Long-time evolution of gas-free disk galaxies in binary systems

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ABSTRACT

We present the results of several detailed numerical N -body simulations of the dynamical interactions of two equal-mass disk galaxies. Both galaxies are embedded in spherical halos of dark matter and contain central bulges. Our analysis of the dynamical evolution of the binary system focuses on the morphological evolution of the stellar distribution of the disks. The satellite galaxy has coplanar or polar disk orientation in relation to the disk of the primary galaxy and their initial orbits are prograde eccentric ($e = 0.1$, $e = 0.4$ or $e = 0.7$). Both galaxies have mass and size similar to the Milky Way. We show that the merger of the two disk galaxies, depending on the relative orientation of the disks, can yield either a disk or lenticular remnant, instead of an elliptical one. These are the first reported simulations that show the formation of S0-like galaxies from protracted binary galaxy interactions. Additionally, we demonstrate that the time to merger increases linearly with the initial apocentric distance between the galaxies, and decreases with the initial orbital eccentricity. We also show that the tidal forces of the disks excite transient $m = 1$ and $m = 2$ wave modes, that is, lopsidedness, spiral arms, and bars. However, after the merging of the disks, these larger instabilities fade completely, and the remnant is thicker and more extended than the original disks. The maximum relative amplitude of these waves is at most about 15 times higher than the control case. The $m = 2$ wave mode is generated mainly by tidal interaction in the outer region of the disks. The $m = 1$ wave mode depends mostly on the interaction of the inner part of the disks, producing an off-centering effect of the wave mode center relative to the center of mass of the disk. These characteristics produce a time lag among the maximum formation of these two wave modes. Finally, the disk settles down quickly after the merger, in less than one outer disk rotation period.

Key words. galaxies: spiral – galaxies: evolution – galaxies: interactions – galaxies: kinematics and dynamics – galaxies: structure

1. Introduction

Seventy percent of the galaxies in the nearby universe are characterized by a disk with prominent spiral arms, but our understanding of the origin of these patterns is incomplete, even after decades of theoretical study (Sellwood 2011, 2013). Several ideas have been proposed to explain the formation of spiral arms.

The latest simulations show that gravitational instabilities in the stars lead to flocculent and multi-armed spirals that persist for many Gyr (Oh et al. 2008; Fujii et al. 2011). However, the mechanism that produces and maintains two-armed grand design galaxies is still ambiguous.

Grand design galaxies, which exhibit symmetric two-armed spiral structures, represent a significant fraction of spiral galaxies. The production of such a spiral galaxy faces two main obstacles: first, inducing the $m = 2$ spiral structure, and secondly, maintaining it.

It is known that the spiral arms of disk galaxies can be excited by tidal interactions with nearby companion galaxies (Oh et al. 2008; Dobbs et al. 2010; Struck et al. 2011).

Oh et al. (2008) have investigated the physical properties of tidal structures in a disk galaxy created by gravitational interactions with a companion using numerical N -body simulations. They have considered a galaxy model consisting of a rigid halo/bulge and an infinitesimally thin stellar disk with Toomre parameter $Q \approx 2$. The perturbing companion was treated as a

pointmass moving on a prograde parabolic orbit, with varying mass and pericenter distance. They have shown that tidal interactions produce well-defined spiral arms and extended tidal features, such as bridge and tail, which are all transients.

Dobbs et al. (2010) modeled the disk galaxy M51 and its interaction with a companion point-mass NGC 5195, focusing primarily on the dynamics of the gas, and secondly on the stellar disk. The halo was represented by a rigid potential. The tidal interaction has produced spiral arms in the stars and in the gas. The resulting spiral structure has shown excellent agreement with that of M51.

Lotz et al. (2010) analyzed the effect of a gas fraction on the morphologies of a series of simulated disk galaxy mergers. Each galaxy was initially modeled as a disk of stars and gas, a stellar bulge, and a dark matter halo, with different number of particles and masses for each component. All the simulated mergers had the same orbital parameters. Each pair of galaxies started on a subparabolic orbit with eccentricity 0.95 and an initial pericentric radius of 13.6 kpc. The galaxies had a roughly prograde-prograde orientation relative to the orbital plane, with the primary galaxy tilted 30° from a pure prograde orientation. Their simulations have predicted that galaxy mergers would exhibit high asymmetries for longer periods of time if they had high gas fractions.

Struck and collaborators (Struck et al. 2011) have discovered long-lived waves in numerical simulations of fast (marginally

bound or unbound) flyby galaxy collisions. The main galaxy had a rigid halo potential, and gas and the companion was modeled as a point mass. They have found that none of the simulations resulted in bar formation. They have also shown that while these waves propagate through the disk, they are maintained by the coherent oscillations initiated by the impulsive disturbance.

Snaith et al. (2012) have studied the properties and evolution of a simulated polar-disk galaxy. This galaxy was composed of two orthogonal disks, one of which contained old stars (old stellar disk) and the other both younger stars and cold gas (polar disk). They have confirmed that the polar disk galaxy is the result of the last major merger, where the angular momentum of the interaction is orthogonal to the angle of the infalling gas.

In one of our previous works in kinematic and morphology of spiral galaxies we have shown a deep interaction between the dynamical and morphological properties of this type of galaxy (Chan & Junqueira 2003). With continuous satellite forcing, the final state was in the form of a slowly evolving wave pattern, as shown by the pattern speeds for stable $m = 1$ and $m = 2$ wave modes. The pattern speeds obtained from the density and the three positive-velocity component distributions are the same. This was also true for the negative velocity components.

