Scattering polarization of Mg I b triplet in solar chromosphere

Hao Li^{1,2} and Zhongquan Qu¹

¹ Yunnan Observatories, Chinese Academy of Sciences, Kunming, Yunnan 650011, PR China
 e-mail: sayahoro@ynao.ac.cn

² Graduate School of University of Chinese Academy of Sciences, Yuquan Road, Shijingshang Block Beijing 100049, PR China

Received 18 April 2014 / Accepted 12 November 2014

ABSTRACT

Aims. The magnetic field in the chromosphere of the quiet sun is too weak to be diagnosed via the general Zeeman effect. To obtain the hidden magnetic field vector, a complete comprehension of the linear polarization produced by scattering and modified by magnetic field is needed. We report a systematic and quantitative investigation of the sensitivity of the fractional linear polarization of the magnesium triplet to a weak magnetic field in the chromosphere, which can be a guide for the diagnostics of a weak magnetic field in the solar chromosphere.

Methods. We focus on the fractional linear polarization of the magnesium triplet induced by scattering. We use the quantum theory under the flat-spectrum approximation developed by Landi Degl'Innocenti for the investigation. A seven-level atomic model of magnesium is adopted, while we do not take the radiative transfer or partial frequency redistribution (PRD) into account.

Results. Polarization is directly related to the anisotropy factor, and modified by the continuum, collision, and magnetic field as well as dichroism effect. In our calculation, the elastic collision rates found are not large enough to destroy the population imbalances of the lower levels of the b_2 and b_1 lines. This is the prerequisite for the lower level Hanle effect and dichroism effect. When considering the dichroism effect, the magnetic field can influence the polarization not only in the line cores but also in the line wings. However, the magnetic field cannot obviously influence the polarization degree in the line wings once the continuum becomes significant. The polarization of the triplet is sensitive to the magnetic field between 0.001 and 0.1 Gauss due to the lower level Hanle effect and between 1 and 10 Gauss due to the upper level Hanle effect. In the lower level Hanle effect dominated region, the Q/I amplitude of the b_4 line increases with magnetic field strength because of the coupling among atomic alignments of the involved levels as described in the statistical equilibrium equations, while the amplitudes of the b_2 and b_1 lines decrease. In the upper level Hanle effect dominated region, the most sensitive line is the b_4 .

Conclusions. The polarized lower levels and the contribution of dichroism have to be considered to fit the observed profiles of Mg I b triplet in the line cores. Therefore the b_2 and b_1 lines can be potential candidates for probing sub-Gauss range magnetic field, because their Q/I amplitudes are sensitive to the magnetic field strength between 0.001 and 0.1 Gauss due to the lower level Hanle effect. The b_4 line is suitable for diagnostics of magnetic field between 1 Gauss and 10 Gauss.

Key words. polarization – Sun: chromosphere – magnetic fields – line: profiles – atomic processes

1. Introduction

Solar chromosphere is a passage where the matter and energy transport from the photosphere to the corona. The magnetic field in the chromosphere outside the active region is usually weaker than 100 Gauss (Trujillo Bueno et al. 2004). Measuring the chromospheric magnetic field vector is strongly desired for understanding essential structures, and energy storage and release for solar eruptions, but it is notoriously difficult and challenging. It is well known that the measurement is feasible via the Zeeman effect for probing a relatively strong magnetic field, and via the Hanle effect for detecting weak magnetic field (see reviews by, e.g., Stenflo 2003; Trujillo Bueno 2003). In fact, diagnosing a weak magnetic field through the Hanle effect is also a very complicated mission and it needs a complete physical comprehension of the polarization and depolarization mechanisms.

As pointed out by Stenflo (2013), the second solar spectrum, originated from the scattering processes close to the solar disk limb, is a playground for the Hanle effect through which one can reveal various types of physical effects like J state interference, continuum polarization, optical pumping, hyperfine structure, and isotope effects. Observation with sensitivities of an order of 10^{-4} – 10^{-5} have shown some great details of the spectral

richness and complexity of the second solar spectrum (Stenflo 1996; Stenflo & Keller 1997), and Gandorfer (2000, 2002, 2005) also compiled serial Atlas of the second solar spectrum from the deep UV to the near infrared. It reveals that the linear polarization signal is mainly produced by the population imbalances of the atomic levels, which is caused by the anisotropic incident radiation through the optical pumping. This is the so-called atomic polarization, and often refers to resonance lines. More importantly, a magnetic field can modify this population imbalance via the Hanle effect and can produce an observable signal. It is believed that in this way the magnetic vector can be diagnosed.

As is well known, the fractional polarization profiles will be significantly changed when the magnetic field strength B locates

$$0.1B_c \leq B \leq 10B_c, \quad (1)$$

with

$$B_c \approx \frac{1.137 \times 10^{-7}}{t_{\text{life}} g_J}, \quad (2)$$

where t_{life} and g_J are the lifetime (in seconds) of upper or lower level and the Landé factor of the J level under consideration, respectively, and B_c indicates the critical magnetic field strength.

We focus on the scattering induced linear polarization and the Hanle effect of Mg I b lines with polarized lower levels. The three lines (b₄ 5168.761 Å, b₂ 5174.125 Å, and b₁ 5185.048 Å) share the same upper level, 3S_1 , and sit on three different metastable lower levels, $^3P_{0,1,2}$ (Kelleher & Podobedova 2008). They are formed in the lower layers of the chromosphere and some of them are proven to be suitable to diagnose the magnetic field in the chromosphere via the Zeeman effect (Lites et al. 1988). However, identifying their suitability to probe the weak magnetic field is the task of this work. Observation of Mg I b triplet close to the solar limb by Stenflo et al. (2000) shows similar Q/I amplitudes. To explain the observation, we need to consider the polarizations of the lower levels and the dichroism contribution (see Trujillo Bueno 1999, 2001, 2009). From Eqs. (1) and (2) along with the values of t_{lifc} and g_u , one can find that the linear polarizations of these lines are very sensitive to the sub-Gauss magnetic field due to the lower level Hanle effect, and to the Gauss magnetic field due to the upper level Hanle effect. Therefore it satisfies the basic requirement as a useful tool to diagnose the weak chromosphere magnetic field like Ca II IR triplet (Manso Sainz & Trujillo Bueno 2010). Based on previous studies, our main target is to study how the fractional linear polarization Q/I and U/I profiles of Mg I b triplet behave with a weak magnetic field and how other mechanisms influence the polarization. We use the quantum theory proposed by Bommier (1980) and developed by Landi Degl'Innocenti (1983, 1984) under the flat-spectrum approximation using a slab scattering model.

The outline of this paper is as follows. In Sect. 2, we briefly skip the theoretical framework of the statistical equilibrium equations in the density matrix formalism. The roles of the anisotropic incident radiation, continuum, and collision in modifying the polarizations of the lower levels are presented in Sect. 3. In Sect. 4, the lower level Hanle effect in both line cores and line wings of the triplet is studied quantitatively. Finally, our main conclusions are summarized in the last section.

2. Models and formulations

In this section, we first describe the atomic model of the neutral magnesium, and then a simplified scattering model used in this paper, as well as the associated statistical equilibrium equations of the density matrix formalism. All of these form the theoretical basis of our calculations.

