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The period-luminosity and period-radius relations of Type II and anomalous Cepheids in the Large and Small Magellanic Clouds⋆

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ABSTRACT

Context. Type II Cepheids (T2Cs) and anomalous Cepheids (ACs) are pulsating stars that follow separate period-luminosity relations.
Aims. We study the period-luminosity (PL) and period-radius (PR) relations for T2Cs and ACs in the Magellanic Clouds.
Methods. In an accompanying paper we determined the luminosities and effective temperatures for the 335 T2Cs and ACs in the LMC and SMC discovered in the OGLE-III survey, by constructing the spectral energy distribution (SED) and fitting this with model atmospheres and a dust radiative transfer model (in the case of dust excess). Building on these results we studied the PL and PR relations of these sources. Using existing pulsation models for RR Lyrae and classical Cepheids we derive the period-luminosity-mass-temperature-metallicity relations and then estimate the pulsation mass.
Results. The PL relation for the T2Cs does not appear to depend on metallicity and is $M_{\rm bol} = +0.12 - 1.78 \log P$ (for $P < 50$ days), excluding the dusty RV Tau stars. Relations for fundamental and first overtone LMC ACs are also presented. The PR relation for T2C also shows little or no dependence on metallicity or period. Our preferred relation combines SMC and LMC stars and all T2C subclasses and is $\log R = 0.846 + 0.521 \log P$. Relations for fundamental and first overtone LMC ACs are also presented. The pulsation masses from the RR Lyrae and classical Cepheid pulsation models agree well for the short period T2Cs, the BL Her subtype, and ACs, and are consistent with estimates in the literature, i.e. $M_{\rm BLH} \sim 0.49\ M_{\odot}$ and $M_{\rm AC} \sim 1.3\ M_{\odot}$, respectively. The masses of the W Vir appear similar to the BL Her. The situation for the pWVir and RV Tau stars is less clear. For many RV Tau the masses are in conflict with the standard picture of (single-star) post-AGB evolution, where the masses are either too large ($\gtrsim$1 $M_{\odot}$) or too small ($\lesssim$0.4 $M_{\odot}$).

Key words. stars: variables: Cepheids – stars: fundamental parameters – Magellanic Clouds

1. Introduction

Type II Cepheids (T2Cs) and anomalous Cepheids (ACs) are pulsating stars located in the instability strip (IS) of the Hertzsprung-Russell diagram (HRD) also occupied by the classical Cepheids (CCs) and RR Lyrae (RRL) variables. Type II Cepheids are classically divided into subgroups based on their period and, following Soszyński et al. (2008b) and Soszyński et al. (2010b), they are the BL Herculis (BLH; 1−4 days), the (peculiar) W Virginis [(p)WVir; 4−20 days], and the RV Tauris (RVT; 20−70 days). The pulsation period of the ACs (from ~0.9 to ~2 days) overlaps with the short period T2Cs. While ACs pulsate in the first overtone (FO) and fundamental mode (FU), T2Cs are known to pulsate in fundamental mode (FU) only.

An important characteristic of T2Cs and ACs is that they follow a period-luminosity (PL) relation and that these objects can be found in globular clusters and galaxies (Catelan & Smith 2015). They can therefore be used in the calibration of the distance scale. In particular, T2Cs are useful where there are too few CCs and the RRL variables are too faint; see for example the review by Sandage & Tammann (2006) and Wallerstein (2002). While ACs fill a space on the PL diagram above the RRL and T2Cs by ~0.5 to ~2 mag (as the period increases), they do not reach into the CC region. Caputo et al. (2004) investigated the possibility that they continue to the PL relation of CCs, but Fiorentino & Monelli (2012) concluded that they are metal-poor stars that would not evolve into the CC IS region.

Period-luminosity relations have been discussed in several papers in recent decades. Nemec et al. (1994) is a rare example where $P - L$−[Fe/H] relations are derived for T2Cs and ACs (as well as RRL and SX Phe stars) in (B, V, K) colours based on objects in globular clusters. Recently, Clementini et al. (2016) presented PL relations for T2Cs and FU and FO ACs in the *Gaia G* band. Marconi et al. (2004) provided a theoretical mass-dependent period-magnitude–colour (PMC), period-Wesenheit (PW), and period-magnitude-amplitude relations for ACs in the metallicity range Z = 0.0001−0.0004. They also give the empirical PW(VI) relation based on ACs observed in seven dwarf spheroidal galaxies. Ripepi et al. (2014), as a part of the VISTA Magellanic Cloud (VMC) Survey (Cioni et al. 2011), provided

⋆ Full Table 3 is only available at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via http://cdsarc.u-strasbg.fr/viz-bin/qcat?J/A+A/604/A29 the PL relation in the K_s band and the PW(V, K) relation for FU and FO ACs in the LMC; these authors also presented the PL relation in the V and I band, a PMC, and the PW(V, I) relation based on the original OGLE-data.

Di Criscienzo et al. (2007) derived theoretical period-magnitude (PM) in the near-infrared (NIR) and PW for various optical and NIR colour combination relations combining pulsation models and evolutionary tracks for stars with periods up to eight days, i.e. BLHs. Matsunaga et al. (2011) presented NIR PL relations and Wesenheit relations for T2C in the SMC, and compared the results to their earlier work on the LMC (Matsunaga et al. 2009) and Galactic centre (GC; Matsunaga et al. 2006). The K-band PL relation for T2C in the GC was also presented by Groenewegen et al. (2008) and the absolute calibration was considered by Feast et al. (2008). More recently, Manick et al. (2017) used the OGLE-III LMC T2Cs to derive the Wesenheit PW relation.

Recent survey work in the NIR allowed a reappraisal of the PL relations, notably, Ripepi et al. (2015) considered VMC data (Cioni et al. 2011) to present several PL, PLC, and PW relations, while Bhardwaj et al. (2017) did a similar study with NIR data from the LMC Synoptic Survey (Macri et al. 2015).

