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Measuring mean densities of δ Scuti stars with asteroseismology

Theoretical properties of large separations using TOUCAN*

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ABSTRACT

Aims. We study the theoretical properties of the regular spacings found in the oscillation spectra of δ Scuti stars.

Methods. We performed a multivariable analysis that covered a wide range of stellar structure, seismic properties and model parameters that are representative of intermediate-mass, main-sequence stars. The workflow was entirely performed using a new Virtual Observatory tool: TOUCAN (the VO gateway for asteroseismic models), which is presented in this paper.

Results. A linear relation between the large separation and the mean density is predicted to be found in the low-frequency domain (i.e. radial orders spanning from 1 to 8, approximately) of the main-sequence, δ Scuti stars' oscillation spectrum. We found that this linear behavior remains the same whatever the mass, metallicity, mixing length, and overshooting parameters considered. The intrinsic error of the method is discussed. This includes the uncertainty in the large separation determination and the role of rotation. The validity of the relation found is only guaranteed for stars rotating up to 40% of their break-up velocity. Finally, we applied the diagnostic method presented in this work to five stars for which regular patterns have been found. Our estimates for the mean density and the frequency of the fundamental radial mode match with those given in the literature within a 20% of deviation.

Conclusions. Asteroseismology has thus revealed an independent direct measure of the average density of δ Scuti stars, which is analogous to that of the Sun. This places tight constraints on the mode identification and hence on the stellar internal structure and dynamics, and allows determining the radius of planets orbiting δ Scuti stars with unprecedented precision. This opens the way for studying the evolution of regular patterns in pulsating stars, and its relation with stellar structure and evolution.

Key words. asteroseismology – stars: evolution – stars: variables: delta Scuti – stars: interiors – stars: oscillations – virtual observatory tools

1. Introduction

Thanks to asteroseismic space missions such as MOST (Walker et al. 2003), CoRoT (Baglin 2003), and *Kepler* (Gilliland et al. 2010), the study of the intrinsic variability of A- and F-type stars is currently changed fundamentally. In particular, the large number of modes detected and the variety of frequency domains covered has given rise to the so-called *hybrid* phenomenon, which imposes a revision of the current observational instability strips of δ Scuti and γ Dor stars (see Uytterhoeven et al. 2011, and references therein). These stars have been found to be located across the entire γ Dor and δ Scuti instability strips, which implies that a review of their pulsation mechanisms is necessary to supplement (or even substitute) the convective blocking effect and the κ mechanisms. Based on stellar energy balance studies, Moya & Rodríguez-López (2010) concluded that δ Scuti stars are predicted to excite hundreds of pulsation modes, whose accumulated pulsation energy is not strong enough to destroy their hydrostatic equilibrium. Likewise, the richness found in the oscillation spectrum of these stars was interpreted by Kallinger & Matthews (2010) as nonradial pulsation superimposed on granulation noise (correlated noise).

We focus here on δ Scuti stars, which are intermediate-mass stars (i.e., from 1.4 to 3 $M_{\odot}$, approximately), located mainly in the main sequence and the subgiant branch, but also in the pre-main sequence. Their spectral types range from A2 to F5, and their luminosity classes from IV to V, approximately. This locates them at the lower part of the Cepheid instability strip. Their pulsations are mainly driven by the so-called κ mechanism (see reviews by Breger 2000; Handler 2009), showing radial and nonradial oscillation modes excited in a frequency domain ranging from a few tens of μHz to about 800−900 μHz (see e.g. Poretti et al. 2011).

Interpreting the oscillation spectra of these stars has never been an easy task. Due to the complexity of their oscillation spectra the identification of detected modes is often difficult. A unique mode identification is often impossible and this hampers seismology studies of these stars. Additional uncertainties arise from the effect of rapid rotation directly on the hydrostatic balance in the star and, perhaps more importantly, through mixing caused by circulation or instabilities induced by rotation (see e.g. Zahn 1992).

However, some decades ago, this scenario started to change because regular patterns were found in the observed oscillation spectra. These patterns were detected in the low radial order modes (mixed modes) frequency domain, in which

* Appendices A and B are available in electronic form at http://www.aanda.org

main-sequence classical pulsators show their strongest oscillation power, that is, between 80 and 800 μHz (hereafter intermediate frequency domain). In this domain, regularities similar to those found in solar-like stars were not expected (for a review on this topic, see Goupil et al. 2005). Great efforts were made in long ground-based multisite campaigns, from which Handler et al. (1997) found regularities (near 26 μHz) in the oscillation spectrum of the young δ Scuti star CD-24 7599 that contained 13 frequencies. These regularities were found using a Fourier transform technique. Breger et al. (1999) also found a regular spacing close to 46 μHz in the oscillation spectrum of the δ Scuti star FG Vir, composed of 24 frequencies; they searched for regular spacings using histograms of frequency differences. Later on, they re-studied the star using data spanning ten years (Breger et al. 2009), in which the number of detected frequencies increased to 68. The frequency spacing was attributed to an observed clustering of certain non-radial modes around radial ones. This clustering was explained by the higher probability of certain $\ell = 1$ modes (the so-called trapped modes, Dziembowski & Krolikowska 1990) to be excited to observable amplitudes than other modes.

Different space missions provided asteroseismic data with which similar studies were undertaken, with many more frequencies detected with very high precision. In 2007, Matthews (2007) found 88 frequencies in the oscillation spectrum of the δ Scuti star HD209775 observed by MOST. In that work, a regular spacing of about 50 μHz was obtained using histograms of frequency differences. This was assumed to be a large separation, although no physical explanation was given to support this. Recently, regular patterns were also found in the oscillation spectra of δ Scuti stars observed by CoRoT (García Hernández et al. 2009; Mantegazza et al. 2012; García Hernández et al. 2013) and *Kepler* (Hernández et al. 2013), which supports that they can be identified with large separation.

Theoretically, large separation is expected to grow from low to high radial orders. In contrast to the well-defined plateau shown by the large separation in the high radial order domain (the so-called asymptotic regime, see e.g. Antoci et al. 2011; Zwintz et al. 2011), it forms a quasi-periodic structure at low radial order regime (García Hernández et al. 2009, from now on GH09). In that work, it was shown that the standard deviation of such structure is roughly 2.5 μHz, and the distance between the mean value of this structure and the high-order regime is on the order of 5 μHz.

We here answer the questions about the physical properties of the periodicities observed in the low frequency domain of δ Scuti stars, and whether they have properties similar to those found in solar-like stars.