2.1. The atomic model

We adopt a seven-level atomic model of neutral ^{24}Mg atom, which consists of the upper levels $3S4P\ ^3P_{0,1,2}$, the middle level $3S4S\ ^3S_1$ (i.e., the upper level of the Mg I b triplet), and metastable lower levels $3S3P\ ^3P_{0,1,2}$. Because of the fact that ^{24}Mg does not have net nuclear spin, we did not take the hyperfine structure into account while a total of six electric dipole transitions are under consideration. The transition probabilities of magnesium given by Kelleher & Podobedova (2008) are listed in Table 1 and compiled at National Institute of Standards and Technology (NIST) atomic spectra database (Kramida et al. 2013). The Landé factors are calculated under LS coupling approximation.

2.2. The scattering model

For simplicity, a plane parallel slab model is adopted, where the atmosphere is simplified to be an optically thin slab and

Table 1. Data for seven-level atomic model of Mg I.

Transition	λ_{vac} (nm)	A_{ul} (10^7 s^{-1})	$C_S^{(0)}(\alpha J, \alpha_u J_u)/N_e$ ($10^{-7}\text{ s}^{-1}\text{ cm}^3$)
$3S4S\ ^3S_1-3S3P\ ^3P_0$	516.876	1.13	0.159
$3S4S\ ^3S_1-3S3P\ ^3P_1$	517.413	3.37	0.476
$3S4S\ ^3S_1-3S3P\ ^3P_2$	518.505	5.61	0.798
$3S4P\ ^3P_2-3S4S\ ^3S_1$	1502.91	1.35	9.41
$3S4P\ ^3P_1-3S4S\ ^3S_1$	1504.44	1.34	9.37
$3S4P\ ^3P_0-3S4S\ ^3S_1$	1505.18	1.34	9.39

Notes. $C_S^{(0)}(\alpha J, \alpha_u J_u)$ is the super-elastic collision rates in the temperature of 5755 K.

composed of magnesium atoms, thus the radiative transfer is not considered. This model was also used by Manso Sainz & Landi Degl'Innocenti (2002) and Belluzzi et al. (2007, 2009) to investigate the theoretical profiles of the second solar spectrum. A reference system with z axis directed along the solar radius is adopted, and the radiation field is described by means of the irreducible tensor of the radiation (Landi Degl'Innocenti 1983),

$$J_Q^K(\nu) = \int \frac{d\Omega}{4\pi} \sum_{i=0}^3 \mathcal{T}_Q^K(i, \Omega) S_i(\nu, \Omega), \quad (3)$$

where $S_i(\nu, \Omega) = I, Q, U, V$ indicate the four Stokes parameters, and $\mathcal{T}_Q^K(i, \Omega)$ is the irreducible spherical tensor to signify the geometry of the incident radiation. Assuming that the incident radiation field is unpolarized and axissymmetric about the radius, it is easily understood that only two components $J_0^0(\nu)$ and $J_0^2(\nu)$ are nonzero in this system,

$$\begin{aligned} J_0^0(\nu) &= \oint \frac{d\Omega}{4\pi} I(\nu, \mu), \\ J_0^2(\nu) &= \oint \frac{d\Omega}{4\pi} \left[\frac{1}{2\sqrt{2}} (3\mu^2 - 1) I(\nu, \mu) \right], \end{aligned} \quad (4)$$

where $I(\nu, \mu)$ is the incident radiation intensity, and $\mu = \cos \theta$ with θ indicating the heliocentric angle. Note that $J_0^0(\nu)$ is the mean radiation intensity, and the atomic polarization comes from $J_0^2(\nu)$ due to the anisotropy of the radiation intensity. For simplicity, we use two dimensionless quantities $\bar{n}(\nu)$, the mean number of photons, and then w_ν , the anisotropy factor describing the incident radiation field. The values of $\bar{n}(\nu)$ and w_ν are related to the radiation field tensor components through the equations,

$$\bar{n} = \frac{c^2}{2h\nu^3} J_0^0(\nu), \quad w_\nu = \sqrt{2} \frac{J_0^2(\nu)}{J_0^0(\nu)}. \quad (5)$$

The mean number of photons can be estimated by

$$\bar{n} = \frac{1}{\exp\left(\frac{h\nu}{kT}\right) - 1}. \quad (6)$$

In the following, we adopt a typical temperature $T = 5755\text{ K}$ from the FLA-C atmosphere model (Fontenla et al. 1993) at a height of 905 km, a mean formation height where the magnesium triplet is roughly considered to be formed. We do not consider the quantum interferences between different J levels in our calculation, because the value of $\bar{n}$ is too small to influence the scattering polarization profile in the typical solar situation (Belluzzi & Trujillo Bueno 2011).

2.3. The statistical equilibrium equations

The density matrix was introduced by [Fano \(1957\)](#) to describe the excitation states of the atomic system. The diagonal matrix elements are the population of the atomic levels, and the nondiagonal matrix elements are the interference term. Here, we adopt the multipolar components of the atom density matrix ([Omolt 1977](#); [Landi Degl'Innocenti 1996](#)):

$$\rho_Q^K(J) = \sum_{MM'} (-1)^{J-M} \sqrt{2K+1} \begin{pmatrix} J & J & K \\ M & -M' & -Q \end{pmatrix} \rho_J(M, M'), \quad (7)$$

where $K = 0, 1, \dots, 2J$, $Q = -K, -K+1, \dots, K-1, K$. The expression in brackets is the Wigner 3j symbol, and $\sqrt{2J+1}\rho_0^0$ indicates the total population of the level. In the situation of weak anisotropy of the radiation field (corresponding to the condition of the solar chromosphere), the tensor of rank 1 indicates the orientation components contributing to the circular polarization and the tensor of rank 2 signifies the alignment components contributing to the linear polarization, while the tensor of rank K larger than 2 is too weak to influence the line profile.

The statistical equilibrium equations without quantum interference between the sublevels pertaining to different J levels were derived by [Landi Degl'Innocenti & Landolfi \(2004\)](#). The equations are valid under the flat-spectrum approximation. In contrast to the partial frequency redistribution (PRD), this approximation is also called the complete frequency redistribution (CRD). It is noteworthy that the PRD theory ([Bommier 1997a,b](#)) causes significantly different results from the CRD theory, but PRD theory becomes dominant only for a limited number of very strong lines (e.g., Ca II, Mg II resonance lines, Ly-alpha), mainly in the line wings. So the CRD theory is still a robust approach to investigate the Mg I b triplet. On the other hand, the collision rates scale can simply be added to the corresponding radiative rates in the statistical equilibrium equation under the impact approximation, which requires that the collision time is much smaller than the relaxation time scale due to radiative rates:

$$\begin{aligned} \frac{d}{dt} \rho_Q^K(\alpha J) = & -2\pi i \nu_L g_{\alpha J} Q \rho_Q^K(\alpha J) \\ & + \sum_{\alpha_\ell J_\ell} \sum_{K_\ell Q_\ell} \rho_{Q_\ell}^{K_\ell}(\alpha_\ell J_\ell) \mathbb{T}_A(\alpha J K Q, \alpha_\ell J_\ell K_\ell Q_\ell) \\ & + \sum_{\alpha_u J_u} \sum_{K_u Q_u} \rho_{Q_u}^{K_u}(\alpha_u J_u) [\mathbb{T}_E(\alpha J K Q, \alpha_u J_u K_u Q_u) \\ & + \mathbb{T}_S(\alpha J K Q, \alpha_u J_u K_u Q_u)] \\ & - \sum_{K' Q'} \rho_{Q'}^{K'}(\alpha J) [\mathbb{R}_A(\alpha J K Q K' Q') + \mathbb{R}_E(\alpha J K Q K' Q') \\ & + \mathbb{R}_S(\alpha J K Q K' Q')] \\ & + \sum_{\alpha_\ell J_\ell} \sqrt{\frac{2J_\ell+1}{2J+1}} C_1^{(K)}(\alpha J, \alpha_\ell J_\ell) \rho_{Q_\ell}^K(\alpha_\ell J_\ell) \\ & + \sum_{\alpha_u J_u} \sqrt{\frac{2J_u+1}{2J+1}} C_S^{(K)}(\alpha J, \alpha_u J_u) \rho_{Q_u}^K(\alpha_u J_u) \\ & - \left[\sum_{\alpha_u J_u} C_1^{(0)}(\alpha_u J_u, \alpha J) + \sum_{\alpha_\ell J_\ell} C_S^{(0)}(\alpha_\ell J_\ell, \alpha J) \right. \\ & \left. + D^{(K)}(\alpha J) \right] \rho_Q^K(\alpha J), \end{aligned} \quad (8)$$

where $D^{(K)}(\alpha J)$ is the depolarizing rate, defined as

$$D^{(K)}(\alpha J) = C_E^{(0)}(\alpha J) - C_E^{(K)}(\alpha J). \quad (9)$$

On the righthand side of Eq. (8), the first term describes the Hanle effect, and ν_L is the Larmor frequency. The coefficients $\mathbb{T}_A$, $\mathbb{T}_E$, $\mathbb{T}_S$, $\mathbb{R}_A$, $\mathbb{R}_E$, $\mathbb{R}_S$ are the radiative rates due to spontaneous emission described with subscript ‘‘E’’, stimulated emission with ‘‘S’’ and absorption with ‘‘A’’, respectively, and C_1^K , C_S^K , and C_E^K indicate the multi-pole components of the inelastic, super-elastic and elastic collision rates, respectively.

The multi-pole components of rank K are related to those of rank 0 by the following equations ([Landi Degl'Innocenti & Landolfi 2004](#)) in the case of inelastic collision:

$$\begin{aligned} C_S^{(K)}(\alpha J, \alpha_u J_u) &= (-1)^K \frac{\begin{Bmatrix} J & J & K \\ J_u & J_u & \tilde{K} \end{Bmatrix}}{\begin{Bmatrix} J & J & 0 \\ J_u & J_u & \tilde{K} \end{Bmatrix}} C_S^{(0)}(\alpha J, \alpha_u J_u), \\ C_1^{(K)}(\alpha J, \alpha_\ell J_\ell) &= (-1)^K \frac{\begin{Bmatrix} J & J & K \\ J_\ell & J_\ell & \tilde{K} \end{Bmatrix}}{\begin{Bmatrix} J & J & 0 \\ J_\ell & J_\ell & \tilde{K} \end{Bmatrix}} C_1^{(0)}(\alpha J, \alpha_\ell J_\ell), \end{aligned} \quad (10)$$

and in the case of elastic collision:

$$C_E^{(K)}(\alpha J) = (-1)^K \frac{\begin{Bmatrix} J & J & K \\ J & J & \tilde{K} \end{Bmatrix}}{\begin{Bmatrix} J & J & 0 \\ J & J & \tilde{K} \end{Bmatrix}} C_E^{(0)}(\alpha J). \quad (11)$$

The expressions in braces are the 6j symbol. It is well known that in the case of electrons with much higher energy than the threshold energy (i.e., under the so-called Born approximation), the interaction Hamiltonian depends on the dynamical variables of the atomic system only through the dipole operator (a tensor of rank $\tilde{K} = 1$).

As pointed out by [Bommier \(2009\)](#), the inelastic collision is mainly originated from the atom colliding with the electrons, and the collision with neutral Hydrogen dominates the elastic collision. Here the elastic collision means the changes of Zeeman substates under collision. Now we estimate the inelastic collision rates according to [van Regemorter \(1962\)](#)'s formula:

$$C_S^{(0)}(\alpha J, \alpha_u J_u) = 20.60 A(\alpha_u J_u \rightarrow \alpha J) \lambda_{\text{cm}}^3 N_e T_e^{-1/2} P \left(\frac{\Delta E_n}{KT} \right), \quad (12)$$

where $A(\alpha_u J_u \rightarrow \alpha J)$ is the Einstein coefficient for spontaneous emission, λ_{cm} is the wavelength in unit of cm, N_e is the density of electron, T_e is the temperature of electron, and $P \left(\frac{\Delta E_n}{KT} \right)$ is a function of $\frac{\Delta E_n}{KT}$.

Assume that the colliding particles obey Maxwellian velocity distribution and the detailed-balance stands, and the following Einstein-Milne relation is valid,

$$C_S^{(0)}(\alpha J, \alpha_u J_u) = \frac{2J+1}{2J_u+1} C_1^{(0)}(\alpha_u J_u, \alpha J) \exp \left[\frac{E(\alpha_u J_u) - E(\alpha J)}{K_B T} \right]. \quad (13)$$

The elastic collision rates $C_E^{(0)}(\alpha J)$ can be obtained in Van der Waals potential approximation ([Lamb & Ter Haar 1971](#); [Landi Degl'Innocenti & Landolfi 2004](#)),

$$C_E^{(0)}(\alpha J) = 1.4 \times 10^{-10} \left[\langle r^2(\alpha J) \rangle \right]^{0.4} \left[T \left(1 + \frac{1}{\mu} \right) \right]^{0.3} n_{\text{H}}. \quad (14)$$

In the above expression, T is the temperature of the atoms, n_H the number density of hydrogen atom in unit of cm^{-3} , μ the statistical weight of the atom, $\langle r^2(\alpha J) \rangle$ the mean squared radius of the electronic cloud in the level (αJ) . The values for $\langle r^2(\alpha J) \rangle$ can be deduced through an approximate expression for neutral atoms,

$$\langle r^2(\alpha J) \rangle = \frac{5}{2} \left(\frac{13.6}{I_{\text{ion}} - E_{\alpha J}} \right)^2. \quad (15)$$

In the above equation, I_{ion} is the ionization potential of the atom, and $E_{\alpha J}$ the excitation energy of the (αJ) level. The ionization energy of Mg I ground state is 7.6462 eV from [Kaufman & Martin \(1991\)](#) and compiled at the NIST atomic spectra database ([Kramida et al. 2013](#)).