Apparently, there has been little recent work carried out on the radii of T2Cs. Burki & Meylan (1986) gave a period-radius (PR) relation based on older data and Balog et al. (1997) derived the radii for 17 Galactic T2Cs with the Baade-Wesselink method. Figure 5 in their article shows a PR relation, but they did not give an equation for their fit.

Masses for the T2Cs were estimated by Bono et al. (1997a) to be in a range between 0.52 and 0.59 $M_\odot$ (for $Z < 0.001$) for stars with periods below 15 days. Quoting the results of Vassiliadis & Wood (1993), Wallerstein (2002) gave the initial mass of the brighter T2Cs (the RVT) to be around 1 $M_\odot$. In the case of ACs, pulsation models have been considered by various authors to find masses in the range 1.3–2.2 $M_\odot$ (for $Z = 0.0001$ and 0.0004; Bono et al. 1997b; Marconi et al. 2004) or specifically 1.2 ± 0.2 $M_\odot$ for the ACs in the LMC (Fiorentino & Monelli 2012). Recently, Martínez-Vázquez et al. (2016) have found ∼1.5 $M_\odot$ for four ACs in the Sculptor dSph galaxy.

In the accompanying paper, Groenewegen & Jurkovic (2017, hereafter GJ17) studied all 335 T2C and AC in the Small and Large Magellanic Clouds (MCs) detected in the OGLE-III data (Soszyński et al. 2008b, 2010b,a)[1]. The spectral energy distributions (SEDs) were constructed via photometry from the literature and fitted with the dust radiative transfer code More of DUSTY (MoD; Groenewegen 2012), which is an extension of the DUSTY radiative transfer code DUSTY (Ivezić et al. 1999). Luminosities and effective temperatures were derived and are given in the Appendix in GJ17. The resulting Hertzsprung-Russell diagram was compared in a qualitative way to modern evolutionary tracks. In agreement with the findings cited above, the BL Her can be explained by stars in the mass range ∼0.5–0.6 $M_\odot$ and the ACs by stars in the mass range ∼1.1–2.3 $M_\odot$. The origin of the (p)WVir is unclear, however; tracks of ∼2.5–4 $M_\odot$ cross the IS at the correct luminosity, along with (some) lower mass stars on the AGB that undergo a thermal pulse when the envelope mass is small, but the timescales make theseunlikely scenarios.

An infrared excess was detected from the SED fitting for ∼60% of the RVT and ∼10% of the W Vir (including the pWVir) objects. We confirmed the results of Kamath et al. (2016) that stars exist with luminosities below those predicted from single-star evolution, which show a clear infrared excess. The light curves of more than 130 systems were investigated to look for the light-travel time (LTT) effect or light-time effect (LITE) (Irwin 1952) in so-called *observed minus calculated* (O–C) diagrams. Twenty possible new binaries and about 40 stars that show a significant period change were identified.

Previous work concentrated almost exclusively on deriving the PL relation in the NIR bands or using the Wesenheit index; the main aim of this paper is to use the stellar luminosity as parameter and in that way study the properties of these stars in a more fundamental way. We used the results published in GJ17 to derive the period-luminosity and PR relations of T2Cs and ACs. In addition, we derive estimates of the masses of these stars, based on theoretical pulsation models of RRL and CCs.

In Sect. 2 we discuss the derived PL relations for T2Cs and ACs. In Sect. 3 the period-radius relation is presented, and in Sect. 4 we estimate the masses. In Sect. 5 we discuss and summarise our findings.

2. Period-luminosity relation

Figure 1 first shows the classical PL relation using the Wesenheit magnitude, $W = I - 1.55 \cdot (V - I)$, combining Fig. 1 in Soszyński et al. (2008b) and Fig. 1 in Soszyński et al. (2010b). In GJ17 we used distances to the LMC and SMC of 50 and 61 kpc in the SED modelling, and therefore we shifted the magnitudes of the SMC objects by 0.432 mag (distance moduli, DM, 18.927–18.495) to put them on the magnitude scale of the LMC. The most prominent outliers are indicated with their identifier.

Wesenheit PL relations were derived for various combinations of subclasses of T2Cs and ACs for both the LMC and SMC and were combined ("@LMC", meaning the SMC objects were placed at the distance of the LMC) and the results are listed in Table 1. Stars showing eclipsing or ellipsoidal variation (as identified by OGLE; the blue crosses in Fig. 1) were excluded in the fitting and iterative 3σ clipping was applied to remove outliers.

Table 1, in addition, includes other, both observational and theoretical, determinations of the Wesenheit PL relation from the literature. The Wesenheit PL relation does not seem to depend on metallicity. At characteristic periods of $\log P = 0.5$ (BLH), 1.5 (RVT) and 1.0 (WVir, and the solutions that include BLH and/or RVT) the magnitude difference between the solutions for the SMC and LMC are within the errors that are consistent with the expected difference of 0.43 mag.

The derived relations are also in agreement with those listed in the literature, although this is not so surprising as they are all based on the same OGLE-III data and only differ in details. The RVT stars that show IR excess are brighter in W than those without IR excess (also remarked by Manick et al. 2017), but excluding those there is well-defined $W(VI)$ relation for BLH, WVIR and non-dusty RVT, as illustrated in Fig. 1.

The agreement with theoretical models is good for the BLH. The comparison with observations requires an adopted distance to the LMC and an assumed metallicity for the models. For [Fe/H] = −1 and DM = 18.50 theory and observations agree within the error. The agreement is less good for the ACs, in particular the theoretical slope differs by almost 3σ from the empirically derived slope.

[1] Soszyński et al. (2010a) originally listed the six SMC ACs with classical Cepheid identification numbers. In the OGLE-III Variable Stars Database (http://ogledb.astrouw.edu.pl/~ogle/CVS/) these sources were subsequently listed under the names that we use in GJ17 and the present paper, OGLE-SMC-ACEP 01...06.