Kinematics studies of spiral galaxies have revealed a remarkable variety of interesting behavior: some galaxies have large-scale asymmetries in their rotation curves as a signature of kinematic lopsidedness (Junqueira & Combes 1996), while in others the inner regions counter-rotate with the respect to the rest of the galaxy (Garcia-Burillo et al. 2000). Most of the spiral galaxies have asymmetric HI profiles and asymmetric rotation curves (Haynes et al. 2000; Andersen & Bershadsky 2013). Such intriguing kinematics might arise if these galaxies are the end-products of minor mergers (Haynes et al. 2000). Minor mergers and weak tidal interactions between galaxies occur with much higher frequency than major ones. By weak interactions between galaxies we mean those that do not destroy the disk of the primary galaxy. However, weak interactions may cause disk heating, and satellite remnants may build up the stellar halo.

Galaxy interactions probably play a key role in determining the morphology and the dynamical properties of disk galaxies. Careful examination shows that most disk galaxies are not truly symmetric but exhibit a variety of morphological peculiarities of which spiral arms and bars are the most pronounced. Disk galaxies currently show significant spiral-generating tidal perturbations by one or more small-mass companions, and nearly all have had tidal interactions at some time in the past.

After decades of efforts, we now know that these features may be driven by environmental disturbances acting directly on the disk, in addition to self-excitation of a local disturbance (e.g., by swing amplification, Toomre 1981). However, the disk is embedded within a halo and, therefore, the luminous disk is not dynamically independent (Combes 2008). The dark matter halo is disturbed by dwarf companions, neighboring galaxies in groups and clusters, and the tidal force from the overall cluster. If the halo can respond globally to these disturbances, it can affect the disk structure. Thus, because most spirals have dwarf companions, there are interactions with these companions and the inward propagation of external perturbations by the halo could be a dominant source of disk structure for all galaxies (Vesperini & Weinberg 2000).

To study the dynamical evolution of two disk galaxies and their morphological evolution, this paper explores the picture as follows: first, we assume a disk galaxy with the characteristics of the Milk Way (disk, bulge, and halo). Second, we let a secondary galaxy orbit on prograde coplanar or polar disk

(orientation in relation to the primary disk galaxy). Although the gas is important in modeling a realistic disk galaxy, in this work we focus only on the morphological stellar properties.

As an example of a work on the evolution of stellar disk galaxies without gas, we mention the recently published work by Baba et al. (2013). Using N -body simulations, they analyzed the physical mechanisms of non-steady stellar spiral arms in disk galaxies, which are those without the gaseous component. They have studied the growing and damping phases of the spiral arms and confirmed that the spiral arms are formed by a swing amplification mechanism that reinforces density enhancement. The main motivation was that all the previous time-dependent simulation works have not been able to prove the existence of stationary density waves in a disk galaxy without external perturbations and a bar structure.

Thus, the main goal of the present work is, by utilising detailed numerical N -body simulations, to study the dynamical interactions of the two disks of the galaxies. In particular, we have investigated whether interactions can induce persistent and stable $m = 1$ or $m = 2$ patterns in disk galaxies.

The paper is organized as follows: in Sect. 2 we describe the numerical method used in the simulations. In Sect. 3 we present the initial conditions. In Sect. 4 we describe the results of the simulations. In Sect. 5 we show power spectra of the instabilities. Finally, in Sect. 6 we discuss and summarize the results.

2. Numerical method

The full N -body code used in the simulations was GADGET (Springel et al. 2001). The code was parallelized and the communication was made by means of the message passing interface (MPI). GADGET evolves self-gravitating collisionless fluids with the traditional N -body approach and a collisional gas by smoothed particle hydrodynamics. But in our case we used only the particle integration, which uses a tree algorithm to compute gravitational forces. The parallel version has been designed to run on massively parallel supercomputers with distributed memory.

Fortin, Athanassoula & Lambert (2011) published a comparison with different codes for galactic N -body simulations, in particular, the GADGET code and the Dehnen algorithm. They have shown that the serial implementation of the Dehnen algorithm is slower in terms of execution time than the parallelized implementation of the GADGET code, with 8 to 128 processors.

The GADGET code does not exactly conserve energy or momentum, but the energy is conserved to better than 0.3% over an entire evolution, and the center of mass moves a distance of at most 82ϵ (the softening parameter) from the initial system center mass, with a time step size $\Delta t = 1.000 \times 10^{-3}$ and $\epsilon = 8.000 \times 10^{-4}$. Too large ϵ reduces the noise but increases the error in the calculation of the force because it fails to resolve the interactions of particles with scale lengths shorter than it. Conversely, too small value for ϵ produces a noisy estimate of the force because of the finite number of particles. The value of the optimal softening scale depends both on the mass distribution and on the number of particles used to represent it. For more particles the optimal softening scale is smaller. More highly concentrated mass distributions necessitate smaller softening scale. Several works have analyzed this problem carefully (Merritt 1996; Athanassoula et al. 2000; Rodionov & Sotnikova 2005). For example, Athanassoula et al. (2000) have shown that for a Dehnen sphere, $\epsilon = 4.000 \times 10^{-3}$. Thus, the softening scale is motivated by a trade-off between accuracy and computational speed. We have run several simulations of the isolated

Table 1. Disk galaxy model properties.

Galaxy	M_d	N_d	R_d	Z_d	R_t	M_b	N_b	M_h	N_h	m	ϵ
G_1	0.871	40 000	1.000	0.100	5.000	0.425	19 538	4.916	225 880	2.176×10^{-5}	8.000×10^{-4}

Notes. M_d is the disk mass in units of mass, N_d the number of particles of the disk, R_d the disk scale radius in units of length, Z_d the disk scale height in units of length, R_t the disk truncation radius in units of length, M_b the bulge mass in units of mass, N_b the number of particles in the bulge, M_h the halo mass in units of mass, N_h the number of halo particles, m the mass of each particle in units of mass, and ϵ is the softening of each particle in units of length.

disk galaxy to find the optimal ϵ and the longest integration time step. The criteria were to find the maximum softening parameter and integration time step such that it could minimize the heating of the disk, without changing the physical disk parameters too much, mainly Z_d . When the scale ϵ was too big the disk heated up, which increased its width. Thus, the chosen softening parameter is smaller than the disk scale height Z_d , so that the disk of the galaxies can be followed accurately.