2.4. Emission coefficients and line profile

The emission and absorption coefficients are the basis of our calculations, and can be found from [Landi Degl'Innocenti & Landolfi \(2004\)](#),

$$\begin{aligned} \epsilon_i(\nu, \Omega) &= \frac{2h\nu^3}{c^2} \eta_i^S(\nu, \Omega), \\ \eta_i &= \eta_i^A - \eta_i^S, \end{aligned} \quad (16)$$

where

$$\begin{aligned} \eta_i^A(\nu, \Omega) &= \frac{h\nu}{4\pi} \mathcal{N} \sum_{\alpha_\ell J_\ell} \sum_{\alpha_u J_u} (2J_\ell + 1) B(\alpha_\ell J_\ell \rightarrow \alpha_u J_u) \\ &\times \sum_{K Q K_\ell Q_\ell} \sqrt{3(2K+1)(2K_\ell+1)} \\ &\times \sum_{M_\ell M'_\ell M_u q q'} (-1)^{1+J_\ell-M_\ell+q'} \\ &\times \begin{pmatrix} J_u & J_\ell & 1 \\ -M_u & M_\ell & -q \end{pmatrix} \begin{pmatrix} J_u & J_\ell & 1 \\ -M_u & M'_\ell & -q' \end{pmatrix} \\ &\times \begin{pmatrix} 1 & 1 & K \\ q & -q' & -Q \end{pmatrix} \begin{pmatrix} J_\ell & J_\ell & K_\ell \\ M_\ell & -M'_\ell & -Q_\ell \end{pmatrix} \\ &\times \text{Re} \left[\mathcal{T}_Q^K(i, \Omega) \rho_{Q_\ell}^{K_\ell}(\alpha_\ell J_\ell) \Phi(\nu_{\alpha_u J_u M_u, \alpha_\ell J_\ell M_\ell - \nu}) \right], \end{aligned} \quad (17)$$

and

$$\begin{aligned} \eta_i^S(\nu, \Omega) &= \frac{h\nu}{4\pi} \mathcal{N} \sum_{\alpha_\ell J_\ell} \sum_{\alpha_u J_u} (2J_u + 1) B(\alpha_u J_u \rightarrow \alpha_\ell J_\ell) \\ &\times \sum_{K Q K_u Q_u} \sqrt{3(2K+1)(2K_u+1)} \\ &\times \sum_{M_u M'_u M_\ell q q'} (-1)^{1+J_u-M_u+q'} \\ &\times \begin{pmatrix} J_u & J_\ell & 1 \\ -M_u & M_\ell & -q \end{pmatrix} \begin{pmatrix} J_u & J_\ell & 1 \\ -M'_u & M_\ell & -q' \end{pmatrix} \\ &\times \begin{pmatrix} 1 & 1 & K \\ q & -q' & -Q \end{pmatrix} \begin{pmatrix} J_u & J_u & K_u \\ M'_u & -M_u & -Q_u \end{pmatrix} \\ &\times \text{Re} \left[\mathcal{T}_Q^K(i, \Omega) \rho_{Q_u}^{K_u}(\alpha_u J_u) \Phi(\nu_{\alpha_u J_u M_u, \alpha_\ell J_\ell M_\ell - \nu}) \right]. \end{aligned} \quad (18)$$

In the above equations, $i = I, Q, U, V$, represents the four Stokes parameters respectively, $\mathcal{N}$ is the number density of atoms, $B(\alpha_\ell J_\ell \rightarrow \alpha_u J_u)$ and $B(\alpha_u J_u \rightarrow \alpha_\ell J_\ell)$ are the Einstein coefficients for absorption and stimulated emission, respectively,

and $\Phi(\nu_{\alpha_u J_u M_u, \alpha_\ell J_\ell M_\ell - \nu})$ indicates the complex profile, which is resulted from the convolution of a Gaussian profile and a Lorentzian profile, given by

$$\begin{aligned} \Phi(\nu_0 - \nu) &= \phi(\nu_0 - \nu) + i\psi(\nu_0 - \nu) \\ &= \frac{1}{\sqrt{\pi}\Delta\nu_D} H(\nu, a) + i \frac{1}{\sqrt{\pi}\Delta\nu_D} L(\nu, a). \end{aligned} \quad (19)$$

In the above expression,

$$\begin{aligned} H(\nu, a) &= \frac{a}{\pi} \int_{-\infty}^{\infty} e^{-y^2} \frac{1}{(\nu - y)^2 + a^2} dy, \\ L(\nu, a) &= \frac{1}{\pi} \int_{-\infty}^{\infty} e^{-y^2} \frac{\nu - y}{(\nu - y)^2 + a^2} dy, \end{aligned} \quad (20)$$

whereas $H(\nu, a)$ is called Voigt function, and $L(\nu, a)$ is called associated dispersion profile. The frequency ν is the reduced frequency, and a indicates the damping constant. The damping constant is given by

$$a = \frac{\Gamma'}{\Delta\nu_D} = \frac{\Gamma + 2\Gamma_c}{\Delta\nu_D} = \frac{\gamma_u + \gamma_\ell}{4\pi\Delta\nu_D} + \frac{f}{2\pi\Delta\nu_D}, \quad (21)$$

where Γ and Γ_c are the natural width and the collision width, respectively, γ_u and γ_ℓ represent the inverse lifetimes of the upper and lower levels, respectively, and f is the frequency of elastic collisions. The Doppler width $\Delta\nu_D$ can be estimated through $\sqrt{2k_B T / \mu M + v_{\text{turb}}^2} \nu_0 / c$, where k_B is the Boltzmann constant, M is the mass of unit atomic weight, and v_{turb} is the microturbulent velocity. In the following, we use $\Delta\lambda_D = 40 \text{ m\AA}$, resulted from a temperature of 5755 K and a micro-turbulent velocity of 1 km s^{-1} .

In the situation of a weakly polarized atmosphere ($\epsilon_\ell \gg \epsilon_Q$, ϵ_U , ϵ_V ; $\eta_I \gg \eta_Q$, η_U , η_V), where ϵ_i^l and η_i^l ($i = I, Q, U, V$) are the line emission and absorption coefficients for Stokes parameters, respectively. The emergent fractional polarization is given by ([Trujillo Bueno 2003](#))

$$\frac{X(\nu, \omega)}{I(\nu, \omega)} \approx \frac{\epsilon_X^l(\nu, \omega)}{\epsilon_I^l(\nu, \omega)} - \frac{\eta_X^l(\nu, \omega)}{\eta_I^l(\nu, \omega)}, \quad (22)$$

where $X = Q, U, V$. The first term in the righthand side of the equation represents the contribution of spontaneous emission and the second term represents the contribution of absorption and dichroism effect. The continuum around the triplet has a fractional linear polarization $Q/I = 0.06\%$, given by [Fluri & Stenflo \(1999\)](#) and [Stenflo \(2005\)](#). It is found that the contribution of the continuum should be added to reproduce the observation profile. For simplicity, we assume that the continuum emission coefficients and absorption coefficients are constant around the triplet, the linear polarization of continuum comes from the emission,

$$\frac{X(\nu, \omega)}{I(\nu, \omega)} \approx \frac{\epsilon_X^c(\nu, \omega) + \epsilon_X^l(\nu, \omega)}{\epsilon_I^c(\nu, \omega) + \epsilon_I^l(\nu, \omega)} - \frac{\eta_X^l(\nu, \omega)}{\eta_I^l(\nu, \omega) + \eta_I^c(\nu, \omega)}. \quad (23)$$

In the above equation, $\epsilon_X^c(\nu, \omega)$, $\epsilon_I^c(\nu, \omega)$, and $\eta_I^c(\nu, \omega)$ are the coefficients for continuum.