Table 1. Wesenheit and bolometric period-luminosity relations.

Sample	Galaxy[a]	Mag	Mag= a	$+b$ log P	dispersion (mag)	$\chi_r^{2\,b}$	N	N outliers	Ref.
BL Her	LMC	W	17.359 ± 0.022	−2.576 ± 0.080	0.089	9.18	55	6	
BL Her	SMC	W	17.558 ± 0.134	−2.429 ± 0.480	0.241	76.0	17	0	
BL Her	@LMC	W	17.347 ± 0.038	−2.669 ± 0.137	0.170	32.8	74	4	
W Vir	LMC	W	17.402 ± 0.064	−2.558 ± 0.063	0.093	9.91	76	2	
W Vir	SMC	W	18.329 ± 0.168	−3.009 ± 0.158	0.091	11.5	10	0	
W Vir	@LMC	W	17.471 ± 0.061	−2.624 ± 0.060	0.098	10.8	86	2	
RV Tau	LMC	W	18.101 ± 0.557	−3.142 ± 0.352	0.249	72.6	41	1	
RV Tau	SMC	W	17.707 ± 1.049	−2.585 ± 0.674	0.164	42.1	7	0	
RV Tau	@LMC	W	18.004 ± 0.494	−3.077 ± 0.313	0.240	66.9	48	1	
BL Her + W Vir	LMC	W	17.363 ± 0.017	−2.522 ± 0.021	0.102	11.6	133	6	
BL Her + W Vir	SMC	W	17.597 ± 0.072	−2.356 ± 0.103	0.209	53.0	26	1	
BL Her + W Vir	@LMC	W	17.335 ± 0.017	−2.496 ± 0.021	0.108	13.2	153	13	
BL Her + W Vir + RV Tau[c]	LMC	W	17.358 ± 0.014	−2.530 ± 0.017	0.089	9.00	136	10	
BL Her + W Vir + RV Tau[c]	SMC	W	17.577 ± 0.073	−2.388 ± 0.097	0.232	64.9	28	0	
BL Her + W Vir + RV Tau[c]	@LMC	W	17.355 ± 0.017	−2.526 ± 0.020	0.118	15.8	162	12	
AC FU	LMC	W	16.612 ± 0.020	−3.158 ± 0.141	0.150	25.7	62	0	
AC FO	LMC	W	16.029 ± 0.058	−3.373 ± 0.247	0.140	24.2	19	0	
BL Her + W Vir	LMC	W	17.364 ± 0.015	−2.521 ± 0.022	0.105		131		1
BL Her + W Vir	SMC	W	17.554 ± 0.083	−2.304 ± 0.107	0.230		27		2
BL Her + W Vir + RV Tau[d]	LMC	W	17.33 ± 0.03	−2.53 ± 0.03					3
BL Her	Theory	W	17.30 ± 0.07	−2.43 ± 0.02					4
AC FU	LMC	W	16.59 ± 0.02	−3.41 ± 0.16	0.15				5
AC FO	LMC	W	16.05 ± 0.05	−3.44 ± 0.22	0.13				5
AC FU	Theory	W	16.55	−2.94					6
BL Her	LMC	$M_{\rm bol}$	+0.141 ± 0.051	−1.749 ± 0.200	0.274	33.0	57	4	
BL Her	SMC	$M_{\rm bol}$	−0.250 ± 0.176	−0.691 ± 0.717	0.302	64.9	15	2	
BL Her	MCs	$M_{\rm bol}$	−0.027 ± 0.065	−1.326 ± 0.257	0.282	89.5	72	6	
W Vir	LMC	$M_{\rm bol}$	0.723 ± 0.115	−2.358 ± 0.119	0.186	36.8	74	5	
W Vir	SMC	$M_{\rm bol}$	0.965 ± 0.318	−2.589 ± 0.319	0.210	33.2	10	0	
W Vir	MCs	$M_{\rm bol}$	0.743 ± 0.109	−2.379 ± 0.112	0.201	37.1	85	4	
RV Tau[d]	LMC	$M_{\rm bol}$	+1.442 ± 1.146	−2.919 ± 0.750	0.301	91.1	15	0	
RV Tau[d]	SMC	$M_{\rm bol}$	−1.088 ± 0.433	−1.367 ± 0.290	0.041	4.6	4	0	
RV Tau[d]	MCs	$M_{\rm bol}$	+0.951 ± 0.974	−2.620 ± 0.639	0.298	78.4	19	0	
BL Her + W Vir	LMC	$M_{\rm bol}$	+0.199 ± 0.035	−1.827 ± 0.042	0.230	40.6	130	10	
BL Her + W Vir	SMC	$M_{\rm bol}$	−0.087 ± 0.100	−1.561 ± 0.182	0.349	256.	26	1	
BL Her + W Vir	MCs	$M_{\rm bol}$	+0.068 ± 0.037	−1.704 ± 0.049	0.267	83.1	159	8	
BL Her + W Vir + RV Tau[c]	LMC	$M_{\rm bol}$	+0.226 ± 0.033	−1.870 ± 0.039	0.233	40.5	136	11	
BL Her + W Vir + RV Tau[c]	SMC	$M_{\rm bol}$	−0.048 ± 0.101	−1.686 ± 0.172	0.370	275.	27	1	
BL Her + W Vir + RV Tau[c,e]	MCs	$M_{\rm bol}$	+0.119 ± 0.036	−1.787 ± 0.044	0.276	81.8	166	9	
AC FU	LMC	$M_{\rm bol}$	−0.436 ± 0.033	−3.122 ± 0.213	0.255	71.3	61	1	
AC FO	LMC	$M_{\rm bol}$	−1.126 ± 0.074	−3.248 ± 0.305	0.244	53.2	20	0	

Notes. [(a)] For the PL relations in the Wesenheit index "@LMC" means the stars in the LMC plus the stars in the SMC placed at the distance of the LMC by a shift of 0.432 mag. [(b)] The reduced χ^2 is based on an assumed "error" in the Wesenheit index and bolometric magnitude of 0.03 mag. [(c)] Excluding RVTs with dust excess and for $P < 50$ days. [(d)] Excluding RVTs with dust excess. [(e)] The preferred solution.