For non-zero tolerance parameter θ , the treecode CPU time scales as $O(N \log N)$, but in the limit $\theta \rightarrow 0$ this method scales as $O(N^2)$, approaching to the direct code. At the very beginning of this work we used the GADGET-1 (Springel et al. 2001). Thus, we assumed for the tolerance parameter θ the value 0.577, to avoid pathological situations when using the treecode as described by (Salmon & Warren 1994). However, this value of θ needs a lot of CPU time but the error in the force calculation is minimized, although many authors studying disk galaxies have used $\theta > 0.7$ (Oh et al. 2008; Minchev et al. 2012).

3. Initial conditions of the simulations

We used the nearly self-consistent disk-bulge-halo galaxy model proposed by (Kuijken & Dubinski 1995) in the simulations. We assumed a model that has mass distributions and rotation curves closely resembling those of the Milk Way, that is, the model MW-A of (Kuijken & Dubinski 1995). This galaxy disk model has a disk-bulge mass ratio of 2:1 and halo-disk mass ratio of 5:1 (see Table 1). The disk is warm with a Toomre parameter $Q = 1.7$ at the disk half-mass radius.

Our simulations are based on fewer particles than other works on disk galaxies. However, since we are the first to attempt to solve the problem proposed here, we have decided to run modest simulations using the computer clusters that were available, to have, at least, an initial dynamical study of these types of binary galaxies.

The disk follows approximately an exponential-sech law described by

$$\rho_d(R, Z) = \rho_0 \exp\left(-\frac{R}{R_d}\right) \text{sech}^2\left(\frac{Z}{Z_d}\right), \quad (1)$$

where ρ_0 is the central density that is related to the total mass of the disk. This approximation was used because the full potential equation obtained by Kuijken & Dubinski (1995) is analytically more complicated.

4. Results of the simulations

In Fig. 1 we show the contour plot of the primary galaxy at the beginning of the simulation ($t = 0$) and at the Hubble time of the simulation ($t = t_H$). We note that the central density in the plane XY has increased slightly after one Hubble time of simulation,

Fig. 1. Contour plot of the primary galaxy at the beginning of the simulation ($t = 0$) and at the Hubble time of the simulation ($t = t_H$). The smoothing was made by averaging the 25 first and second neighbors of each pixel. Hereafter, the density levels in the planes XY and XZ at $t = 0$ will be used in all the contour plots, in the planes XY and XZ, respectively.

since the contour levels are the same for the two instants of time. In the XZ plane the scale height apparently has increased because of the two-body relaxation heating, but this quantity has changed very little (see Fig. 5).

From comparing Figs. 2 and 3, we note from the quantity $\langle V_z^2 \rangle^{1/2}$ that the self-heating of the initial disk and the particle halo adds another significant source of heating in the disk. The gravitational softening can also cause the disk to puff up; this is the reason why we chose a such small softening parameter, 125 times smaller than the scalar disk height. Moreover, the total rotation curves V_c and the angular momentum in the Z direction have not changed after one Hubble time of simulation.

In Figs. 4 and 5 we present the time evolution of the scale radius (R_d) and the scale height (Z_d). As expected, because of the heating of the disk the first quantity diminishes with the time while the second increases with the time. The linear fitting parameters of these two quantities are presented in the captions of these figures. Since the scale height has increased by less than 0.2%, we assumed throughout that this scale has not changed when we analyzed the data of the simulations.

The units used in the simulations are $G = 1$, [length] = 4.500 kpc, [mass] = $5.100 \times 10^{10} M_\odot$, [time] = 1.993×10^7 years ($H_0 = 100 \text{ km s}^{-1} \text{ Mpc}$) and [velocity] = $220.730 \text{ km s}^{-1}$. All the physical quantities will be referred to these units. The particle softening radius was assumed to be 0.0008 or 1/125 of the disk

Fig. 2. Rotation curve at the time $t = 0$ of the disk V_c , the main component of the angular momentum per unit of mass J_z and the velocity dispersion in the Z direction $\langle V_z^2 \rangle^{1/2}$. The coordinate R is the radius in cylindrical coordinates. The dotted line denotes the disk, the long-dashed line denotes the bulge, the short-dashed line denotes the halo, and the solid line denotes the total rotation curve.

Fig. 3. Rotation curve at the time $t = t_H$ of the disk V_c , the main component of the angular momentum per unit of mass J_z and the velocity dispersion in the Z direction $\langle V_z^2 \rangle^{1/2}$. The coordinate R is the radius in cylindrical coordinates. The dotted line denotes the disk, the long-dashed line denotes the bulge, the short-dashed line denotes the halo, and the solid line denotes the total rotation curve.

scale height. The critical opening angle was set to $\theta = 0.577$ and the forces between the cells and particles used the quadrupole correction. The longest integration step time was assumed to be 0.001 in units of simulation time. The Hubble time t_H corresponds to 490 time units.

We ran several simulations, without satellite to check the initial instabilities of the galaxy model. The initial galaxy simulations were run in a SUN FIRE 6800 cluster with 16 CPU processors. Each simulation took about 50 days of

Fig. 4. Time evolution of the scale radius (R_d). The projected particle number density on the XY plane was fitted using the approximation given by the Eq. (1). The coordinate R is the radius in cylindrical coordinates. The fitting parameters are $R_d = (-0.7042 \times 10^{-1} \pm 0.2840 \times 10^{-1})[t/t_H] + (0.8819 \pm 0.1620 \times 10^{-1})$.

