The compactness of the isolated neutron star RX J0720.4–3125[★]

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ABSTRACT

Aims. To estimate the compactness of the thermally-emitting isolated neutron star (INS) RX J0720.4–3125, an X-ray spin-phase-resolved spectroscopic study is conducted. In addition, to identify the genuine spin-period, an X-ray timing analysis is performed.

Methods. The data from all observations of RX J0720.4–3125 conducted by *XMM-Newton* EPIC-pn with the same instrumental setup in 2000–2012 were reprocessed to form a homogenous dataset of solar barycenter corrected photon arrival times registered from RX J0720.4–3125. A Bayesian method for the search, detection, and estimation of the parameters of an unknown-shaped periodic signal was employed. A number of single- and double-peaked complex models of light curves from pulsating neutron stars were statistically analyzed. The distribution of phases for the registered photons was calculated by folding the arrival times with the derived spin-period and the resulting distribution of phases, which was approximated with a mixed von Mises distribution, and its parameters were estimated by using the expected maximization method. Spin-phase-resolved spectra were extracted, a number of highly magnetized atmosphere models of an INS were used to perform simultaneous fits, and the results were verified via an Markov chain Monte Carlo approach.

Results. The phase-folded light curves in different energy bands with high signal-to-noise ratio show high complexity and variations that depend on time and energy. They can be parameterized with a mixed von Mises distribution, meaning with a double-peaked light curve profile that shows a dependence of the estimated parameters, such as the mean directions, concentrations, and proportion upon the energy band, indicating that radiation emerges from at least two emitting areas.

Conclusions. We derive a most-likely genuine spin-period of the isolated neutron star RX J0720–3125 that is twice that reported in the literature, 16.78 s instead of 8.39 s. We determine the gravitational redshift of RX J0720.4–3125 to be $z = 0.205^{+0.006}_{-0.003}$ and estimate the compactness to be $(M/M_{\odot})/(R/\text{km}) = 0.105 \pm 0.002$.

Key words. X-rays: individuals: RX J0720.4-3125 – stars: neutron – X-rays: general – X-rays: stars – pulsars: general – methods: data analysis

1. Introduction

A comprehensive understanding of neutron stars is one of the key challenges facing astrophysics and astronomy. In particular, the observational determination of the gravitational redshift of an isolated neutron star (INS) can be used to put constraints on the theoretical models of superdense matter. This goal can be achieved, for instance, by modeling and analyzing the observed emergent spectrum of a thermally-emitting, pulsating INS via rotational phase-resolved X-ray spectroscopy (Zavlin et al. 1995; Motch et al. 2003; Pérez-Azorín et al. 2006; Zane & Turolla 2006; Ho et al. 2007; Suleimanov et al. 2010; Hambaryan et al. 2011, 2014, and also developed and successfully applied by us).

This spectroscopy involves simultaneous fitting of high quality spectra¹ from different spin-phases with models of highly magnetized atmospheres of INSS. This allows us to constrain a

number of physical properties of the X-ray emitting areas, including their temperatures, magnetic field strengths at the poles, and their distribution parameters. In addition, we may place some constraints on the viewing geometry of the emerging X-ray emission and the gravitational redshift and, hence, the compactness of the INS. This model of spectra is based on various local models, such as blackbody or condensed (iron or other heavy element) surface, covered by a thin hydrogen layer.

On the other hand, it is clear that for such kind of study the true spin parameter of the rotating INS must be known at different epochs, meaning that X-ray timing analysis is also needed for the application of advanced methods (see Sect. 3.1).

At the beginning of 1990s, the ROSAT X-ray observatory discovered a small group, so far seven, of nearby, thermally-emitting and radio-quiet INSS (e.g., see reviews by Haberl 2007; Mereghetti 2008; Turolla 2009, and references therein). Meanwhile, multi-epoch X-ray observations using the *XMM-Newton* and *Chandra* telescopes allowed the detection of X-ray pulsations and the uncovering of absorption features in the

phases can be achieved by combining observations performed at different epochs.

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¹ High signal-to-noise ratio spectra, for which relative errors in each spectral bin are on the order of a few percent, for different rotational

Table 1. *XMM-Newton* EPIC-pn full frame mode observations of RX J0720.4–3125.

Obs. ID	Date begin	MJD [start]	Exposure [ks]	Effective exposure [ks]	Number of counts	Group No.
0124100101	2000–05–13 01:42:22	51 677.104103	65.9	33.6	205 200	I
0156 960201	2002–11–06 17:51:49	52 584.761786	30.2	19.7	125 305	"
0156 960401	2002–11–08 19:25:02	52 586.826653	32.0	30.0	189 113	"
016456 0501	2004–05–22 10:15:22	53 147.442236	52.0	25.4	166 528	II
030052 0201	2005–04–28 08:41:05	53 488.377978	53.3	30.0	193 632	"
030052 0301	2005–09–22 23:44:23	53 636.004188	53.0	35.2	230 808	"
0400140301	2006–05–22 04:44:47	53 877.228785	21.9	16.2	103 291	III
0400140401	2006–11–05 11:19:29	54 044.489372	21.9	20.0	127 835	"
050 2710201	2007–05–05 17:01:25	54 225.732373	21.9	15.0	94 214	"
050 2710301	2007–11–17 05:14:32	54 421.236398	24.9	23.0	142 855	"
055 4510101	2009–03–21 14:14:24	54 911.669656	21.9	14.8	92 506	IV
0601170301	2009–09–22 04:27:35	55 096.200867	30.8	13.2	81 991	"
0650 920101	2011–04–10 23:59:26	55 662.016898	21.9	16.1	99 286	"
0670700201	2011–05–02 23:25:17	55 684.067143	28.8	10.1	62 887	V
0670700301	2011–10–01 03:47:26	55 835.184239	26.9	24.1	150 050	"
0690070201	2012–09–18 08:50:53	56 188.369444	25.1	24.9	159 712	"

spectra of some of them (Haberl et al. 2003; Hambaryan et al. 2009; Borghese et al. 2015).

The isolated neutron star RX J0720.4–3125 is a special case among the nearby thermally-emitting INSs because it shows long-term variations in its timing and spectral parameters. A clear variation in the spectrum has been detected by the *XMM-Newton* high spectral resolution reflection grating spectrometers (RGS; de Vries et al. 2004). This discovery was subsequently confirmed and analyzed by imaging-spectroscopic EPIC-pn observations (Haberl et al. 2006; Hohle et al. 2009).

