1})	3.34×10^{38}	5.09×10^{37}	3.1×10^{39}	8.11×10^{38}	6.51×10^{37}	8.69×10^{37}
τ_0 (yr)	2201	3950	730	220	11 208	2997
M_{ej} ($M_\odot$)	23.0	8.0	9.5	10.0	13.0	8.0
E_{SN} (10^{51} erg)	2.0	1.0	1.0	1.0	1.0	1.0
n_{H} (cm^3)	0.01	0.1	0.1	0.1	0.01	0.01
d (kpc)	49	4.1	2.0	5.8	7.0	2.0
Soft photon parameters						
T_{FIR} (K)	88.0	35.0	70.0	30.0	40.0	40.0
U_{FIR} (eV cm^{-3})	1.0	1.4	0.5	1.2	0.3	0.3
T_{NIR} (K)	5000.0	3500.0	5000.0	3000.0	4000.0	4000.0
U_{NIR} (eV cm^{-3})	1.0	5.0	1.0	2.2	0.3	0.3
Injection parameters						
E_b (10^5 MeV)	150.00	1.52	6.26	2.13	6.50	0.61
α_1	1.75	1.00	1.59	1.55	1.06	1.03
α_2	2.31 ± 0.19	2.63 ± 0.01	2.42 ± 0.01	2.09 ± 0.03	2.71 ± 0.06	3.07 ± 0.02
η_B	0.0014 ± 0.0004	0.046 ± 0.006	0.040 ± 0.001	0.0019 ± 0.0004	0.0017 ± 0.0002	0.061 ± 0.008
ϵ	0.30	0.32 ± 0.04	0.14 ± 0.01	0.58 ± 0.23	0.12	0.064 ± 0.045
Chi-squared χ^2	0.20	1.35	5.73	2.08	4.88	1.91
Derived parameters						
E_{max} (10^8 MeV)	$7.06^{+0.95}_{-1.09}$	$20.78^{+4.07}_{-3.83}$	$30.91^{+2.62}_{-2.57}$	$3.76^{+2.02}_{-1.74}$	$1.83^{+0.10}_{-0.12}$	$4.25^{+3.44}_{-3.07}$
B_{pwn} (μG)	$45.22^{+14.81}_{-9.65}$	$84.35^{+7.13}_{-7.52}$	$116.34^{+0.98}_{-1.02}$	$14.18^{+3.33}_{-2.69}$	$13.16^{+0.90}_{-0.91}$	$23.92^{+1.71}_{-1.81}$
R_{pwn} (pc)	0.68	0.86	1.95	0.99	1.32	3.05
R_{rs} (pc)	4.64	4.04	4.00	4.12	6.58	10.03
R_{snr} (pc)	5.52	4.81	4.76	4.91	7.83	11.93
R_{ts} (pc)	0.05	0.06	0.20	0.05	0.09	0.23

and NIR (U_{NIR} and T_{NIR}) can be different for PWNe. We refer to these parameters as soft photon parameters. The emissivity of IC scattering used here is given by [Zhang et al. \(2008\)](#).

To fit the observed spectral energy distribution (SED) for each PWN, pulsar and ejecta parameters and soft photon parameters are fixed, and injection parameters are considered as fitting parameters (see Tables 1–3). The Levenberg-Marquardt (LM) method of the χ^2 minimization fitting procedure is used to find the best-fitting values of injection parameters and their uncertainties. However, because of the lack of data (from radio to optical) for most PWNe, the best-fitting values and their uncertainties of α_1 and E_b cannot be easily obtained. Therefore, at first, visual fitting is used to determine the values of α_1 and E_b , and then the LM method is used to obtain the values of α_2 , η_B , and ϵ . With this procedure, we fit 14 PWNe, except for N 158A, G310.6-1.6, G292.0+1.8, and HESS J1303-631. Since the observed data at GeV–TeV and from radio to X-ray band are upper limits for these four PWNe, the LM method can only determine the values of α_2 and η_B for N 158A, G310.6-1.6, and G292.0+1.8, and α_2 for HESS J1303-631; other injection parameters are estimated via visual fitting. These values are shown in Tables 1–3.

In addition to the above parameters, six derived parameters (E_{max} , B_{pwn} , R_{pwn} , R_{rs} , R_{snr} , and R_{ts}) are also given (see Tables 1–3). We note that E_{max} and B_{pwn} , which relate to ϵ and η_B , are calculated. Because the slight difference in the injection spectrum has little effect on the dynamical properties of PWNe, the uncertainties of R_{pwn} , R_{rs} , R_{snr} , and R_{ts} are not calculated here.

3. Applications to individual PWNe

The model described in Sect. 2 is now applied to explain the observed SEDs of nonthermal photons for 18 PWNe. The detailed descriptions of observed and derived properties of each PWN are summarized in Appendix B. The PWN sample is divided into three groups based on possible ages T_{age} of PWNe. The first group consists of the PWNe with $T_{\text{age}} \leq 2400$ yr (we call them young PWNe), the second group with $2400 < T_{\text{age}} < 5000$ yr (called evolved PWNe), and the third group with $T_{\text{age}} \geq 5000$ yr (called mature/old PWNe). Although the division of the three groups is arbitrary, it is convenient for us to study the properties of PWNe that lie in different age ranges.

Fig. 1. Comparisons of predicted SEDs and observed data for young PWNe (from upper left panel for N 158A to bottom right panel for 3C 58). In each panel, the black line represents synchrotron SED; the magenta, blue, green, and cyan lines represent the SEDs of inverse Compton scatterings with the synchrotron photons, IR, CMB, and starlight, respectively; and the total SED is shown by the red line. See text for the descriptions of the observed data; the relevant parameters are listed in Table 1.

Some of the 18 PWNe have been already investigated (e.g., Fang & Zhang 2010a; Tanaka & Takahara 2011; Torres et al. 2014; Martin et al. 2014; Zhu et al. 2015; Martin et al. 2016). Two important quantities that influence the electron energy loss and photon SED of a PWN are $R_{\text{pwn}}(t)$ and $B_{\text{pwn}}(t)$. In Torres et al. (2014), $R_{\text{pwn}}(t) \propto t^{6/5}$ is calculated during the SNR's free expansion phase (van der Swaluw et al. 2001, 2003) and $B_{\text{pwn}}(t)$ is estimated with a method similar to that used in Gelfand et al. (2009). In Tanaka & Takahara (2011), $B_{\text{pwn}}(t)$ is calculated by using the magnetic field energy conservation, i.e., $(4\pi/3)R_{\text{pwn}}^3(t)B_{\text{pwn}}^2(t)/(8\pi) = \int \eta L(t')dt'$, and R_{pwn} is estimated by reproducing photon SED.

3.1. Group 1: young PWNe

There are six young PWNe in this group, N 158A, G21.5-0.9, Crab nebula, Kes 75, G310.6-1.6, and 3C 58, whose ages range from 700 yr to 2400 yr. The parameters of pulsar and ejecta, electron injections, and soft photon fields for these young PWNe are listed in Table 1, and the details for each PWN is given in Appendix B. Using the parameters in Table 1, the SEDs of these young PWNe are calculated. The comparisons of the calculated and observed SEDs are shown in Fig. 1 and derived parameters are listed in Table 1. The multiband SEDs of these young PWNe have been studied (e.g., Tanaka & Takahara 2011; Torres et al. 2014; Martin et al. 2014). These PWNe are in the SNR's free

Table 2. Parameters for evolved PWNe.

PWN	HESS J1813-178	G54.1+0.3	G292.0+1.8	G0.9+0.1	MSH 15-52	N 157B
Pulsar and ejecta parameters						
P (ms)	44.7	136.85	135.31	52.2	150	16.12
$\dot{P}$ (s s^{-1})	1.26×10^{-13}	7.51×10^{-13}	7.47×10^{-13}	1.56×10^{-13}	1.50×10^{-12}	5.13×10^{-14}
n	3.0	3.0	3.0	3.0	1.4	3.0
τ_c (yr)	5624	2889	2872	5305	1585	4982
$L(t)$ (erg s^{-1})	5.56×10^{37}	1.16×10^{37}	1.19×10^{37}	4.32×10^{37}	1.75×10^{37}	4.83×10^{38}
T_{age} (yr)	2500	2600	2700	3000	4000	4600
L_0 (erg s^{-1})	1.80×10^{38}	1.15×10^{39}	3.32×10^{39}	2.29×10^{38}	1.18×10^{39}	8.21×10^{40}
τ_0 (yr)	3124	289	172	2305	3927	382
$M_{\text{ej}}(M_{\odot})$	8.0	20.0	18.0	14.0	6.0	20.0
$E_{\text{SN}} (10^{51} \text{ erg})$	1.0	1.0	1.0	1.0	1.0	1.0
$n_{\text{H}} (\text{cm}^3)$	1.0	1.0	0.5	0.01	0.01	0.03
d (kpc)	4.7	7.0	6.0	13.3	4.4	53.7
Soft photon parameters						
T_{FIR} (K)	40.0	25.0	25	30.0	30.0	50.0
$U_{\text{FIR}} (\text{eV cm}^{-3})$	0.1	0.8	1.0	3.8	1.2	5.0
T_{NIR} (K)	4000.0	3000.0	2800	3000.0	3000.0	5000.0
$U_{\text{NIR}} (\text{eV cm}^{-3})$	0.5	1.1	0.7	25.0	2.2	4.0
Injection parameters						
$E_{\text{b}} (10^5 \text{ MeV})$	5.92	5.45	1.60	0.45	6.65	1.62
α_1	1.09	1.22	1.56	1.10	1.17	1.15
α_2	2.76 ± 0.02	2.67 ± 0.03	2.62 ± 0.18	2.52 ± 0.02	2.28 ± 0.02	2.58 ± 0.01
η_{B}	0.0057 ± 0.0008	0.0045 ± 0.0009	0.046 ± 0.007	0.029 ± 0.004	0.45 ± 0.03	0.03 ± 0.002
ϵ	0.26 ± 0.05	0.26 ± 0.09	0.42	0.25 ± 0.08	0.56 ± 0.39	0.33 ± 0.05
Chi-squared χ^2	1.58	0.65	14.98	0.30	0.60	2.34
Derived parameters						
$E_{\text{max}} (10^8 \text{ MeV})$	$7.59^{+2.08}_{-1.91}$	$3.07^{+1.46}_{-1.27}$	$16.13^{+1.19}_{-1.28}$	$14.52^{+6.00}_{-5.35}$	$81.42^{+61.23}_{-57.52}$	$65.18^{+12.35}_{-11.75}$
$B_{\text{pwn}} (\mu\text{G})$	$8.11^{+0.72}_{-0.75}$	$9.48^{+1.21}_{-1.26}$	$23.98^{+2.35}_{-2.23}$	$20.29^{+1.86}_{-1.93}$	$18.72^{+0.35}_{-0.37}$	$26.47^{+0.94}_{-0.99}$
$R_{\text{pwn}} (\text{pc})$	3.67	2.41	3.12	3.51	11.21	10.02
$R_{\text{rs}} (\text{pc})$	4.97	5.00	5.73	10.27	14.67	11.17
$R_{\text{snr}} (\text{pc})$	6.97	6.16	6.97	12.23	17.88	13.29
$R_{\text{ts}} (\text{pc})$	0.28	0.11	0.13	0.22	0.53	0.72

expansion phases and the dynamical evolution in the free expansion phase used here is similar that used in [Torres et al. \(2014\)](#).

The fluxes, $F_X \equiv F_{1-10 \text{ keV}}$ and $F_\gamma \equiv F_{1-10 \text{ TeV}}$, in X-ray (1–10 keV) and γ -ray (1–10 TeV) bands for each PWN are calculated.

At an age of 760 yr N 158A was not detected in GeV and TeV γ -ray bands, so the sensitivity curves of the Cherenkov Telescope Array (CTA) is used as an upper limit to restrict soft photon parameters (see Table 1). Our results are shown in Table 1 and Fig. 1, and give that $R_{\text{pwn}} = 0.68 \text{ pc}$ and $B_{\text{pwn}} = 45.22 \mu\text{G}$, which is roughly consistent with the values given in [Martin et al. \(2014, \$R_{\text{pwn}} = 0.7 \text{ pc}\$ and \$B_{\text{pwn}} = 32 \mu\text{G}\$ \)](#). The observed SED can be reproduced and predicted as $F_X/F_\gamma \sim 118$.