Fig. 5. Time evolution of the scale height (Z_d). The projected particle number density on the XZ plane was fitted using the approximation given by the Eq. (1). The fitting parameters are $Z_d = (0.1791 \times 10^{-2} \pm 0.6320 \times 10^{-3})[t/t_H] + (0.9006 \times 10^{-1} \pm 0.3563 \times 10^{-3})$.

CPU time. For the simulations with the primary and satellite galaxies we used several clusters with a variety of CPU processors: SUN BLADE X6250, SUN FIRE X2200, SGI ALTIX ICE 8200, SGI ALTIX 450/1350, SGI ALTIX-XE 340, IBM P750, INTEL PENTIUM QUAD CORE and INTEL PENTIUM DUAL CORE. The number of CPU processors varied from at least 8 to at most of 128. Each simulation took 90 days of CPU time in average.

Very often (Oh et al. 2008; Dobbs et al. 2010; Lotz et al. 2010; Struck et al. 2011; Bois et al. 2011), simulations of interactions between two disk galaxies have used initial

Table 2. Primary and secondary galaxy initial conditions.

EXP	Θ	R_p	e	R_a	V_a
00					
01	0	30	0.1	36.67	0.5521
02	0	30	0.4	70.00	0.3263
03	0	30	0.7	170.00	0.1480
04	0	40	0.1	48.89	0.4782
05	0	40	0.4	93.33	0.2826
06	0	40	0.7	226.67	0.1282
07	90	30	0.1	36.67	0.5521
08	90	30	0.4	70.00	0.3263
09	90	30	0.7	170.00	0.1480
10	90	40	0.1	48.89	0.4782
11	90	40	0.4	93.33	0.2826
12	90	40	0.7	226.67	0.1282
13	0	15	0.1	18.33	0.7808
14	0	15	0.4	35.00	0.4614
15	0	15	0.7	85.00	0.2094
16	0	20	0.1	24.44	0.6762
17	0	20	0.4	46.67	0.3996
18	0	20	0.7	113.33	0.1813
19	90	15	0.1	18.33	0.7808
20	90	15	0.4	35.00	0.4614
21	90	15	0.7	85.00	0.2094
22	90	20	0.1	24.44	0.6762
23	90	20	0.4	46.67	0.3996
24	90	20	0.7	113.33	0.1813
25	0	23	0.1	28.11	0.6306
26	0	23	0.4	53.67	0.3726
27	0	10	0.7	56.67	0.2564
28	0	23	0.1	28.11	0.6306
29	0	23	0.4	53.67	0.3726
30	0	10	0.7	56.67	0.2564
31	90	23	0.1	28.11	0.6306
32	90	23	0.4	53.67	0.3726
33	90	10	0.7	56.67	0.2564
34	90	23	0.1	28.11	0.6306
35	90	23	0.4	53.67	0.3726
36	90	10	0.7	56.67	0.2564

Notes. G_1 is the primary galaxy, $G_2 = G_1$ the secondary galaxy, Θ the angle between the two planes of the disks in units of degree, R_p the pericentric distance in units of length, M_1 the primary galaxy mass in units of mass, e the eccentricity, R_a the apocentric distance in units of length, V_a the velocity at the apocentric distance in units of velocity, M_1 the primary galaxy mass, and $M_2 = M_1 = 0.621$ is the secondary mass galaxy in units of mass.

conditions such as parabolic or hyperbolic orbits. However, the bounded eccentric orbits in an interval of a Hubble time have not been studied because is very expensive in terms of CPU time. This is mainly because in this kind of simulation we must adequately integrate the internal dynamic of the disk galaxy during a very long cosmological time interval. We decided not to use cosmologically consistent initial conditions from publicly available simulations as done, e.g., by (Ruszkowski & Springel 2009) for two reasons. First, we would like to know what happened with the binary galaxies in bound orbits and, secondly, it would cost much more (in number of experiments and in CPU time) if we had begun our simulations from cosmologically unbound galaxies to arrive at eccentrically bound galaxies.

All the initial conditions of the numerical experiments are presented in Table 2. The orbits of the initial galaxies are eccentric ($e = 0.1, 0.4$ or 0.7) and the orientations of the disks are coplanar ($\Theta = 0$) or polar ($\Theta = 90$) to each other. The simulations began with the two galaxies at the apocentric positions.

Fig. 6. Time of merging with the fitted straight lines, for each eccentricity, where R_a is the apocentric distance. The open circles represent the simulations with $e = 0.1$. The open triangles represent the simulations with $e = 0.4$. The open squares denote the experiments with $e = 0.7$. The best-fit parameters are: $[t_M/t_H] = (0.039 \pm 0.002)R_a + (-0.508 \pm 0.056)$ (for $e = 0.1$), $[t_M/t_H] = (0.049 \pm 0.005)R_a + (-1.285 \pm 0.020)$ (for $e = 0.4$) and $[t_M/t_H] = (0.038 \pm 0.006)R_a + (-1.635 \pm 0.038)$ (for $e = 0.7$) (the far two points were obtained by extrapolating the time evolution of the distance between the two disks, using the simulation EXP17).

In Fig. 6 we show the dependence of the time of merging (T_M) (see Table 3) with the eccentricity (e) and the initial apocentric distance (R_a). The time of merging is defined as the time when the centers of mass of the two disks (primary and secondary galaxies) overlap (Chan & Junqueira 2001). The merging time increases linearly with the initial apocentric distance. The T_M for different eccentricities is obtained by extrapolating the linear approximation for each eccentricity and determining T_M for a fixed value of R_a . We obtained that the time of merging decreases with eccentricity.