To do this, we analyzed the properties of the predicted regularities from a large collection of models and physical variables. We present here the virtual observatory tool, TOUCAN[1], which we developed to easily handle stellar and seismic models, examine their properties, compare them with observational data and find models representative of the studied star(s).

2. Method

We examined a dense sample of asteroseismic models representative of A-F main-sequence stars, that is, stars that cover the area in the HR diagram where classical pulsations for these stars are expected. This modeling approach requires an efficient computing procedure as well as the capability of managing and analyzing large sets of models. For the former, we performed most of the computation using the Grid computing service provided at IAA-CSIC as one of the nodes of the *Ibergrid* virtual organization.

For the latter, we have developed TOUCAN, a VO tool able to easily compare different and heterogeneous collections of asteroseismic models (equilibrium models and their corresponding synthetic oscillation spectra). Details about the workflow followed and/or the different TOUCAN services can be found in Appendices B.4 and B.3. The model collection we used is described in the next section.

Table 1. Stellar modeling parameters.

Parameter	Lowest	Highest	Step
$M/M_\odot$	1.25	2.20	0.01
[Fe/H]	−0.52	+0.08	0.20
$\alpha_{\rm ML}$	0.50	1.50	0.50
$d_{\rm OV}$	0.10	0.30	0.10

Notes. Ranges of the four parameters used to construct the current model dataset representative of intermediate-mass stars.

2.1. Model grid

We constructed a model collection composed of approximately 5×10^5 models of intermediate-mass stars (δ Scuti and γ Dor stars). For the sake of homogeneity and precision of the asteroseismic mode sample, models were computed following the prescriptions suggested by the ESTA/CoRoT[2] working group (Moya et al. 2008; Lebreton et al. 2008).

The equilibrium models were computed with the evolutionary code CESAM (Morel 1997; Morel & Lebreton 2008). Oscillation frequencies were computed with GraCo (Moya et al. 2004; Moya & Garrido 2008), which uses the perturbative approach, to provide adiabatic and non-adiabatic quantities related to pulsation and includes the convection-pulsation interaction using the time-dependent convection theory (TDC, Dupret et al. 2005).

The model grid is composed of evolutionary tracks, evolved from the zero-age main sequence (ZAMS) to the subgiant branch. Each track contains about 250 equilibrium models with their corresponding oscillation spectra. The equilibrium models were computed by varying four quantities: two global stellar parameters, mass and metallicity (M and [Fe/H]), and two modeling parameters often used in asteroseismology, the convection efficiency, $\alpha_{\rm ML} = l/H_{\rm p}$, where l is the mixing length and $H_{\rm p}$ is the pressure scale height, and the overshoot parameter $d_{\rm OV} = l_{\rm ov}/H_{\rm p}$ ($l_{\rm ov}$ is the penetration length of the convective elements). The variation range for each parameter in this dataset is listed in Table 1. The physical parameters of the models were chosen to be those typically adopted for δ Scuti stars.

Although δ Scuti stars have also been observed in the pre- and post-main-sequence evolutionary stages, we focus here on the main sequence (i.e. hydrogen-burning phase in the convective core) where the low-order periodicity has been found (see previous section).

[1] http://svo.cab.inta-csic.es/theory/sisms3/index.php?

[2] http://www.astro.up.pt/corot/

2.2. Large frequency separation

The frequency difference defined as

$$\Delta\nu_\ell = \nu_{n,\ell} - \nu_{n-1,\ell}, \tag{1}$$

is known as a large separation, where $\nu_{n,\ell}$ is the frequency of a mode with radial order n and degree ℓ, for all azimuthal orders m. This frequency difference is nearly constant for p modes in the asymptotic regime, and obeys, in a first approximation, the following physical dependence (Tassoul 1980; Gough 1990):

$$\Delta\nu = 2\left[\int_0^R \frac{\mathrm{d}r}{c_\mathrm{s}}\right]^{-1} = \tau^{-1}, \tag{2}$$

where c_s is the sound speed and τ is the acoustic time between 0 and R (i.e. the stellar center and surface, respectively) which define the stellar region in which the oscillation mode is propagating.

TOUCAN calculates $\Delta\nu$ everywhere in the theoretical oscillation spectrum for any combination of angular degree (up to $\ell = 3$), frequency, and radial-order domains. In practice, the calculation is performed in two steps. First, for a given angular degree ℓ, that is,

$$\Delta\nu_\ell = \frac{1}{N_n}\sum_{n=n_j}^{n_{k-1}} (\nu_{n+1} - \nu_n), \quad j < k \tag{3}$$

where n is the radial order, and $N_n = n_k - n_j$ is the number of frequencies found in the interval $[n_j, n_k]$ for the given ℓ. Then TOUCAN calculates the average of $\Delta\nu_\ell$ over all the angular degrees calculated in the model, that is,

$$\Delta\nu = \langle\Delta\nu_\ell\rangle = \frac{1}{N_\ell}\sum_{\ell=\ell_i}^{\ell_j} \Delta\nu_\ell, \tag{4}$$

where $N_\ell = \ell_j - \ell_i$ is the total number of angular degrees considered. Here we are interested in studying the physical properties of such separations for the intermediate-frequency domain. But before analyzing the TOUCAN results, we need to take into account some considerations with respect to the rotation and the presence of mixed modes.

2.3. Rotation and mixed modes

The A-F type stars are typically fast rotators, and rotation effects on both the stellar structure and the oscillation frequencies cannot be neglected, particularly those caused by stellar distortion (Soufi et al. 1998; Suárez et al. 2006), even for $m = 0$ modes, with which $\Delta\nu$ are calculated here (Suárez et al. 2010).

Rotation effects on oscillations are commonly taken into account through the perturbation approximation (Dziembowski & Goode 1992), which is limited to slow-to-moderate rotation, that is, small stellar deformations (see e.g. Suárez et al. 2005; Reese et al. 2006, for semi-empirical and theoretical studies of the limitations of the perturbation approximation). Therefore, for moderate-to-rapid rotators one needs a non-pertubative approach to calculate of the oscillation modes on a deformed star (e.g. Lignières et al. 2006). However, today this calculation is available for polytropic models and for some more realistic fully deformed 2D stellar models on the ZAMS based on the self-consistent field (SCF) method (Jackson et al. 2005; MacGregor et al. 2007; Reese et al. 2009), which considers the models to be chemically homogenous with an angular velocity assumed to be dependent only on the distance from the axis of rotation. Furthermore, these latter models together with the calculation of non-perturbative oscillations require a significant amount of computing resources as well as time of computation. Therefore a proper modeling for rapidly rotating stars would be unpractical for the present work.