The spin-phase-averaged X-ray spectra of RX J0720.4–3125 can formally be modeled by a blackbody, $kT \sim 84\text{--}94\text{ eV}$, plus a broad, $\sim 10\text{--}70\text{ eV}$ wide, absorption feature centered at $\sim 300\text{ eV}$ (Haberl et al. 2004; Hohle et al. 2009, 2012b) undergoing somewhat sensible changes over a timescale of a few years. However, Viganò et al. (2014) state that the observed broad absorption feature might be partially owing to the X-ray spectral distortions that are induced by the inhomogeneous temperature distributions of the neutron star surface. A narrow, $\sim 5\text{ eV}$ wide, absorption feature was identified in *XMM-Newton* RGS data at $\sim 570\text{ eV}$, possibly due to highly ionized oxygen of circumstellar origin (Mendez et al. 2004; Hambaryan et al. 2009; Hohle et al. 2012a). In this paper, we perform a rotational phase-resolved spectral re-analysis of the *XMM-Newton* EPIC-pn data from the available multi-epoch X-ray observations of RX J0720.4–3125.

2. Observations and data reduction

The isolated neutron star RX J0720.4–3125 was observed many times by *XMM-Newton* with different instrumental setups (Table 1). Here we focus mainly on the high quality data collected with EPIC-pn in full frame mode (Strüder et al. 2001) from sixteen *XMM-Newton* observations with the same instrumental setup positioned on-axis. These were distributed over 12.4 yr, in total presenting about 350 ks of effective exposure time and consisting of approximately 2.2 million registered source photons.

The data were reduced using standard threads from the *XMM-Newton* data analysis package SAS version 14.0.0. We

reprocessed all observations listed in Table 1 with the standard metatask epcchain. To determine good time intervals that were free of background flares, we applied a filter to the background light curves and performed a visual inspection. This reduced the total exposure time by $\sim 35\%$. Solar-barycenter-corrected source and background photon events files were produced from the cleaned single-pixel events, using an extraction radius of $45\text{--}60''$ that depended on the brightness of RX J0720.4–3125 in the corresponding pointed observation.

We extracted light curves of RX J0720.4–3125 and the corresponding background from nearby, source-free regions. We then used the SAS task `epicccorr` to correct the observed count rates for various sorts of detector inefficiencies, such as vignetting, bad pixels, dead time, and effective areas etc., that can appear in different energy bands for each pointed observation made during uninterrupted good time intervals that exceed 900 s (see Fig. 1).

3. Data analysis

To extract spin-phase-resolved spectra, one has to determine the rotational phase of each registered photon, meaning that an accurate estimate of the most probable value of the spin-period is required for each epoch. In the case of RX J0720.4–3125, apart from the clear variation detected in the spectrum, a significant variation of the spin-period has also been reported. A number of published papers are devoted to the timing analysis and behavior of spectral variations of this enigmatic object (Cropper et al. 2001, 2004; van Kerkwijk et al. 2007; Hohle et al. 2009). To explain significant phase residuals from a steady spin-down model, different interpretations have been suggested; free precession and glitch possibilities are the most-debated (see, e.g., Hohle et al. 2009, 2012a, possible glitch ($\Delta\nu \sim 4.1(12)\text{ nHz}$ at $\text{MJD} = 52\,866 \pm 73\text{ d}$)). However, sometimes an unambiguous identification of the true spin-period of a spinning compact star, for which the pulsed emission is dominated by emitting areas that have slightly different physical characteristics in comparison to the other parts of the surface or atmosphere such as around the magnetic poles, is very difficult from a simple analysis of a

Fig. 1. Observed count rates of RX J0720.4–3125 in the energy ranges 0.16–1.6 keV, 0.16–0.38 keV, and 0.38–1.6 keV in different *XMM-Newton* EPIC-pn full frame mode observations.

periodogram (Bucccheri et al. 1983; Ögelman & van den Heuvel 1989)², meaning by assigning the frequency corresponding to the most significant peak in a periodogram to the true rotational period.

The problem is that, the genuine spin frequency in a periodogram highly depends on the unknown true light curve shape, such as single-, double-, or multiple-peaked profile. This in turn depends on the physical characteristics of the emitting areas, such as temperatures, sizes, and locations, meaning the ratios and locations of the maxima and minima in a light curve, and the viewing geometry, such as the angles between rotation, magnetic axes, line of sight, and gravitational redshift etc. (for example see Poutanen & Beloborodov 2006).

On the other hand, it is clear that if there is a periodic signal in the dataset with frequency ν , then there is also a less significant signal at 0.5ν . Therefore, the most significant peak in the periodogram unequivocally corresponds to the genuine spin frequency for a single-peaked light curve shape. The situation is not simple for the case of a signal from a double- or multiple-peaked light curve shape. In this case, depending on the parameters mentioned above, the periodogram can be identical to a single-peaked one. Moreover, the dependence of an applied statistic such as Z_m^2 (see Appendix A) from a trial frequency will

² A commonly used statistics in X-ray astronomy is:

$$Z_m^2 = \frac{2}{N} \sum_{k=1}^m \left\{ \left(\sum_{i=1}^N \sin(2k\pi\phi_i) \right)^2 + \left(\sum_{i=1}^N \cos(2k\pi\phi_i) \right)^2 \right\} \text{ with phases } \phi_i \text{ at}$$

trial frequency. Here, $\phi_i = \nu(t_i - t_0) + \dot{\nu}(t_i - t_0)^2/2$ is the phase, ν and $\dot{\nu}$ are the trial frequency and its derivative, t_i ($i = 1, \dots, N$) are the photon arrival times, and t_0 is the epoch of zero phase. We note that Z_m^2 is a sufficient statistic for detection of sinusoidal signals in comparison to the arrival times, if they are distributed uniformly (obeying constant-rate Poissonian counting statistic, i.e., a periodic component is absent).

show a highly significant detection only at twice the true spin frequency, 2ν , if the two emitting areas have identical physical characteristics and favorable viewing geometry such as the orthogonal rotator (see expression for Z_1^2 and Fig. 2 in Hambaryan et al. 2015).