There are two different kinds of merged remnants in our simulations. The first (coplanar disks) is a disk galaxy with scale radius and height very similar to the initial disk galaxy (see Table 3), but with a tidal radius that is at least five times larger than that of the initial galaxy (see Fig. 7). The second (polar disks) resembles a lenticular galaxy, but again with a tidal radius that is larger than the initial galaxy radius (see Fig. 7).

In Figs. 7 and 8 we present the contour snapshots of the result of the merger of the primary and secondary galaxies in the planes XY and XZ at the Hubble time of the simulation ($t = t_H$). We show the simulations EXP13, 14, 16, 17, 19, 20, 22, 23, 25, 27, 28, 30, 31, 33, 34, and 36. In the simulations with coplanar disk orbits (EXP13, 14, 16, 17, 25, 27, 28, and 30) the resulting fused galaxies are still disk galaxies. Their fitted scale radii ($R_{d(12)}$) and heights ($Z_{d(12)}$) using Eq. (1) are presented in Table 3. The merged disk galaxies are thicker and larger than the initial ones.

However, for the simulations with polar disk orbits (EXP19, 20, 22, 23, 31, 33, 34, and 36) the resulting fused galaxies are no longer disk galaxies. In both planes, XY and XZ , the galaxies resemble lenticular galaxies. The outer contour level of the merged galaxy in EXP23 is clearly deformed, differently from other merged polar disks, maybe because of the number of orbits

Table 3. Characteristics of the final stage of the orbits and merged disks.

EXP	Disk Interaction	Number of Orbits	T_M	$R_{d(12)}$	$Z_{d(12)}$	R_f
01	Open	1.5				
02	Open	1.0				
03	Open					
04	Open	1.0				
05	Open	0.5				
06	Open					
07	Open	1.5				
08	Open	1.0				
09	Open					
10	Open	1.0				
11	Open	0.5				
12	Open					
13	Merge	1.0	0.21	0.867 ± 0.041	0.116 ± 0.006	10
14	Merge	1.5	0.42	0.946 ± 0.051	0.142 ± 0.010	10
15	Graze	1.0	1.60*			
16	Merge	1.5	0.42	0.771 ± 0.042	0.128 ± 0.009	10
17	Merge	2.5	1.00	0.682 ± 0.025	0.199 ± 0.018	10
18	Open	0.5				
19	Merge	1.0	0.25			
20	Merge	1.5	0.43			
21	Graze	1.0	1.61*			
22	Merge	1.5	0.42			
23	Merge	2.5	1.00			
24	Open	0.5				
25	Merge	1.5	0.63	0.791 ± 0.038	0.116 ± 0.006	10
26	Open	1.5				
27	Merge	1.5	0.50	0.871 ± 0.044	0.465 ± 0.079	10
28	Merge	1.5	0.60	0.791 ± 0.038	0.116 ± 0.006	10
29	Open	1.5				
30	Merge	1.5	0.50	0.809 ± 0.041	0.175 ± 0.007	10
31	Merge	1.5	0.60			
32	Open	1.5				
33	Merge	1.5	0.54			
34	Merge	1.5	0.60			
35	Open	1.5				
36	Merge	1.5	0.54			

Notes. *Open* means that the two disks do not touch each other during the time of the experiment (t_H). *Graze* means that the two disks touch each other for a while and then separate. *Merge* means that the two disks fuse. Number of orbits is the total angular excursion of the companion until the merger, relative to its starting point. T_M is the time of merging in units of t_H when the two disks fuse (the symbols * in the times of merging of the simulations EXP15 and EXP21 denote that these times are estimates, using EXP17). $R_{d(12)}$, $Z_{d(12)}$ and R_f are the fitted scale radius, height and cutoff fitting radius of the only merged coplanar disk in units of length, using Eq. (1).

(see Table 3). This is the only simulation among all our experiments with the most orbits (2.5), with the longest time interval with tidal interaction between the two disk galaxies.

In Figs. 9 and 10 we show what happened to these galaxies using the simulation EXP31 at $t = 0.5t_H$ and at $t = t_H$. These figures show the disks of the primary galaxy G_1 and secondary G_2 . The polar characteristic of the G_2 is still there at $t = 0.5t_H$ but this is lost at $t = t_H$. At this time the polar disk is completely disrupted and its debris forms a stellar halo. From overlapping the contours of G_1 and G_2 , we obtain Fig. 8 for the simulation EXP31.

5. Power spectrum analysis

The power spectrum analysis provides a useful and objective tool for studying the induced waves. This analysis uses the amplitude and phase of the Fourier components of the surface density of the stars, and allows us to show the spiral modes in our simulations. This analysis give us the pattern speed of all spiral perturbations and their relative positions.

The power spectrum method, known historically as periodogram, was used to search for periodicities in sparse, noisy, unevenly spaced data (Junqueira & Combes 1996).

If we take an N -point sample of the function $c(t)$ at equal intervals of time t and compute its discrete Fourier transform (Press et al. 1992), we obtain the power spectrum $P(\Omega)$ of $c(t)$.

We used the grid-expansion method to analyze the density distribution ($128 \times 128 \times 128$ pixels) to obtain the power spectrum.

Firstly, using a radial binning of 0.0125, we obtained the amplitude and the phase of the Fourier components. Secondly, using the snapshots of the slab of the disk density in a time interval $0.01t_H$, we calculated the superposition of the entire Fourier amplitude and phase for each radial bin and for each time interval. Finally, we obtained the power spectrum as a plot of the number density for each radial bin. The power spectrum analysis was made by studying the primary galaxy just before the merging time. The orientation of the disk was not followed dynamically because it has not deviated from the initial orientation angle, as we can see in Fig. 10.

Fig. 7. Contour snapshot of the merger of the primary and secondary galaxies (flat disk merged remnants) together in the planes XY and XZ at the Hubble time of the simulation ($t = t_H$). Simulations EXP13, 14, 16, 17, 25, 27, 28 and 30.