References. (1) Matsunaga et al. (2009); (2) Matsunaga et al. (2011); (3) Manick et al. (2017); (4) Di Criscienzo et al. (2007) for [Fe/H] = −1, $l/H_{\rm p}$ = 1.5, and LMC distance modulus 18.50; (5) Ripepi et al. (2014); (6) Marconi et al. (2004) for M = 1.3 $M_\odot$ and LMC distance modulus 18.50.

Figure 2 shows the bolometric version of the PL relation, using the luminosities derived in GJ17. The bottom part of Table 1 gives the corresponding fits to the PL relation. What is immediately noticeable is that the scatter in the bolometric PL relations is significantly larger than in the corresponding Wesenheit relations. There could be several reasons for this. First, the Wesenheit relations are based on two intensity-mean magnitudes, while the luminosities are derived based on a fit to the entire SED that is based on non-contemporaneous photometry. Second, if there are issues related to blending or binarity then certain combinations of the parameters involved may still yield a Wesenheit index that is close to the mean relation, but the fitting of the entire SED more likely yields deviant results.

For comparison we refer to two other systems throughout the discussion, namely the best studied of the known CCs in an eclipsing binary in the LMC (OGLE-LMC-CEP-0227), and

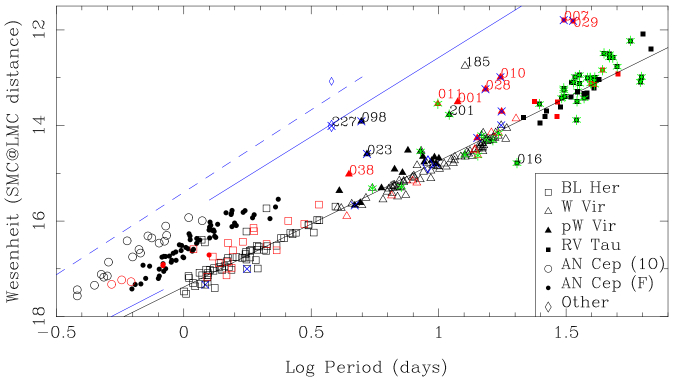

Fig. 1. Wesenheit PL relation. Stars in the SMC are plotted in red and shifted to the distance of the LMC. Some outliers are indicated with their identifier. Stars with an IR excess according to GJ17 are indicated by a green plus sign. Stars that show eclipsing or ellipsoidal variations according to OGLE are indicated by a blue cross. Stars plotted as a blue diamond are OGLE-LMC-CEP-0227 (the classical Cepheid in an eclipsing binary system, at P = 3.79 days) and the Galactic T2C κ Pav (at P = 9.08 days) scaled to the distance of the LMC (see text). For −0227 both the system value (the smaller, brighter point) and the Wesenheit magnitude of the Cepheid in the system (from Pilecki et al. 2013) are plotted. The black solid line shows the relation derived from the BLH + WVIR + non-dusty RVTs with periods below 50 days in the LMC (see Table 1), plotted over the entire period range. The blue lines indicate the Wesenheit relation for CC in the LMC from Soszyński et al. (2008a) for FU (solid line) and FU (dashed line) pulsators and the relation for RRab stars (at $\log P < -0.08$) from Soszynski et al. (2003).

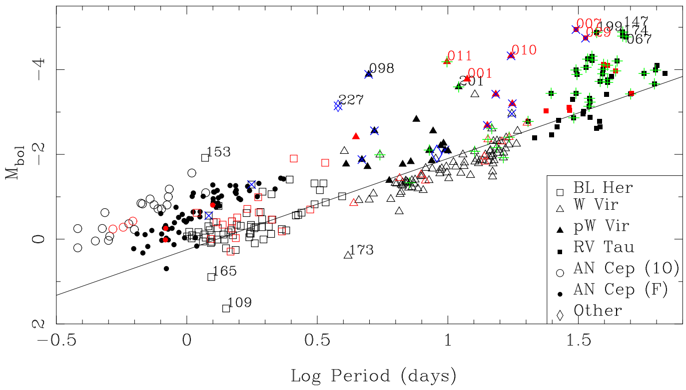

Fig. 2. Bolometric PL relation. Stars in the SMC are plotted in red. The error in M_{bol} is smaller than the plot symbol. Some outliers are plotted with their identifier. Stars with an IR excess according to GJ17 are indicated by a green plus sign. Stars that show eclipsing or ellipsoidal variations according to OGLE are indicated by a blue cross. The classical Cepheid in the eclipsing binary OGLE-LMC-CEP-0227 (at P = 3.79 days) and the Galactic T2C κ Pav (at P = 9.08 days) are plotted as blue diamonds. The black solid line shows the relation derived from the BLH + WVIR + non-dusty RVTs with periods below 50 days in the LMC (see Table 1) plotted over the entire period range.

one of the best studied T2Cs in our Galaxy, κ Pav (WVir type, P = 9.09 days). The latter has a *Hubble* Space Telescope (HST)-based distance of 180 ± 9 pc (Benedict et al. 2011) and a metallicity of [Fe/H] = 0.0 (Luck & Bond 1989). This source is not listed in the 1st *Gaia* data release (Gaia Collaboration 2016). Time-series photometry in V, I is available from Berdnikov (2008), from which we derived the mean magnitudes. In Fig. 1 it κ Pav is plotted as if it were located at the adopted distance to the LMC. Breitfelder et al. (2015) quote an effective temperature of $T_{\rm eff}$ = 5739 ± 107 K, implying L = 508 ± 65 $L_\odot$. Pilecki et al. (2013) have derived the V, I magnitudes of the two components in OGLE-LMC-CEP-0227, and the Wesenheit magnitude of the Cepheid in the system and of the total binary system are plotted in Fig. 1 (at P = 3.79 d), together with the Wesenheit relation for FU and FO CCs in the LMC from Soszyński et al. (2008a). Pilecki et al. (2013) also derived $\log L$ = 3.158 ± 0.049 $L_\odot$ and $T_{\rm eff}$ = 6050 ± 160 K, which is in agreement with Marconi et al. (2013; $\log L$ = 3.16 ± 0.02 $L_\odot$, $T_{\rm eff}$ = 6100 ± 50 K).

