

Dielectronic satellite lines and double layers in solar flares

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ABSTRACT

Context. Particle acceleration during solar flares results in departures of the distribution of particle energies from the Maxwellian distribution. Apart from the high-energy tail, the bulk of the distribution was recently also found to be significantly affected, due, e.g., to the presence of double layers.

Aims. We investigate the influence of several proposed non-Maxwellian distribution functions on the X-ray flare line spectra. The distribution functions considered are sharply peaked and include the n -distribution, the moving Maxwellian distribution, and the distribution formed in strong double layers in the flaring plasma.

Methods. Synthetic Si XIIId–Si XIV spectra involving allowed and dielectronic transitions at 5–6 Å are calculated numerically. The parameters chosen for the calculations correspond to the impulsive phase of solar flares, as inferred by previous authors.

Results. The Si XIIId λ 5.56/Si XIII λ 5.68 and Si XIIId λ 5.82/Si XIII λ 5.68 ratios depend on the relative number of electrons at energies corresponding to the formation of the Si XIIId lines. Therefore, these ratios increase with the increasing narrowness of the peak of the electron distribution function. The highest ratios are achieved for the distribution formed in double layers, while the moving Maxwellian distribution is less likely to reproduce the observed enhancement of Si XIIId intensities. However, the ratio of the allowed Si XIV λ 5.22/Si XIII λ 5.68 transitions depends on the ionization equilibrium. This ratio is very small for the double-layer distribution. Combination of the double-layer distribution with a Maxwellian distribution with the same mean energy significantly enhances this ratio, while keeping the Si XIIId intensities sufficiently increased to explain the characteristics of the observed spectra.

Conclusions. These results support the presence of double layers in the plasma during impulsive phase of solar flares.

Key words. Sun: X-rays, gamma rays – Sun: flares – atomic processes – radiation mechanisms: non-thermal

1. Introduction

Solar flares are dynamic manifestations of magnetic energy release accompanied by particle acceleration (e.g., Fletcher et al. 2011; Holman et al. 2011; Lin 2011; Zharkova et al. 2011; Raymond et al. 2012). The most common consequence of electron acceleration is the presence of high-energy power-law tail (e.g., Lin & Hudson 1971; Holman et al. 2003; Veronig et al. 2010), i.e., an enhanced number of electrons with energies up to several tens of MeV. However, departures from the equilibrium Maxwellian distribution in solar flares were also observed at much lower energies, several keV, corresponding to the excitation energies of X-ray lines, e.g., of He-like Fe (Seely et al. 1987) or Si (Dzifčáková et al. 2008).

That the departures from Maxwellian distribution affect line intensities, especially of X-ray lines, has already been proposed by, e.g., Gabriel & Phillips (1979). These authors showed that the high-energy tail would lead to a decrease in the observed Fe XXIVd/Fe XXV ratio because of the enhanced excitation of the Fe XXV resonance line due to the high-energy tail. However, an opposite situation was reported by Seely et al. (1987). These authors proposed that a distribution with a slope steeper than the Maxwellian distribution at energies corresponding to the formation of Fe XXIVd dielectronic satellite lines could explain the observed enhanced excitation of these lines. To do that, they introduced an analytical, parametric n -distribution

$$f(\mathcal{E}, n)d\mathcal{E} = \mathcal{B}_n \frac{2}{\sqrt{\pi}(k_B T)^{3/2}} \left(\frac{\mathcal{E}}{k_B T} \right)^{\frac{n-1}{2}} \mathcal{E}^{1/2} e^{-\mathcal{E}/k_B T} d\mathcal{E}, \quad (1)$$

where $\mathcal{B}_n = \sqrt{\pi}/(2\Gamma(n/2 + 1))$ is the normalization constant, k_B is the Boltzmann constant, and $n \in \langle 1, \infty \rangle$ and T are the parameters of the distribution. For $n = 1$, the distribution is Maxwellian. The mean energy $\langle \mathcal{E} \rangle = 3/2 k_B \tau$, where $\tau = (n/2 + 1)T$, is the pseudo-temperature.