These arguments motivated us to perform a complete X-ray timing re-analysis of the observational datasets of RX J0720.4–3125 for the identification of the true pulsation period by using a more advanced analysis, such as a Bayesian analysis of the search, estimate, and testing of the hypothesis following (Gregory & Loredo 1992, hereafter GL). We were also motivated to explore some effects, such as folding into other possible periods, as well half of the frequency of the most significant peak in the periodogram, and checking of the presence of possible asymmetries in the folded light curves over time and various energy ranges, meaning peak-to-peak and minimum-to-minimum ratios and phase differences (for details of methodical concerns, see Hambaryan et al. 2015). Moreover, to solve this challenging problem we perform a study of the distribution of the spin-phases that result from the folding of the photon arrival times into the various frequencies mentioned above (see Appendix A).

3.1. X-ray timing analysis

First of all, for each pointed observation (Table 1), we used the cleaned photon arrival times (see Sect. 2) to apply the Z_m^2 test for periodicity detection in the frequency range of 0.05–0.5 Hz in the broad energy band (0.16–1.6 keV). We note that this range includes, as reported in the literature, the spin frequency 0.119 Hz of RX J0720.4–3125 and the second harmonic and subharmonic frequencies. As already mentioned in Sect. 3, the periodograms reveal a very significant peak at the frequency 0.119 Hz. Apart from that they also show a less significant second peak at the frequency of 0.0596 Hz, in other words close to half the frequency of the most significant peak. In Fig. 2, we present the Z_m^2 statistic periodogram of two observations (Obsids: 0124100101 and 0164560501, see Table 1 and Fig. 1) that corresponds to the first and the brightest phase of RX J0720.4–3125 in the long-term light curve.

If we apply the H -test developed by de Jager et al. (1989), de Jager & Büsching (2010)³ to the datasets, we identify a second peak in all pointed observations of RX J0720.4–3125 in different energy bands. Moreover, for some energy bands, the pointed observations show a more significant peak corresponding to 0.0596 Hz instead of 0.119 Hz.

These results led us to consider the frequency 0.0596 Hz as the most probable true spin-period of RX J0720.4–3125 and to perform a detailed analysis of the light curve shapes in different energy bands. For this purpose, we applied the GL method, together with Z_m^2 and H -statistics, to the different datasets in various energy bands for the frequency ranges 0.05–0.07 Hz and 0.10–0.14 Hz separately. In general, the GL method first tests if a constant, variable, or periodic signal is present in a dataset, given photon arrival times of events or a binned dataset obeying Poissonian counting statistics, when we have no specific prior information about the shape of the signal.

In the GL method, periodic models are represented by a signal of folded photon-arrival times into trial frequencies with a light curve shape that is a stepwise function with m phase bins

³ The H -test $H \equiv Z_m^2 - 4m + 4$ is a modified Z_m^2 test that includes higher harmonics and finds an optimal number of them to be included in the search and detection of a periodic signal.

Fig. 2. Z_m^2 test statistic (Z_1^2 left and Z_4^2 right panel, respectively) vs. trial period of two pointed *XMM-Newton* EPIC-pn observations (Obsids: 0124100101 and 0164560501, see Table 1 and Fig. 1) in the 0.24–0.28 keV narrow energy band of RX J0720.4–3125.

per period plus a noise contribution. With this kind of a model we are able to approximate a phase-folded light curve of any shape. Hypotheses for detecting periodic signals represent a class of stepwise, periodic models with the following parameters: trial period, phase, noise parameter, and number of bins (m). The most probable model parameters are then estimated by marginalization of the posterior probability over the prior specified range of each parameter. In Bayesian statistics, posterior probability contains a term that penalizes complex models unless there is no significant evidence to support that hypothesis. Therefore we calculate the posterior probability by marginalizing over a range of models corresponding to a prior range of number of phase bins, m , from two to twenty. For all cases that have strong support for the presence of a periodic signal, using posterior probability density functions we estimated the most probable values and uncertainties of all parameters, meaning the frequency⁴,

⁴ In the GL method, a posterior probability density function is evaluated for any parameter of the model by application of Bayes theorem, e.g., for the trial frequency (ν):

$$p(\nu|D, M_m) = \frac{C}{\nu} \int_0^{2\pi} d\phi \frac{1}{W_m(\nu, \phi)}.$$

C and $W_m(\nu, \phi) = \frac{N!}{n_1! n_2! \dots n_m!}$ are the normalization constant and number of ways the binned distribution could have arisen “by chance”. $n_j(\nu, \phi)$ is the number of events falling into the j th of m phase bins given the frequency, ν , and phase, ϕ . N is the total number of photons

phase, and the most probable number of phase bins of a periodic signal. Results are presented in Table 2 and Figs. 3 and 4. Using these results, we created phase-folded light curves for all observations of RX J0720.4–3125 in three different energy bands: soft, 0.16–0.38 keV; hard, 0.38–1.6 keV; and broad, 0.16–1.6 keV, Figs. B.1–B.4).

In addition, we constructed combined phase-folded light curves (Figs. 5, B.5) for all observations and different data groups (see Sect. 3.2) that clearly show the dependence of the double-peak light curve profile upon the energy band (cf., e.g., Hambaryan et al. 2011). Indeed, the relative strength of the modulation⁵ for all the combined data groups are 0.07 ± 0.02 , 0.05 ± 0.02 , 0.07 ± 0.02 , and 0.12 ± 0.04 in

(for details, see Gregory & Loredo, 1992, 1993). For its uncertainty, the highest posterior density (HPD) interval is determined as a probabilistic region around a posterior mode or moment, and is similar to a confidence interval in classical statistics.

⁵ We have employed the relative strength of modulation

$$A = \frac{\sum_i |CR_i - CR_{\text{mean}}|}{\sum_i CR_i}, \quad (1)$$

instead of the pulsed-fraction $PF = (CR_{\text{max}} - CR_{\text{min}})/(CR_{\text{max}} + CR_{\text{min}})$ as an appropriate descriptor of the pulsed emission for complex shape light curves, in which the strengths of variations are independent of the light curve shape, i.e., single or double pulse profile. Here CR is the count-rate. We note that our defined relative strength of the modulation for a single-peaked sinusoidal light curve differs by a factor of $2/\pi$ from the definition of semi-amplitude or pulsed-fraction.

Table 2. Results of the application of the GL method in the frequency range of 0.05–0.07 Hz for the broad energy band of 0.16–1.60 keV.