κ Pav, which is close to the estimated PL relation in the Wesenheit index, is brighter than the relation in bolometric magnitude. The derived radius and effective temperature (Breitfelder et al. 2015) imply $M_{\rm bol}$ = −2.01 ± 0.13, while the various PL relations give values in the range −1.53 to −1.59. This could be due to the intrinsic width of the IS; the assumed distance, although the HST-based distance is accurate to 5%; a metallicity dependence of the PL relation, which is not obvious from a comparison of LMC and SMC objects; or the fact that this object has a binary companion. In fact, Matsunaga et al. (2009) discusses κ Pav in detail and suggests that it should be classified as a pWVir object.

The outliers that are indicated in both figures are mostly pWVir type stars, some of which have been classified as binaries by the OGLE team: OGLE-LMC-T2CEP-098, and -023, OGLE-SMC-T2CEP-007, -010, -028, or where the LITE was tentatively detected in GJ17, OGLE-SMC-T2CEP-001, and -029. The other outliers are mostly dusty objects with infrared excess detected in GJ17, namely some pWVir objects (OGLE-LMC-T2CEP-201 and OGLE-SMC-T2CEP-011), and mostly RVT (OGLE-LMC-T2CEP-016, -067, -147, -174, -199).

3. Period-radius relation

Figure 3 shows the PR relation based on the derived effective temperatures and luminosities in GJ17. The resulting radii with error bars are given in Table 3.

We derived PR relations for various combinations of the T2C subclasses in the SMC, LMC, and both, and for the FU and FO ACs in the LMC; the results are listed in Table 2. Stars showing eclipsing or ellipsoidal variation have been excluded in the fitting and iterative 3σ clipping was applied to remove outliers. Contrary to the PL relations, where the dusty RVT stars deviated significantly and were excluded, this is not the case here.

Marconi et al. (2015) have presented the latest non-linear, time-dependent convective hydrodynamical models of RRL stars for different metallicites and masses. Specifically they present period-mass-radius-metallicity (PMRZ) relations for fundamental and first-overtone pulsators (their Eqs. (7) and (8)). As they were concerned with RRL these authors have excluded "the sequence D models" (see Marconi et al. 2015, for details) in their fitting procedure, since these luminosity levels were considered too bright for typical RRLs. However, these luminosities are typical for T2C and therefore we rederived the PMRZ relation for all models with $\log L > 1.65$ $L_\odot$ (and that reach up to $\log L \sim 2.0$, and periods up to ~2.4 days) using their dataset. We find

$$\log R = (0.763 \pm 0.003) - (0.037 \pm 0.001) \log Z + (0.560 \pm 0.004) \log P \quad (N = 195) \tag{1}$$

for FU pulsators, and

$$\log R = (0.855 \pm 0.005) - (0.034 \pm 0.001) \log Z + (0.585 \pm 0.007) \log P \quad (N = 63) \tag{2}$$

for FO pulsators. These relations are plotted in Fig. 3 at the average metallicity of RRL in the LMC (Gratton et al. 2004) of

Table 2. Period-radius relations.

Sample	Galaxy	$\log R =$ ($R_\odot$)	a	$+b \log P$	dispersion	χ^2_r	N	N outliers	Solution	Ref.
BL Her	LMC	0.830 ± 0.013		0.564 ± 0.049	0.047	2.60	57	4	(1)	
BL Her	SMC	0.852 ± 0.028		0.574 ± 0.117	0.056	12.4	17	0	(2)	
BL Her	MCs	0.847 ± 0.013		0.551 ± 0.052	0.058	5.43	76	2	(3)	
W Vir	LMC	0.823 ± 0.020		0.541 ± 0.021	0.037	1.74	77	2	(4)	
W Vir	SMC	0.709 ± 0.079		0.620 ± 0.071	0.038	4.11	10	0	(5)	
W Vir	MCs	0.828 ± 0.020		0.531 ± 0.020	0.037	2.25	87	2	(6)	
RV Tau	LMC	0.848 ± 0.141		0.528 ± 0.088	0.076	5.72	41	1	(7)	
RV Tau	SMC	0.977 ± 0.188		0.440 ± 0.124	0.039	2.54	7	0	(8)	
RV Tau	MCs	0.864 ± 0.112		0.517 ± 0.071	0.072	5.16	48	1	(9)	
BL Her + W Vir	LMC	0.837 ± 0.007		0.528 ± 0.008	0.041	2.09	134	6	(10)	
BL Her + W Vir	SMC	0.869 ± 0.015		0.480 ± 0.022	0.050	9.81	27	0	(11)	
BL Her + W Vir	MCs	0.852 ± 0.006		0.508 ± 0.008	0.044	3.46	161	6	(12)	
BL Her + W Vir + RV Tau	LMC	0.833 ± 0.007		0.535 ± 0.007	0.050	2.82	174	8	(13)	
BL Her + W Vir + RV Tau	SMC	0.861 ± 0.013		0.501 ± 0.016	0.050	8.75	34	0	(14)	
BL Her + W Vir + RV Tau[a]	MCs	0.846 ± 0.006		0.521 ± 0.006	0.053	3.91	209	7	(15)	
AC FU	LMC	0.972 ± 0.005		0.692 ± 0.034	0.045	2.39	61	1	(16)	
AC FO	LMC	1.113 ± 0.016		0.733 ± 0.073	0.054	3.65	20	0	(17)	
T2C	Galactic	0.87		0.54						(1)

Notes. [a] The preferred solution.