Our results for G21.5-0.9 are shown in Table 1 and Fig. 1. Here the age is assumed to be $T_{\text{age}} = 900 \text{ yr}$, which is close to $T_{\text{age}} = 870 \text{ yr}$ in [Torres et al. \(2014\)](#). Our results give $R_{\text{pwn}} = 0.86 \text{ pc}$ and $B_{\text{pwn}} \approx 84 \mu\text{G}$, which are consistent with those in [Torres et al. \(2014; \$R_{\text{pwn}} = 0.9 \text{ pc}\$ and \$B_{\text{pwn}} = 71 \mu\text{G}\$ \)](#) and those in [Tanaka & Takahara \(2011; \$R_{\text{pwn}} = 1.0 \text{ pc}\$ and \$B_{\text{pwn}} = 64\$, and \$47 \mu\text{G}\$ \)](#). We note that R_{pwn} is less than that given in [Abdalla et al. \(2017; \$<4 \text{ pc}\$ \)](#). The predicted $F_X/F_\gamma \sim 130$.

For Crab nebulae, the results are the same as those given in [Zhu et al. \(2015\)](#). As mentioned above, the main differences

between our results and those in [Tanaka & Takahara \(2011\)](#) and [Torres et al. \(2014\)](#) come from the use of different synchrotron emissivity. Our results give $R_{\text{pwn}} \approx 1.95 \text{ pc}$ ($<3 \text{ pc}$ in [Abdalla et al. 2017](#)) and $B_{\text{pwn}} \approx 116 \mu\text{G}$. The predicted $F_X/F_\gamma \sim 460$.

For Kes 75, its age is assumed to be 1000 yr, which is slightly older than the ages given in [Tanaka & Takahara \(2011\)](#) and [Torres et al. \(2014\)](#), and its SED is calculated with the same values of pulsar and ejecta parameters and soft photon parameters as those in [Torres et al. \(2014\)](#) and can reproduce the observed SED well with $F_X/F_\gamma \sim 13$ (see Fig. 1). However, our injection parameters are different from those given in [Torres et al. \(2014\)](#) because the formula of synchrotron emissivity used here (see Eq. (12) of [Zhang et al. 2008](#)) is different from that used in [Torres et al. \(2014, see Eqs. \(27\) and \(28\) of \[Martin et al. 2012\]\(#\)\)](#), resulting in different values of $R_{\text{pwn}} = 0.99 \text{ pc}$ and $B_{\text{pwn}} \approx 14 \mu\text{G}$ of Table 1 in comparison with those ($R_{\text{pwn}} = 0.9 \text{ pc}$ and $B_{\text{pwn}} = 19 \mu\text{G}$) in [Torres et al. \(2014\)](#) and those ($R_{\text{pwn}} = 0.29 \text{ pc}$ and $B_{\text{pwn}} = 20 \mu\text{G}$) in [Tanaka & Takahara \(2011\)](#).

The HESS observation gives the upper limit of the flux at TeV band for G310.6-1.6, which can roughly restrict model parameters. With $T_{\text{age}} = 1500 \text{ yr}$, which is less than the age of $<1900 \text{ yr}$ found in [Renaud et al. \(2010\)](#), $R_{\text{pwn}} = 1.32 \text{ pc}$

Fig. 2. Comparisons of predicted SEDs and observed data for evolved PWNe (from upper left panel for HESS J1813-178 to bottom right panel for N 157B). In each panel, the black line represents the synchrotron SED; magenta, blue, green, and cyan lines represent the SEDs of inverse Compton scatterings with the synchrotron photons, IR, CMB, and starlight, respectively; and the total SED is shown by the red line. See text for the descriptions of the observed data; the relevant parameters are listed in Table 2.

and $B_{\text{pwn}} \approx 13 \mu\text{G}$ are obtained here. We note that Tanaka & Takahara (2013) gave $R_{\text{pwn}} = 1.3 \text{ pc}$ and $B_{\text{pwn}} = 17 \mu\text{G}$ with $T_{\text{age}} = 600 \text{ yr}$, and Martin et al. (2014) obtained $R_{\text{pwn}} = 1.3 \text{ pc}$ and $B_{\text{pwn}} = 8.2 \mu\text{G}$ with $T_{\text{age}} = 1100 \text{ yr}$. The predicted $F_X/F_\gamma \sim 11$.

3C 58 has an age of $\sim 2400 \text{ yr}$ (Chevalier 2005; Tanaka & Takahara 2013; Torres et al. 2013). With $T_{\text{age}} = 2400 \text{ yr}$, $R_{\text{pwn}} = 3.05 \text{ pc}$ and $B_{\text{pwn}} \approx 24 \mu\text{G}$ are obtained, which can compare with the values in Tanaka & Takahara (2013, $R_{\text{pwn}} = 2.0 \text{ pc}$ and $B_{\text{pwn}} = 17 \mu\text{G}$) and those in Torres et al. (2013, $R_{\text{pwn}} = 3.7 \text{ pc}$ and $B_{\text{pwn}} = 35 \mu\text{G}$). We note that the distance of the PWN is assumed to be 2 kpc here, which is the same

as that in Tanaka & Takahara (2013), but is different from that in Torres et al. (2013; 3.2 kpc). The predicted $F_X/F_\gamma \sim 20$.

3.2. Group 2: evolved PWNe

The group of evolved PWNe consists of HESS J1813-178, G54.1+0.3, G292.0+1.8, G0.9+0.1, MSH 15-52, and N 157B whose ages range from 2500 yr to 4600 yr. Since the SN explosion energy E_{sn} are unknown for these PWNe, here we assume $E_{\text{sn}} = 1.0 \times 10^{51} \text{ erg}$. The related parameters for the evolved PWNe are listed in Table 2 and the comparisons of the modeled and observed SEDs are shown in Fig. 2.

Table 3. Parameters for mature/old PWNe.

PWN	HESS J1356-645	CTA 1	HESS J1418-609	HESS J1420-607	HESS J1119-614	HESS J1303-631
Pulsar and ejecta						
P (ms)	166	316.0	110.6	68.0	408.0	184
$\dot{P}$ (s s^{-1})	3.6×10^{-13}	3.614×10^{-13}	1.69×10^{-13}	8.3×10^{-14}	4.0×10^{-12}	2.65×10^{-13}
n	3.0	3.0	3.0	3.0	1.2	2.5
τ_c (yr)	7310	13 863	10 376	12 990	1617	11 008
$L(t)$ (erg s^{-1})	3.10×10^{36}	4.52×10^{35}	4.93×10^{36}	1.04×10^{37}	2.32×10^{36}	1.68×10^{36}
T_{age} (yr)	6500	7500	8000	8500	9000	13 000
L_0 (erg s^{-1})	2.52×10^{38}	2.14×10^{36}	9.39×10^{37}	8.70×10^{37}	1.78×10^{40}	2.64×10^{38}
τ_0 (yr)	810	6363	2376	4490	7170	1678
M_{ej} ($M_{\odot}$)	7.0	8.0	10.0	15.0	35.0	9.0
E_{SN} (10^{51} erg)	1.0	1.0	1.0	1.0	1.0	1.0
n_{H} (cm^{-3})	0.05	0.1	0.2	0.5	0.06	0.02
d (kpc)	2.5	1.4	5.0	5.6	8.4	6.6
Soft photon parameters						
T_{FIR} (K)	25.0	25.0	40.0	40.0	25.0	40.0
U_{FIR} (eV cm^{-3})	0.4	0.3	0.4	0.3	0.3	6.0
T_{NIR} (K)	5000.0	3000.0	4000.0	4000.0	3300.0	4000.0
U_{NIR} (eV cm^{-3})	0.5	0.6	1.0	0.3	0.7	2.0
Injection parameters						
E_{b} (10^5 MeV)	5.02	2.00	6.00	7.00	7.70	36.00
α_1	1.05	1.20	1.15	1.35	1.50	1.50
α_2	2.68 ± 0.02	2.11 ± 0.11	2.52 ± 0.08	2.73 ± 0.06	3.08 ± 0.09	2.19 ± 0.02
η_{B}	0.11 ± 0.03	0.49 ± 0.02	0.058 ± 0.015	0.011 ± 0.002	0.0010 ± 0.0001	0.07
ϵ	0.17 ± 0.08	0.20 ± 0.12	0.38 ± 0.35	0.34 ± 0.16	0.69 ± 0.01	0.12
Chi-squared χ^2	1.39	4.25	0.92	0.40	2.48	3.94
Derived parameters						
E_{max} (10^8 MeV)	$5.15^{+3.39}_{-2.83}$	$4.88^{+3.08}_{-2.97}$	$10.54^{+12.17}_{-9.82}$	$5.97^{+3.57}_{-3.11}$	$1.72^{+0.12}_{-0.11}$	2.13
B_{pwn} (μG)	$4.31^{+0.48}_{-0.59}$	$5.66^{+0.27}_{-0.23}$	$4.38^{+0.60}_{-0.63}$	$3.73^{+0.38}_{-0.40}$	$1.34^{+0.07}_{-0.07}$	1.63
R_{pwn} (pc)	10.34	6.24	9.43	7.73	14.67	20.44
R_{rs} (pc)	13.06	10.84	0	0	0	0
R_{snr} (pc)	18.72	17.67	15.90	13.59	17.00	30.17
R_{ts} (pc)	0.56	0.28	0.47	0.34	0.16	1.22

HESS J1813-178 is observed to have a distance $d \approx 4.7$ kpc (Brogan et al. 2005; Halpern et al. 2012) and $R_{\text{pwn}} = 4.0 \pm 0.3$ pc (Abdalla et al. 2017). With $T_{\text{age}} = 2500$ yr, $R_{\text{pwn}} \approx 3.7$ pc and $B_{\text{pwn}} = 8.11 \mu\text{G}$ are obtained here. The predicted $F_X/F_\gamma \sim 6$. We note that Fang & Zhang (2010b) investigated the PWN in detail, and obtained $R_{\text{pwn}} = 1.7$ pc and $B_{\text{pwn}} = 5 \mu\text{G}$ with $T_{\text{age}} = 1200$ yr.

The age of G54.1+0.3 is uncertain, which ranges from 1500 to 6000 yr (Camilo et al. 2002a), or from 2100 to 3600 yr (Gelfand et al. 2015). Because the characteristic age is ~ 2872 yr, $T_{\text{age}} = 2600$ yr is assumed, and our results give $R_{\text{pwn}} \sim 2.4$ pc, $B_{\text{pwn}} \approx 9.5 \mu\text{G}$, and $F_X/F_\gamma \sim 3$. With different ages, Tanaka & Takahara (2011) gave $R_{\text{pwn}} = 1.8$ pc and $B_{\text{pwn}} = 10 \mu\text{G}$ ($6.7 \mu\text{G}$) for $T_{\text{age}} = 1700$ yr (2300 yr) and Torres et al. (2014) obtained $R_{\text{pwn}} = 1.4$ pc and $B_{\text{pwn}} = 14 \mu\text{G}$ for $T_{\text{age}} = 1700$ yr.

Although the age of G292.0+1.8 is uncertain, $R_{\text{pwn}} \approx 3.0$ pc is given in Bhalerao et al. (2015). With $T_{\text{age}} = 2700$ yr (Tanaka & Takahara 2013), which is larger than 1600 yr given in Murdin & Clark (1979) and 2500 yr in Martin et al. (2014), our results show that $R_{\text{pwn}} = 3.12$ pc and $B_{\text{pwn}} \approx 24 \mu\text{G}$, which are consistent with $R_{\text{pwn}} = 3.5$ pc and $B_{\text{pwn}} = 16 \mu\text{G}$ obtained by Tanaka & Takahara (2013) and $R_{\text{pwn}} = 3.5$ pc and $B_{\text{pwn}} = 21 \mu\text{G}$ by Martin et al. (2014). The predicted $F_X/F_\gamma \sim 14$.

The age (from 2000 yr to 3000 yr, see Camilo et al. 2009) and distance (from 8.0 kpc to 16 kpc, see Dubner et al. 2008)

of G0.9+0.1 are both uncertain. Here $T_{\text{age}} = 3000$ yr and $d = 13.3$ kpc are assumed; the results show that $R_{\text{pwn}} = 3.51$ pc, $B_{\text{pwn}} \approx 20 \mu\text{G}$, and $F_X/F_\gamma \sim 4$. Tanaka & Takahara (2011) obtained $R_{\text{pwn}} = 2.3$ pc (3.8 pc) and $B_{\text{pwn}} = 15 \mu\text{G}$ ($12 \mu\text{G}$) for $d = 8.0$ kpc (13 kpc) and $T_{\text{age}} = 2000$ yr (4500 yr). Torres et al. (2014) obtained $R_{\text{pwn}} = 2.5$ pc (3.8 pc) and $B_{\text{pwn}} = 14 \mu\text{G}$ ($15 \mu\text{G}$) for $d = 8.5$ kpc (13 kpc) and $T_{\text{age}} = 2000$ yr (3000 yr).