Obsid	MJD* [mid-time]	Frequency (Hz)	HPD _{lo} (68%) [Hz]	HPD _{hi} (68%) [Hz]	Most probable number of phase bins
0124100101	51 677.485469	0.0595869	0.0595863	0.0595873	12
0156960201	52 584.936555	0.0595867	0.0595858	0.0595873	10
0156960401	52 587.011838	0.0595868	0.0595859	0.0595875	11
0164560501	53 147.743162	0.0595923	0.0595919	0.0595928	12
0300520201	53 488.686427	0.0595867	0.0595863	0.0595873	12
0300520301	53 636.310901	0.0595867	0.0595860	0.0595872	11
0400140301	53 877.355 521	0.0595876	0.0595870	0.0595882	12
0400140401	54 044.616108	0.0595868	0.0595864	0.0595876	12
0502710201	54 225.859 109	0.0595874	0.0595851	0.0595878	12
0502710301	54 421.380495	0.0595865	0.0595859	0.0595868	10
0554510101	54 911.796392	0.0595857	0.0595844	0.0595868	11
0601170301	55 096.379108	0.0595869	0.0595858	0.0595876	11
0650920101	55 662.143634	0.0595851	0.0595841	0.0595858	4
0670700201	55 684.233810	0.0595872	0.0595864	0.0595877	11
0670700301	55 835.339910	0.0595873	0.0595866	0.0595877	12
0690070201	56 188.51 4699	0.0595870	0.0595865	0.0595874	10

Notes. (*) The mid-time of the pointed observation.

Fig. 3. Estimated most probable spin frequencies of RX J0720.4–3125 (for details, see text and Table 2) obtained by application of the GL method for periodicity searching in the broad energy band for ObsIds 0124100101 and 0164560501 (*XMM-Newton* EPIC-pn Full Frame mode). The highest posterior density (HPD) intervals, 68% and 95%, are determined as a probabilistic region around a posterior mode and depicted as vertical dashed lines in black and red, respectively.

the energy bands of 0.16–0.5 keV, 0.5–0.6 keV, 0.6–0.7 keV, 0.7–2.0 keV, respectively. Thus, the X-ray phase-folded light curves of RX J0720.4–3125 (cf., RBS1223 with the spin-period of 10.31 s, Hambaryan et al. 2011) have a markedly double-humped shape with different ratios of maxima in different spectral bands. Moreover, they can be parameterized with a mixed von Mises distribution (see Appendix A, Fig. A.1, Table A.1), in other words with a double-peaked light curve profile that shows a dependence of the estimated parameters, such as mean directions, concentrations, and proportion upon the energy band and indicates that radiation emerges from at least two emitting areas. These spin-phase-folded light curves and the estimated parameters from the distribution of phases may serve as rough estimates of the viewing geometry and physical characteristics of the emitting areas (see below).

3.2. Rotational phase-resolved X-ray spectral analysis

To create high signal-to-noise (S/N) ratio spin-phase-resolved spectra of RX J0720.4–3125 we coadded spectra for some sequential pointed observations where it shows approximately similar brightness in the broad energy band (0.16–1.6 keV). Namely, the total observational interval that spanned almost 12.5 yr was divided into five groups (see, Fig. 1 vertical dashed-lines). The second group represents the brightest phase of RX J0720.4–3125 in the hard energy band.

Firstly, we simultaneously fit the phase-averaged, high S/N ratio spectral data collected from different observations (Table 1), which were greater than thirty with each spectral bin containing at least 1000 counts. We used a combination of an absorbed blackbody and a Gaussian absorption line,

Table 3. Simultaneous fitting results of combined, spin-phase averaged X-ray spectra of RX J0720.4–3125 with a spectral model of an absorbed blackbody with and without a Gaussian absorption line (in XSPEC `tbabs*bbbodyrad*gabs`).

Fit parameter**	Groups*					Remark
	I	II	III	IV	V	
Energy range	0.16–1.2 keV					
$nH \times 10^{20}$ [cm ⁻²]	1.06 ± 0.06	1.09 ± 0.11	0.87 ± 0.10	1.02 ± 0.10	0.92 ± 0.07	free
kT [eV]	85.2 ± 0.4	91.3 ± 0.4	91.5 ± 0.6	89.5 ± 0.5	88.2 ± 0.4	free
E_{line} [eV]	329.0 ± 3.3	ibid	ibid	ibid	ibid	linked
σ_{line} [eV]	18.3 ± 8.0	70.0 ± 6.7	77.2 ± 8.7	70.4 ± 8.6	69.2 ± 8.4	free
D_{line} **	0.011 ± 0.002	0.076 ± 0.009	0.074 ± 0.010	0.059 ± 0.096	0.044 ± 0.072	free
Reduced χ^2	$\chi^2/\text{d.o.f.} = 853.07/581 = 1.47$					
$nH \times 10^{20}$ [cm ⁻²]	1.04 ± 0.04	ibid	ibid	ibid	ibid	linked
kT [eV]	85.2 ± 0.4	90.6 ± 0.3	91.1 ± 0.3	89.1 ± 0.3	88.0 ± 0.3	free
E_{line} [eV]	328.0 ± 6.2	323.0 ± 3.1	340.7 ± 3.7	329.9 ± 4.0	342.9 ± 4.3	free
σ_{line} [eV]	26.3 ± 10.9	82.7 ± 3.5	74.4 ± 5.3	76.8 ± 5.3	63.7 ± 5.6	free
D_{line} **	0.012 ± 0.002	0.095 ± 0.006	0.069 ± 0.007	0.066 ± 0.006	0.039 ± 0.004	free
Reduced χ^2	$\chi^2/\text{d.o.f.} = 835.15/587 = 1.42$					

Notes. (*) See Sect. 3.2, Table 1 and Fig. 1 for definition of the data groups. (**) Parameter uncertainty corresponds to the 90% of the confidence interval. D_{line} presents the strength of a Gaussian line (for details see XSPEC manual). **Fig. 4.** Spin frequency history of RX J0720.4–3125 (for details, see text and Table 2) obtained by application of the GL method for periodicity searching in the broad energy band for different *XMM-Newton* EPIC-pn full frame mode observations.

a multiplicative component, in the model (in XSPEC `tbabs*bbbodyrad*gabs`, see Fig. 6 and Table 3). During the fitting, the interstellar absorption and the Gaussian line energy were kept linked or free (see, Table 3), whereas other parameters such as blackbody temperature, Gaussian width, and normalizations were left free for different data groups. We note that if the linked parameters were also left free, the fitting results remained almost unchanged. It is also worthwhile to note that these spectra cannot be fit with the model of fixed parameters of the absorption line feature at the pre-brightening phase.