References. (1) Burki & Meylan (1986).

[Fe/H] = −1.5 (or $\log Z = -3.23$). The theoretical relation lies above the observed relation. The slope agrees within the error bar with the observed relation for BLH (see Table 2), but the zero point is slightly larger.

In a similar way, Bono et al. (2000) present non-linear pulsation models for CCs for various masses and metallicities. Period-radius relations for FU pulsators at three different metallicities were already presented in Bono et al. (1998). Here we rederived the PMRZ relations for FU and FO pulsators from the Bono et al. (2000) dataset; following this work, we combine the canonical and non-canonical models, and we find

$$\log R = (1.115 \pm 0.012) - (0.039 \pm 0.005) \log Z + (0.653 \pm 0.003) \log P \quad (N = 202) \qquad (3)$$

for FU pulsators, and

$$\log R = (1.257 \pm 0.028) - (0.003 \pm 0.014) \log Z + (0.706 \pm 0.016) \log P \quad (N = 27) \qquad (4)$$

for FO pulsators. These relations are plotted in Fig. 3 at the average metallicity of Cepheids in the LMC (Romaniello et al. 2008) of [Fe/H] = −0.33 (or $\log Z = -2.06$).

For comparison we plotted objects with known radii. The values for the Cepheid in OGLE-LMC-CEP-0227 and Galactic T2C κ Pav are plotted as blue diamonds. Based on a Baade-Wesselink type analysis, Breitfelder et al. (2015) derived a projection factor of $p = 1.26 \pm 0.04 \pm 0.06$, and a radius of $R = 22.83 \pm 1.14\ R_\odot$. The big light blue stars represent Galactic Type II Cepheids that had their radii derived by Balog et al. (1997) with the Baade-Wesselink method. In case of κ Pav ($R = 19 \pm 5\ R_\odot$) their result is in good agreement with Breitfelder et al. (2015). Since the article was published in 1997, some of the objects have been reclassified, which explains why they scatter so much. Looking at the classification by the General Catalog of Variable Stars (GCVS)[2] and the International Variable

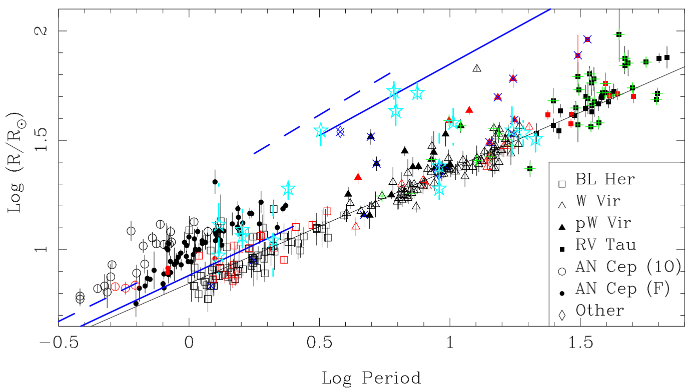

Fig. 3. Period-radius relation. Stars in the SMC are plotted in red. The black solid line is a fit to the BL Her, W Vir, and RV Tau stars in the LMC (excluding 3σ outliers). The blue solid and dashed lines are the theoretical PR relations for FU and FO RRL (shorter periods) and CC in the LMC, respectively (see text). The values for the Cepheid in the eclipsing binary system OGLE-LMC-CEP-0227 (at $P = 3.79$ days) and the Galactic T2C κ Pav (at $P = 9.08$ days) are plotted as blue diamonds. Light blue stars represent the stars from Balog et al. (1997). Details about these stars are given in the text.

Star Index (VSX)[3] it appears that KL Aql ($P = 6.1$ day), V733 Aql ($P = 6.2$), BB Her ($P = 7.5$), and DR Cep ($P = 19.1$) are CCs. DQ And ($P = 3.2$) has a questionable classification, but it is more likely than not a CCs as well. The cases of TX Del ($P = 6.2$) and IX Cas ($P = 9.1$) are different because their radius from the Baade-Wesselink analysis might have been influenced by the fact that they are in binary systems. AU Peg ($P = 2.4$) is hard to interpret because it was suggested that it might not be a T2C and the radius of $19 \pm 4\ R_\odot$ puts it above the PR relation for T2Cs. It is also a binary, so, again, that could have had an

[2] http://www.sai.msu.su/gcvs/gcvs/

[3] https://www.aavso.org/vsx/

influence on the determined radius. BL Her (P = 7.5), XX Vir (P = 1.3), SW Tau (P = 1.6), NW Lyr (P = 1.6), V553 Cen (P = 2.1, a C-rich object), κ Pav, AL Vir (P = 10.3), W Vir (P = 17.3), and V1181 Sgr (P = 21.3) are T2Cs, and they follow our PR relation.

There is no obvious dependence of the PR relation on metallicity or on subclass. Within the error bars, all T2C can be represented by a single PR relation (solutions 13–15). For this type of relation the slope between the solution for SMC and LMC stars differ by 2σ, but slope and zero point are not independent. At the characteristic period of 10 days the predicted radii for an SMC and LMC T2C are identical. Combining both galaxies and all periods, solution (15) becomes our preferred PR relation for T2Cs.

Table 2 also includes the old relation presented in Burki & Meylan (1986). They did not give error bars, but at face value the relation is similar to the preferred relation we derive for the MC T2Cs. Together with the T2C from Balog et al. (1997) this supports the suggestion that there is no strong dependence of the PR relation on metallicity.