For MSH 15-52, its radius is $R_{\text{pwn}} = 11.1 \pm 2.0$ pc (Abdalla et al. 2017). Our results give $R_{\text{pwn}} = 11.21$ pc, $B_{\text{pwn}} \approx 19 \mu\text{G}$, and $F_X/F_\gamma \sim 10$ with $T_{\text{age}} = 4000$ yr. We note that younger age in previous studies is used; for example, $T_{\text{age}} \sim 1600$ yr, $R_{\text{pwn}} = 3.0$ pc, and $B_{\text{pwn}} = 21 \mu\text{G}$ were obtained by Torres et al. (2014), and $R_{\text{pwn}} = 3.6$ pc and $B_{\text{pwn}} = 19.0 \mu\text{G}$ by Fang & Zhang (2010a).

N 157B has an estimated distance of 48 kpc (Macri et al. 2006), and its radius is about 10.6 pc (Lazendic et al. 2000), or 3.5 pc (Chen et al. 2006), or <94 pc (Abdalla et al. 2017). Here, $T_{\text{age}} = 4600$ yr is used, and the results show that $R_{\text{pwn}} = 10.02$ pc, $B_{\text{pwn}} \approx 26 \mu\text{G}$, and $F_X/F_\gamma \sim 5$. With $T_{\text{age}} = 4600$ yr, Martin et al. (2014) obtained $R_{\text{pwn}} = 10.6$ pc and $B_{\text{pwn}} = 13 \mu\text{G}$.

3.3. Group 3: mature/old PWNe

The third group of mature/old PWNe consists of HESS J1356-645, CTA 1, HESS J1418-609, HESS J1420-607, HESS J1119-614, and HESS J1303-631 whose ages range from 6500 yr to

Fig. 3. Comparisons of predicted SEDs and observed data for mature/old PWNe (from upper left panel for HESS J1356-645 to bottom right panel for HESS J1303-631). In each panel, the black line represents the synchrotron SED; magenta, blue, green, and cyan lines represent the SEDs of inverse Compton scatterings with the synchrotron photons, IR, CMB, and starlight, respectively; and the total SED is shown by the red line. See text for the descriptions of the observed data; the relevant parameters are listed in Table 3.

13 000 yr. The related parameters for these mature/old PWNe are listed in Table 3 and the comparisons of modeled and observed SEDs are shown in Fig. 3.

HESS J1356-645 is a TeV PWN with a distance of 2.5 kpc (Cordes & Lazio 2002; Chang et al. 2012) and a radius of $R_{\text{pwn}} = 10.1 \pm 0.9$ pc (Abdalla et al. 2017). With $T_{\text{age}} = 6500$ yr, the values of $R_{\text{pwn}} = 10.34$ pc, $B_{\text{pwn}} \approx 4$ μG , and $F_X/F_\gamma \sim 0.9$ are obtained. With $T_{\text{age}} = 6000$ (8000) yr, Torres et al. (2014) obtained $R_{\text{pwn}} = 9.5$ (9.9) pc and $B_{\text{pwn}} \approx 3.1$ (3.5) μG .

CTA 1 has a radius of $R_{\text{pwn}} = 6.6 \pm 0.5$ pc (Abdalla et al. 2017). With $T_{\text{age}} = 7500$ yr, the values of $R_{\text{pwn}} = 6.24$ pc, $B_{\text{pwn}} \approx 5.66$ μG , and $F_X/F_\gamma \sim 4$ are obtained here. There have been several studies of CTA 1 (e.g., Zhang et al. 2009;

Torres et al. 2014; Martin et al. 2016). Torres et al. (2014) found $R_{\text{pwn}} = 8$ pc and $B_{\text{pwn}} \approx 4.1$ μG with $T_{\text{age}} = 7500$ yr. Martin et al. (2016) studied both radiative and dynamical properties of CTA 1; they obtained that $T_{\text{age}} = 9200$ (11 400) yr, $R_{\text{pwn}} = 6.7$ (6.7) pc, and $B_{\text{pwn}} \approx 4.3$ (1.8) μG in the free expansion (compression) phase of CTA 1.

For HESS J1418-609, its radius is $R_{\text{pwn}} = 9.4 \pm 0.9$ pc (Abdalla et al. 2017). In the current paper $R_{\text{pwn}} = 9.4$ pc, $B_{\text{pwn}} \approx 4.4$ μG , and $F_X/F_\gamma \sim 1$ are obtained with $T_{\text{age}} = 8000$ yr.

The radius of HESS J1420-607 is $R_{\text{pwn}} = 7.9 \pm 0.6$ pc (Abdalla et al. 2017). Here the values of $R_{\text{pwn}} = 7.73$ pc, $B_{\text{pwn}} \approx 3.7$ μG , and $F_X/F_\gamma \sim 0.8$ are obtained with $T_{\text{age}} = 8500$ yr.

Table 4. Pair multiplicities and wind Lorentz factors in the sample.

PWN	$\dot{Q}$ (s ⁻¹)	κ_{multi}	γ_w
Young PWNe			
N158A	4.48×10^{41}	1.02×10^8	4.01×10^2
G21.5-0.9	6.76×10^{38}	3.21×10^5	5.83×10^4
Crab nebula	5.93×10^{41}	7.73×10^7	8.97×10^2
Kes 75	4.02×10^{39}	3.87×10^6	2.49×10^3
G310.6-1.6	4.42×10^{38}	1.71×10^5	1.40×10^5
3C 58	1.88×10^{39}	1.00×10^6	1.64×10^4
Evolved PWNe			
HESS J1813-178	6.82×10^{38}	2.52×10^5	9.91×10^4
G54.1+0.3	4.34×10^{38}	3.52×10^5	3.25×10^4
G292.0+1.8	2.42×10^{40}	1.93×10^7	6.33×10^3
G0.9+0.1	4.13×10^{39}	1.73×10^6	5.73×10^2
MSH 15-52	1.11×10^{38}	7.31×10^4	1.06×10^5
N157B	2.38×10^{40}	2.98×10^6	2.41×10^4
Mature/old PWNe			
HESS J1356-645	2.75×10^{37}	4.30×10^4	1.23×10^6
CTA 1	4.25×10^{36}	1.74×10^4	6.63×10^4
HESS J1418-609	7.46×10^{37}	9.26×10^4	7.61×10^4
HESS J1420-607	1.25×10^{39}	1.07×10^6	1.01×10^4
HESS J1119-614	1.57×10^{39}	2.84×10^6	1.81×10^3
HESS J1303-631	1.96×10^{38}	4.17×10^5	9.75×10^3

Table 5. Parameters of the central pulsars in the sample.

PWN	B_s	Φ	B_{LC}
Young PWNe			
N158A	9.95×10^{12}	2.56×10^{16}	7.12×10^5
G21.5-0.9	7.15×10^{12}	1.23×10^{16}	2.79×10^5
Crab nebula	7.61×10^{12}	4.47×10^{16}	1.88×10^6
Kes 75	9.69×10^{13}	6.05×10^{15}	2.63×10^4
G310.6-1.6	2.23×10^{12}	1.50×10^{16}	6.78×10^5
3C 58	7.21×10^{12}	1.09×10^{16}	2.34×10^5
Evolved PWNe			
HESS J1813-178	4.79×10^{12}	1.57×10^{16}	4.95×10^5
G54.1+0.3	2.05×10^{13}	7.19×10^{15}	7.38×10^4
G292.0+1.8	2.03×10^{13}	7.29×10^{15}	7.57×10^4
G0.9+0.1	5.77×10^{12}	1.39×10^{16}	3.74×10^5
MSH 15-52	3.03×10^{13}	8.85×10^{15}	8.29×10^4
N157B	1.84×10^{12}	4.64×10^{16}	4.05×10^6
Mature/old PWNe			
HESS J1356-645	1.56×10^{13}	3.73×10^{15}	3.15×10^4
CTA 1	2.16×10^{13}	1.42×10^{15}	6.32×10^3
HESS J1418-609	8.75×10^{12}	4.69×10^{15}	5.96×10^4
HESS J1420-607	4.81×10^{12}	6.82×10^{15}	1.41×10^5
HESS J1119-614	8.17×10^{13}	3.22×10^{15}	1.11×10^4
HESS J1303-631	1.41×10^{13}	2.74×10^{15}	2.09×10^4

HESS J1119-614 has a radius of $R_{\text{pwn}} = 14 \pm 2$ pc (Abdalla et al. 2017). The values reported here are $R_{\text{pwn}} = 14.67$ pc, $B_{\text{pwn}} \approx 1.3$ μG , and $F_X/F_\gamma \sim 0.01$ with $T_{\text{age}} = 9000$ yr here. Torres et al. (2014) gave $R_{\text{pwn}} = 13$ pc and $B_{\text{pwn}} \approx 4$ μG with $T_{\text{age}} = 4200$ yr.

HESS J1303-631 has a radius of $R_{\text{pwn}} = 20.6 \pm 1.7$ pc (Abdalla et al. 2017). In the current paper $R_{\text{pwn}} = 20.44$ pc, $B_{\text{pwn}} \approx 1.6$ μG and $F_X/F_\gamma \sim 0.02$ are obtained with $T_{\text{age}} = 13\,000$ yr.

3.4. Estimate of pair multiplicity and wind Lorentz factor

Pair multiplicity, κ_{multi} , and the Lorentz factor, γ_w , are calculated for a PWN based on our model. The pair multiplicity can be expressed as

$$\kappa_{\text{multi}} = \frac{\dot{Q}}{\dot{N}}, \quad (6)$$

where $\dot{Q}$ is the injection rate by integrating Eq. (2),

$$\dot{Q} = \int_{E_{\text{min}}}^{E_{\text{max}}} \dot{Q}(E, t) dE, \quad (7)$$

and $\dot{N}$ is the injection rate with electrodynamic minimum suggested by Goldreich & Julian (1969), i.e., $\dot{N} = (\frac{cI\Omega\dot{\Omega}}{e^2})^{1/2}$. The values are shown in Table 5. The pair multiplicity ranges from $\sim 10^5$ to $\sim 10^8$. The Lorentz factor of the pulsar wind can be estimated by

$$\langle E \rangle = \frac{(1 - \eta_B)L(t)}{\int \dot{Q}(E, t) dE} = \gamma_w m_e c^2, \quad (8)$$

where $\langle E \rangle$ is the average energy for the SED of each PWN. The values of γ_w are listed in Table 4. Clearly, E_b is larger than $\langle E \rangle$ by up to several orders of magnitude.

3.5. Brief summary of calculated results

Here, the main results in the above calculations are briefly summarized.

The observed SEDs of 18 PWNe can be reproduced well in the frame of the model. For the electron injection, five free parameters ($\alpha_1, \alpha_2, E_b, \eta_B, \epsilon$) are used. Our results give $\alpha_1 \approx 1.0$ – 1.8 , $\alpha_2 \approx 2.0$ – 3.1 , $E_b \approx 10^4$ – 10^7 MeV, $\eta_B \approx 0.1\%$ – 11% for the PWNe except for MSH 15-52 ($\eta_B \approx 45\%$) and CTA 1 ($\eta_B \approx 49\%$), and $\epsilon \approx 0.1$ – 0.70 . The results for spectral indices are consistent with previous studies (e.g., Bucciantini et al. 2011; Tanaka & Takahara 2011; Torres et al. 2014). According to Sironi & Spitkovsky (2011, 2012, 2013), these low-energy electrons are accelerated by relativistic magnetic reconnection in a PWN; its spectral slope in the range at 1.0–2.0. Therefore, the low-energy electron component possibly originates from relativistic magnetic reconnection. The high-energy electron component with spectral slope from 2.0–3.0 are consistent with that produced by the Fermi-type process (e.g., Achterberg et al. 2001); thus, high-energy electrons are from the Fermi-type process in the termination shock. The PWN magnetization is weak, which means that most PWNe are particle-dominated. This conclusion is consistent with result of Torres et al. (2014). The values of ϵ are in the range 0.1–0.45, except for HESS J1119-614, and of the break energy E_b are in the range 10^4 – 10^7 MeV.

The fluxes, $F_X \equiv F_{1-10 \text{ keV}}$ and $F_\gamma \equiv F_{1-10 \text{ TeV}}$, in X-ray (1–10 keV) and γ -ray (1–10 TeV) bands for each PWN are obtained from our calculations. Our results show that $F_X/F_\gamma > 10$ for

Table 6. Calculated luminosity at different wave bands and synchrotron cooling break energies in the sample.