The relationship between the continuum emission coefficient $\epsilon_I^c(\nu, \omega)$ and continuum absorption coefficient η_I^c can be estimated through the equation under local thermodynamic equilibrium (LTE),

$$\epsilon_I^c = \eta_I^c B(T), \quad (24)$$

where $B(T)$ is the Planck function at the frequency resulted from the transition under consideration.

3. Nonmagnetic field case

The polarization can originate from many mechanisms. In this paper, we emphasize the scattering induced polarization, because in the atmosphere like the chromosphere, scattering plays a very important role in line formation, and can be anisotropic to produce polarization. Besides the anisotropic scattering, many other processes can also produce or change the polarization degree. To see the individual contribution of continuum and collision to the linear polarization clearly, we firstly investigate their roles in the absence of magnetic field, and then discuss the influence of the magnetic field in the next section.

3.1. The role of continuum

The anisotropic incident radiation can induce the atomic polarization. In the scattering geometry close to the solar limb, the polarization produced by the atomic alignment can be observed. It is found that if we input anisotropy factors $\omega_1 = 0.1506$, $\omega_2 = 0.1504$, and $\omega_3 = 0.1501$ for the b_4 , b_2 , and b_1 lines, respectively, which were obtained with the limb darkening coefficients of the continuum given by Allen (1973), the fractional linear polarization amplitudes at the line core of the triplet are -4.43% , 5.98% , and 4.90% . This result shows a great difference compared with the observation by Stenflo et al. (2000). Obviously, the anisotropy factors derived from the limb darkening will deviate from that obtained through solving radiative transfer equations in the line formation region. Thus these values are not suitable to simulate the fractional linear polarization of strong lines. To fit the observation, we adopt the three anisotropy factors $\omega_1 = 0.0615$, $\omega_2 = 0.016$, and $\omega_3 = 0.009$ for the triplet, respectively, and then obtain the fractional linear polarizations $Q/I = 0.165\%$, 0.172% , and 0.192% for the triplet, respectively, in absence of collision or magnetic field. These values are very close to the results presented by Stenflo et al. (2000). As pointed out by Trujillo Bueno (1999), to get a similar fractional linear polarization for the triplet, the dichroism contribution needs to be considered. In our calculation, the atomic alignment coefficients (ρ_0^2/ρ_0^0) of lower levels of the b_2 and b_1 lines are found to be 0.481% and -0.281% , and that of the upper level is 0.156% . It is shown in the lower levels of the b_2 and b_1 lines that the atomic alignment is larger than that of the upper level. This result is close to the atomic polarization presented by Trujillo Bueno (2001). The lower levels of Mg I b lines are metastable and actually connect with the ground level by forbidden transitions. These forbidden transitions will influence the atomic alignments. As has been seen, the anisotropic illumination in the atmosphere can lead to large population imbalances in the lower levels of atoms, which are also sensitive to the weak magnetic field.

The fractional polarization profiles can be obtained when we take the contribution of the continuum into account according to Eq. (11), and we adopt the degrees of continuum polarization according to the theory of Fluri & Stenflo (1999). We manipulate the continuum absorption coefficient as a free parameter and the ratios $\eta_i^c/\eta_i^l(b_2) = 0, 10^{-7}, 10^{-5}$, and 10^{-1} , where $\eta_i^l(b_2)$ is the absorption coefficient at line core of the b_2 line. The profiles are shown in Fig. 1. It is obvious that the continuum reduces the polarization at line wings greatly, even though the ratio is low (e.g., $\eta_i^c/\eta_i^l(b_2) = 0, 10^{-7}$). As the continuum absorption coefficient becomes larger and larger, the polarization amplitudes in the line wings decrease quickly, while those at the line cores keep unchanged. When the ratio $\eta_i^c/\eta_i^l > 0.1$, the amplitudes

Fig. 1. Influence of continuum on Mg I b triplet Q/I profile. The absorption coefficient ratio of the continuum to the line $\eta_i^c/\eta_i^l = 0, 10^{-7}, 10^{-5}$, and 10^{-1} , corresponding to solid, dotted, dash, and dash dotted lines, respectively.

of line core decline markedly. Evidently, the ratio η_i^c/η_i^l gives a serious contribution to the widths of the fractional polarization profiles. As is well known, the absorption coefficients depend on the number density of atoms, and the ratio can be determined through radiative transfer simulations in a realistic model of the quiet solar photosphere. We found with a trial experience that when $\eta_i^c/\eta_i^l = 0.15$, the widths of observed profiles can be reproduced in the following.

3.2. The influence of collision

To investigate the influence of collision, we multiply the collision rates for the triplet by a factor ε' . The factor is set to be 1, 100, 500, 1000, and 2000 for the inelastic collision, and this set results in the ratios of $C_S^{(0)}(\alpha_l J_l, \alpha_u J_u)/A_{ul} = 1.73 \times 10^{-5}$, 1.73×10^{-3} , 8.65×10^{-3} , 1.73×10^{-2} , and 2.46×10^{-2} , respectively. For the elastic collision, the factor is set to be 0, 0.1, 1, 5, 15, and 30. Figures 2 and 3 show the variation of ϵ_Q/ϵ_l , η_Q/η_l , and Q/I profiles with inelastic and elastic collision rates, respectively. In the case of inelastic collision, as shown in Fig. 2, when ε' enhances, both the ϵ_Q/ϵ_l and η_Q/η_l profiles decrease, and so do the Q/I profiles. The inelastic collision obviously reduces the atomic alignments of both the upper and lower levels only when the super-elastic collision rates are large enough ($\varepsilon' \gtrsim 10^2$). This condition is satisfied by a higher electron density, thus a higher temperature. In contrast to the inelastic collision, we can see from Fig. 3 that the elastic collision can affect the Q/I profile even if the elastic collision rates are small. In detail, when $\varepsilon' < 10$, the lower levels of the b_2 and b_1 lines are still polarized. Theoretically, the elastic collision will depolarize both the upper level and the lower levels of these lines, but in fact the atomic alignment of the upper level is increased, as evidenced in Fig. 3. That is because the atomic alignments of the involved levels are coupled as reflected in the statistic equilibrium equations, while the alignments of the lower levels have a feedback on the upper level and make the alignment of the upper level increase. Thus the Q/I amplitude of the b_4 line increases with the collision rates, while the Q/I amplitudes of the b_2 and b_1 lines decrease. When $\varepsilon' > 10$, the atomic alignments of the lower levels are almost destroyed, which can be seen from η_Q/η_l profiles. Then the dichroism effect will not influence the line profiles and the unpolarized lower level approximation is valid. Thus the atomic polarization of the upper level declines as the elastic collision rates increase. When the factor is equal to unit, the polarizations of the lower levels remain and should be taken into account. This is the basis for the lower level Hanle effect and simulation of the second solar spectrum of Mg I triplet.

Fig. 2. Inelastic collision influence on Q/I profiles of the Mg I triplet. The solid, dotted, dashed, dot-dashed, and triple-dot-dashed lines result from the factor $\epsilon' = 1, 100, 500, 1000$, and 2000 , respectively. In the *top panels* ϵ_Q/ϵ_I profiles are plotted, η_Q/η_I profiles are depicted in the *middle panels*, and in the *bottom panels* $\epsilon_Q/\epsilon_I - \eta_Q/\eta_I$ profiles are presented. The continuum intensity corresponds to $\eta_I^c/\eta_I^f(b_2) = 0.15$.