Following this result, Dzifčáková et al. (2008) and Kulinová et al. (2011) showed that the Si XIIId/Si XIII ratios observed during impulsive phases of several solar flares observed by the RESIK instrument (Sylwester et al. 2005, also Fig. 1) can be explained neither by an iso-thermal nor by a multi-thermal plasma with a Maxwellian distribution. They diagnosed values of $n = 11$ or higher. Furthermore, Kulinová et al. (2011) showed that highest values of n are observed during the impulsive phase and coincide with the presence of type III radio bursts. This means that the n -distributions should be connected with the beam and return-current system in the flare plasma (Dzifčáková & Karlický 2008; Karlický et al. 2012). Including the n -distribution also significantly improves the goodness-of-fit of the RHESSI data at energies of 4 keV or more (Kulinová et al. 2011). During the decay phase, the RESIK spectra can be well fitted by the Maxwellian distribution of isothermal plasma (Fig. 1 bottom), indicating equilibration and cooling of the plasma after cessation of the particle acceleration processes.

Karlický et al. (2012) investigated the physical meaning of the n -distributions in connection to the beam – return-current system in solar flares. They found that the Maxwellian distribution with a velocity drift v_0 (“moving Maxwellian”) closely mimics the high-energy slope of the n -distribution

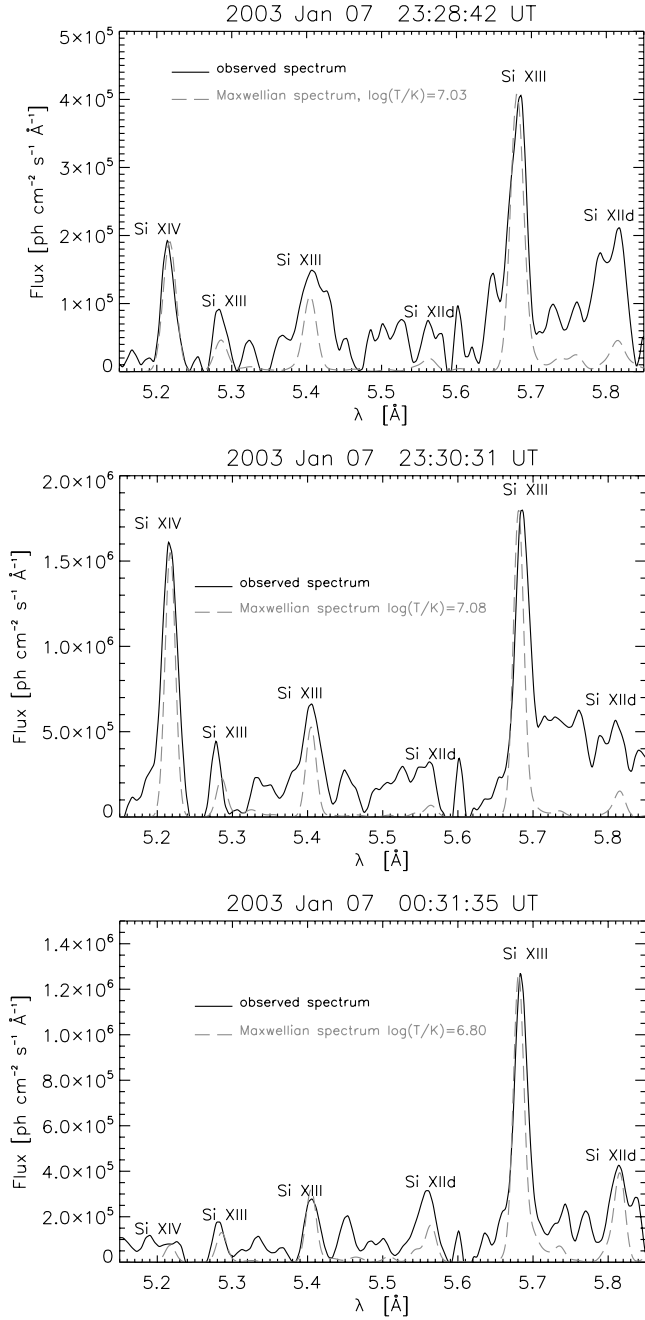


Fig. 1. X-ray spectra observed by the RESIK instrument during impulsive (*top* and *middle*) and decay (*bottom*) phase of an M4.9 flare on January 07, 2003 (Dzifčáková et al. 2008). Synthetic Maxwellian isothermal Si spectra having the same Si XIV $\lambda 5.22$ /Si XIII $\lambda 5.68$ ratio are indicated by the grey dashed lines. Note that the RESIK observations contain a number of unidentified lines.

when expressed as a function of electron energy,

$$f(\mathcal{E}, v_0) d\mathcal{E} = \frac{\sinh\left(2\frac{v_0}{v_T} \sqrt{\frac{\mathcal{E}}{k_B T}}\right)}{\sqrt{\pi} k_B T v_0 / v_T} e^{-\mathcal{E}/k_B T - (v_0/v_T)^2} d\mathcal{E}, \quad (2)$$

where v_T is the thermal velocity of the plasma. To reach values of $n = 11$ or higher, v_0/v_T has to be greater than 1. This means that the moving Maxwellian distribution is unstable, mainly due to the Buneman instability. That the observed departures from the Maxwellian distribution last several minutes (Kulinová et al. 2011, Figs. 4, 7, and 9 therein) means that the true distribution