Secondly, we extracted high S/N ratio spectral data, which were greater than ten with each spectral bin containing at

least 100 counts, corresponding to the eight rotational phase bins, meaning the maxima, minima, and intermediate stages, of the double-peaked pulse profile and collected from different observations (see, Figs. 5, B.5). These forty spectra for the five data groups were subject to simultaneous fitting with a number of highly magnetized INS surface/atmosphere models developed by us (Suleimanov et al. 2010) and implemented into the X-ray spectral fitting package XSPEC (Hambaryan et al. 2011). They are based on various local surface models and compute rotational-phase-dependent integral emergent spectra of INS using analytical approximations. The basic model includes temperature and magnetic field distributions over the INS surface, viewing geometry, and gravitational redshift. Three local radiating surface models are also considered: a naked condensed iron surface with a partially ionized hydrogen model atmosphere that is either semi-infinite or finite on top of the condensed surface. To compute an integral spectrum, the model uses an analytical expression for the local spectra, an emitting condensed iron surface, and a diluted blackbody spectrum transmitted through the thin magnetized atmosphere with an absorption feature. The center of the line depends on the local magnetic field strength, representing either the ion cyclotron line for completely ionized iron and/or the blend of the proton-cyclotron and bound-bound atomic transitions in neutral hydrogen. In the latter case, the center of the line is parameterized by half of a Gaussian line with the following parameters: E_{line} , optical depths $\tau = \tau^0 \exp\left(-\frac{(E-E_{\text{line}})^2}{2\sigma_{\text{line}}^2}\right)$, and the widths σ_{line} (for details and further references, see Suleimanov et al. 2010; Hambaryan et al. 2011).

Given the large number of free parameters in the model, we performed a preliminary analysis to get rough estimates, or constraints, on some of them. From the observed double-peaked profile light curve shape in different energy bands (see, right panel Fig. 5 and Fig. B.5), it was already clear that the two emitting areas have slightly different spectral and geometrical characteristics, such as the relatively cooler one has a larger size. Secondly, some constraints, in other words initial values for the fitting, on magnetic field strengths at the

Fig. 5. Combined phase-folded light curves for RBS 1223 (left panel Hambaryan et al. 2011) and RX J0720.4–3125 (right panel) in different energy bands: 0.16–0.5 keV, 0.5–0.6 keV, 0.6–0.7 keV, and 0.7–2.0 keV.

Fig. 6. Combined spin-phase averaged spectra and simultaneously-fit spectral model of an absorbed blackbody and a Gaussian absorption line for RX J0720.4–3125. We note that the observed flux is almost the same before and after the variation at $E_0 \sim 0.38$ keV.

poles may be used on the base of the period and its derivative values⁶, assuming magnetic dipole braking as a main mechanism

⁶ A steady spin-down of RX J0720.4–3125 was derived (Hambaryan & Neuhäuser 2016) based on the spin frequency history (Fig. 4) by robust linear fitting, i.e., eliminating the outliers: $P_0 = 16.78215$ s, $\dot{P} = 1.86 \times 10^{-13}$ ss⁻¹, MJD = 51 677.104103, and $\Delta\nu \sim 10^{-6}$ – 10^{-5} Hz at MJD = 53 147.442236

of the spin-down of RX J0720.4–3125 (see, e.g., Hohle et al. 2012b; Hambaryan & Neuhäuser 2016). Moreover, from the peak-to-peak separation and the ratio of the minima in this double-humped light curve, we may locate the cooler one at an offset angle with respect to the magnetic axis and azimuth (Schwope et al. 2005).

Having these general constraints on temperatures and magnetic field strengths at the poles, we simulated a large number of photon spectra, absorbed blackbody with Gaussian absorption line, folded with the response of the *XMM-Newton* EPIC-pn camera with the same observing mode of RX J0720.4–3125. We also included the interstellar absorption (for parameters see Table 3), and used a characteristic magnetic field strength of $B \sim 3.0 \times 10^{13}$ Gauss. The predicted phase-folded light curves in four spectral ranges, 0.16–0.5 keV, 0.5–0.6 keV, 0.6–0.7 keV, and 0.7–2.0 keV, were cross-correlated with the observed ones (Fig. 5). The predicted curves have a purely dipolar magnetic field configuration that depends on the relative orientation between the magnetic axis, the rotation axis, the observer line of sight, and the gravitational redshift (see upper left panel of Fig. A.1), and were normalized to the mean of the brightness. We inferred some constraints on the parameters from their unimodal distributions when cross-correlation coefficients exceeded 0.9 in the mentioned four energy bands simultaneously. We then used the parameters as the initial input values and as the lower and upper bounds for fitting purposes. For example, gravitational redshift cannot exceed 0.3 because it is not possible to obtain the observed relative strength of modulation at larger z due to strong light bending, and the antipodal shift angle must be less than 5° due to the observed phase separation between the two peaks in the light curve. It is also evident that the sum of the inclination angle of the line of sight and magnetic poles relative to the rotational axis are already constrained by the light curve class (see, Poutanen & Beloborodov 2006, class III).

Having the abovementioned crude constraints and input values of the free parameters, we performed fitting with the models implemented in the XSPEC package of all the combined spectra of RX J0720.4–3125 (see Table 1 and Fig. 1) simultaneously, meaning that each of those phase-resolved spectra were

Fig. 7. Box-plots of the MCMC fit parameters kT_{p1} , kT_{p2} (upper panel) and τ , σ (bottom panel) in the energy band of 0.16–2.0 keV of RX J0720.4–3125 for different groups combined from multi-epoch pointed *XMM-Newton* EPIC-pn observations with spectral model of a condensed iron surface and a partially ionized hydrogen atmosphere (for details see text and Table 4).

considered as a different dataset with fixed spin-phase and with linked parameters to the other datasets.