Fig. 8. Contour snapshot of the merger of the primary and secondary galaxies (oblate disk merged remnants) together in the planes XY and XZ at the Hubble time of the simulation ($t = t_H$). Simulations EXP19, 20, 22, 23, 31, 33, 34, and 36.

Since we have a 3D particle disk, we limited the number of the particles within the planes $Z = -Z_{\max}$ and $Z = Z_{\max}$ to simplify the application of the grid expansion method (Chan & Junqueira 2003). We considered this thin slab between these two planes as the plane $Z = 0$ for the grid expansion. We denote this thin slab as $Z = 0$ in the equations. The chosen quantity $Z_{\max} = 0.1$ is the value of the scale height of the disk (Z_d). There are approximately 40% of the total disk particles (N_d) within these two planes. We assumed a largest radius of 5 length units throughout since we have 95% of the mass of the disk within this radius.

Fig. 9. Contour of the snapshot of the merger of the primary and secondary galaxies plotted separately in the plane XY and XZ at 50% of the Hubble time ($t = 0.5 t_H$). Simulation EXP31.

Fig. 10. Contour of the snapshot of the merger of the primary and secondary galaxies plotted separately in the plane XY and XZ at 60% of the Hubble time ($t = 0.6 t_H$). From overlapping the contours of G_1 and G_2 , at $t = t_H$, we obtain the contours of Fig. 8 for the simulation EXP31.

The basic assumption of the density wave theory is that spiral arms are not always composed of the same stars, but instead are the manifestation of the excess matter associated with the crest of a rotating-wave pattern. Two other assumptions were introduced from the onset, the linearity and quasi-stationarity of the wave. These assumptions allow us to write any perturbation of the axisymmetric background as superposition waves given by

$$\rho_d(R, \phi, Z = 0, t) = \sum_m \rho_m(R) e^{i[\Omega(m)t - m\phi]}, \quad (2)$$

where ρ_d is the density. The summation index indicates the symmetry of the component: $m = 0$ corresponds to the axisymmetric background; $m = 1$ corresponds to the lopsided perturbation,

Fig. 11. Wave modes $m = 1$ and $m = 2$ for simulation EXP15 at two different instants of time $0.65t_H$ and t_H . The density levels for these plots are the same as used in $m = 1$ ($t = 0.65t_H$).

and $m = 2$ corresponds to the symmetric two-arm perturbation (spiral, bar). $\Omega(m)$ is the pattern speed of the component m .

We can rewrite Eq. (2) in the usual wave notation

$$\rho_d(R, \phi, Z = 0, t) = \sum_m p_m(R) e^{i[\Psi_m(R) + \Omega(m)t - m\phi]}, \quad (3)$$

where $p_m(R)$ is the amplitude of the wave and $\Psi_m(R)$ is the phase angle of the mode m .

Now the density mode m can be obtained from

$$\rho_d^m(R, \phi, Z = 0, t) = p_m(R) \cos[\Psi_m(R) + \Omega(m)t - m\phi]. \quad (4)$$

We interpret the $m = 1$ and $m = 2$ plots in Figs. 11 and 12. They do not look like a clear one-armed spirals, or a clear asymmetry in the two arms, like in Fig. 13 of (Junqueira & Combes 1996), because in this present work they represent the Fourier analysis of a transient wave. Junqueira & Combes analyzed only the gaseous disk, but the analysis is similar to a stellar disk.

In Fig. 11 we show the transient wave modes $m = 1$ and $m = 2$ for the simulation EXP15 at two different instants of time $0.65t_H$ and t_H . Transient $m = 1$ wave modes are mostly present in the outer part of the disks. Furthermore, transient spiral arms ($m = 2$) are formed in the outer regions of the disks, and bars are present in the inner regions. Since they are transient $m = 2$ wave modes, the power spectrum for EXP15 (see Fig. 14) shows an undefined angular velocity for this mode.

In Fig. 12 we show the transient wave modes $m = 1$ and $m = 2$ for the simulation EXP31 at two different instants of time $0.5t_H$ and t_H . The transient $m = 1$ wave mode at $t = 0.5t_H$ is mostly present in the outer part of the disks, except at $t = t_H$. There is a large transient spiral arm ($m = 2$) at $t = 0.5t_H$ in the outer region of the disk and a prominent bar in the inner region of the disk at $t = 0.6t_H$. As in EXP15, here we have transient $m = 2$ wave modes seen in the power spectrum for EXP31 (see Fig. 14). If we compare Figs. 12 and 13, the Fourier decompositions are very similar with the snapshots of the disks.

Fig. 12. Wave modes $m = 1$ and $m = 2$ for simulation EXP31 at two different instants of time $0.5t_H$ and $0.6t_H$. The density levels for these plots are the same as were used in EXP15 ($m = 1$, $t = 0.65t_H$).

Fig. 13. Snapshots of the slabs $Z = 0$ for the simulation EXP15 at two different instants of time $0.65t_H$ and t_H . We also show the snapshots of the slabs $Z = 0$ for the simulation EXP31 at two different instants of time $0.5t_H$ and $0.6t_H$.

Figure 14 shows the power spectra for the $m = 2$ wave mode for simulations EXP00, 13, 14, 15, 16, 17, 19, 20, 21, 25, 28, and 31. We show only the $m = 2$ wave mode because we have not detected $m = 1$ wave mode in any simulation. Other experiments that presented an $m = 2$ wave mode and that are not shown in this figure are EXP22, 23, 27, and 33. These simulations were similar to that in Fig. 14. Other simulations did not show any sign of an $m = 2$ wave mode mostly because the primary and secondary halo did not touch each other during their evolution time (open disk interaction: see Table 3).