Figure 4 shows the observed and fitted profiles of the triplet. The dashed lines present the observed profiles by [Stenflo et al. \(2000\)](#), while the solid lines are obtained from the scattering theory using the collision rates ($\epsilon' = 1$) in absence of magnetic field. One can see that these amplitudes and widths are matched in the line cores when we input $\eta_I^c/\eta_I^f(b_2) = 0.15$, and the anisotropy factors $\omega_1 = 0.131$, $\omega_2 = 0.0575$, and $\omega_3 = 0.0156$. Because the collisions and the continuum play a depolarization role in the triplet, the anisotropy factors are set to be larger than the above values to reproduce the profiles. This simulation leads to the results that ρ_0^2/ρ_0^0 of the upper level is equal to 0.216%, and those of the lower levels of the b_2 and b_1 lines are 0.569% and -0.291% , respectively. It is noticeable that these results differ a little from the calculation by [Trujillo Bueno \(2001\)](#), and the difference originates from the fact that we input a large continuum absorption coefficient, which is not considered by [Trujillo Bueno \(2001\)](#).

4. The Hanle effect in the Mg I triplet

In this section, we focus on the role of the magnetic field. It is well known that the magnetic field can modify the polarization via the Hanle effect, which can be used for diagnostics of a weak

magnetic field. Especially, the lower level Hanle effect is used for a very weak magnetic field ([Landolfi & Landi Degl'Innocenti 1986](#)). Since the lower levels of Mg I b triplet are polarized and the collision cannot destroy the polarization, it satisfies another requirement as a tool to diagnose very a weak magnetic field. In the following, we investigate the Hanle effect on the triplet to see how the weak magnetic field modifies the emergent linear polarization of the Mg I b triplet. Because of the outstandingly different influences on the line cores and wings, we study them separately.

4.1. The Hanle effect at line core

Figure 5 shows the variations of Q/I amplitudes at the line cores of the triplet with the magnetic field strengths as well as the inclination θ_B close to the solar limb ($\mu = 0.1$). The anisotropy factors and collision rates are the same as those producing Fig. 4. The solid, dotted, and dashed lines correspond to $\theta_B = 30^\circ, 60^\circ$, and 90° , respectively. The top panels are obtained with the magnetic azimuth $\chi_B = 0^\circ$, and the bottom panels are resulted from $\chi_B = 60^\circ$.

From Fig. 5, it is easily seen that the Q/I amplitudes at the line cores of the triplet are very sensitive to the sub-Gauss range magnetic field between 0.001 and 0.1 Gauss due to the

Fig. 3. Elastic collision influence on Q/I profiles of the Mg I triplet. The solid, dotted, dashed, dot-dashed, triple-dot-dashed, and long dashed lines correspond to the factor $\epsilon' = 0, 0.1, 1, 5, 15$, and 30 . In the *top panels* ϵ_0/ϵ_i profiles are plotted, η_Q/η_I profiles are depicted in the *middle panels*, and $\epsilon_0/\epsilon_i - \eta_Q/\eta_I$ profiles in the *bottom panels*. The continuum intensity corresponds to $\eta_I^c/\eta_I^f(b_2) = 0.15$.

Fig. 4. Fractional linear polarization Q/I profiles of Mg I b triplet close to solar limb ($\mu = 0.1$) in absence of magnetic field. The dashed lines were measured by [Stenflo et al. \(2000\)](#). The solid lines show the theoretical profiles obtained with the anisotropy factor $\omega_1 = 0.131$, $\omega_2 = 0.0575$, and $\omega_3 = 0.0156$ for the triplet, respectively. The continuum intensity corresponds to $\eta_I^c/\eta_I^f(b_2) = 0.15$.

lower level Hanle effect, and to the Gauss range between 1 and 10 Gauss due to the upper level Hanle effect. One can see immediately that the three lines behave differently in the region dominated by the lower level Hanle effect. The polarization amplitude of the b_4 line increases with magnetic field

strength, while those of the b_2 and b_1 lines experience reduction. Especially, the b_2 line can even get a negative amplitude. Since the lower level of the b_4 line has a zero total angular momentum ($J = 0$), the Hanle effect loses its influence on this level, and this line should not be influenced by a magnetic field between 0.001 and 0.1 Gauss. However, like the influence of elastic collision discussed above, the alignments of the lower levels of the b_2 and b_1 lines can feed back to the upper level, and thus make the Q/I amplitude of the b_4 increase. When the magnetic field strength locates between 1 and 10 Gauss, the upper level Hanle effect dominates, while the lower level Hanle effect becomes saturated. The magnetic sensitivity of these lines are proportional to the dimensionless coefficient $w_{J_u J_\ell}^{(2)}$ introduced by [Landi Degl'Innocenti \(1984\)](#), and $w_{J_u J_\ell}^{(2)} = 1, -0.5$ and 0.1 for the triplet, respectively. The atomic alignment of the upper level declines with the increasing magnetic field strength. The b_4 line is the most sensitive in this region and then followed by the b_2 and b_1 lines. It is found that the fractional linear polarization of the b_2 will increase with the magnetic field strength, as $\omega_{1,1}^{(2)} = -0.5$ is negative. When the magnetic field become stronger than about 10 Gauss, the upper level Hanle effect becomes saturated.

In general, another main result from the Hanle effect is that the polarization plane rotates about the magnetic vector field,

Fig. 5. Variation of the emergent Q/I amplitudes with magnetic field vector at the line cores of the Mg I triplet calculated at $\mu = 0.1$. The lines in the *top panels* are obtained with the magnetic azimuth $\chi_B = 90^\circ$, and those in the *bottom panels* with the azimuth $\chi_B = 0^\circ$. The solid, dotted, and dashed lines result from the magnetic inclination of 30° , 60° , and 90° , respectively. The other parameters are the same as those producing Fig. 4.

Fig. 6. Hanle effect on the line-core polarization of the Mg I triplet. The lines are obtained close to solar limb ($\mu = 0.1$) in presence of a horizontal magnetic field of various strengths, 0 G (solid lines), 0.005 G (dotted lines), 0.1 G (dashed lines), 1 G (dot-dashed lines), 10 G (triple-dot-dashed lines), and 100 G (long dashed lines). The *top six panels* show the Q/I and U/I profiles with the azimuth $\chi_B = 30^\circ$, and the *bottom six panels* show the Q/I and U/I profiles with the azimuth $\chi_B = 0^\circ$. The other parameters are the same with those producing Fig. 4.

Fig. 7. Fractional polarization profiles obtained in the presence of a horizontal magnetic field with $\chi_B = 60^\circ$. The upper panel shows the whole pattern without continuum, and the bottom panels indicate the line-core profiles with continuum. The pattern varies with the magnetic field of 0 G (solid lines), 0.005 G (dotted lines), 0.01 G (dashed lines), 0.1 G (dot-dashed lines), and 1 G (triple-dot-dashed lines). The anisotropy factors are the same as those resulting in Fig. 1.

and then the observable Stokes U signal appears. When a magnetic field with well-defined inclination and azimuth exists, the Stokes U can be observed due to the rotation. Figure 6 shows both the fractional linear polarization Q/I and U/I profiles of the Mg I b triplet. The profiles in the top six panels are obtained with the magnetic field inclination $\theta_B = 90^\circ$ and azimuth $\chi_B = 30^\circ$, and those in the bottom six are resulted from $\theta_B = 90^\circ$ and $\chi = 0^\circ$. They show quantitatively how Q/I and U/I profiles vary with magnetic field.