The model that was fit is presented in Table 4 (see also, upper left panel of Fig. A.1). The model had: a chi-squared fit statistic value of 6070 for 4910 degrees of freedom; a dipolar field configuration with magnetic field strengths at the poles of $B_{p1} \approx B_{p2} \sim (0.3 \pm 0.02) \times 10^{14}$ G; an orthogonal rotator with an angle between the magnetic and the rotation axis of $\theta = (89 \pm 5)^\circ$; an inclination angle with an angle between the rotation axis and the observer line of sight of $i = (42 \pm 3)^\circ$; and a shift angle of $\kappa \sim 0^\circ$. To assess the degree of uniqueness and to estimate confidence intervals of the determined parameters, we additionally performed a Markov chain Monte Carlo (MCMC) fitting as implemented in XSPEC. The parameters of the model that was fit are presented in Table 4 and Fig. 7. In Fig. 8 we present the probability density distributions of the gravitational redshift. We note that independent initial input values of parameters of the MCMC approach converged to the same values in different chains.

4. Discussion

Our comprehensive X-ray timing analysis of RX J0720.4–3125, based on the *XMM-Newton* EPIC-pn long-term pointed observations, shows that the most probable and plausible spin frequency of this INS is 0.0596 Hz, instead of 0.119 Hz as reported in the literature. The phase-folded light curves of this period show a markedly double-peaked shape, which can be formally parameterized with a mixed von Mises distribution that contains significantly different components, such as concentration and mixing proportion parameters (see Appendix A).

We had seen a similar picture with RBS 1223, another INS. The pulsation discovery paper reports on a spin-period 5.16 s (Hambaryan et al. 2002); however, a more detailed study shows that true period is 10.31 s (Haberl et al. 2003; Haberl 2004; Schwöpe et al. 2005, 2007). Namely, this period with its double-peaked profile is suggested by the presence of a second peak with lower significance in the power spectrum (Haberl 2004). Further analysis of light curves in different energy bands and phase-resolved spectroscopic study (Schwöpe et al. 2007;

Table 4. Simultaneous fitting results of combined, spin-phase resolved X-ray spectra of RX J0720.4–3125 using a condensed iron surface and hydrogen atmosphere model.

Fit parameter**	Groups*					Remark
	I	II	III	IV	V	
$nH \times 10^{20}$ [cm ⁻²]	1.20 ± 0.06	1.17 ± 0.06	1.17 ± 0.06	1.18 ± 0.06	1.18 ± 0.06	free
kT_{p1} [eV]	104.0 ± 4.3	110.2 ± 4.1	108.3 ± 4.5	107.5 ± 4.0	106.6 ± 3.6	free
kT_{p2} [eV]	102.5 ± 4.2	107.7 ± 3.9	109.5 ± 4.8	105.8 ± 3.8	105.6 ± 3.4	free
$B_{p12} \times 10^{13}$ [G]	3.00 ± 0.15	ibid	ibid	ibid	ibid	linked
θ [°]	88.9 ± 4.8	ibid	ibid	ibid	ibid	linked
i [°]	42.8 ± 2.2	ibid	ibid	ibid	ibid	linked
z	0.205 ± 0.001	ibid	ibid	ibid	ibid	linked
σ_{line} [ev]	268.3 ± 50.9	313.4 ± 38.6	329.0 ± 56.9	302.7 ± 60.3	310.6 ± 60.4	free
τ_{line}	0.80 ± 0.24	1.56 ± 0.26	1.38 ± 0.26	1.37 ± 0.26	1.18 ± 0.22	free
Reduced χ^2	$\chi^2/\text{d.o.f.} = 6070/4910 = 1.24$					
C-statistic	6050 using 5050 PHA bins and 4910 d.o.f.					

Notes. (*) See Sect. 3.2, Table 1, Fig. 1, and Fig. A.1 for definition of the data groups and parameters. (**) Parameter uncertainty corresponds to the 95% of the HPD interval.

Fig. 8. Probability density distributions of gravitational redshift by MCMC fitting with the model of a neutron star with a condensed surface and a partially ionized hydrogen layer above it. The most probable parameter value is indicated by the vertical solid line. Vertical dashed lines indicate the highest probability interval (68% and 95%, for details see text).

Hambaryan et al. 2011) provides additional confirmation of the plausibility of the model of RBS 1223 in which emergent emission is dominated by two emitting areas with slightly different parameters. Yet another case, Speagle et al. (2011), notes that without the radio period, they might have actually identified the pulse period of PSR J0726 as half of the true value. Cropper et al. (2001) performed spin pulse profile analysis of RX J0720.4–3125 based on earlier *XMM-Newton* observations and folded photon-arrival times that are twice the detected period, via Fourier periodogram, that produces a double-humped light curve with different count rates at the different peaks. However, having no indication of subharmonics in the periodogram, they do not really consider twice the period as correct. Another possible indication of the double-peaked light curve shape

of RX J0720.4–3125 may be the non-sinusoidal light curve shape and the reported phase lag in different energy bands (Haberl et al. 2006; Hohle et al. 2009). Indeed, slightly different effective temperatures and sizes of two emitting areas at the magnetic poles that are not located exactly in podal and antipodal directions may cause this kind of a lag, if the photon arrival times fold into phases with half of the value of the genuine spin-period (see Fig. A.1).

The combined phase-resolved spectra of RX J0720.4–3125 can also be fit by emergent radiation of a spectral model of an iron condensed surface with a partially ionized hydrogen atmosphere above it. Formally they can also be fit by a blackbody spectrum with a proton-cyclotron absorption Gaussian line and a peaked angular distribution of the emergent radiation that is typical for an electron scattering atmosphere. In both cases, two emitting areas with slightly different characteristics are required (see Table 4). It is worth noting that the resulting fit parameters are very similar for different spectral models, and were also confirmed by an MCMC approach with different input parameters.

However, we believe that the emission properties that are due to the condensed surface model with a partially ionized, optically thin hydrogen layer above it, including vacuum polarization effects, are more physically motivated. Moreover, semi-infinite atmospheres have somewhat fan-beamed emergent radiation (see Suleimanov et al. 2010; Hambaryan et al. 2011, and references therein) and it seems impossible to combine a proton-cyclotron line with a pencil-beamed emergent radiation.

Emission spectra based on realistic temperature and magnetic field distributions with strongly magnetized hydrogen, or other light element, atmospheres are formally still an alternative⁷, but they are unphysical because the absorption line is added by hand.