On the other hand, because periodicities are indeed observed, it might be concluded that rotation effects are not sufficient to break the regularities. Indeed, non-perturbative calculations of the oscillation spectra for rapidly rotating polytropic models indicate that as rotation increases, the asymptotic structure of the non-rotating frequency spectrum is replaced by a new form of organization (Reese et al. 2008, 2009). This new mode frequency organization also shows regular structures, including the large separation (see Lignières et al. 2010, for a recent theoretical work on regular patterns in rapidly rotating stars), whose variation (normalized by the density) from the non-rotating case is negligible (Lignières et al. 2006). Furthermore, calculations of non-perturbative oscillation frequencies on SCF models show a maximum variation of the large separation of around 2.3 μHz for stars rotating up to 40% of the Keplerian velocity (Reese, priv. comm.), which is small compared with the precision with which the periodicities are predicted at both low- and high-frequency domains (10 μHz approx., see e.g. Mantegazza et al. 2012). Considering all the above theoretical arguments, we used non-rotating models for the present study.

This supports the hypothesis that identifies the observed periodicities with the large separation, as well as the hypothesis that these appear in a new distribution of modes predicted by the non-perturbative theory. To check these hypotheses, it is necessary to analyze the oscillation spectra of many stars with different rotational velocities. Thanks to space missions like CoRoT, *Kepler*, or PLATO, this study can be undertaken in the near future.

Another problem that might hamper the detection of periodicities, in particular of the large separation, is the presence of mixed modes. This phenomenon is implicitly considered here because the selected models cover the whole main sequence. In contrast, this cannot be properly studied within the non-perturbative approach, since both the polytropic and the SCF models in ZAMS are not expected to properly show the avoided crossing phenomenon. To properly understand how avoided-crossing may affect the detection of periodicities, the present analysis needs to be performed for a more evolved SCF model (work in progress).

On the other hand, all these results were obtained assuming no dependence on the visibility of the modes, which may introduce artificial periodicities that may potentially be confused with the large separation. Recently, this has been studied in the framework of the non-perturbative theory. Specifically, it has been found (Reese et al. 2013) that acoustic modes with the same (ℓ, $|m|$) values tend to have similar amplitude ratios, although this effect is not systematic. Because the global visibility of the modes decreases while the mode degree increases, we do not expect this effect to affect the large separation determination significantly. We currently work on the influence of mode visibilities in the study of periodicities for rapid rotators.

2.4. Computation of large separations

In GH09, regular spacings were found using the Fourier transform (FT). Here we study whether this technique is equivalent to the average of the periodicity used by TOUCAN.

To do so, we used a subset of asteroseismic models within typical values for T_eff and $\log g$ of δ Scuti stars with their

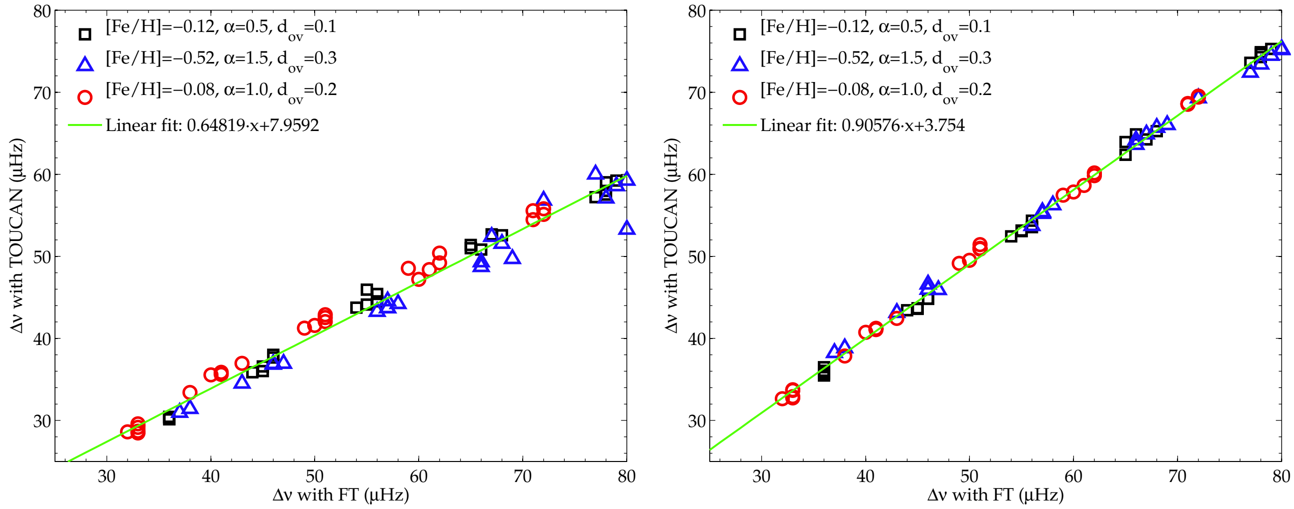

Fig. 1. *Left panel*: spacings found using the FT method are compared with large separation values computed with TOUCAN in the range of the observed modes for δ Scuti stars. *Right panel*: similar to *left panel*, but only p modes with radial order $n \geq 1$ were considered in the calculation of TOUCAN $\Delta\nu$. Spacings obtained with the FT method and TOUCAN computations are only equivalent under the conditions adopted in the *right panel*. (This figure is available in color in electronic form.)

corresponding uncertainties. For completeness, we considered the following extreme combinations of the physical parameters listed in Table 1 divided into three sets:

#1 [Fe/H] = −0.12, $\alpha_{\rm ML}$ = 0.5, and $d_{\rm ov}$ = 0.1
#2 [Fe/H] = −0.52, $\alpha_{\rm ML}$ = 1.5, and $d_{\rm ov}$ = 0.3
#3 [Fe/H] = 0.08, $\alpha_{\rm ML}$ = 1, and $d_{\rm ov}$ = 0.2.

For each set of parameters, we selected five models that covered the extreme values (four models) and the central value (one model) of effective temperature and gravity considered in the selected model subset.

For each set of five models, we computed the large separation using both methods in the intermediate frequency domain, that is, no restrictions in the radial orders, but only in the frequency range were considered to mimic the observations. Independently of the combination of model parameters the relation between the large separations obtained with the FT method and those obtained by TOUCAN was found to be linear (Fig. 1, left panel).