Fig. 14. Power spectrum for the mode $m = 2$ for the primary galaxy (G_1) and for simulations EXP00, 13, 14, 15, 16, 17, 19, 20, 21, 25, 28, and 31 until the time of merging. The density levels are three times higher than that of the experiment EXP00.

In Fig. 14, the first plot shows the power spectrum for the mode $m = 2$ for the simulation EXP00, without the secondary galaxy. This was made to analyze the existence of self-excited gravitational instabilities of the $m = 2$ wave mode in the disk. Evidently, there are not any wave modes.

Note the fuzzy small perturbations in the outer radii of the disks in Fig. 14 (note also that the density levels are three times higher than that used in the experiment EXP00). Most of the experiments in this figure have shown merged disks (see Table 3), except for the simulations EXP15 and 21 (grazing disks). There we can also see partial $m = 2$ wave modes in the outer radii of the disk with high clumpy density regions that do not extend to the inner part of the disks. Therefore, we cannot classify them as $m = 2$ stable wave modes because of these characteristics of the power spectra.

There are no discernable and significant substructures in Fig. 14. If there were a real stable wave we should have a plot like Fig. 16 of that (Junqueira & Combes 1996), a straight line parallel to the radial axis giving the pattern speed of the mode. Instead, since we have a transient wave, we obtain a fuzzy plot, more concentrated in the outer radial regions.

We integrated the wave amplitudes radially to derive the global Fourier amplitudes $|p_m|$ (Harsono et al. 2011), where m is the Fourier mode:

$$|p_m|(t) = \frac{1}{N_{\text{disk}}} \sum_{R=0}^{R=5} |p_m(R) e^{i[\Psi_m(R) + \Omega(m)t - m\phi]}|, \quad (5)$$

where $p_m(R)$ is giving by Eq. (3) and N_{disk} is total number of particles in the disk within $R = 5$. The global amplitude analysis was made in two distinct ways: a) by studying the primary galaxy until the merging time, and b) by analyzing the compound galaxies after the merger, when they occur. The orientation of the

Fig. 15. Global integrated ratio amplitudes of the modes $m = 1$ (solid lines) and $m = 2$ (dotted lines) for the primary galaxy (G_1) for the simulations 15 and 17 during a Hubble time. The $|p_{00}|$ denotes the global amplitude of the control simulation EXP00. For simulations EXP13, 16, 25, and 28 we show the global ratio amplitudes for G_1 until the merger time, denoted by the dashed lines. After the merge we show the global ratio amplitude of the compound galaxy ($G_1 + G_2$). For experiments EXP19, 20, and 31 (polar disk), we plot the evolution of G_1 until the merger of G_1 and G_2 , represented again by the dashed lines.

disk was not followed dynamically because of the deformation of the disk in some simulations. In Fig. 15 we show the temporal evolution of the relative global integrated amplitudes for simulations EXP13, 14, 15, 16, 17, 19, 20, 21, 25, 28, and 31.

Figure 15 shows that there is no possible kick due to uncological initial conditions. The reason is simply that all the simulations have begun at the apocentric distance, where the tidal interaction between the two binary galaxies is weaker.

Furthermore, all the waves are mostly driven shortly before merger, because the separations decreased, even in the cases with no merger.

We analyzed the special case EXP17 in Fig. 15. The simulation EXP17 presents three maxima for $m = 2$ wave mode. The first maximum is due to the formation of a bar, when the two disks are still far apart. The second and third ones are due to the formation of a two-arm spiral, when the disks already touch and deform each other.

Lopsided features are preferentially observed in the distribution of gas in late-type spiral galaxies. In several cases these features can be identified as one-armed spirals ($m = 1$ mode). More frequently, nuclei of galaxies are observed displaced with respect to the gravity center, as in M33 and M101. The nucleus of M31 reveals such an off-centering, which has been interpreted in terms of an $m = 1$ perturbation. Miller & Smith (1992) have studied through N -body simulations of disk galaxies a peculiar oscillatory motion of the nucleus with respect to the rest of the

Fig. 16. G_1 halo contours of the EXP15 in the plane XY at three different times: $t = 0$, $t = 0.68t_H$ at the time of the maximum amplitude of the disk $m = 2$ component (see Fig. 15), and $t = t_H$. The G_1 disk contour of the EXP15 in the plane XY at the time $t = 0.68t_H$. The two outer halo contour density levels correspond approximately to 40% of the total halo mass at the radius $R \approx 4$ and 90% of the total halo mass at the radius $R \approx 11$. The halo and the disk of the secondary galaxy G_2 can be obtained by reflecting the respective contour image relative to the Y axis at $X = 0$, since the two galaxies have the same mass distribution.

Fig. 17. G_1 halo contours of the EXP15 in the plane XZ at three different times: $t = 0$, $t = 0.68t_H$ at the time of the maximum amplitude of the disk $m = 2$ component (see Fig. 15), and $t = t_H$. The G_1 disk contour of the EXP15 in the plane XZ at the time $t = 0.68t_H$. The two outer halo contour density levels correspond approximately to 40% of the total halo mass at the radius $R \approx 4$ and 90% of the total halo mass at the radius $R \approx 11$. The halo and the disk of the secondary galaxy G_2 can be obtained by reflecting the respective contour image relative to the Y axis at $X = 0$, since the two galaxies have the same mass distribution.

axisymmetric galaxy. They interpreted the phenomenon as an $m = 1$ instability, a density wave in orbital motion around the center of mass of the galaxy. Moreover, [Junqueira & Combes \(1996\)](#) have shown that stars and gas are off-centered with respect to the center of mass of the system. Figure 15 shows that the $m = 1$ and $m = 2$ modes are excited at different times. This is because the mode excitation comes from the outer region to the inner region of the disk. Thus, there is a time lag for this excitation to reach the inner disk region. Since the $m = 1$ mode needs an off-centered disk mode with respect to the center of mass and since the $m = 2$ mode is mostly excited from the outer disk region, this produces a time delay among the maxima of these two modes.