4. Masses from evolutionary models

Following Sect. 3 we derived the following equation for FU pulsators from the models in Marconi et al. (2015) with $\log L > 1.65$,

$$\begin{aligned}\log P = (11.468 \pm 0.049) + (0.8627 \pm 0.0028) \log L \\ - (0.617 \pm 0.015) \log M - (3.463 \pm 0.012) \log T_{\rm eff} \\ + (0.0207 \pm 0.0013) \log Z \quad (N = 195, \sigma = 0.0044). \quad (5)\end{aligned}$$

Similarly, we used the models in Bono et al. (2000) to find for FU pulsators (cf. their Table 6),

$$\begin{aligned}\log P = (10.649 \pm 0.085) + (0.9325 \pm 0.0053) \log L \\ - (0.799 \pm 0.020) \log M - (3.282 \pm 0.022) \log T_{\rm eff} \\ + (0.0393 \pm 0.0026) \log Z \quad (N = 202, \sigma = 0.0085). \quad (6)\end{aligned}$$

Equations (5) and (6) allow us to derive the current mass if we know period, luminosity, effective temperature, and metallicity. The procedure seems to give sensible results. For the stellar parameters derived for the CC -0227 (Pilecki et al. 2013), and its metallicity of Z = 0.004 (Marconi et al. 2013), the equation based on the Cepheid models gives a mass of $4.41 \pm 0.44\ M_\odot$, in agreement with the masses found by Pilecki et al. (2013; $4.165 \pm 0.032\ M_\odot$) and Marconi et al. (2013; $4.14 \pm 0.06\ M_\odot$). However, the equation from the RRL pulsation modes also gives an estimate that is correct within the error bar, namely $5.86 \pm 1.18\ M_\odot$.

We used Eqs. (5) and (6) with Z = 0.004 (for both LMC and SMC) to estimate the masses. For the overtone AC the equations were evaluated at their fundamental period $\log P_{\rm FU} = \log P_{\rm FO} + 0.127$. Errors were estimated from the error in $T_{\rm eff}$ and L, and the intrinsic scatter in the relation. The results are given in Table 3. The Cepheid and RRL-based masses agree within 3× the combined error bars (denoted as "OK" in Table 3) or within 15% irrespective of the error bars (denoted as "ok" in Table 3) in 90% for the BLH (72/80), 82% for the FU ACs (53/65), 74% for the FO ACs (17/23), 58% for the pWVir (17/24), but only 22% for the WVir (20/90) and 18% (9/51) for the RVT.

The effect of changing the metallicity was investigated. As a test it was lowered to Z = 0.0012, which would give a mass for the Cepheid -0227 from the Cepheid models in exact agreement with the observations (see above). This improved the agreement between the two mass estimates for the BLH and WVir slightly reduced the agreement for the FU ACs and left the other percentages unchanged.

The geometric mean of the two estimates was taken for the estimates based on Z = 0.004. For the various pulsation classes, we find the following range in masses (listed are the 10, 50, 90% percentiles), where known eclipsing and ellipsoidal variables were excluded; i.e. BL Her (0.36, 0.49, 0.87 $M_\odot$), W Vir (0.31, 0.41, 0.57 $M_\odot$), pW Vir (0.37, 0.74, 1.29 $M_\odot$), RV Tau (0.25, 0.43, 0.82 $M_\odot$), and ACs (0.89, 1.29, 1.90 $M_\odot$), with the same range for FU and FO pulsators. Assuming Z = 0.0012 would lower these mass estimates by about 5%. For the RVT it does not matter significantly whether one separates them into the dusty or non-dusty objects.

Taking only the stars where the two mass estimates agree (the "OK" and "ok" from Table 3) leaves these ranges essentially unchanged for the BLH and the ACs. For the WVir the range becomes 0.43, 0.50, and 0.66 $M_\odot$ based on 20 stars. The pWVir are suggested to be in binary systems. Including eclipsing/ellipsoidal stars, and taking the stars where the two mass estimates agree, the mass range is increased significantly to 0.72, 1.22, and 1.77 $M_\odot$, for 14 stars. The number of RVT for which the two mass estimates agree (and are non-eclipsing/ellipsoidal) is only seven and the median mass is 0.82 $M_\odot$.

The classes of objects for which the mass estimates agree best and for most stars are the BLH and ACs. The estimates also agree with previous estimates in the literature for these classes. For the WVir, the situation is slightly less clear but the mass estimates are similar to those of the BLH. The most confusing picture is presented by the pWVir and the RVT. The mass estimate for the former classes is definitely larger than for the BLH and W Vir.

Some of the known binaries have a (spurious) large mass assigned: for example, LMC -098 (3 $M_\odot$, pWVir), SMC -007 (1.9 $M_\odot$, RVT), -010 (2.3 $M_\odot$, pWVir), -028 (1.6 $M_\odot$, pWVir), and -029 (2.5 $M_\odot$, RVT). Based on this, the following stars (non-ACs) could also be binaries: the LMC objects -032 (RVT, 1.9 $M_\odot$), -123 (BLHer, 2.2 $M_\odot$), -136 (BLHer, 2.0 $M_\odot$), -153 (BLHer, 1.6 $M_\odot$), -185 (WVir, 4.5 $M_\odot$), and SMC objects -001 (pWVir, 1.7 $M_\odot$), and -011 (pWVir, 1.8 $M_\odot$). The first five stars listed were also removed as outliers in the PR relation and -153 and -185 were also removed as outliers in the $P - M_{\rm bol}$ relation. The star -185 was an outlier in the amplitude-magnitude diagram (Fig. 10 in GJ17). None of these stars were indicated as possible binaries based on the LTT effect in GJ17, however. The star -153 has been indicated by the OGLE team as blended and the finding chart for -156 on the OGLE-III Variable Stars Database[4] suggests that it is blended as well, so for those stars that could easily be the cause of their brighter appearance. On the other hand, based on the mass estimate, period, and the fact that it is brighter and larger than the other BLH, -123 could be classified as a FU AC.