On the other hand, the corresponding linear fit does not have a slope equal to one, which would be expected if the two methods were equivalent. To understand this, we first attempted to explain these discrepancies by the presence of gravity modes (g modes). However, when removing g modes from the TOUCAN computations (not from the FT ones), the slope of the trend was improved but still not equal to one.

We then considered the well-known ratio between the period of the fundamental radial mode and the period of its first overtone (which is 0.77 for population I, main-sequence stars). This ratio, which is constant for each δ Scuti star, implies a relation between these two radial modes that does not necessarily follow the average behavior of $\Delta\nu$. To avoid this contribution in the calculation of the large spacings by TOUCAN, we also restricted the oscillation modes to those with a radial order $n \geq 1$.

With all these requirements, the FT and TOUCAN large separations are found to be formally equivalent (Fig. 1, right panel). These requisites were thus adopted throughout the whole work.

Note that we always used the same range of frequencies for computing the FT. This test emphasizes the robustness of the FT method to detect the large separation even when g modes and the fundamental radial mode are not distinguished. Detailed analysis of the slopes obtained in the two cases indicates that an uncertainty of 10% in the equivalence should be considered if other constraints, namely in the frequency range, are not taken into account.

3. Large separation vs. mean density

One of the main characteristics of large separations observed at high frequency (e.g. in solar-like stars) is that they are proportional to the stellar mean density through Eq. (2), which, together with another regularity, the so-called small separation, provides a direct measure of the mass, radius, and evolutionary stage of those stars (see e.g., Christensen-Dalsgaard 1988). We therefore investigated whether a dependence is also predicted at low frequencies, in particular in the frequency domain where δ Scuti stars pulsate.

To do so, we followed the workflow described in Appendix B.4 to select a set of models to work with. In particular, we considered all the models contained in the model database described in the previous sections, that is, those whose parameters are described in Table 1. For the sake of simplicity, the study was restricted to models in the main sequence, that is, between zero-age to turn-off main sequence stages. For the asteroseismic content, we adopted the prescriptions deduced in the previous section, that is, we computed the large separation in the intermediate-frequency domain using radial and non-radial modes with $n \geq 1$, and $\ell \leq 3$. Indeed, modes with $\ell > 3$, whose presence is suggested by spectroscopic studies (Mantegazza et al. 2012), would hamper the determination of the large separation. Because our oscillation spectra obtained from photometry and the FT method (details in GH09) uses the modes with the largest amplitudes, it is plausible to consider that these modes correspond to $\ell = [0, 3]$ modes.

Analysis of the large separation predicted in the intermediate frequency domain as a function of the mean density reveals (Fig. 2) a clear relation that can be expressed mathematically as

$$\Delta\nu/\Delta\nu_{\odot} = 0.776\,(\rho/\rho_{\odot})^{0.46}, \tag{5}$$

obtained by performing a linear regression of the complete set of models. This gives us an approximation of the overall behavior

Table 2. Mean density estimates of δ Scuti stars found in the literature.

No.	Star (μHz)	$\Delta\nu$ (μHz)	u (g cm^{-3})	$\bar{\rho}$ (g cm^{-3})	$\sigma(\bar{\rho})$ (μHz)	ν_0 (g cm^{-3})	$\bar{\rho}^{\rm T}$ (g cm^{-3})	$\sigma(\bar{\rho}^{\rm T})$ (mag)	$\langle J-K\rangle$
1	HD 50870[6]	45[b]	5	– (0.20)	– (0.17)	80.09(162.352)	0.68	2.55	0.14
2	FG Vir[2]	46[a]	...	0.16(0.24)	0.10(–)	140.62(157.05)	0.78	2.54	0.17
3	HD 209775[5]	50[a]	...	0.25 (0.29)	– (0.17)	... (168.55)	0.53	2.56	0.09
4	HD 174936[3]	52[b]	5	0.33(0.31)	0.10(0.17)	... (174.72)	0.52	2.56	0.09
5	XX Pyx[4]	54[b]	2.3	0.25(0.33)	0.02(0.12)	... (180.87)	0.98	2.55	0.23
6	HD 174966[1]	65[b]	1	0.51(0.48)	0.11(0.17)	200.23 (212.14)	0.59	2.56	0.11

Notes. Sample selected to have mean densitities derived from regularities in their their power spectra, which were associated with a large spacing. From left to right, columns represent the star identification, the observed $\Delta\nu$, its uncertainty u, determinations of the mean density $\bar{\rho}$ with their corresponding errors $\sigma(\bar{\rho})$, and determinations of the fundamental radial mode frequency. Quantities in parentheses were calculated in this work. The last three columns give the estimates of the mean density of each star using the Tingley method, together with their errors, and average $J-K$ magnitude differences used for its calibration.

References. (a) Histograms of frequency differences; (b) Fourier transform of the oscillation powerspectrum; (1) García Hernández et al. (2013); (2) Breger et al. (2009); (3) GH09; (4) Handler et al. (1997); (5) Matthews (2007); (6) Mantegazza et al. (2012).

of the relation between $\Delta\nu$ and $\bar{\rho}$ in the intermediate-frequency domain, which is predicted here to be proportional to the mean density of the star to the power 0.46, which is quite close to $\Delta\nu \propto \rho^{1/2}$, predicted for high radial orders. This opens the way for studying the evolution of regular patterns in pulsating stars throughout the HR diagram, and its relation with stellar evolution. This might also provide new insights into the hybrid phenomenon, that is, the unexplained excitation modes in a wide frequency range covering typically the γ Doradus, δ Scuti and even solar-like oscillation modes (Uytterhoeven et al. 2011).

We present no errors in the coefficients of the fitting because none of the theoretical $(\Delta\nu_i, \bar{\rho}_i)$ are independent of each other, and therefore regression error estimates are meaningless. Nevertheless, it is possible to examine the domain of validity of the method. In Appendix A we provide estimates of intrinsic errors of the method and their relation with the domain of validity of the present method.

These results imply that Fig. 2 can be used as a powerful diagnostic tool for the study of A-F stars, such as δ Scuti stars and/or hybrid stars that are being observed by the space missions. Not only does it provide a direct estimate of the mean density of the stars, but also an estimate of the frequency of the fundamental radial mode. Up to date, this latter asteroseismic observable has not been fully exploited in δ Scuti stars (with the exception of HADS, see e.g. Poretti et al. 2005; Suárez et al. 2007) because of the well-known difficulties for the mode identification. Therefore our results also represent an additional help for the mode identification for this type of pulsator.