PWN	L_R (1.4 GHz) erg/s	L_X (1–10 keV) erg/s	L_γ (1–10 TeV) erg/s	L_{bol} erg/s	E_s MeV	L_s erg/s
Young PWNe						
N158A	$3.32^{+1.13}_{-1.40} \times 10^{32}$	$1.10^{+0.27}_{-0.27} \times 10^{37}$	$9.28^{+2.75}_{-4.64} \times 10^{34}$	$3.94^{+0.05}_{-0.98} \times 10^{37}$	$5.68^{+5.98}_{-3.25} \times 10^{-5}$	$2.35^{+0.33}_{-0.95} \times 10^{36}$
G21.5-0.9	$2.38^{+0.53}_{-0.44} \times 10^{32}$	$3.65^{+0.30}_{-0.31} \times 10^{35}$	$2.81^{+0.50}_{-0.39} \times 10^{33}$	$3.67^{+0.35}_{-0.36} \times 10^{36}$	$6.24^{+2.02}_{-1.35} \times 10^{-6}$	$2.79^{+0.27}_{-0.29} \times 10^{35}$
Crab nebula	$6.90^{+0.20}_{-0.20} \times 10^{33}$	$1.81^{+0.06}_{-0.06} \times 10^{37}$	$3.94^{+0.13}_{-0.13} \times 10^{34}$	$1.34^{+0.03}_{-0.03} \times 10^{38}$	$2.18^{+0.06}_{-0.05} \times 10^{-6}$	$9.18^{+0.02}_{-0.09} \times 10^{36}$
Kes 75	$2.06^{+1.42}_{-0.96} \times 10^{31}$	$1.73^{+0.37}_{-0.67} \times 10^{35}$	$1.37^{+0.55}_{-0.51} \times 10^{34}$	$5.76^{+1.97}_{-2.38} \times 10^{35}$	$1.06^{+0.94}_{-0.50} \times 10^{-3}$	$5.72^{+1.02}_{-1.75} \times 10^{34}$
G310.6-1.6	$1.45^{+0.15}_{-0.16} \times 10^{31}$	$1.93^{+0.80}_{-0.60} \times 10^{35}$	$1.89^{+0.40}_{-0.34} \times 10^{34}$	$1.12^{+0.29}_{-0.24} \times 10^{36}$	$5.92^{+1.42}_{-1.06} \times 10^{-4}$	$9.06^{+3.19}_{-2.59} \times 10^{34}$
3C 58	$2.18^{+0.31}_{-0.28} \times 10^{32}$	$5.16^{+1.03}_{-1.55} \times 10^{33}$	$2.58^{+0.56}_{-0.48} \times 10^{32}$	$1.85^{+0.28}_{-0.28} \times 10^{35}$	$3.84^{+1.03}_{-0.72} \times 10^{-5}$	$6.68^{+1.56}_{-1.41} \times 10^{33}$
Evolved PWNe						
HESS J1813-178	$3.43^{+0.72}_{-0.61} \times 10^{31}$	$1.55^{+0.31}_{-0.30} \times 10^{35}$	$2.78^{+0.40}_{-0.34} \times 10^{34}$	$1.06^{+0.19}_{-0.18} \times 10^{36}$	$9.10^{+3.05}_{-2.06} \times 10^{-4}$	$6.28^{+1.34}_{-1.24} \times 10^{34}$
G54.1+0.3	$3.39^{+1.01}_{-0.84} \times 10^{31}$	$4.91^{+1.91}_{-2.01} \times 10^{34}$	$1.79^{+0.42}_{-0.32} \times 10^{34}$	$4.22^{+1.12}_{-1.02} \times 10^{35}$	$5.26^{+2.82}_{-1.59} \times 10^{-4}$	$2.30^{+0.77}_{-0.76} \times 10^{34}$
G292.0+1.8	$3.92^{+0.60}_{-0.70} \times 10^{32}$	$5.54^{+9.73}_{-3.64} \times 10^{34}$	$4.02^{+4.86}_{-2.25} \times 10^{33}$	$5.82^{+4.98}_{-2.45} \times 10^{35}$	$3.01^{+1.03}_{-0.74} \times 10^{-5}$	$3.54^{+3.26}_{-1.90} \times 10^{34}$
G0.9+0.1	$5.60^{+1.36}_{-1.13} \times 10^{32}$	$2.11^{+0.39}_{-0.37} \times 10^{35}$	$5.47^{+1.20}_{-0.94} \times 10^{34}$	$1.85^{+0.32}_{-0.29} \times 10^{36}$	$4.03^{+1.41}_{-0.93} \times 10^{-5}$	$9.31^{+1.67}_{-1.55} \times 10^{34}$
MSH 15-52	$6.88^{+3.01}_{-2.10} \times 10^{31}$	$4.89^{+0.57}_{-0.57} \times 10^{35}$	$4.88^{+0.72}_{-0.66} \times 10^{34}$	$3.14^{+0.49}_{-0.54} \times 10^{36}$	$2.89^{+0.18}_{-0.16} \times 10^{-5}$	$1.69^{+0.19}_{-0.18} \times 10^{35}$
N157B	$1.33^{+0.11}_{-0.10} \times 10^{34}$	$4.31^{+0.31}_{-0.30} \times 10^{36}$	$8.64^{+0.70}_{-0.65} \times 10^{35}$	$5.52^{+0.32}_{-0.32} \times 10^{37}$	$7.72^{+0.92}_{-0.77} \times 10^{-6}$	$3.19^{+0.22}_{-0.22} \times 10^{36}$
Mature/old PWNe						
HESS J1356-645	$5.67^{+1.14}_{-1.15} \times 10^{30}$	$7.18^{+2.23}_{-3.37} \times 10^{33}$	$8.16^{+1.69}_{-1.33} \times 10^{33}$	$8.80^{+1.36}_{-1.54} \times 10^{34}$	$8.96^{+5.04}_{-2.43} \times 10^{-4}$	$3.10^{+0.87}_{-1.21} \times 10^{33}$
CTA 1	$2.22^{+2.94}_{-0.17} \times 10^{29}$	$4.97^{+8.26}_{-2.55} \times 10^{33}$	$1.31^{+1.57}_{-0.21} \times 10^{33}$	$1.85^{+2.98}_{-0.64} \times 10^{34}$	$2.97^{+0.27}_{-0.27} \times 10^{-4}$	$1.42^{+1.81}_{-0.33} \times 10^{33}$
HESS J1418-609	$9.85^{+14.11}_{-5.73} \times 10^{30}$	$4.49^{+4.05}_{-4.32} \times 10^{34}$	$3.12^{+2.16}_{-1.37} \times 10^{34}$	$3.33^{+2.45}_{-1.92} \times 10^{35}$	$5.61^{+3.33}_{-3.86} \times 10^{-4}$	$1.69^{+1.37}_{-1.44} \times 10^{34}$
HESS J1420-607	$2.35^{+1.56}_{-0.93} \times 10^{31}$	$4.15^{+3.16}_{-2.25} \times 10^{34}$	$4.84^{+2.18}_{-1.45} \times 10^{34}$	$4.68^{+2.19}_{-1.53} \times 10^{35}$	$8.08^{+3.32}_{-2.06} \times 10^{-4}$	$1.79^{+1.27}_{-0.87} \times 10^{34}$
HESS J1119-614	$9.76^{+0.82}_{-0.84} \times 10^{31}$	$1.30^{+0.71}_{-0.36} \times 10^{32}$	$1.79^{+0.67}_{-0.49} \times 10^{34}$	$3.72^{+0.36}_{-0.29} \times 10^{35}$	$1.55^{+0.29}_{-0.22} \times 10^{-2}$	$3.38^{+3.61}_{-3.26} \times 10^{28}$
HESS J1303-631	$1.00^{+0.08}_{-0.08} \times 10^{30}$	$1.92^{+0.01}_{-0.02} \times 10^{33}$	$1.26^{+0.05}_{-0.06} \times 10^{35}$	$4.26^{+0.22}_{-0.25} \times 10^{35}$	4.17×10^{-3}	$5.09^{+0.05}_{-0.05} \times 10^{32}$

young PWNe, $1 < F_X/F_\gamma \leq 10$ for evolved PWNe except for G292.0+1.8 ($F_X/F_\gamma \sim 20.6$), and $F_X/F_\gamma \leq 1$ for mature/old PWNe except for CTA 1 ($F_X/F_\gamma \sim 5$). The results indicate that F_X/F_γ decreases with increasing PWN age.

4. Correlation analysis of the PWNe

In this section, the correlations of various physical quantities in our model are studied. To this end, the best linear fit $y = p_1 x + p_0$ to the data with the minimum χ^2 technique is used. For each fit, the values of p_1 and p_0 are given, and the Pearson's correlation coefficient r and the probability of the null hypothesis P_{null} are calculated. Here, it is assumed that the correlation between two physical quantities is noneffective if $P_{null} \geq 0.05$. In correlation analysis, the sample of 14 PWNe is used in which α_2 , η_B , and ϵ are obtained through the LM method. All results have been recalculated and various luminosities with uncertainties are listed in Table 6. The uncertainties of the various physical quantities are included. We note that although four PWNe (N 158A, G310.6-1.6, G292.0+1.8, and HESS J1303-631) are not included in the best linear fits, the values of relevant physical quantities of these four PWNe are listed in Tables 4–6 and shown in Figs. 4–10.

Here, the 2σ confidence band for each panel shown in Figs. 4–10 are from the dispersion of the fit to the points.

4.1. Correlations between pulsar parameters with injection spectrum parameters and with derived parameters

For a given central pulsar with a period P (s) and a period derivative $\dot{P}$ (s s⁻¹) in a PWN, it is commonly assumed that the magnetic field around it can be approximated as a dipole, so the surface magnetic field B_s is given by

$$B_s = \left(\frac{3c^3 I P \dot{P}}{2\pi^2 R^6} \right)^{1/2} \approx 6.4 \times 10^{19} (P \dot{P})^{1/2} \text{ G}, \quad (9)$$

where c is the light speed, $I = 10^{45} \text{ g cm}^2$ is the inertia moment, and $R = 10^6 \text{ cm}$ is the surface radius of the pulsar. At the light cylinder with a radius $R_{LC} = (cP)/(2\pi) \approx 4.77 \times 10^9 P \text{ cm}$, the magnetic field is described as

$$B_{LC} = B_s (R/R_{LC})^3 \approx 5.9 \times 10^8 P^{-2.5} \dot{P}^{1/2} \text{ G}. \quad (10)$$

Fig. 4. Correlations between $E_{\max}$ (upper panels) and B_{pwn} (bottom panels) and pulsar parameters B_{LC} , Φ , and $L(t)$. The solid lines represent the best linear fits (from top left to bottom right): $\log E_{\max} = (0.42 \pm 0.10) \log B_{\text{LC}} + (6.59 \pm 0.54)$, $\log E_{\max} = (0.88 \pm 0.19) \log \Phi - (5.31 \pm 2.97)$, $\log E_{\max} = (0.44 \pm 0.09) \log L(t) - (7.65 \pm 3.46)$, $\log B_{\text{pwn}} = (0.59 \pm 0.08) \log B_{\text{LC}} - (1.72 \pm 0.47)$, $\log B_{\text{pwn}} = (1.11 \pm 0.12) \log \Phi - (16.47 \pm 2.04)$, and $\log B_{\text{pwn}} = (0.56 \pm 0.06) \log L(t) - (19.44 \pm 2.42)$; the correlation coefficient $r = 0.71, 0.76, 0.76, 0.88, 0.91$, and 0.91 ; and the probability of the null hypothesis $P_{\text{null}} = 8.95 \times 10^{-4}, 2.26 \times 10^{-4}, 2.24 \times 10^{-4}, 1.14 \times 10^{-6}, 1.23 \times 10^{-7}$, and 1.71×10^{-7} . The dashed lines are the 2σ confidence bands for the sample.

Fig. 5. Correlations between $L(t)$ and L_{τ} , L_X , and L_{bol} . The solid lines represent the best linear fits (from left to right): $\log L_{\tau} = (1.21 \pm 0.17) \log L(t) - (13.20 \pm 6.38)$, $\log L_X = (1.54 \pm 0.12) \log L(t) - (22.50 \pm 4.68)$, and $\log L_{\text{bol}} = (1.24 \pm 0.12) \log L(t) - (10.15 \pm 4.57)$; the correlation coefficients $r = 0.88, 0.95$, and 0.93 ; and the probability of the null hypothesis $P_{\text{null}} = 1.87 \times 10^{-6}, 1.03 \times 10^{-9}$, and 1.52×10^{-8} . The dashed lines are the 2σ confidence bands for the sample.