Table 1. Strongest Si lines observed in the fourth channel of RESIK.

Ion	λ [Å]	Transition
Si XIV	5.22	$1s^2 2s^2 2p^2 P_{1/2,3/2} - 3p^2 P_{1/2,3/2}$
Si XIII	5.28	$1s^2 1S_0 - 1s 5p^1 P_1$
Si XIII	5.40	$1s^2 1S_0 - 1s 4p^1 P_1$
Si XIII	5.68	$1s^2 1S_0 - 1s 3p^1 P_1$
Si XIII	5.56	$1s^2 2p^2 P_{1/2,3/2} - 1s 2s 4p^2 D_{3/2,5/2}$
Si XIII	5.82	$1s^2 2p^2 P_{1/2,3/2} - 1s 2s (3P) 3p^2 D_{3/2,5/2}$

in the flaring plasma should be stable. By using the 3D particle-in-cell simulations of a flare loop containing electron beam, reflected beam, and background plasma with return current, Karlický (2012) showed that the formation of double layers (DL) in flare plasma can lead to a stable peak of the distribution function, closely mimicking the n -distribution at the required energy range. This peak is superimposed on the broken power-law distribution. The DL is a spatially very narrow structure of the strong electric field parallel to the magnetic field lines. Based on the numerical simulations (Karlický 2012), it was found that only electrons entering into this layer can be accelerated. Thus, the Maxwellian distribution (on the low-potential side of DL) after an acceleration in this layer results in the distribution peak (on the high-potential side of DL), which can be approximated by a moving half-Maxwellian distribution, i.e., a distribution that is Maxwellian in v_x and v_y , and also $v_z > 0$, but zero for $v_z < 0$, where z is the coordinate along the flare loop. The moving half-Maxwellian distribution is hereafter referred to as the distribution in DL or simply the “DL-distribution”.

While having the same gradient $df/d\mathcal{E}$ at the energies of few keV, where the dielectronic satellite lines are formed, the n -distribution, moving Maxwellian, and DL-distributions differ by orders of magnitude in the low-energy end (Fig. 2 top). These differences can affect the recombination rates and subsequently the diagnostics of non-Maxwellian plasma during the impulsive phase of solar flares. In this paper, we investigate the influence of these distributions on the formation of flare X-ray spectra containing the dielectronic satellite lines of Si.