The strength of this diagnostic tool is that it is almost model independent, since all the models contained in the heterogenous dataset follow the same trend. In the following sections, we discuss different details of this finding and its consequences.

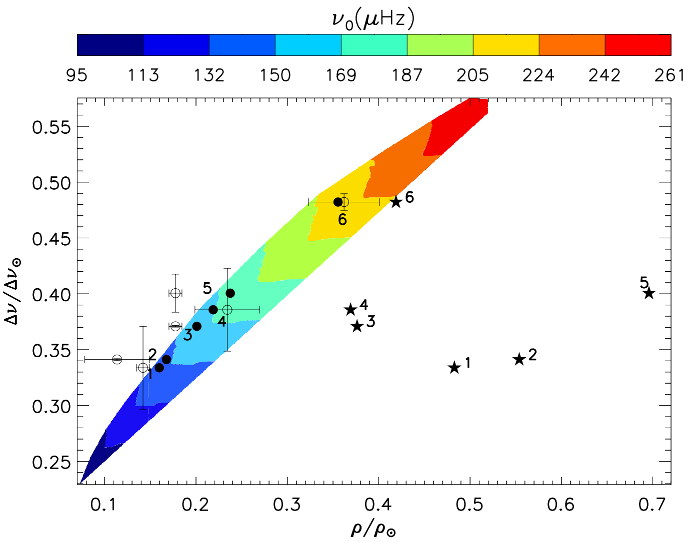

Fig. 2. Predicted large separation as a function of the mean density of the star, normalized to their solar values, $\Delta\nu_\odot$ = 134.8 μHz (Kjeldsen et al. 2008) and $\bar{\rho}_\odot$ = 1.48 g cm^{-3} (Haynes et al. 2012), respectively. Contours in grey scale indicate the predicted frequency of the fundamental radial mode. Filled dots, empty dots, and star symbols represent mean densities found in this work, in the literature, and using Tingley's calibration, respectively. Stars are labeled as in Table 2. For the sake of clarity, the error bars in star symbol estimates are omitted, since they are larger than the abscissa range. Colors are only available in the online version of the paper. (This figure is available in color in electronic form.)

4. Some real examples

For a first rough quality check of the diagnostic tool presented here, we applied it to some known δ Scuti stars: FG Vir, HD 174936, HD 174966, XX Pyx, HD 50870, and HD 209775 (see references in Table 2), for which the regular patterns have been found and analyzed. The comparison of the results found using Fig. 2 and/or Eq. (5) with those published in the literature is given in Table 2.

In general, predicted values are found within the uncertainties provided in the literature. As an example, a value of 52 μHz was proposed as possible large spacing for the CoRoT δ Scuti star HD 174936 by GH09. For such a value of the large spacing, and assuming no error on this value, a mean density of about ρ = 0.31 g cm^{-3} is predicted here. Our agree well with those of GH09, taking into account that an uncertainty in $\Delta\nu$ of about 5 μHz was considered in the latter.

We compared the above results with an independent method for estimating the mean density of stars without making use of models or asteroseismic information. More specifically, we used the method that uses a color-density calibration for main-sequence stars (see details in Tingley et al. 2011). This calibration is independent of factors such as age and metallicity, although it may suffer from significant error if extinction is unknown or poorly estimated. The method applied to our selected stars yields the values given in Table 2. The uncertainties in the density measurements are significantly larger than those obtained by asteroseismology (works cited in Table 2) and those obtained in the present work. This prevents us concluding about a possible trend between the density and the large separation.

To date, the number of A-F stars whose regularities have been studied is very small. Fortunately, the asteroseismic space missions will constantly increase the number of A-F stars observed with high precision. With this diagnostic tool it will be possible to better study asteroseismic properties of A-F stars, including the large spacing itself.

5. Conclusions

We have studied the theoretical properties of the regular spacings in the oscillation spectra of δ Scuti stars. A comprehensive dataset of models representative of A-F stars that covered a wide range of stellar physical magnitudes (e.g. effective temperature, gravity, metallicity) and model parameters was performed. We analyzed the behavior of the predicted large spacings (calculated in the intermediate-frequency domain, that is, the frequency domain in which δ Scuti stars pulsate) and their possible dependencies upon other physical magnitudes and/or model parameters. The workflow was entirely performed using TOUCAN, a virtual observatory tool for managing and analyzing asteroseismic models that we have developed within the Spanish Virtual Observatory (see Appendix B).

Firstly, we have shown that for the frequency domain where δ Scuti stars pulsate, the regular spacings obtained using the FT technique (as in GH09) and the regular mathematical definition of large separation computed by TOUCAN are only equivalent when selecting oscillation modes with radial order $n \geq 1$.

Secondly, we have found a linear relation between large spacing and the mean density (in a logarithmic scale), for models in the main sequence, for all the metallicity, mass, and convection parameters ($\alpha_{\rm ML}$ and $d_{\rm OV}$) considered (within the values typically used for δ Scuti stars). With the use of a large model grid, we constructed a diagnostic diagram whose properties were also studied in detail. The most important error source in determining the mean density and other quantities such as the frequency of the fundamental radial mode, comes from the uncertainty in the large spacing measure for rotating stars. In particular, our results are only valid for stars rotating at most at 40% of the break-up velocity. Nevertheless, these uncertainties are expected to be drastically lowered when additional observational constraints are included (as shown by GH09).

Finally we applied our diagnostic method to five stars for which regular patterns have been found. Our estimates for the mean density and the frequency of the fundamental radial mode match those given in the literature using asteroseismology reasonably well (within 20% of deviation). The comparison with other independent (non-asteroseismic) methods for obtaining the mean density of stars reveals that asteroseismology provides, by now, the more precise estimates.

This is a significant step towards the level of precision achieved in studies of solar-like stars for asteroseismic studies of A-F pulsating stars. This is particularly relevant, not only to understand the structure and evolution of these stars, but also for the study of planetary systems. Moreover, the observation of solar-like oscillations in stars that host planets provides a significant help for characterizing the planetary systems, through the precise knowledge of the mass, radius, mean density, and age of the host star (e.g. Christensen-Dalsgaard et al. 2010). Through our work, we can extend these capabilities to the study of planets orbiting around A-F pulsating stars. In fact, most of the planets discovered by direct imaging are found to be orbiting such stars. These systems are critical for understanding the spin-orbital interactions between the planets and the hosting star (Winn et al. 2009; Wright et al. 2011). For these, an estimate of the age of the system is crucial to determine the mass of the discovered planets. Thanks to the large amount of A-F stars observed by CoRoT and Kepler, the number of applications of the relations derived here open a new and important window to study the structure and evolution of these stars (Uytterhoeven et al. 2011; Moya et al. 2010).