For such a pulsar, the potential difference at the polar cap, Φ , can be expressed as

$$\Phi = \frac{2\pi^2 B_s R^3}{(cP)^2} = 4.2 \times 10^{20} (P^{-3} \dot{P})^{1/2} \text{ stat volts.} \quad (11)$$

We note that the spin-down power $L(t)$ at time t (see Eq. (3)) can be also expressed in terms of P and $\dot{P}$ as

$$L(t) = 4\pi^2 I P^{-3} \dot{P} \approx 4.0 \times 10^{46} P^{-3} \dot{P} \text{ erg s}^{-1}. \quad (12)$$

For each pulsar, the $L(t)$ and τ_c are shown in Tables 1–3, and B_s , B_{LC} , and Φ are listed in Table 5.

The correlation analysis indicate that (1) there are no correlations between E_b , η_B , α_1 , α_2 , ϵ and the pulsar parameters such as B_s , B_{BL} , Φ , $L(t)$, and τ_c ; (2) two derived parameters $E_{\max}$ and B_{pwn} have positive correlations with B_{BL} , Φ , and $L(t)$; and (3) there are no correlations between B_s and $E_{\max}$ and B_{pwn} . The correlations between $E_{\max}$ and B_{BL} , Φ , and $L(t)$ are shown in the upper panel of Fig. 4, and the correlations between B_{pwn} and B_{BL} , Φ , and $L(t)$ in the bottom panel. Since $E_{\max} \sim L(t)^{1/2}$,

Fig. 6. Correlations between $L(t)$ and L_r/L_y and L_X/L_y . The solid lines represent the best linear fits (from left to right): $\log L_r/L_y = (1.28 \pm 0.28) \log L(t) - (50.38 \pm 10.53)$ and $\log L_X/L_y = (1.41 \pm 0.22) \log L(t) - (50.21 \pm 8.33)$; the correlation coefficients $r = 0.76$, and 0.85 ; and the probability of the null hypothesis $P_{\text{null}} = 2.81 \times 10^{-4}$ and 7.81×10^{-6} . The dashed lines are the 2σ confidence bands for the sample.

Fig. 7. Correlation between L_s , E_s and $L(t)$, L_s , and E_s . The solid lines are the best linear fits, which are $\log L_s = (1.64 \pm 0.09) \log L(t) - (26.39 \pm 3.55)$, $\log E_s = -(0.78 \pm 0.11) \log L(t) + (24.46 \pm 3.99)$, and $\log L_s = -(1.15 \pm 0.11) \log E_s + (30.47 \pm 0.64)$; the correlation coefficients $r = 0.98, 0.88$, and 0.93 ; and the probability of the null hypothesis $P_{\text{null}} = 5.59 \times 10^{-12}, 1.70 \times 10^{-6}$, and 2.27×10^{-8} (from left to right). The dashed lines are the 2σ confidence bands for the sample.

$L(t) \sim (P\dot{P})^{1/2}$, $B_{\text{LC}} \sim P^{5/2}\dot{P}^{1/2}$, and $\Phi \sim (P^{-3}\dot{P})^{1/2}$, these correlations are only consistent with the logical conclusions.

4.2. Correlations between luminosities at various bands and $L(t)$

The possible correlations between the luminosities at radio (1.4 GHz), L_r ; X-ray (1–10 keV), L_X ; VHE γ -ray (1–10 TeV), L_γ ; bolometric luminosity at all wave bands, L_{bol} (which can be obtained from our model results, the corresponding values and errors are listed in Table 5); and the pulsar’s spin-down power, $L(t)$, were analyzed. The results show that there are significant positive correlations between L_r and $L(t)$, between L_X and $L(t)$, and between L_{bol} and $L(t)$ (see Fig. 5), and the best linear fits yield $L_r \sim L(t)^{1.21 \pm 0.17}$, $L_X \sim L(t)^{1.54 \pm 0.12}$, and $L_{\text{bol}} \sim L(t)^{1.24 \pm 0.12}$, respectively. The correlation between L_X and $L(t)$ is in agreement with the recent conclusions of [Mattana et al. \(2009\)](#), [Kargaltsev & Pavlov \(2010\)](#), and [Kargaltsev et al. \(2013\)](#). As

expected, there is no correlation between L_γ and $L(t)$, which is consistent with [Mattana et al. \(2009\)](#), [Kargaltsev & Pavlov \(2010\)](#), and [Kargaltsev et al. \(2013\)](#). This phenomenon may be due to the differences in the IR background and uncertain distance.

Since L_r/L_γ and L_X/L_γ are distance-independent, the correlations between L_r/L_γ and $L(t)$ and between L_X/L_γ and $L(t)$ are analyzed. Our results are shown in Fig. 6 where the best linear fits give $L_R/L_\gamma \sim L(t)^{1.28 \pm 0.28}$, and $L_X/L_\gamma \sim L(t)^{1.41 \pm 0.22}$. In fact, the correlation between L_X/L_γ and $L(t)$ is found in [Mattana et al. \(2009\)](#).

4.3. Correlations between synchrotron luminosity and $L(t)$

Because synchrotron radiation dominates over the radiation from radio to X-ray bands, it is important to study the correlations of synchrotron cooling break energy (E_s) and synchrotron radiation luminosity (L_s) with $L(t)$. Following [Tanaka & Takahara \(2011\)](#),

Fig. 8. Correlations between $L(t)$ and S_r , S_X , and S_γ . The solid lines represent the best linear fits (from left to right): $\log S_r = (1.78 \pm 0.20) \log L(t) - (36.89 \pm 7.60)$, $\log S_X = (1.89 \pm 0.32) \log L(t) - (37.79 \pm 12.42)$, and $\log S_\gamma = (0.71 \pm 0.17) \log L(t) + (5.89 \pm 6.59)$; the correlation coefficients $r = 0.91, 0.82$, and 0.71 ; and the probability of the null hypothesis $P_{\text{null}} = 1.24 \times 10^{-7}, 2.57 \times 10^{-5}$, and 9.05×10^{-4} , respectively. The dashed lines are the 2σ confidence bands for the sample.

Fig. 9. Correlations between R_{pwn} and L_X , L_X/L_γ , and L_s . The solid lines are the best linear fits, which are $\log L_X = -(1.36 \pm 0.68) \log R_{\text{pwn}} + (37.03 \pm 0.38)$, $\log L_X/L_\gamma = -(2.54 \pm 0.52) \log R_{\text{pwn}} + (2.69 \pm 0.31)$, and $\log L_s = -(1.90 \pm 0.75) \log R_{\text{pwn}} + (37.48 \pm 0.24)$; the correlation coefficients $r = 0.45, 0.77$, and 0.54 ; and the probability of the null hypothesis $P_{\text{null}} = 6.40 \times 10^{-2}, 1.73 \times 10^{-4}$, and 2.18×10^{-2} (from left to right). The dashed lines are the 2σ confidence bands for the sample.

the inverse Compton cooling is assumed to be ineffective in most of the evolutionary phases, and the synchrotron cooling break frequency ν_s is given by

$$\nu_s = 7.32 \times 10^{20} \left(\frac{B_{\text{PWN}}}{10^{-6} \text{ G}} \right)^{-3} \left(\frac{T_{\text{age}}}{10^3 \text{ yr}} \right)^{-2} \text{ Hz.} \quad (13)$$

Using Eq. (13) and based on our model calculation, $E_s = h\nu_s$ and L_s are calculated for each PWN in our sample. The results show that there is a strong positive correlation between L_s and $L(t)$, the best linear fit yields $L_s \sim L(t)^{1.64 \pm 0.09}$, and that there are strong negative correlation between E_s and $L(t)$, where $E_s \sim L(t)^{-0.78 \pm 0.11}$, and E_s and L_s , where $L_s \sim E_s^{-1.15 \pm 0.11}$. The best linear fit results are shown in Fig. 7.

4.4. Correlations between surface brightness S and $L(t)$

The surface brightness is defined as (e.g., Abdalla et al. 2017)

$$S = \frac{L}{4\pi R_{\text{own}}^2} \approx \frac{F}{\sigma^2}, \quad (14)$$

where L is the luminosity at a given band, R_{PWN} is the radius of the PWN, F is the energy flux at a given band, and σ is its angular extent as seen from the Earth. Using our model and Eq. (14), the surface brightness, S_{R} , S_{X} , and S_{γ} , at radio (1.4 GHz), X-ray (1–10 keV), and γ -ray (1–10 TeV) bands are calculated. The results are shown in Fig. 8 and show that there are correlations between $L(t)$ and S_{R} , S_{X} , and S_{γ} . The best linear fits give $S_{\text{R}} \sim L(t)^{1.78 \pm 0.20}$, $S_{\text{X}} \sim L(t)^{1.89 \pm 0.32}$, and $S_{\gamma} \sim L(t)^{0.71 \pm 0.17}$. We note that $S_{\gamma} \sim L(t)^{0.81}$ has been given in Abdalla et al. (2017).

4.5. Correlations between luminosity and R_{pwn} and T_{age}

The correlations between the luminosities at various bands and R_{pwn} are analyzed here. In our model, the PWN radius for each PWN can be calculated (see Tables 1–3). The results show that there are no correlations between R_{pwn} and L_r , L_γ , and L_{bol} , but there are correlations between R_{pwn} and L_X , L_s , and L_X/L_γ . The best linear fits give that $L_X \sim R_{\text{pwn}}^{-1.36 \pm 0.68}$, $L_s \sim R_{\text{pwn}}^{-1.90 \pm 0.75}$, and $L_X/L_\gamma \sim R_{\text{pwn}}^{-2.54 \pm 0.52}$, respectively. We note that the correlation

Fig. 10. Correlations between T_{age} and L_X , L_X/L_γ , and L_s . The solid lines are the best linear fits, which are $\log L_X = -(2.44 \pm 0.59) \log T_{\text{age}} + (44.23 \pm 1.89)$, $\log L_X/L_\gamma = -(3.37 \pm 0.29) \log T_{\text{age}} + (12.47 \pm 0.94)$, and $\log L_s = -(2.83 \pm 0.53) \log T_{\text{age}} + (45.37 \pm 1.59)$; the correlation coefficients $r = 0.72, 0.95$, and 0.80 ; and the probability of the null hypothesis $P_{\text{null}} = 7.81 \times 10^{-4}, 3.15 \times 10^{-9}$, and 7.22×10^{-5} . The dashed lines are the 2σ confidence bands for the sample.

between L_X and R_{pwn} is marginal here. The results are shown in Fig. 9.

The same procedure is performed for the correlations between T_{age} and the luminosities at various bands, and almost the same correlations are found. We found that there are correlations between T_{age} and L_X , L_s , and L_X/L_γ . These correlations mean that $L_X \sim T_{\text{age}}^{-2.44 \pm 0.59}$, $L_s \sim T_{\text{age}}^{-2.83 \pm 0.53}$, and $L_X/L_\gamma \sim T_{\text{age}}^{-3.37 \pm 0.29}$. The best linear fit results are shown in Fig. 10.

5. Discussions and conclusions

In this paper, the leptonic model with a broken power-law injection for the electrons/positrons is applied to the sample of 18 PWNe. The sample is divided into three groups: young, evolved, and mature/old PWNe. Observed SEDs of all 18 PWNe can be reproduced well in this model (see Figs. 1–3). The model parameters obtained in our calculations are listed in the Tables 1–3, and the relevant discussion are described in Sect. 3.

Using a time-dependent modeling of PWNe given by Martin et al. (2012), Torres et al. (2014) modeled the SEDs of nine PWNe and studied their statistical properties. These nine PWNe are included in our sample. As mentioned in Sect. 1, the model given in Torres et al. (2014) does not include the dynamical evolution of a PWN, but our model does. Therefore, some model parameters obtained in these two models are different. Meanwhile, our sample is larger than some previous works. So it is worth comparing our correlation results with the results of Torres et al. (2014) and some previous works.

For the relation between the derived parameters and pulsar's parameter, our results indicate that the maximum electron energy has positive correlations with the magnetic field at the light cylinder, the potential difference at the polar cap, and the pulsar's spin-down power and that the magnetic field in the PWN is positively correlated with the magnetic field at the light cylinder, the potential difference at the polar cap, and the pulsar's spin-down power (see Fig. 4). Other parameters that describe pulsar properties and electron injection have no correlations. These results are consistent with those of Torres et al. (2014).