4.2. The Hanle effect in the line wings

Now, we analyze the Hanle effect in the line wings in the presence of a magnetic field. Figure 7 shows the variations of the overall pattern (top panel) and line core profiles (bottom panels) with the magnetic field in the absence of a collision. As proven by Landi Degl’Innocenti & Landolfi (2004), under the assumptions of unpolarized and infinitely sharp lower level in the absence of a collision, the Hanle effect vanishes in the line wings. From the top panel in Fig. 7, however, it is easily seen that magnetic field can influence the whole Q/I profile not only in the line cores with the polarized lower level. The main contribution to the line wings comes from the dichroism effect. This can be explained as follow.

When the dichroism contribution is not considered, the line profile depends on ϵ_i , and we have (Landi Degl’Innocenti & Landolfi 2004):

$$\epsilon_i \propto \Phi_Q^{KK'}(\alpha_u J_u) \times \rho_Q^K(\alpha_u J_u), \quad (25)$$

where $\Phi_Q^{KK'}(\alpha_u J_u)$ is the generalized profile. In the far wings, we have $\Phi_Q^{KK'} \propto (1 + iQH'_u)$ and $\rho_Q^K(\alpha_u J_u) \propto 1/(1 + iQH'_u)$, where $H_u = 2\pi\nu_L g_u/(\gamma_u + \gamma_\ell)$ and $H'_u = 2\pi\nu_L g_u/\gamma_u$. Because

the lifetime of the lower level is much longer than that of the upper level, we have $H_u \sim H'_u$, and the Larmor frequency dependent of the magnetic field will cancel out in the expression of ϵ_i . Therefore, the magnetic field does not affect the line wings. However, when the dichroism contribution is considered, η_i^A will contribute to the line profile. In this case, we have a similar relation,

$$\eta_i^A \propto \Phi_Q^{KK'}(\alpha_\ell J_\ell) \times \rho_Q^K(\alpha_\ell J_\ell). \quad (26)$$

In the far wings, we also have $\Phi_Q^{KK'}(\alpha_\ell J_\ell) \propto (1 + iQH_\ell)$ and $\rho_Q^K(\alpha_\ell J_\ell) \propto 1/(1 + iQH'_\ell)$, where $H_\ell = 2\pi\nu_L g_\ell/(\gamma_u + \gamma_\ell)$ and $H'_\ell = 2\pi\nu_L g_\ell/\gamma_\ell$. However, H_ℓ is now one hundred times bigger than H'_ℓ in the case of Mg I b triplet. The magnetic term remains in the dichroism contribution, so the line wings will be influenced by magnetic field via the dichroism effect. When considering the contribution of the continuum, the influence on the line wings would not be seen as shown in the bottom panels of 7.

4.3. Hanle-effect diagrams

Hanle-effect diagram can intuitively show the variations of Q/I and U/I amplitudes at line cores with the strength, inclination, and azimuth of the magnetic field. It is a useful tool to determine the magnetic field vector. Figure 8 presents the Hanle-effect diagrams corresponding to the geometry close to the solar limb ($\mu = 0.1$). Each panel corresponds to a fixed magnetic field inclination. The thick lines in all panels correspond to a definite strength, and the azimuth ranges from $\chi_B = 0^\circ$ to $\chi_B = 360^\circ$. The magnetic field strength is sampled as 0.008, 0.02, 4, and 8 Gauss. The thick solid lines are produced by the azimuth from 0° to 180° , and the thick dotted lines are resulted from 180° to 360° . The thin lines result from the magnetic field strength ranging from 0 to 300 Gauss with a fixed χ_B . These lines show the increase and decrease of Q/I and U/I amplitudes close to the solar limb, as well as the rotation of polarization plane induced by the Hanle effect. In contrast to the case of the unpolarized lower levels, it is easily seen from Fig. 8 that there will be two interlaced loops corresponding to the lower and upper level Hanle effect. The two loops can be overlapped in the Hanle-effect diagram. Because the loops of the b_2 and b_1 lines due to the lower level Hanle effect are more conspicuous, especially the b_2 line, these two lines are suitable for diagnostic of a sub-Gauss range magnetic field. Another interesting thing is that in a two-level atom model, the lower level Hanle effect does not occur in the transition from $J_u = 1$ to $J_\ell = 0$, while in a multi-level magnesium atom model the lower level Hanle effect of the b_4 line takes place because of the coupling among atomic alignment with those of the other levels. This line has the most conspicuous upper level Hanle effect loops, thus it is suitable for diagnosing the Gauss range magnetic field.

5. Conclusions

We investigate the dependence of scattering induced fractional linear polarization of the magnesium triplet on the continuum, collision, dichroism, and weak magnetic field with the scattering geometry close to the solar limb. It is found that the simplified scattering model in our calculation can fit the main features, such as the amplitudes and widths of the observed Q/I profiles by inputting proper anisotropy factors and continuum, but the inputting values of these parameters are not unique to reproduce these features. This is noticeable when the diagnostic is applied.

Fig. 8. Hanle diagrams show the Q/I and U/I amplitudes in the presence of magnetic field at the line cores of the triplet calculated with $\mu = 0.1$. The top, middle, and bottom panels correspond to $\theta_B = 90^\circ$, $\theta_B = 60^\circ$, and $\theta_B = 30^\circ$, respectively. The thick lines correspond to constant strength magnetic field of 0.008, 0.02, 4, and 8 Gauss with the azimuth $0^\circ < \chi_B < 180^\circ$ (thick solid line) and $180^\circ < \chi_B < 360^\circ$ (thick dotted line). The thin lines correspond to the azimuths $\chi_B = 0^\circ$ and $\chi_B = 180^\circ$ thin (dot-dashed line), $\chi_B = 30^\circ$ and $\chi_B = 150^\circ$ (thin dashed line), $\chi_B = 60^\circ$ and $\chi_B = 120^\circ$ (thin dotted line) and $\chi_B = 90^\circ$ (thin solid line). The anisotropy factors, collision rates and continuum are the same as those producing Fig. 4.

The anisotropic illumination in the atmosphere can lead to large population imbalances in levels of the atoms, which are sensitive to the weak magnetic field. In our calculation, the collision rates are not large enough to destroy the population imbalances of the lower levels of the b_2 and b_1 lines, this is the prerequisite for the lower level Hanle effect and dichroism effect. To get close Q/I amplitudes of triplet to fit the observation, the dichroism effect has to be considered.

In the absence of magnetic field, only Q/I signals are nonzero, and the amplitudes mainly depend on the anisotropy factors. The continuum also can greatly degrade the fractional linear polarization outside the line core, thus significantly influence the widths of the Q/I profiles. On the other hand, the inelastic collision can depolarize in both the line cores and wings. As the inelastic collision rates increase, the atomic polarization of the each level becomes smaller and smaller, and finally disappears. Like the Hanle effect, the elastic collision can influence the atomic alignments. Their difference indicates that the collision cannot lead to any rotation of polarization plane. Theoretically, the elastic collision should decrease the atomic alignments of the upper and lower levels. However, in a multi-level atom model, the atomic alignments can increase in the case of the Mg I triplet. This is because the alignments are coupled in the statistical equilibrium equations. If the elastic collision rates are greater than $5 \times 10^6 \text{ s}^{-1}$, the atomic alignments of lower

levels of these lines will be completely destroyed and then the unpolarized lower level approximation is valid.