In a forthcoming study, we examine the statistical properties of the frequency spectra by analyzing their autocorrelation functions and by looking at the cumulative distribution functions of the frequency separations. Results on mode visibility (Reese et al. 2013) together with the use of multicolor photometry will help us to better asses how large separation is affected by rotation, and to improve the accuracy of the relation found here.

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Appendix A: Estimate of intrinsic errors of the method

To estimate the domain of validity of the method, it is necessary to determine how sensitive the distribution of $(\Delta\nu, \bar{\rho})_i$ points in Fig. 2 is to variations of the physical parameters used to construct the model grid. Note that the linear fit given by Eq. (5) was obtained considering each model as an independent *measurement* without uncertainties. Indeed, the whole dataset is composed of independent subsets of models: the evolutionary tracks. As a consequence, the standard procedure for calculating coefficient errors and/or the goodness of the fit are meaningless here. Instead, we sought to estimate the errors committed by studying how variations of the physical parameters with which the model dataset was constructed (Table 1) modify the shape or the thickness of the strip shown in Fig. 2.

In particular, we examined the $\Delta\nu$-$\bar{\rho}$ relation by analyzing the maximum variation of a given parameter p at once (Table 1), leaving the remaining parameters free to vary. Then, we calculated the size of the model strip at a given value of $\Delta\nu_i$ with a given uncertainty $\pm u_i$, defined as

$$S_p(\bar{\rho}) = |\bar{\rho}(\Delta\nu_i + u_i) - \bar{\rho}(\Delta\nu_i - u_i)|, \quad \text{(A.1)}$$

and the intrinsic error committed in the estimate of $\bar{\rho}$ from Fig. 2 for a given value $\Delta\nu_i$ with an uncertainty of u_i is given by

$$\epsilon_p(\bar{\rho}) = \max\{S_{p_{\max}}(\bar{\rho}), S_{p_{\min}}(\bar{\rho})\}. \quad \text{(A.2)}$$

That is, we considered the largest possible error for a given parameter variation.

These error estimates are necessarily dependent on the uncertainty in the observed value of the large spacings. We studied this dependence by calculating $\epsilon_p(x)$ for a set of u_i values (in μHz) ranging from 0 to 10 μHz. Figure A.1 shows the evolution of the errors with u_i, which is roughly linear. Note that for $u_i \lesssim 2\,\mu$Hz, values lower than 0.12 g cm^{-3} are predicted for $\epsilon_p(\bar{\rho})$. Therefore, very low uncertainties in $\Delta\nu$ are required for a good determination of the mean density. For instance, to derive an uncertainty of ± 0.02 in $\bar{\rho}$, one would need to measure $\Delta\nu$ with a precision u_i lower than 1 μHz. Note that this is the precision reached by García Hernández et al. (2013) in the study of periodicities of the δ Scuti star HD174966. Indeed, ϵ_p are intrinsic errors of our method. The total error σ (see Table 2) on the mean density also depends on all the constraints considered to model the studied star.

For an ideal perfect measurement of the large spacing, the lowest precision is given by the strongest rotation effect (due to the star deformation) on $\Delta\nu$ (see Sect. 2.3). For very rapidly rotating objects (stars rotating faster than 40% of the Keplerian velocity) the effect of rotation on the large spacing is stronger than 2.3 μHz, which means an intrinsic error on $\bar{\rho}$ of approximately 0.12gcm^{-3}. For slower stars, this intrinsic error becomes smaller. The largest errors predicted for the estimate of the mean density range from 11% to 21% of the total variation of $\bar{\rho}$ in the main sequence.

We recall that such variations correspond to the worst case, and therefore they must be regarded as an upper limit. When other observational constraints are considered (e.g., metallicity, gravity, effective temperature), the errors in the diagnostics proposed here can drop drastically (GH09).

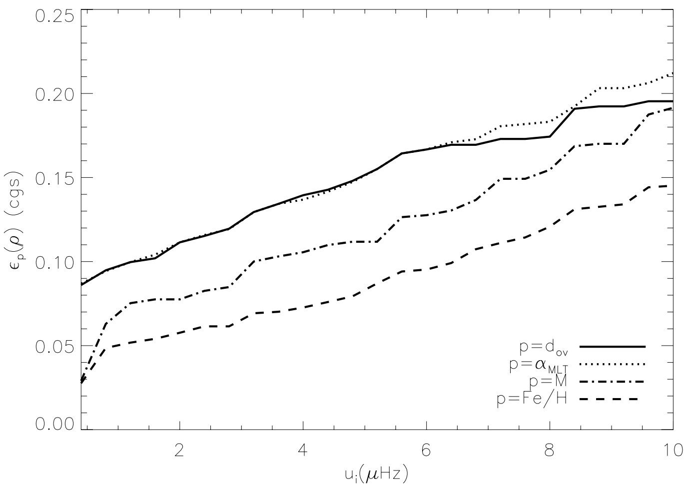

Fig. A.1. Dependence of $\sigma_p(\bar{\rho})$ errors (Eq. (A.2)) with the uncertainty in $\Delta\nu$ for the four quantities considered. A value of $\Delta\nu = 60\,\mu$Hz (about the middle of the main sequence) is assumed.

Appendix B: TOUCAN

Here we present TOUCAN, the first virtual observatory tool for asteroseismology developed by the Spanish Virtual Observatory (SVO)[3], with which performed the entire workflow of our study. In this section we show the tool in the context of necessities of the asteroseismic community, describe its main objectives and characteristics, and detail its current workflow. Note that such an application is constantly evolving and some of the snapshots provided here might be different in the future. But the main purpose and ultimate objectives will remain the same.