The results presented in this paper indicate that the spin-down power $L(t)$ is correlated with L_r , L_X , and L_{bol} (see Fig. 5). The results of $L(t)$ versus L_r and $L(t)$ versus L_X are consistent with those in Torres et al. (2014). Meanwhile, our results show that the $L(t)$ correlates with the ratio L_r/L_γ and the ratio L_X/L_γ

(see Fig. 6). These correlations are consistent with those given in previous papers (e.g., Mattana et al. 2009; Kargaltsev & Pavlov 2010; Torres et al. 2014).

In this paper, the correlations between $L(t)$ and surface brightness at different bands are presented here (see Fig. 8). Moreover, the correlations between $R_{\text{pwn}}(T_{\text{age}})$ and L_X , L_X/L_γ , and L_s are analyzed (see Figs. 9 and 10). For example, the results show that $L_X/L_\gamma \sim R_{\text{pwn}}^{-2.54}$ and $L_X/L_\gamma \sim T_{\text{age}}^{-3.37}$, which means that old PWNe have smaller values of L_X/L_γ than do young PWNe. In fact, our results show that the $L_X/L_\gamma \geq 10$ for young PWNe, $1 < L_X/L_\gamma \lesssim 10$ for evolved PWNe, and $L_X/L_\gamma \leq 1$ for mature/old PWNe. This result may provide a new tool for classifying the evolution state of PWNe. It should be pointed out that the results presented in this paper require confirmation from further observations.

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Appendix A: Calculations of $R_{\text{pwn}}(t)$ and $B_{\text{pwn}}(t)$

In this appendix, the calculations of R_{pwn} and B_{pwn} are briefly described in the frame of the model for dynamical and radiative evolution of a PWN inside an SNR presented in [Gelfand et al. \(2009\)](#).

A.1. Calculation of $R_{\text{pwn}}(t)$

To compute the radius of the PWN during the evolution process of the PWNe, the following initial conditions are required ([Gelfand et al. 2009](#)). The first is the equation of motion of the PWN for the pressure of the SNR ejecta $P_{\text{snr}}(R_{\text{pwn}}) \equiv 0$,

$$M_{\text{sw,pwn}} \frac{dv_{\text{pwn}}}{dt} = 4\pi R_{\text{pwn}}^2 [P_{\text{pwn}} - \rho_{\text{ej}}(R_{\text{pwn}}) \times (v_{\text{pwn}} - v_{\text{ej}}(R_{\text{pwn}}))^2], \quad (\text{A.1})$$

where $v_{\text{pwn}} = dR_{\text{pwn}}/dt$, $M_{\text{sw,pwn}}$, P_{pwn} , and R_{pwn} are the velocity, mass, pressure, and radius of the PWN shell; ρ_{ej} and v_{ej} are the density and velocity of SNR ejecta and

$$\frac{dE_{\text{pwn}}}{dt} = \dot{E} - P_{\text{pwn}} 4\pi R_{\text{pwn}}^2 v_{\text{pwn}}. \quad (\text{A.2})$$

In the above treatment, the adiabatic losses dominate and the PWN is expanding with a constant velocity.

The second is expression of R_{pwn} is

$$R_{\text{pwn}}(t) = 1.44 \left(\frac{E_{\text{sn}}^3 \dot{E}_0^2}{M_{\text{ej}}^5} \right)^{1/10} t^{6/5}, \quad (\text{A.3})$$

where E_{sn} is the explosion energy of the supernova, $\dot{E}_0 \approx \dot{E}$, and M_{ej} is the mass of the SNR ejecta.

The PWN expands outwards due to the energy provided by the central pulsar and sweeps out the material of the SNR ejecta, so a thin shell is formed around the PWN. Because there is a pressure P_{pwn} inside the PWN and a pressure $P_{\text{snr}}(R_{\text{pwn}})$ outside the PWN, the net force suffered by the PWN is $F_{\Delta P} = 4\pi R_{\text{pwn}}^2 [P_{\text{pwn}} - P_{\text{snr}}(R_{\text{pwn}})]$, and the change rate of the momentum is

$$\frac{d}{dt}(M_{\text{sw,pwn}} v_{\text{pwn}}) = F_{\Delta P} = 4\pi R_{\text{pwn}}^2 [P_{\text{pwn}} - P_{\text{snr}}(R_{\text{pwn}})]. \quad (\text{A.4})$$

In Eq. (A.4), the pressure of the PWN P_{pwn} is calculated as follows. Assuming the PWN is mainly composed of a relativistic nonthermal lepton (electron/positron) plasma and magnetic field $E_{\text{pwn}} = E_{\text{pwn,e}} + E_{\text{pwn,B}}$ (the spin-down power is described as $L(t) = \dot{E} = \eta_B \dot{E} + \eta_e \dot{E}$), where $E_{\text{pwn,e}}$ is the total energy contained in the leptons and is given by $E_{\text{pwn,e}} = \int_0^{E_{\text{max}}} N(E, t) E dE$, and $E_{\text{pwn,B}}$ is the energy stored in the magnetic field of the PWN and reads $E_{\text{pwn,B}} = (B_{\text{pwn}}^2/8\pi) V_{\text{pwn}}$. The corresponding pressures contributed by the leptons and stored in the magnetic field of the PWN are $P_{\text{pwn,e}} = [(\gamma_{\text{pwn}} - 1) E_{\text{pwn,e}}]/V_{\text{pwn}} = E_{\text{pwn,e}}/(4\pi R_{\text{pwn}}^3)$ and $P_{\text{pwn,B}} = B_{\text{pwn}}^2/8\pi$, where $\gamma_{\text{pwn}} = 4/3$ and $V_{\text{pwn}} = (4/3) R_{\text{pwn}}^3$ is the volume of the PWN. Therefore, $P_{\text{pwn}} = P_{\text{pwn,e}} + P_{\text{pwn,B}}$. $P_{\text{snr}}(R_{\text{pwn}})$ is the pressure of the shocked material at $r = R_{\text{pwn}}$ and its calculation (see Appendix A in [Gelfand et al. 2009](#)).

If the density, velocity, pressure of the ejecta, and the inner pressure of the PWN at some time t are given, then the radius and velocity of the PWN shell are calculated using above expressions. The calculation process here is the same as that in [Gelfand et al. \(2009\)](#), see their Sect. 2.2, in details).

A.2. Calculation of $B_{\text{pwn}}(t)$

From the relation of B_{pwn} and $L(t)$ is given by

$$\frac{dE_{\text{pwn,B}}(t)}{dt} = \eta_B L(t) - E_{\text{pwn,B}}(t) \frac{1}{R_{\text{pwn}}(t)} \frac{dR_{\text{pwn}}(t)}{dt},$$

the following equation can be derived:

$$\frac{d[E_{\text{pwn,B}}(t) R_{\text{pwn}}(t)]}{dt} = \eta_B R_{\text{pwn}}(t) L(t). \quad (\text{A.5})$$

Integrating Eq. (A.5) from $t = 0$ to t and noting that $E_{\text{pwn,B}}(t) = 0$ at $t = 0$, the magnetic field $B_{\text{pwn}}(t)$ is given by

$$B_{\text{pwn}}(t) = \left(\frac{1}{R_{\text{pwn}}(t)} \right)^2 \sqrt{6\eta_B \int_0^t L(t') R_{\text{pwn}}(t') dt'}. \quad (\text{A.6})$$

Appendix B: Basic properties of PWNe in the sample

B.1. Group 1: young PWNe

N 158A. It is also known as G279.7-31.5, and is powered by an energetic pulsar PSR B0540-69 with a rotation period of 50.5 ms, a period derivative of $4.79 \times 10^{-13} \text{ s s}^{-1}$ ([Seward et al. 1984](#); [Livingstone et al. 2005b](#); [Ferdman et al. 2015](#)), and a braking index n for the PSR B0540-69 is 2.08 ([Kaaret et al. 2001](#)), very recent measurement shows $n = 2.129 \pm 0.012$ ([Ferdman et al. 2015](#)), which is used here. Therefore, the pulsar has a characteristic age of ~ 1672 yr, and a spin-down luminosity $L(t) = 1.47 \times 10^{38} \text{ erg s}^{-1}$. The system lies at a distance of 49 kpc ([Seward et al. 1984](#); [Taylor & Cordes 1993](#)). Meanwhile, according to [Manchester et al. \(1993\)](#), the physical size of the N 158A is 0.7 pc. Observationally, N 158A has been detected in radio ([Manchester et al. 1993](#)), IR and optical ([Mignani et al. 2012](#)), and X-ray bands ([Kaaret et al. 2001](#); [Campana et al. 2008](#)), but it was not detected in GeV and TeV γ -ray bands. The age of the system is ~ 760 yr by measurements of the expansion velocity of the SNR shell in the optical spectral band ([Reynolds 1985](#); [Kirshner et al. 1989](#); [Chevalier 2005](#)), and the mass of 20–25 $M_{\odot}$ for the progenitor star is inferred by [Williams et al. \(2008\)](#). The parameters used in our calculation are $E_{\text{sn}} = 2.0 \times 10^{51} \text{ erg}$, $M_{\text{ej}} = 23.0 M_{\odot}$, and $n_{\text{H}} = 0.01 \text{ cm}^{-3}$. In [Martin et al. \(2014\)](#), $U_{\text{IR}} = 5.0 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 0.2 \text{ eV cm}^{-3}$ were used to compute the energy density needed for the PWN to be detected by CTA. Here, $U_{\text{IR}} = 1.0 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 1.0 \text{ eV cm}^{-3}$ are used in our calculation.

PWN G21.5-0.9 or HESS J1833-105. It is powered by an energetic PSR J1833-1034. The pulsar has a 61.8 ms rotation period, a period derivative of $2.02 \times 10^{-13} \text{ s s}^{-1}$ ([Gupta et al. 2005](#); [Camilo et al. 2006](#)), and $n = 3.0$ is assumed. Thus, the pulsar has a characteristic age $\tau_c = 4850$ yr and spin-down power $L(t) = 3.4 \times 10^{37} \text{ erg s}^{-1}$. According to [Camilo et al. \(2006\)](#), the distance of the system is 4.7 ± 0.4 kpc; the same value was obtained by [Tian & Leahy \(2008\)](#). Here the distance of 4.1 kpc ([Abdalla et al. 2017](#)) is used. Observationally, PWN G21.5-0.9 is detected in radio ([Salter et al. 1989](#); [Morsi & Reich 1987](#); [Wilson & Weiler 1976](#); [Becker & Kundu 1976](#)), IR ([Gallant & Tuffs 1998, 1999](#)), X-ray ([De Rosa et al. 2009](#); [Tsujimoto et al. 2011](#); [Nynka et al. 2014](#)), and TeV γ -ray bands by HESS ([Djannati-Atai et al. 2008](#)). Although [Wang et al. \(1986\)](#) suggested that the age of PWN G21.5-0.9 is about 2000 yr because it might be the historical supernova of

48 BC, [Bietenholz & Bartel \(2008\)](#) estimated $T_{\text{age}} \sim 900$ yr through the observation of the expansion rate of the PWN. Following [Bietenholz & Bartel \(2008\)](#) or [Tanaka & Takahara \(2011\)](#), $T_{\text{age}} = 900$ yr is assumed here. The parameters used in our calculation are $n_{\text{H}} = 0.1 \text{ cm}^{-3}$, which is constrained to be between 0.1 and 0.4 cm^{-3} by [Matheson & Safi-Harb \(2005\)](#), and $M_{\text{ej}} = 8.0 M_{\odot}$. The values $U_{\text{IR}} = 1.4 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 5.0 \text{ eV cm}^{-3}$ given in the GALPROP code of [Porter et al. \(2006\)](#) are used.

Crab nebula. It is a famous PWN that has been widely studied. The details of the observed and derived properties for the Crab nebula can be found in many papers (e.g., [Torres et al. 2014](#); [Zhu et al. 2015](#)).