In the investigation of magnetic field, it is found that the lower level Hanle effect can influence polarization in the line wings when the dichroism effect is taken into account. When the continuum becomes strong, the polarization in the line wings cannot be detected because the continuum greatly reduce the polarization amplitudes of line wings. In the line cores, the fractional linear polarizations of the triplet are sensitive to the magnetic field between 0.001 and 0.1 Gauss due to the lower level Hanle effect, and between 1 and 10 Gauss due to the upper level Hanle effect. The most interesting thing is that in the lower level Hanle effect region, the Q/I amplitude of the b_4 line increases with magnetic field strength, and that of the b_2 can turn from being positive to negative at a definite magnetic field vector. This phenomenon means that the weak magnetic field can not only reduce, but also enhance the atomic polarization because the atomic alignments of different levels are coupled in the statistical equilibrium equations. The b_2 and b_1 lines, especially the b_2 line, are more suitable for the diagnostics of a sub-Gauss range magnetic field than the b_4 line. In the upper level Hanle effect dominated region, the b_4 line is the most sensitive and then followed by the b_2 and b_1 lines, since their $w_{J_u J_l}^{(2)} = 1, -0.5$, and 0.1 , respectively. Therefore, the Mg I b triplet can be used for probing the Gauss range magnetic field due to the upper

level Hanle effect, and sub-Gauss range magnetic field due to the lower level Hanle effect.

Acknowledgements. The authors are grateful to Prof. Stenflo for supplying the data and to E. Landi Degl'Innocenti for a careful reading of the manuscript and helpful suggestions. We also thank Dr. Fei Teng for providing the code to compute the complex profile. This work is sponsored by Natural Science Foundation of China (NSFC) under grant numbers 11078005 and 11373065, and the 973 project under grant number G2011CB811400.

References

- Allen, C. W. 1973, *Astrophysical quantities*, 3rd edn. (University of London, London: The Athlone Press)
- Belluzzi, L., & Trujillo Bueno, J. 2011, *ApJ*, 743, 3
- Belluzzi, L., Trujillo Bueno, J., & Landi Degl'Innocenti, E. 2007, *ApJ*, 666, 588
- Belluzzi, L., Landi Degl'Innocenti, E., & Trujillo Bueno, J. 2009, *ApJ*, 705, 218
- Bommier, V. 1980, *A&A*, 87, 109
- Bommier, V. 1997a, *A&A*, 328, 706
- Bommier, V. 1997b, *A&A*, 328, 726
- Bommier, V. 2009, in *Solar Polarization 5*, in Honor of Jan Stenflo, eds. S. V. Berdyugina, K. N. Nagendra, & R. Ramelli, *ASP Conf. Ser.*, 405, 335
- Fano, U. 1957, *Rev. Mod. Phys.*, 29, 74
- Fluri, D. M., & Stenflo, J. O. 1999, *A&A*, 341, 902
- Fontenla, J. M., Avrett, E. H., & Loeser, R. 1993, *ApJ*, 406, 319
- Gandorfer, A. 2000, *The Second Solar Spectrum: A high spectral resolution polarimetric survey of scattering polarization at the solar limb in graphical representation*, Vol. I: 4625 Å to 6995 Å
- Gandorfer, A. 2002, *The Second Solar Spectrum: A high spectral resolution polarimetric survey of scattering polarization at the solar limb in graphical representation*, Vol. II: 3910 Å to 4630 Å
- Gandorfer, A. 2005, *The Second Solar Spectrum: A high spectral resolution polarimetric survey of scattering polarization at the solar limb in graphical representation*, Vol. III: 3160 Å to 3915 Å
- Kaufman, V., & Martin, W. C. 1991, *J. Phys. Chem. Ref. Data*, 20, 83
- Kelleher, D. E., & Podobedova, L. I. 2008, *J. Phys. Chem. Ref. Data*, 37, 1285
- Kramida, A., Yu. Ralchenko, Reader, J., & NIST ASD Team. 2013, *NIST Atomic Spectra Database* (ver. 5.1), available: <http://physics.nist.gov/asd>, 2014, February 21, National Institute of Standards and Technology, Gaithersburg, MD
- Lamb, F. K., & Ter Haar, D. 1971, *Phys. Rep.*, 2, 253
- Landi Degl'Innocenti, E. 1983, *Sol. Phys.*, 85, 3
- Landi Degl'Innocenti, E. 1984, *Sol. Phys.*, 91, 1
- Landi Degl'Innocenti, E. 1996, *Sol. Phys.*, 164, 21
- Landi Degl'Innocenti, E., & Landolfi, M. 2004, *Polarization in Spectral Lines* (Springer), *Astrophys. Space Sci. Lib.*, 307
- Landolfi, M., & Landi Degl'Innocenti, E. 1986, *A&A*, 167, 200
- Lites, B. W., Skumanich, A., Rees, D. E., & Murphy, G. A. 1988, *ApJ*, 330, 493
- Manso Sainz, R., & Landi Degl'Innocenti, E. 2002, *A&A*, 394, 1093
- Manso Sainz, R., & Trujillo Bueno, J. 2010, *ApJ*, 722, 1416
- Omont, A. 1977, *Prog. Quant. Electr.*, 5, 69
- Stenflo, J. O. 1996, *Nature*, 382, 588
- Stenflo, J. O. 2003, in *Solar Polarization*, eds. J. Trujillo-Bueno, & J. Sanchez Almeida, *ASP Conf. Ser.*, 307, 385
- Stenflo, J. O. 2005, *A&A*, 429, 713
- Stenflo, J. O. 2013, *A&ARv*, 21, 66
- Stenflo, J. O., & Keller, C. U. 1997, *A&A*, 321, 927
- Stenflo, J. O., Keller, C. U., & Gandorfer, A. 2000, *A&A*, 355, 789
- Trujillo Bueno, J. 1999, in *Polarization*, eds. K. N. Nagendra, & J. O. Stenflo, *Astrophys. Space Sci. Lib.*, 243, 73
- Trujillo Bueno, J. 2001, in *Advanced Solar Polarimetry – Theory, Observation, and Instrumentation*, ed. M. Sigwarth, *ASP Conf. Ser.*, 236, 161
- Trujillo Bueno, J. 2003, in *Solar Polarization*, eds. J. Trujillo-Bueno, & J. Sanchez Almeida, *ASP Conf. Ser.*, 307, 407
- Trujillo Bueno, J. 2009, in *Solar Polarization 5: In Honor of Jan Stenflo*, eds. S. V. Berdyugina, K. N. Nagendra, & R. Ramelli, *ASP Conf. Ser.*, 405, 65
- Trujillo Bueno, J., Shchukina, N., & Asensio Ramos, A. 2004, *Nature*, 430, 326
- van Regemorter, H. 1962, *ApJ*, 136, 906