A pure proton-cyclotron absorption line scenario can be excluded owing to the width of the observed absorption spectral feature in the X-ray spectrum of RX J0720.4–3125. Magnetized

⁷ We failed in our attempt to fit the combined, phase-averaged spectrum of RX J0720.4–3125 by a partially ionized, strongly magnetized hydrogen or a mid-Z element plasma model (XSPEC nsmax, Mori & Ho 2007; Ho et al. 2008), or two spots or purely condensed iron surface models. It is worth noting that an acceptable fit is obtained by the nsmax model with an additional, multiplicative Gaussian absorption line component (model gabs).

semi-infinite atmospheres predict too low an equivalent width of the proton-cyclotron line in comparison with the observed one. This result of spectral modelling, meaning the fitting with the condensed surface model with a partially ionized, optically thin hydrogen atmosphere above it, including vacuum polarization effects, suggests a true radius of RX J0720.4–3125 of 13.3 ± 0.5 km for a standard neutron star of 1.4 solar mass, and indicates a stiff equation of state of RX J0720.4–3125 (for similar results, see also Ho et al. 2007; Heinke et al. 2006; Suleimanov & Poutanen 2006; Suleimanov et al. 2011).

Our model fits provide the angles between our line of sight and both the rotation axis and the magnetic field axes, for which we obtain $42 \pm 3^\circ$ and $47 \pm 6^\circ$, respectively. The direction of motion should be similar to the former angle in case of spin-orbit almost (see, e.g., Ng & Romani 2007, $\theta < 15^\circ$) alignment.

In Tetzlaff et al. (2011), the direction of motion was determined to be $62 \pm 9^\circ$, by tracing back the motion of RX J0720.4–3125 to its possible birth association Trumpler-10. The direction of motion determined kinematically is thus roughly consistent with an almost-alignment of the spin and orbit.

5. Conclusions

We carried out a complete X-ray timing re-analysis of the multi-epoch observational datasets of RX J0720.4–3125 for an identification of the true spin-period. The genuine spin-period of the isolated neutron star RX J0720.4–3125 is twice of the previously reported value in the literature, 16.78 s instead of 8.39 s, with a markedly double-peaked light curve profile that depends on time and energy.

The observed phase-resolved spectra of the INS RX J0720.4–3125 are satisfactorily fit with slightly different physical and geometrical characteristics of two emitting areas, using the model of a condensed iron surface, with a partially ionized, optically thin hydrogen atmosphere above it, including vacuum polarization effects, and an orthogonal rotator. The fit also suggests the absence of a strong toroidal magnetic field component. Moreover, the determined mass-radius ratio, $(M/M_\odot)/(R/\text{km}) = 0.105 \pm 0.002$, suggests a very stiff equation of state of RX J0720.4–3125.

It will be certainly worth doing more work to develop a detailed spectral model (also with quadrupole magnetic field configuration) computation in the near future and apply it to the phase-resolved spectra of other INSs and magnetars, in particular high resolution spectra observed by *XMM-Newton* and *Chandra* which possibly have other absorption and emission features (Hambaryan et al. 2009; Potekhin 2010; Hambaryan & Neuhäuser 2016).

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Table A.1. Results of the fitting of spin-phases with a two-component mixed von Mises distribution.

Groups	Energy range [keV]	N [counts]	μ_1/μ_2	κ_1/κ_2	p
GRI	0.16–0.5	294516	0.507 ± 0.003	3.124 ± 0.367	0.349
"	0.50–0.6	30943	0.508 ± 0.007	2.236 ± 0.880	0.415
"	0.60–0.7	16125	0.479 ± 0.011	2.032 ± 0.843	0.408
"	0.70–1.2	14338	0.495 ± 0.013	3.286 ± 1.714	0.311
GRII	0.16–0.5	359528	0.494 ± 0.002	3.101 ± 0.282	0.352
"	0.50–0.6	51604	0.496 ± 0.005	2.622 ± 0.732	0.392
"	0.60–0.7	30956	0.514 ± 0.008	1.333 ± 0.108	0.511
"	0.70–1.2	29568	0.502 ± 0.006	2.159 ± 0.483	0.402
GRIII	0.16–0.5	234977	0.502 ± 0.003	3.138 ± 0.271	0.347
"	0.50–0.6	32629	0.500 ± 0.011	2.397 ± 1.151	0.381
"	0.60–0.7	19045	0.520 ± 0.009	1.363 ± 0.294	0.512
"	0.70–1.2	17789	0.524 ± 0.012	1.454 ± 0.200	0.514
GRIV	0.16–0.5	210538	0.508 ± 0.003	2.796 ± 0.330	0.360
"	0.50–0.6	27712	0.498 ± 0.008	1.911 ± 0.728	0.505
"	0.60–0.7	15814	0.508 ± 0.009	1.467 ± 0.476	0.507
"	0.70–1.2	14840	0.508 ± 0.007	1.346 ± 0.368	0.506
GRV	0.16–0.5	352642	0.497 ± 0.002	3.130 ± 0.260	0.342
"	0.50–0.6	43187	0.505 ± 0.005	1.611 ± 0.461	0.507
"	0.60–0.7	23725	0.498 ± 0.006	1.709 ± 0.451	0.484
"	0.70–1.2	21436	0.508 ± 0.011	1.439 ± 0.636	0.507
Student's t-test			H0: $\mu_1/\mu_2 \equiv 0.5$	H0: $\kappa_1/\kappa_2 \equiv 1.0$	H0: $p \equiv 0.5$
P-value			0.14	9.8×10^{-9}	0.005
Wilcoxon test					
P-value			0.08	1.9×10^{-6}	0.008

Appendix A: True spin-period of RX J0720.4–3125

Most of the methods for the search and detection of a periodicity in X-ray astronomy are based on the analysis of the distribution of the phases ($\theta_i = \nu(t_i - t_0) + \dot{\nu}(t_i - t_0)^2/2$), which are the result of folding photon arrival times (t_i ($i = 1, \dots, N$)) with a trial frequency (ν) and its derivative ($\dot{\nu}$) for the epoch of zero phase (t_0 , e.g., Z_m^2 , H -test, the GL method, etc.). This procedure transforms the time variable to phases distributed on a circle.

To disentangle between a single- or multiple-peaked profile of a light curve shape the phases have to be analyzed statistically. For this purpose, commonly-used circular distributions are the von Mises distribution, the wrapped normal distribution, and the wrapped Cauchy distribution, etc. The von Mises distribution ($f(\theta; \mu, \kappa) = \frac{1}{I_0(\kappa)} e^{\kappa \cos(\theta - \mu)}$) has two parameters: the mean direction and concentration of the data distributed on a circle ($I_0(\kappa)$ is the modified Bessel function of order 0).