Kes 75. It is also known as HESS J1846-029 or G29.7-0.3, which is a typical composite supernova remnant. The central pulsar is PSR 1846-0258 ([Gotthelf et al. 2000](#)), and it has a rotational period $P = 324$ ms, a period derivative $\dot{P} = 7.08 \times 10^{-12} \text{ s s}^{-1}$, and a braking index $n = 2.65 \pm 0.01$ ([Livingstone et al. 2006](#)). Very recently, the braking index $n = 2.19 \pm 0.03$ was measured by [Archibald et al. \(2015\)](#); this value is applied in our paper. Thus, the pulsar has a characteristic age $\tau_c = 730$ yr and its spin-down power $L(t) = 8.21 \times 10^{36} \text{ erg s}^{-1}$. [Leahy & Tian \(2008\)](#) estimated a distance for the system between 5.1 to 7.5 kpc; the distance of 5.8 kpc ([Abdalla et al. 2017](#)) is adopted in our model. Observationally, Kes 75 has been detected in radio band ([Salter et al. 1989](#); [Bock & Gaensler 2005](#)), X-ray band ([Helfand et al. 2003](#)), and TeV γ -ray band by HESS ([Djannati-Atai et al. 2008](#)). The ambient medium density of 1 cm^{-3} is estimated by [Safi-Harb & Kumar \(2012\)](#). Recently, the ambient medium density is constrained to be between 0.005 and 0.1 cm^{-3} by [Gelfand et al. \(2014\)](#); here, 0.1 cm^{-3} is used. The PSR 1846-0258 is very young; the actual age was constrained to be between 980 and 1770 yr by [Mereghetti et al. \(2002\)](#). Here, the age of 1000 yr and $M_{\text{ej}} = 10.0 M_{\odot}$ are used. The values $U_{\text{IR}} = 1.2 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 2.0 \text{ eV cm}^{-3}$ are taken from [Tanaka & Takahara \(2011\)](#).

G310.6-1.6. It is powered by an energetic pulsar PSR J1400-6325, which has a rotation period of 31.18 ms, a period derivative $3.89 \times 10^{-14} \text{ s s}^{-1}$ ([Renaud et al. 2010](#)), and $n = 3.0$ is assumed. Thus, these parameters derive the characteristic age $\tau_c = 12700$ yr and spin-down power $L(t) = 5.1 \times 10^{37} \text{ erg s}^{-1}$. The system lies at a distance of 7 kpc ([Renaud et al. 2010](#); [Tanaka & Takahara 2013](#)). Observationally, G310.6-1.6 has been measured in radio band ([Murphy et al. 2007](#); [Griffith & Wright 1993](#); [Condon et al. 1993](#); [Duncan et al. 1995](#)), X-ray band ([Renaud et al. 2010](#)), and TeV γ -ray band by HESS (e.g., [Khélifi et al. 2008](#)). The parameters used here are $M_{\text{ej}} = 13.0 M_{\odot}$, and $n_{\text{H}} = 0.01 \text{ cm}^{-3}$. The values $U_{\text{IR}} = 0.3 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 0.3 \text{ eV cm}^{-3}$ are taken from [Tanaka & Takahara \(2013\)](#).

3C 58. It is also known as SNR G130.7+3.1 or SN 1181, which is a composite SNR ([Weiler & Panagia 1978](#)). Its central pulsar is PSR J0205+6449, which is detected in radio, X-ray, and γ -ray bands. The pulsar has a rotation period 65.7 ms, a period derivative of $1.93 \times 10^{-13} \text{ s s}^{-1}$ ([Murray et al. 2002](#); [Camilo et al. 2002b](#); [Livingstone et al. 2009](#)), and $n = 3.0$ is assumed. These parameters derive a characteristic age of ~ 5397 yr and a spin-down luminosity $L(t) = 2.68 \times 10^{37} \text{ erg s}^{-1}$. According to [Roberts et al. \(1993\)](#), 3C 58 is located at a distance of 3.2 kpc ([Roberts et al. 1993](#)) or 2 kpc ([Kothes et al. 2010](#); [Kothes 2013](#)). Here, the distance of 2 kpc is adopted. Observationally, 3C 58 has been detected in radio ([Green 1986](#); [Morsi & Reich 1987](#); [Salter et al. 1989](#)), IR ([Green 1994](#); [Slane et al. 2008](#)),

X-ray ([Torii et al. 2000](#)), and GeV by *Fermi*-LAT ([Abdo et al. 2013](#); [Ackermann et al. 2013](#)) and TeV bands by MAGIC ([Aleksić et al. 2014](#)). [Chevalier \(2005\)](#) estimated that the age of the system is $\sim 2400 \pm 500$ yr by the PWN evolution and energetics, so 2400 yr is used in our paper. The parameters used in our calculation are $M_{\text{ej}} = 8.0 M_{\odot}$, and $n_{\text{H}} = 0.01 \text{ cm}^{-3}$. The values $U_{\text{IR}} = 0.3 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 0.3 \text{ eV cm}^{-3}$ are taken from [Tanaka & Takahara \(2013\)](#).

B.2. Group 2: evolved PWNs

HESS J1813-178 or G12.8-0.0. It is powered by an energetic pulsar PSR J1813-1749, which is the third most energetic pulsar in the Galaxy ([Halpern et al. 2012](#)). The pulsar has a rotation period of 44.7 ms ([Gotthelf & Halpern 2009](#)), a period derivative of $1.265 \times 10^{-13} \text{ s s}^{-1}$ ([Halpern et al. 2012](#)), and $n = 3.0$ is assumed. These parameters lead to a characteristic age of ~ 5600 yr and a spin-down luminosity $L(t) = 5.6 \times 10^{37} \text{ erg s}^{-1}$. According to [Brogan et al. \(2005\)](#) and [Halpern et al. \(2012\)](#), the distance of the PWN is 4.7 kpc, and the corresponding radius of 4.0 ± 0.3 pc ([Funk et al. 2007](#); [Abdalla et al. 2017](#)). Observationally, [Brogan et al. \(2005\)](#) presented the discovery of nonthermal radio emission of the radio shell (SNR G12.8-0.0) that is spatially coincident with HESS J1813-178, so we assume that the radio emission comes from HESS J1813-178, which was observed in X-ray band ([Funk et al. 2007](#); [Ubertini et al. 2005](#)), GeV band by *Fermi*-LAT ([Acero et al. 2013](#)), and TeV γ -ray band by HESS ([Aharonian et al. 2006b](#)). In our model, we assume an age of 2500 yr. The parameters used in our calculation are the density of ambient medium of 1.0 cm^{-3} ([Brogan et al. 2005](#)), and $M_{\text{ej}} = 8.0 M_{\odot}$. For the soft seed photons, $U_{\text{IR}} = 1.0 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 1.5 \text{ eV cm}^{-3}$ are used in [Fang & Zhang \(2010b\)](#). Here, $U_{\text{IR}} = 0.1 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 0.5 \text{ eV cm}^{-3}$ are used to reproduce observed γ -ray SED.

G54.1+0.3. It is also known as VER J1930+188, which was first discovered by [Reich et al. \(1985\)](#). The PWN is powered by an energetic pulsar PSR J1930+1852, which was observed in radio ([Camilo et al. 2002a](#)) and X-ray bands ([Lu et al. 2007](#)). The pulsar has a period of 136 ms, a period time derivative $7.51 \times 10^{-13} \text{ s s}^{-1}$ ([Camilo et al. 2002a](#)), and $n = 3.0$ is assumed. These parameters lead to a characteristic age of ~ 2872 yr, and the spin-down power $L(t) = 1.16 \times 10^{37} \text{ erg s}^{-1}$. PWN G54.1+0.3 has a distance of $6.2^{+1.0}_{-0.6}$ kpc ([Leahy et al. 2008](#)). Here, the distance of 7.0 kpc ([Abdalla et al. 2017](#)) is used. Observationally, the PWN has been observed in radio band ([Green 1985](#); [Velusamy et al. 1986](#); [Velusamy & Becker 1988](#); [Lang et al. 2010](#)), X-ray band ([Lu et al. 2001, 2002](#); [Bocchino et al. 2010](#)), and TeV band by HESS ([Acciari et al. 2010](#)). The age of G54.1+0.3 is estimated to be between 1500 and 6000 yr ([Camilo et al. 2002a](#)). Recently, the age is constrained to be between 2100 and 3600 yr ([Chevalier 2005](#); [Bocchino et al. 2010](#); [Gelfand et al. 2015](#)). Here $T_{\text{age}} = 2600$ yr is used. The ejected mass $M_{\text{ej}} = 20 M_{\odot}$, which is constrained between 16 and $27 M_{\odot}$ ([Temim et al. 2010, 2017](#)) and $n_{\text{H}} = 1.0 \text{ cm}^{-3}$ are used here. The values $U_{\text{IR}} = 0.8 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 1.1 \text{ eV cm}^{-3}$ are taken from the GALPROP code of [Porter et al. \(2006\)](#).

G292.0+1.8. It is also known as MSH 11-54 or HI 122-59. The system is powered by an energetic pulsar PSR J1124-5916 with a rotation period of 135.31 ms and a period derivative of $7.47 \times 10^{-13} \text{ s s}^{-1}$ ([Camilo et al. 2002a](#)); $n = 3.0$ is assumed. Thus, the pulsar has a characteristic age of ~ 2872 yr and a spin-down luminosity $L(t) = 1.19 \times 10^{37} \text{ erg s}^{-1}$. The system lies at a distance of 6.0 kpc ([Winkler et al. 2009](#)). The

radius of the PWN is about 3 pc (Bhalerao et al. 2015). Observationally, G292.0+1.8 has been observed in radio band (Gaensler & Wallace 2003), X-ray band (Hughes et al. 2001), and GeV band with an upper limit (Ackermann et al. 2011). The age of the system is uncertain, and ranges from ≤ 1600 yr (Murdin & Clark 1979) to ~ 3000 yr (Winkler et al. 2009); here $T_{\text{age}} = 2700$ yr is used, which is consistent with Tanaka & Takahara (2013). The values of $n_{\text{H}} = 0.5 \text{ cm}^{-3}$ and $M_{\text{ej}} = 18.0 M_{\odot}$ are adopted. For the soft seed photons, $U_{\text{IR}} = 0.3 \text{ eV cm}^{-3}$, $U_{\text{OPT}} = 0.3 \text{ eV}$ and $U_{\text{IR}} = 0.42 \text{ eV cm}^{-3}$, $U_{\text{OPT}} = 0.7 \text{ eV}$ are applied in Tanaka & Takahara (2013) and Martin et al. (2014), respectively. Here, $U_{\text{IR}} = 1.0 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 0.7 \text{ eV cm}^{-3}$ are used.

G0.9+0.1 or HESS J1747-281. It is powered by an energetic pulsar PSR J1747-2809 with a rotation period of 52.2 ms and a period derivative of $1.56 \times 10^{-13} \text{ s s}^{-1}$ (Camilo et al. 2009), and $n = 3.0$ is assumed. This pulsar has a characteristic age $\tau_c = 5305$ yr and the spin-down power $L(t) = 4.32 \times 10^{37} \text{ erg s}^{-1}$. The distance of the system is likely in the range from 8.0 kpc to 16 kpc, due to the uncertainty of the electron density model toward the distant inner Galactic regions (Dubner et al. 2008). Here, the distance of 13.3 kpc (Abdalla et al. 2017) is used. The PWN was detected in radio (Dubner et al. 2008), X-ray (Porquet et al. 2003), and TeV γ -ray bands by HESS (Aharonian et al. 2005b). Camilo et al. (2009) estimated the age of the system in the range between 2000 yr and 3000 yr. Here the age of the PWN is assumed to be 3000 yr; the density of the ambient medium of 0.01 cm^{-3} and $M_{\text{ej}} = 14.0 M_{\odot}$ are used in our calculation. The values $U_{\text{IR}} = 3.8 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 25.0 \text{ eV cm}^{-3}$ given in Torres et al. (2014) are used.

MSH 15-52. A typical composite supernova remnant, which is also known as HESS J1514-591 or SNR G320.4-1.2 (Caswell et al. 1981). The system was first discovered as an extended nonthermal radio source by Mills et al. (1961). The PWN is powered by an energetic pulsar PSR B1509-58 (Gaensler et al. 2008), and it has a 150 ms rotation period, a period derivative of $1.5 \times 10^{-12} \text{ s s}^{-1}$, and a braking index $n = 2.839 \pm 0.003$ (Livingstone et al. 2005a). Therefore, the pulsar has a characteristic age $\tau_c \sim 1585$ yr and spin-down power $L(t) = 1.75 \times 10^{37} \text{ erg s}^{-1}$. This system is located at a distance of 5.2 ± 1.4 kpc based on the HI absorption measurement (Gaensler et al. 1999). It is consistent with the value of 4.2 ± 0.6 kpc derived from the dispersion measure Cordes & Lazio (2002). Here, the distance of 4.4 kpc (Abdalla et al. 2017) is used. Observationally, MSH 15-52 has been detected in radio band (Gaensler et al. 1999, 2008), X-ray band (Mineo et al. 2001; Forot et al. 2006), GeV band by *Fermi*-LAT (Abdo et al. 2010), and TeV γ -ray band by HESS (Aharonian et al. 2005a). Although the spin-down age is ~ 1600 yr, the true age of the system could be much older than the spin-down age (Seward et al. 1983; Gvaramadze 2001). To reconcile the age of the system, the braking index has to be < 2.0 . Here $T_{\text{age}} = 4000$ yr and $n = 1.4$ are assumed. The density of ambient medium $n_{\text{H}} = 0.01 \text{ cm}^{-3}$ and the ejected mass $M_{\text{ej}} = 6.0 M_{\odot}$ are adopted; $U_{\text{IR}} = 1.2 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 2.2 \text{ eV cm}^{-3}$ are taken from the GALPROP code of Porter et al. (2006).