Figures 16 and 17 show the evolution of some halo contours containing, for example, about 40% and 90% the halo mass (EXP15). The early merger paper of [Barnes & Hernquist \(1996\)](#), which first discussed the halo mergers, has given attention only to the remnant halos at the end of the simulation. Our figures show that the strongest halo contour deformation (90%) coincides with the strongest disk deformation. After the passage of the secondary galaxy through the primary at the Hubble time, the halos settle down and their contours resemble the initial ones.

6. Discussion

We have dynamically evolved two disk galaxies with halo and bulge using N -body simulations. The initial disk model is stable against any self-excited $m = 1$ or $m = 2$ wave modes. The satellite galaxy has coplanar or polar disk orientation with respect to the disk of the primary galaxy and their initial orbits are prograde eccentric ($e = 0.1$, $e = 0.4$ or $e = 0.7$). Both galaxies have a similar mass and size as the Milky Way.

Most recent papers that studied the tidal interaction between two galaxies have used a fixed potential for the halo ([Oh et al. 2008](#); [Dobbs et al. 2010](#); [Struck et al. 2011](#)). This condition can be misleading because the halos are very important in transmitting angular momentum to the disk of the primary galaxy. The halo of the primary galaxy can respond globally to disturbance of the halo of the secondary galaxy, thus it can affect the disk structure in an inward effect. These effects can be clearly seen in the analysis of the power spectra (see Fig. 14).

This is the first published work, as far as we know, that has studied the secular evolution of bound disk binary galaxies. Nevertheless, we only compare our results with the global results of similar papers, since the numerical methods, initial conditions, time of integration, etc. are different from ours.

We showed that the merger of two coplanar ($\Theta = 0$) pure stellar disk galaxies can result in a disk galaxy instead of an elliptical one, as was shown in other papers ([Bournaud et al. 2005](#); [Bois et al. 2011](#)). If we have merger of two polar ($\Theta = 90$) disk galaxies we can also have the formation of lenticular-like galaxies. These results are new to our knowledge.

In fact, none of our simulations resulted in elliptical galaxies. In a recent work, [Bois et al. \(2011\)](#) studied the formation of early-type galaxies through mergers with a sample of high-resolution numerical simulations of binary mergers of disk galaxies. The initial galaxy model had a real halo, bulge, disk, and gas. The orbits used in the merge simulations were all parabolic or hyperbolic, corresponding to initially unbound galaxy pairs, which is different from our simulations where the galaxy pairs were, from the very beginning, bound in eccentric orbits.

Furthermore, we demonstrated that the time of merging increases linearly with the initial apocentric distance of the

galaxies and decreases with the eccentricity (see Fig. 6). Boylan-Kolchin & Quataert (2008) have studied the merging time of extended dark matter haloes using N -body simulations. Each of their simulations consisted of a host halo and a satellite halo; the ratio of the satellite to the host mass varied from 0.025 to 0.3 and the initial circularity of the satellite varied from 0.33 to 1, or in other words, the initial eccentricity varied from 0 to 0.67. They found that the merging time decreases exponentially with the eccentricity. This result partially agrees with our findings since the T_M decreases with the eccentricity. However, we do not have enough simulations with different eccentricities to confirm the exponential behavior.

We also showed that the tidal forces of the disks can excite wave mode $m = 1$ and the wave mode $m = 2$, but they are not stable, meaning that they are transient wave modes (see Fig. 14). In a previous work (Chan & Junqueira 2003) we have shown that tidal interaction of a secondary point-mass galaxy could excite stable $m = 1$ and $m = 2$ wave modes in the density distribution as well as in the velocity distribution. In contrast to our previous paper, here we began the simulations with an apocentric distance where the halos did not touch. However, after the merging of the disks, when this occurred, these instabilities have faded away completely and the fused disk has become thicker and more extended.

Many authors (Oh et al. 2008; Lotz et al. 2010; Dobbs et al. 2010; Struck et al. 2011; Snaith et al. 2012) have shown that tidal interaction can trigger gravitational instabilities, such as spiral arms or lopsidedness. Our results confirmed these results: it was possible to create spiral arms, bars or lopsidedness through the tidal force, but they were transient phenomena.

The simulations with merger remnants the waves abruptly disappear after the merger are completed (in less than one outer disk rotation period). This point, illustrated in Fig. 15, shows that it is almost the opposite result reported by Struck et al. (2011), who have found that weak flybys induce waves that take a long time to nonlinearly break. The maximum relative amplitude of these waves is at most about 15 times larger than that of the control case. The $m = 2$ wave mode is generated mainly by tidal interaction in the outer region of the disks. The $m = 1$ wave mode depends mostly on an interaction of the inner part of the disks, producing an off-centering effect of the wave mode center with respect to the center of mass of the disk. These characteristics produce a time lag among the maximum formation of these two wave modes. The disk settles down quickly after the merger, in less than one outer disk rotation period. Furthermore, although the two disks may spend a long time in orbit, waves are only induced in the short time they are close together. The stellar disks can survive gentle merging, even with a massive companion, and the waves abruptly disappear after the merger is completed.

Finally, galaxy disks are born gas-rich, and the key to S0 formation is the evolutionary path from such progenitors. It is theoretically interesting that some form of disk can be preserved through some types of major merger. Practically, however, it is not likely that too many S0s are result of S0+S0 or early Sa+Sa mergers. A related, and more important point is that if

stellar disks can survive some gas-free major mergers, then they are also likely to survive multiple minor mergers, which may play a more important role in finishing the formation of S0s. The idea that minor mergers play such a role in ellipticals is very well known today, which makes it much more enlightening for S0s.

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