2. Method

We calculate the Si synthetic spectra at 5–6 Å, corresponding to the fourth channel of the RESIK instrument. The relevant Si transitions are listed in Table 1. To calculate the intensities of these lines for the considered distributions, we use the method described by Dzifčáková et al. (2008). The ionization, recombination, and excitation rates are given by the integral product of the distribution function and the corresponding cross sections (e.g., Dzifčáková 2000; Dzifčáková & Tóthová 2007). We used the atomic data available in the CHIANTI database (Dere et al. 1997, 2009; Landi et al. 2012). The ionization rates are obtained from the work of Dere (2007), while the radiative and dielectronic recombination rates for the ions studied here are obtained from the works of Colgan et al. (2004), Badnell (2006a,b), and Bautista & Badnell (2007). The excitation rates are calculated using the method of Dzifčáková & Mason (2008) with the atomic data of Wiese et al. (1966), Lin et al. (1977), Armour et al. (1979), Vainshtein & Safronova (1978, 1980), Martin & Zalubas (1983), Sampson et al. (1983), Drake (1986), Zhang & Sampson (1987), Zhang et al. (1990), Aggarwal & Kingston (1992), Chen et al. (1993), Martin et al. (1993, 1995), and Phillips et al. (2005), implemented in the CHIANTI database.

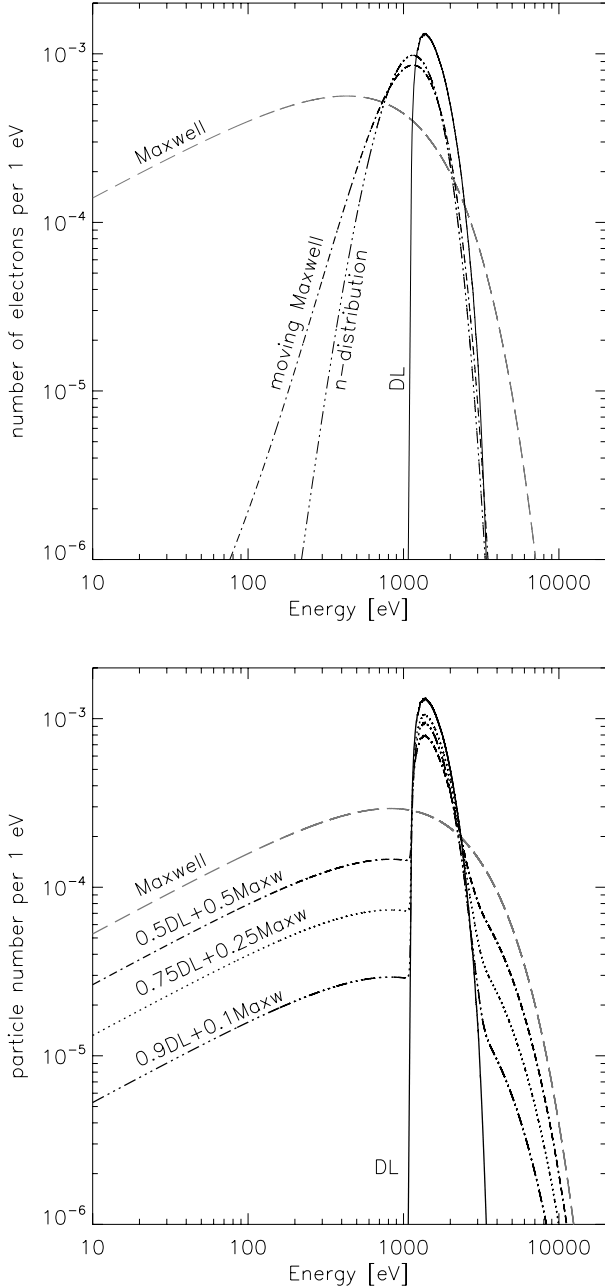


Fig. 2. Distribution functions. *Top:* Maxwellian, n -distribution, moving Maxwellian and DL-distributions. *Bottom:* combination of the DL-distribution ($\log(\tau) = 7.28$) with Maxwellian distribution ($\log(T) = 7.28$). The relative weights are indicated.