During the refereeing process, Pilecki et al. (2017) was published, which analysed this system in detail and derived a mass of $1.51 \pm 0.09\ M_\odot$, $T_{\rm eff} = 5300 \pm 100$ K, and $L = 450 \pm 40\ L_\odot$ for the pulsating star, and $6.8 \pm 0.4\ M_\odot$, $T_{\rm eff}$ = 9500 ± 500 K, and $L = 5000 \pm 1100\ L_\odot$ for the secondary. Fitting the SED as a single object GJ17 found $T_{\rm eff} = 7375 \pm 312$ K and $L = 2857 \pm 169\ L_\odot$. Using the luminosity and effective temperature (and errors) from Pilecki et al. (2017) the mass estimate based on the Cepheid,

[4] http://ogledb.astrouw.edu.pl/~ogle/CVS/

Table 3. Basic parameters and mass estimates.

Name	Type	Period (d)	Radius ($R_\odot$)	$\mathrm{Mass_{Cep}}$ ($M_\odot$)	$\mathrm{Mass_{RRL}}$ ($M_\odot$)	Dusty?	Binary?	Agree?
OGLE-LMC-ACEP-001	F	0.85	7.86 ± 0.48	0.891 ± 0.016	1.023 ± 0.029	0	0	ok
OGLE-LMC-ACEP-002	F	0.98	9.32 ± 0.56	1.127 ± 0.022	1.314 ± 0.045	0	0	ok
OGLE-LMC-ACEP-003	1O	0.51	5.94 ± 0.43	0.925 ± 0.020	1.063 ± 0.041	0	0	ok
OGLE-LMC-ACEP-004	F	1.86	13.12 ± 2.99	1.091 ± 0.202	1.202 ± 0.383	0	0	OK
OGLE-LMC-ACEP-005	F	0.93	8.53 ± 0.50	0.960 ± 0.017	1.106 ± 0.032	0	0	ok

Notes. This table contains the OGLE name, the type (F or 1O for the ACs, and BLHer, WVIR, pWVir, or RVTau for the T2C), the period (from OGLE), the radius with error bar derived from the luminosity and effective temperature given in GJ17, the mass estimates and error bars based on the Cepheid and RRL pulsation models (Eqs. (5) and (6)). For reference it is also indicated if the star has an IR excess (Dusty = 1) or is a known binary (Binary = 1). If the two mass estimates agree within 3× the combined error bars the last column has an "OK" listed, and otherwise, if the two mass estimates still agree within 15% (without considering the errors) the last column has an "ok" listed. The table is available in electronic form at the CDS. The first five entries are shown for guidance regarding its form and content.

respectively, RRL pulsation models is 1.37 ± 0.04 $M_\odot$, respectively, 1.52 ± 0.07 $M_\odot$, in agreement with Pilecki et al. (2017). The parameters these authors derived, in particular the mass, depend on the adopted, so-called projection factor, p. We find that the geometric mean of the Cepheid and RRL pulsation mass and their derived mass agrees best for p = 1.32 ± 0.03, which is in excellent agreement with their adopted p = 1.30 ± 0.04. Interestingly, their derived values of $M_{\rm bol}$ = −1.88 ± 0.09 and R = 25.2±0.4 $R_\odot$ which do not depend very much on the adopted value of p, still make the Cepheid overluminous and oversized with respect to our preferred solutions of Tables 1 and 2, which give $M_{\rm bol}$ = −1.12 ± 0.05 and R = 16.2 ± 0.3 $R_\odot$.

The mass estimates for the RVT show both very high and very low values. As indicated above, many RVT have mass estimates that are well above those expected for a post-AGB object (0.55–1.1 $M_\odot$). In addition, of the about 30 stars that have a mass estimate below 0.35 $M_\odot$, 10 are RVT and 6 of those have dust excesses. Such low masses are also not expected from single-star evolution and, as remarked in in GJ17, the shape of the dust excess in these SEDs points to a disk-like structure that is thought to result from binary evolution. Some are possibly related to the so-called binary evolutionary pulsators (BEP), which are binary stars that appear in the IS after significant mass transfer. Recently, Karczmarek et al. (2017) did extensive simulations to find contaminations of genuine RRL and classical Cepheids of 0.8 and 5% by BEP, respectively. In GJ17 we estimated that a contamination of several percent is plausible for T2C as well.

5. Summary and conclusions

The luminosities and effective temperatures derived in GJ17 for 335 T2Cs and ACs in the SMC and LMC were used to study the period-Wesenheit and, for the first time to our knowledge in the 21st century, the period-bolometric luminosity, and PR relations for these classes of stars.

The $P - M_{\rm bol}$ relation shows more scatter than the PW relation. This is likely because the fits to the SEDs presented in GJ17 are based on non-contemporaneous photometry over a large wavelength region. This introduces some natural scatter, but likely reveals the effect of binarity or blending more easily than when using only the OGLE mean V, I magnitudes. The period-bolometric luminosity and PR relations do not significantly depend on metallicity, as probed by the T2C in the SMC and LMC and supported by the limited data for Galactic T2Cs; excluding the dusty RVTs, the T2C can be described by single relationships.

We used the published results of theoretical pulsation models for classical Cepheids and RRL to derive period-luminosity-mass-temperature-metallicity relations. Assuming a metallicity these relations allow us to derive the pulsation mass for all objects based on both types of models. For the BLH and ACs the masses from the RRL and CC pulsation models agree well, and agree with those found in the literature of ∼0.5 $M_\odot$ and ∼1.3 $M_\odot$, respectively. For the RVT the agreement between the two mass estimates is poorest and often indicates masses that are inconsistent with single-star evolution of a post-AGB star, either above ∼1 $M_\odot$ or well below ∼0.5 $M_\odot$.

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