B.1. Context

Stellar physics experiments today have significantly progress because of the rapid development of one of its main laboratories: stellar seismology, which is the only technique that allow probing the interior of stars to gain detailed knowledge of the internal structure and the physical processes occurring there. In the last decades we have witnessed a significant development of this technique, mainly because of the increase of the quantity and quality of the observations, particularly from space and ground-based multisite campaigns. From space, a significant amount of high-quality asteroseismic data is available from MOST[4](Walker et al. 2003), CoRoT[5](Baglin 2003), and *Kepler*[6](Gilliland et al. 2010), launched in 2009. Other missions such as GAIA (Perryman 2003) and PLATO (Catala 2009), will increase the available datasets by a factor of several hundreds. From the ground, dedicated photometric and spectroscopic follow-up observations for the above-mentioned space missions (e.g. Poretti et al. 2009; Uytterhoeven et al. 2010, for CoRoT and Kepler missions, respectively) are necessary for a better characterization of the stars observed by the satellites.

3 http://svo.cab.inta-csic.es
4 http://www.astro.ubc.ca/MOST/
5 Convection, Rotation, and planetary Transits. The CoRoT space mission was developed and is operated by the French space agency CNES, with participation of ESA's RSSD and Science Programmes, Austria, Belgium, Brazil, Germany, and Spain: http://corot.oamp.fr/
6 http://kepler.nasa.gov/

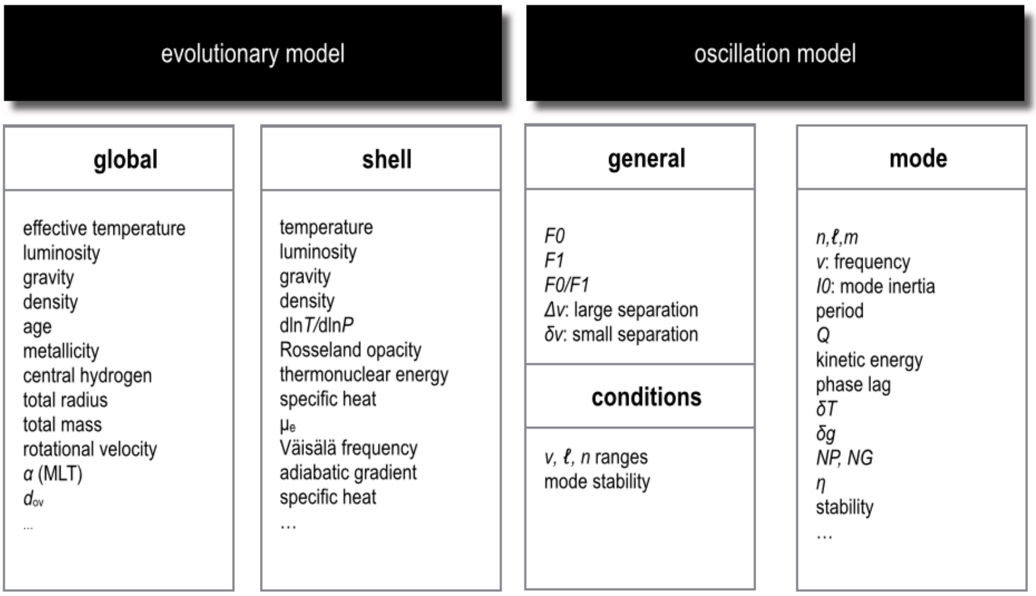

Fig. A.2. Overall scheme of the basic data model adopted in TOUCAN. The physical variables from equilibrium models and the asteroseismic variables are listed on the *left and right panels*.

A proper understanding of this huge amount of information requires a similar leap forward on the theoretical side (see Suárez 2010, for a recent review on this topic). Today, simulations of complex systems produce huge amounts of information that are difficult to manipulate, analyze, extract, and publish. Significant advance has been made in this regard, for instance, the Asteroseismic Modeling Portal (AMP, Metcalfe et al. 2009) or MESA (Paxton et al. 2011) codes. However, the main problem comes from the necessity of dealing with theoretical models developed by different groups, with different codes, numerical approximations, physical definitions, etc. This lack of homogeneity makes it difficult to design automatic tools to simultaneously work with different models and/or applications able to use the models on the fly.

On the observational side, these problems have been successfully solved thanks to Virtual Observatory (VO), which is an international initiative whose main objective is to guarantee easy access and analysis of the information residing in astronomical archives and services. Nineteen VO projects are now funded through national and international programs, all projects working together under the IVOA[7] to share expertise and best practices and develop common standards and infrastructures for the VO.

In this context, the Spanish VO (SVO), which joined IVOA in June 2004, is deeply involved in the development of standards that guarantee a full interoperability between theory and observations and among theoretical collections themselves. In particular, SVO actively participated in the development of the VO access protocol for theoretical spectra[8] and is currently working on a more general protocol called S3[9]. Examples of theoretical models published in the VO framework can be found at the SVO theoretical model server[10].

[7] http://www.ivoa.net
[8] http://ivoa.net/Documents/latest/SSA.html
[9] http://ivoa.net/Documents/latest/S3TheoreticalData.html
[10] http://svo.cab.inta-csic.es/theory/db2vo4/

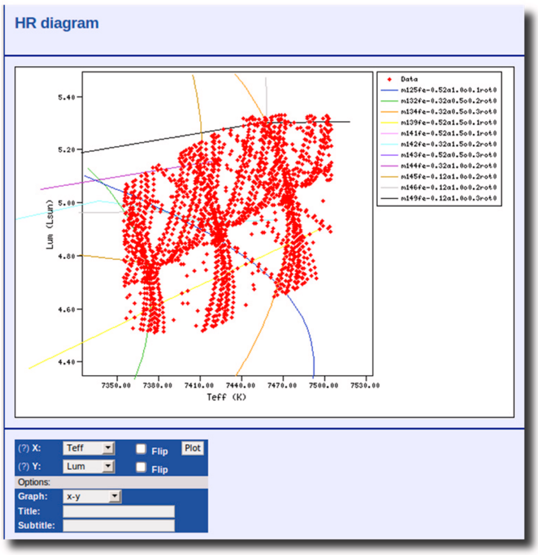

Fig. B.1. Illustration of a Herzprung-Russel diagram of the models matching all the input criteria simultaneously. Dots represent the effective temperature and luminosity of all the valid models. Lines represent evolutionary tracks. For clarity, only some of the models used in this work are depicted. A color version of the plot is accessible in the online version of the paper. (This figure is available in color in electronic form.)

B.2. Characteristics

TOUCAN is a tool conceived to work with VO-compliant models. In the VO, models are described according to the same data model and accessed using the same access protocol which solves all the problems of data discovery, data access, and data representation of non-VO tools.