We do not use the newest available excitation rates or A values for Si XIII and Si XIV (Aggarwal & Keenan 2010), since these contain calculations only for $\log(T/K) \leq 7$, which is less than the $\log(\tau/K)$ inferred from RESIK observations. Furthermore, the calculations of Aggarwal & Keenan (2010) differ by only several percent from the ones implemented in the CHIANTI database.

To calculate the synthetic spectra, we consider the Maxwellian distribution, the n -distribution (Eq. (1)), moving Maxwellian distribution (Eq. (2)) and the DL-distribution. The DL-distribution (moving half-Maxwellian distribution) is computed numerically. Based on the physical considerations for formation of the DL-distributions and the results of numerical simulations of DL, the Maxwellian distribution of 10^6 numerical electrons in the 3D velocity space is produced first, using

the random-number generator. Then to all numerical electrons with the velocity component $v_z > 0$, the drift velocity v_0 is added. Finally, this moving half-Maxwellian 3D velocity distribution is recalculated into the energy distribution, giving thus the DL-distribution.

The considered distributions, normalized to unity, are shown in Fig. 2 top. With the exception of the Maxwellian distribution, they have the same $df/d\mathcal{E}$. The DL-distribution is the narrowest and thus has the highest peak. The n -distribution with the same $df/d\mathcal{E}$, corresponding to $n = 16.3$, exhibits lower mean energy, lower peak, and more low-energy electrons. The moving Maxwellian distribution has the peak even lower, with further increased number of low-energy electrons.

In addition to these distributions, we consider a weighted combination of Maxwellian and DL-distributions. I.e., we want to mimic the results of the particle-in-cell simulations of Karlický (2012). For simplicity, we assume that the DL-distribution and the background Maxwellian distribution have the same mean energies, corresponding to the $\log(T_{\text{Maxw}}) = \log(\tau_{\text{DL}}) = 7.28$. This value is the highest one derived from RESIK observations by Dzifčáková et al. (2008) and Kulinová et al. (2011).

We note again that the observed RESIK spectra are already well-fitted by the n -distribution (Dzifčáková et al. 2008). Therefore, in this paper we only focus on identifying the physical process forming the distribution function, which in turn produces the same type of flare spectra as the observed ones.

In the calculation of the synthetic spectra, the flaring plasma is assumed to be in the ionization equilibrium. This is because the ionization timescales are shorter than the observed changes in τ and the distribution function (Kulinová et al. 2011).

3. Results

First, we studied the effect of the distribution on the ratio of allowed Si XIV $\lambda 5.22$ /Si XIII $\lambda 5.68$ lines together with the ratios involving dielectronic Si XIIId $\lambda 5.56$ /Si XIII $\lambda 5.68$ and Si XIIId $\lambda 5.82$ /Si XIII $\lambda 5.68$ lines. The results are shown in Fig. 3, using full black lines for the considered distribution. The dashed grey lines denote the corresponding Maxwellian spectrum having the same Si XIV/Si XIII ratio. The value of $\log(T)$ for this corresponding Maxwellian spectrum is indicated in each figure.

For the DL-distribution, the Si XIIId/Si XIII ratios are very high (full line in Fig. 3 top). This is caused by the narrow peak of the DL-distribution, resulting in an increased number of particles at the energies of ≈ 2 keV where the dielectronic Si XIIId lines are excited. We note that the Si XIIId/Si XIII ratios are independent of the relative ion abundance. This is not true for the Si XIV/Si XIII ratio. This ratio is strongly dependent on the ionization equilibrium. The calculated ratio for the DL-distribution is very low (Fig. 3 top), which is caused mainly by the ionization equilibrium, since the narrow peak of the DL-distribution results in narrow peaks of the ion abundances as function of $\langle \mathcal{E} \rangle$ or τ (cf. Dzifčáková 1998).