N 157B. It was the first extragalactic PWN to be detected in TeV γ -rays with HESS (Abramowski et al. 2012a, 2015). Its central pulsar is PSR J0537-6910 (Marshall et al. 1998). The pulsar has a rotation period of 16.12 ms and a period derivative of $5.13 \times 10^{-14} \text{ s s}^{-1}$ (Spyrou & Stergioulas 2002; Manchester et al. 2005b); $n = 3.0$ is assumed. Thus, the pulsar has a characteristic age of ~ 4982 yr, and a spin-down luminosity

$L(t) = 4.82 \times 10^{38} \text{ erg s}^{-1}$. The system has an estimated distance of 48 kpc (Macri et al. 2006), and the radius of the PWN is 10.6 pc (Lazendic et al. 2000). However, this radius is not very well defined, and it could include parts of the remnant. Following Abdalla et al. (2017), the distance of 53.7 kpc is used here. Observationally, this PWN has been detected in radio band (Lazendic et al. 2000), X-ray band (Chen et al. 2006), and TeV γ -ray band by HESS (Abramowski et al. 2012a, 2015). For the whole system, the Sedov age of ~ 5000 yr was estimated by Wang & Gotthelf (1998). Here, $T_{\text{age}} = 4600$ yr, which is consistent with Martin et al. (2014), and $n_{\text{H}} = 0.03 \text{ cm}^{-3}$ and $M_{\text{ej}} = 20.0 M_{\odot}$, which is consistent with Chen et al. (2006), are used. For far-IR photons, according to the OB association LH99 and the nearby star-forming region 30 Doradus, using observations from *Spitzer* (Indebetouw et al. 2009), the infrared photon fields are modeled as blackbody radiation with a temperature of 80 K and an energy density of 8.9 eV cm^{-3} for LH99, and a temperature of 88 K and an energy density of 2.7 eV cm^{-3} for 30 Doradus. These are only upper limits to the infrared fields since the distances of N157B to these objects are unknown. Here, $U_{\text{IR}} = 5.0 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 4.0 \text{ eV cm}^{-3}$ are used.

B.3. Group 3: mature/old PWNe

HESS J1356-645 or G309.9-2.51. It is powered by an energetic pulsar PSR J1357-6429 with a 166 ms rotation period, a $3.6 \times 10^{-13} \text{ s s}^{-1}$ period derivative (Camilo et al. 2004; Lorimer et al. 2006), and $n = 3.0$ is assumed. So the characteristic age of this pulsar is ~ 7310 yr and the spin-down luminosity is $L(t) = 3.1 \times 10^{36} \text{ erg s}^{-1}$. Its distance is estimated to be 2.5 kpc (Cordes & Lazio 2002; Chang et al. 2012) and its radius is 10.1 ± 0.9 pc (Abdalla et al. 2017). Observationally, the PWN has been observed at radio band (Duncan et al. 1995; Griffith & Wright 1993; Murphy et al. 2007), X-ray band (Abramowski et al. 2011), GeV γ -ray band by *Fermi*-LAT (Acero et al. 2013), and TeV γ -ray band by HESS (Abramowski et al. 2011). Because the age of the system is not clear, $T_{\text{age}} = 6500$ yr is used here, which is similar to Torres et al. (2014). The values $M_{\text{ej}} = 7.0 M_{\odot}$ and $n_{\text{H}} = 0.05 \text{ cm}^{-3}$ are assumed; $U_{\text{IR}} = 0.4 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 0.5 \text{ eV cm}^{-3}$ are the same as those in Torres et al. (2014).

CTA 1. It is also known as VER J0006+727 or G 119.5 + 10.2, which was first discovered by Harris & Roberts (1960). The PWN is powered by PSR J0007+7303 with a rotation period of 316.86 ms, a period derivative of $3.614 \times 10^{-13} \text{ s s}^{-1}$ (Abdo et al. 2008), and $n = 3.0$ is assumed. Thus, the pulsar has a characteristic age of ~ 13900 yr and a spin-down luminosity $L(t) = 4.48 \times 10^{35} \text{ erg s}^{-1}$. CTA 1 has a distance of 1.4 ± 0.3 kpc (Pineault et al. 1993). The radius of the CTA 1 is estimated to be $R_{\text{pwn}} \sim 7.2$ pc because of the large synchrotron nebula observed by Slane et al. (1997, 2004), Halpern et al. (2004). Here, $R_{\text{pwn}} = 6.6 \pm 0.5$ (Abdalla et al. 2017) is used. Observationally, CTA 1 has been detected in radio band (Aliu et al. 2013), X-ray band (Slane et al. 1997, 2004), GeV γ -ray band by *Fermi*-LAT (Abdo et al. 2012), and TeV γ -ray band by VERITAS (Aliu et al. 2013). The parameters used here are $T_{\text{age}} = 7500$ yr (which is between the values of 5000 yr and 15000 yr found by Pineault et al. 1993; Slane et al. 2004), $M_{\text{ej}} = 8.0 M_{\odot}$, and $n_{\text{H}} = 0.1 \text{ cm}^{-3}$. The values $U_{\text{IR}} = 0.3 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 0.6 \text{ eV cm}^{-3}$ are taken from the GALPROP code of Porter et al. (2006).

HESS J1418-609. It is also known as Rabbit or G313.3+0.1. The PWN is powered by PSR J1418-6058 (Acero et al. 2013). The pulsar has a rotation period of 110.6 ms, a period derivative of $1.69 \times 10^{-13} \text{ s s}^{-1}$ (Abdo et al. 2013), and $n = 3.0$ is

assumed. This pulsar has a characteristic age of $\sim 10\,380$ yr and a spin-down luminosity $L(t) = 4.93 \times 10^{36} \text{ erg s}^{-1}$. The distance of the system is 5.0 kpc (Ng et al. 2005; Aharonian et al. 2006a), which is used here. The radius of the system was estimated as 6.5 ± 0.3 pc (Kishishita et al. 2012) or 9.4 ± 0.9 (Abdalla et al. 2017). HESS J1418-609 has been detected in radio band (Roberts et al. 1999), X-ray band (Roberts et al. 2001; Ng et al. 2005; Kishishita et al. 2012), GeV γ -ray band by *Fermi*-LAT (Acero et al. 2013), and TeV γ -ray band by HESS (Aharonian et al. 2006a). So far, the age of HESS J1418-609 has not been clearly determined. The parameters used in our calculation are $T_{\text{age}} = 8000$ yr, $M_{\text{ej}} = 10.0 M_{\odot}$, and $n_{\text{H}} = 0.2 \text{ cm}^{-3}$; $U_{\text{IR}} = 0.4 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 1.0 \text{ eV cm}^{-3}$ are used to reproduce observed γ -ray SED here.

HESS J1420-607. It is also known as K3 PWN. The central pulsar is PSR J1420-6048 with a rotation period of 68 ms, a period derivative of $8.3 \times 10^{-14} \text{ s s}^{-1}$ (D'Amico et al. 2001; Roberts et al. 2001; Ng et al. 2005; Kishishita et al. 2012), and $n = 3.0$ is assumed. Therefore, the characteristic age of $\sim 13\,000$ yr and the spin-down luminosity $L(t) = 1.0 \times 10^{37} \text{ erg s}^{-1}$ are derived. The distance of the system is estimated as $d = 5.6$ kpc (Cordes & Lazio 2002), and the PWN radius is 7.9 ± 0.6 pc (Abdalla et al. 2017), which is consistent with Kishishita et al. (2012). HESS J1420-607 has been detected in radio band (Van Etten & Romani 2010), X-ray band (Van Etten & Romani 2010; Kishishita et al. 2012), GeV γ -ray band by *Fermi*-LAT Acero et al. (2013), and TeV γ -ray band by HESS Aharonian et al. (2006a). The age of the system is not clear. The parameters used here are $T_{\text{age}} = 8500$ yr, $n_{\text{H}} = 0.5 \text{ cm}^{-3}$, and $M_{\text{ej}} = 15.0 M_{\odot}$; $U_{\text{IR}} = 0.3 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 0.3 \text{ eV cm}^{-3}$ are used to reproduce observed γ -ray SED here.

HESS J1119-614 or G292.2-0.5. The PWN associates with the pulsar J1119-6127 (Caswell et al. 2004), which was discovered by Camilo et al. (2000). The pulsar has a rotational period of 408 ms, a period derivative of $4 \times 10^{-12} \text{ s s}^{-1}$, and the braking index $n = 2.684 \pm 0.002$ (Weltevrede et al. 2011), which derives the characteristic age $\tau_c = 1617$ yr and spin-down power $L(t) = 2.32 \times 10^{36} \text{ erg s}^{-1}$. It lies at a distance of 8.4 ± 0.4 kpc (Caswell et al. 2004). The radius of the system is 14 ± 2 pc (Abdalla et al. 2017). Observationally, this PWN is detected in

X-ray band (Gonzalez & Safi-Harb 2003; Safi-Harb & Kumar 2008) and the X-ray unabsorbed flux between 0.5 and 7 keV is $2.5 \times 10^{-14} \text{ erg cm}^{-2} \text{ s}^{-1}$ and spectral index $1.4 \pm_{0.9}^{0.8}$. It is detected in GeV band by *Fermi*-LAT (Acero et al. 2013), and TeV band by HESS (e.g., Abdalla et al. 2017). Here the whole SNR G292.2-0.5 flux measurement at 1.4 GHz flux density of 5.6 ± 0.3 Jy and 2.5 flux density of 1.6 ± 0.1 Jy (Crawford et al. 2001) are taken as a safe upper limit for the PWN radio emission. The age of HESS J1119-614 lies between 4200 yr (free expansion phase) and 7100 yr (Sedov phase; Kumar et al. 2012). We note that $T_{\text{age}} = 4200$ yr and $n = 1.7$ are used in Torres et al. (2014). These ages are older than the characteristic age ~ 1617 yr. Here $T_{\text{age}} = 9000$ yr and $n = 1.2$ are assumed. The parameters used in our calculation are $M_{\text{ej}} = 35.0 M_{\odot}$ and $n_{\text{H}} = 0.06 \text{ cm}^{-3}$; $U_{\text{IR}} = 0.3 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 0.7 \text{ eV cm}^{-3}$ are taken from the GALPROP code of Porter et al. (2006).

HESS J1303-631. It was first discovered in the VHE band by Aharonian et al. (2005c) and is associated with the pulsar J1301-6305 (Abramowski et al. 2012b). The pulsar has a 184 ms rotation period, a $2.65 \times 10^{-13} \text{ s s}^{-1}$ period derivative, and $n = 3.0$ is assumed. So a characteristic age of $\sim 11\,000$ yr and a spin-down luminosity $L(t) = 1.7 \times 10^{36} \text{ erg s}^{-1}$ are derived (Manchester et al. 2005a; Abramowski et al. 2012b). The pulsar's distance was estimated to be 15.8 kpc by Taylor & Cordes (1993) and 6.6 kpc by Cordes & Lazio (2002). The distance of 6.6 kpc is used here. The radius of the system was estimated as 20 pc by (Abramowski et al. 2012b) and 20.6 ± 1.7 (Abdalla et al. 2017). Observationally, HESS J1303-631 has been detected in radio band (Condon et al. 1993), X-ray band (Abramowski et al. 2012b), GeV γ -ray band by *Fermi*-LAT (Acero et al. 2013), and TeV γ -ray band by HESS (Abramowski et al. 2012b); the X-ray unabsorbed flux between 2 and 10 KeV is 1.6×10^{-13} and the spectral index is unknown. Here, the value is assumed as a safe upper limit. The multiband spectrum of HESS J1303-631 was first studied by a simple stationary leptonic model (Abramowski et al. 2012b). The age of the system is not clear; here $T_{\text{age}} = 13\,000$ yr and $n = 2.5$ are assumed. In our calculations, $M_{\text{ej}} = 9.0 M_{\odot}$ and $n_{\text{H}} = 0.02 \text{ cm}^{-3}$; $U_{\text{IR}} = 6.0 \text{ eV cm}^{-3}$ and $U_{\text{OPT}} = 2.0 \text{ eV cm}^{-3}$ are used to reproduce observed γ -ray SED.