The tool is intended to have a wider applicability in asteroseismology, and more generally in stellar physics and in any other field for which stellar models are required. To summarize, the main characteristics are:

- Efficiency. TOUCAN typically queries multiple model databases in seconds.

- Collections of models are handled easily and with user-friendly web interfaces.
- The only software required is a web browser.
- Tables, figures, and model collections are fully downloadable.
- Designed for the easy and fast comparison of very different and heterogenous models.
- Visualization tools are available. Some of the plots presented here were built with the TOUCAN graphic tools (see Appendix B.4).
- The tool offers new scientific potential, which is otherwise technically impossible or time consuming. The multivariable analysis performed in this work only required a systematic query to TOUCAN for the different parameters needed.

Furthermore, TOUCAN has also been designed for an easy and quick interpretation of the asteroseismic data of running space missions such as MOST, CoRoT, and *Kepler*, as well as future missions such as the PLAnetary Transits and Oscillations (PLATO), currently an M3 candidate in the ESA Cosmic Vision program, or the Transiting Exoplanet Survey Satellite (TESS), a new NASA space mission scheduled for launch in 2017. For this purpose, the next steps in the development of TOUCAN will be:

- The inclusion of new collections of models, namely solar-like and giant-like asteroseismic models. This will be achieved by calculating new model datasets with our own codes, and by adapting other model databases, built with differently codes and different physics, to TOUCAN.
- Implementation of a direct link between TOUCAN and other existing VO services, allowing the search for observed physical parameters stored in VO-compliant databases (those of the space missions), and using them as inputs in TOUCAN.

B.3. VO service

TOUCAN has been designed following the Virtual Observatory standards and requirements. This means that in parallel to the web interface, the system can also be accessed from other VO applications using the S3 protocol to obtain in a standard way information about:

- The available combinations of evolutionary and seismological models.
- The query parameters, their physical description, and the available range of values.
- The list of models that match the query criteria and their properties.
- The stellar shell structure and the oscillation spectrum for each model.

From a technical point of view, this feature is very important, since it allows the tool to work with multiple model databases, no matter where they are located physically. Moreover, this opens the possibility of interconnecting TOUCAN with existing astronomical archives, catalogs, etc., in particular, those being constructed using asteroseismic space data that are already in VO-compliant form.

B.4. Workflow

The TOUCAN workflow we describe here is general, and was therefore applied for the present work. It is composed of three main steps:

- Input parameter specification

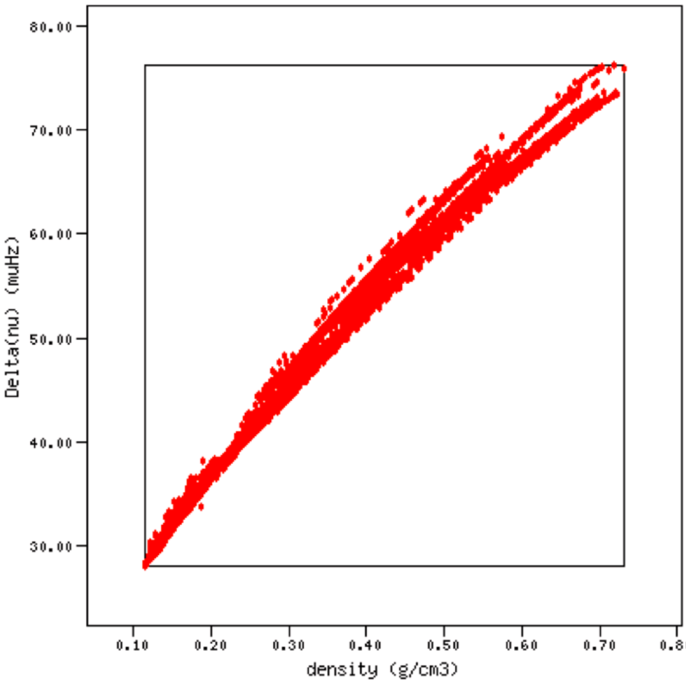

Fig. B.2. Large spacing, $\Delta\nu$, as a function of the mean density for a large set of models described in Sect. 3. Large spacings were calculated using all the frequencies available per ℓ, up to $\ell = 3$ (more details in the text). Time evolution reads from right to left, as the mean density of stars in the main sequence decreases with time. Plots were obtained using TOUCAN graphical utilities. A color version of the plot is accessible in the online version of the paper. (This figure is available in color in electronic form.)

- Summary of the results and check-out
- Model selection and online analysis.

One of the most critical steps when building a tool to handle different theoretical models in a compatible way is the identification of the mandatory parameters to represent the physics involved, and their mapping into a common set of variables. For this, we developed a prototype data model (Fig. A.2) for asteroseismology that contains 17 star global properties (effective temperature, surface gravity, luminosity, etc.), 44 star shell variables (density, pressure, temperature, etc.), and 35 seismic properties (frequency ranges, fundamental radial mode, large and small separation, etc.). For a maximum interoperability, we used the most common definitions in the field for these variables, with the aim of setting the basis of VO standards for asteroseismology.

After selecting the model parameters, TOUCAN queries the user-specified model database. Here we used our own model database described in Sect. 2.1. The results obtained from these queries are shown to the user in different formats, with the possibility of managing them and, more importantly, of using TOUCAN online graphic tools, which allow the researcher to easily do *online asteroseismology*, for instance by:

- Visually examining the resulting models, with some statistics and sorting possibilities for an efficient handling of the results.
- Selecting individual or multiple files to be analyzed (including a shell variables analysis for equilibrium models) with the graphic tools, which allows the user to download the generated plots (an example of HR diagram built with the TOUCAN graphic tools is shown in Fig. B.1).
- Allowing the user to select individual or multiple files to be downloaded (e.g. complete evolutionary tracks) in the

original codes' output formats and in VO table formats. This provides compatibility with other VO-compliant visualization tools like TOPCAT[11].
- To download plots in "png" format, by placing the mouse pointer on the plot window and clicking the mouse's right button.

All these characteristics make it possible to easily perform a quick *on-the-fly* analysis of a large set of models, and/or comparisons of different and heterogeneous models, or even model collections.

Moreover, these tools allow the user to perform statistical works on theoretical properties of multiple variables at the same time. The present work is an example of such a work: the relation between the large spacings and the mean density for δ Scuti stars shown in the contour plot (Fig. 2) was built using the data obtained from Fig. B.2 during this research workflow.

[11] http://www.star.bris.ac.uk/~mbt/topcat/