For demonstration purposes, in Fig. A.1 we show a sample of the light curve shape that is modeled by an exponentiated Fourier series or generalized von Mises distribution (Connors 1997) for two components ($r(t_i) \propto e^{\sum_{j=1}^2 [-\kappa_j \cos(\frac{2\pi t_i}{P} + \varphi_j)]}$, with the phases $\varphi_{1,2}$ of photon arrival times (t_i , $i = 1, N$) of a periodic signal with the period of P). The distribution of phases, indicated using a rose diagram that is a circular histogram plot of a mixed two von Mises distribution (Mardia & Jupp 2009; Connors 1997), mean that $f(\theta; \mu_1, \mu_2, \kappa_1, \kappa_2, p) = \frac{p}{I_0(\kappa_1)} e^{\kappa_1 \cos(\theta - \mu_1)} + \frac{1-p}{I_0(\kappa_2)} e^{\kappa_2 \cos(\theta - \mu_2)}$,

where μ_1, μ_2 are the mean directions of the phases, κ_1, κ_2 are concentrations, and p is a mixing proportion. These parameters determine the locations, peakedness, and relative contribution of each component of a double-profile light curve, meaning that they are expressing the effective temperatures and sizes of the dominating areas from which the emergent radiation arises.

To fit the double-peaked light curve profile we coded an expectation-maximization algorithm, an iterative approach to find the maximum of the likelihood applied to a mixture models (see, e.g., Mardia et al. 2007, 2008), that is used to model the rotational phases of all registered photons (column N in Table A.1) as a mixture of a two component von Mises distribution. The results of the fit are presented in Table A.1, where for the estimation of uncertainties, an additional, bootstrap algorithm was implemented and applied. We generate a bootstrap sample of thirty datasets for each data group by randomly sampling from original spin-phases without replacement. Each of these bootstrap datasets were fit with a mixture of a two component von Mises distribution. The standard deviations of the derived parameters are accepted as an uncertainty measure.

An application of the so-called one sample t-test to the difference of the locations of the peaks (column μ_1/μ_2 , Table A.1, or two-sample paired t-test to the μ_1 and μ_2) of the phase-folded light curve for the double-peaked profile is consistently approximately π , regardless of the energy band. Hence, the null hypothesis on the equality of the ratio of the mean directions of 0.5 (i.e., podal and antipodal directions) is accepted. In contrary, for

Fig. A.1. Upper panel: example of a simulated double-peaked light curve $r(t) \propto e^{\sum_{j=1}^2 [f_j \cos(\frac{2\pi t}{P} + \varphi_j)]}$ with pulsed fractions of each component $f_{1,2}$ of a periodic signal with the period of P , (right panel) and the geometry of the model (left panel). Lower panel: distribution of phases (rose diagram—a circular histogram plot of a mixed two von Mises distribution (Mardia & Jupp 2009; Connors 1997) $f(\theta; \mu_1, \mu_2, \kappa_1, \kappa_2, p) = \frac{p}{I_0(\kappa_1)} e^{\kappa_1 \cos(\theta - \mu_1)} + \frac{1-p}{I_0(\kappa_2)} e^{\kappa_2 \cos(\theta - \mu_2)}$, with the modes of $\mu_1 = \pi/3, \mu_2 = \pi + \pi/3 + \pi/18$, concentrations $\kappa_1 = 1.2, \kappa_2 = 1.5$, and proportion $p = 0.45$) for approximately 2500 photon arrival times for an ideal Poissonian process (right panel) for a periodic signal with a count rate similar to that observed from RX J0720.4–3125. The histogram plot displays directional data and the frequency of each class. Distribution of the phases assumes that the period is equal to half of the true one (left panel).

the parameters of concentration and proportion, κ_1/κ_2 and p , the null hypothesis is rejected, κ_1 and κ_2 are nonidentical populations and the p mean value is not equal to 0.5 (see, Table A.1). We arrive to similar conclusions by an application of a non-parametric Wilcoxon test.

Appendix B: Phase-folded light curves

Here (Fig. B.1), we present rotational phase-folded light curves for all pointed observations of RX J0720.4–3125 performed by *XMM-Newton* EPIC-pn in different energy bands, 0.16–0.38 keV, 0.38–1.6 keV, and 0.16–1.6 keV, with spin frequencies (Table 2), zero-phases derived by an application of the GL method, and using twenty phase bins. All double-peaked

profile light curves are normalized to the mean count rate of the pointed observations in the corresponding energy band.

Appendix C: Phase-resolved spectra with MCMC

As already mentioned, to assess the degree of uniqueness and to estimate the confidence intervals of the determined parameters for the model of an emitting condensed iron surface with a partially ionized hydrogen model atmosphere, (see, Sect. 3.2), we additionally performed a Markov chain Monte Carlo (MCMC) fitting using the Goodman-Wear chain-generating algorithm, which is the default in XSPEC. The results for the basic parameters of the fitted model for two data groups I and II are presented as a matrix plot (see, Fig. C.1).

Fig. B.1. Phase-folded light curves in different energy bands, 0.16–0.38 keV, 0.38–1.6 keV, and 0.16–1.6 keV *left, middle, and right panels*, accordingly, of RX J0720.4–3125 for pointed *XMM-Newton* EPIC-pn observations 0124100101, 0156960201, 0156960401, and 0164560501.

Fig. B.2. For ObsIds 0300520201, 0300520301, 0400140301, and 0400140401 (see Fig. B.1).

Fig. B.3. For ObsIds 0502710201, 0502710201, 0554510101, and 0601170301 (see Fig. B.1).

Fig. B.4. For ObsIds 0650920101, 0670700201, 0670700301, and 0690070201 (see Fig. B.1).

Fig. B.5. Combined phase-folded light curves of different data groups GR I, II, III (left panel, from top to bottom), IV, V and all datasets (right panel) in different energy bands, 0.16–0.50 keV, 0.50–0.60 keV, 0.60–0.70 keV, and 0.70–2.0 keV.

Fig. C.1. Fit spectral model parameters, kept free for all data groups, and MCMC verification results for data groups before the variation (GR I, upper panel), and at the brightest stage (GR II, lower panel) of RX J0720.4–3125. Derived probability density functions of the basic parameters of the fit model are presented in the diagonal. The lower triangular part depicts samplings from the posteriors of two parameters, whereas the upper one depicts their mutual correlations.