For these reasons, the wider peak of the n -distribution causes a decrease in the Si XIIId/Si XIII ratios and an increase in the Si XIV/Si XIII ratio (Fig. 3 middle). The even wider peak of the moving Maxwellian distribution results in further changes in all of the considered ratios (Fig. 3 bottom). Nevertheless, the spectra for all considered distributions show increased Si XIIId/Si XIII ratios with respect to the Maxwellian spectra having the same Si XIV/Si XIII ratio. However, the increase of the dielectronic satellite lines is most pronounced with the increasing

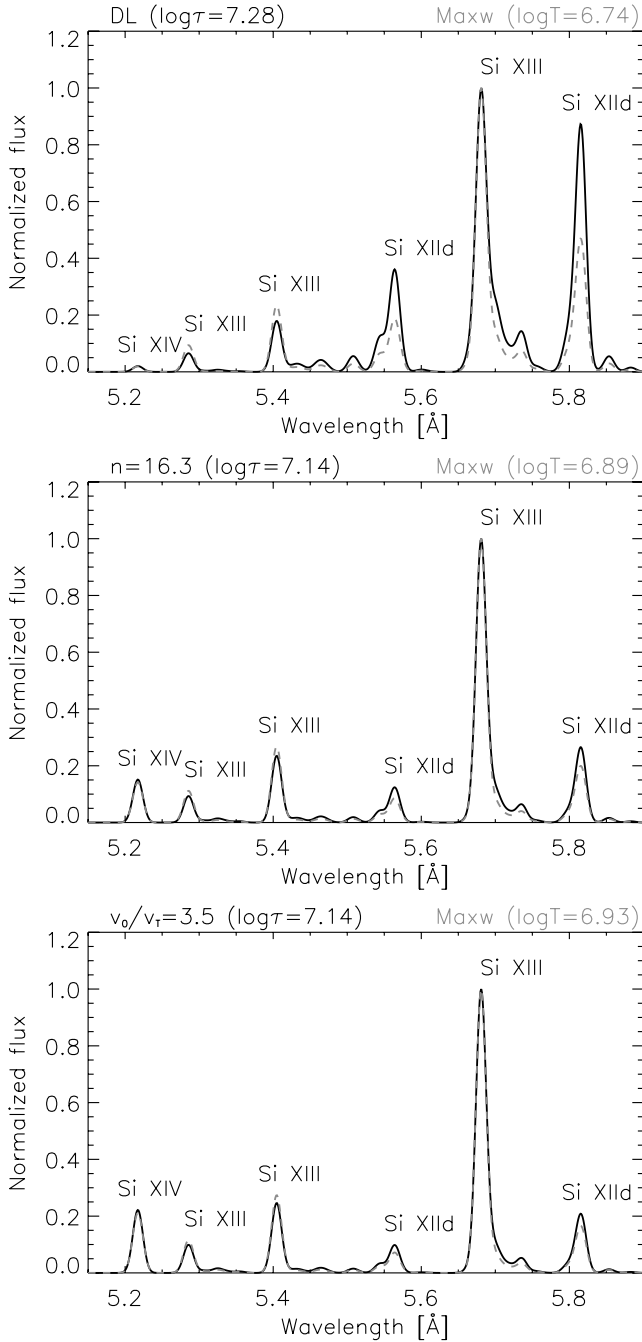


Fig. 3. Synthetic spectra (full black lines) for the DL-distribution (*top*), the n -distribution (*middle*) and the moving Maxwellian distribution (*bottom*). Dashed grey lines show Maxwellian spectra having the same Si XIV/Si XIII ratio. The parameters of each distribution are indicated in each figure.

narrowness of the peak of the distribution, with largest departures from Maxwellian spectra arising for the DL-distribution. Therefore, it is unlikely that the moving Maxwellian distribution would produce a good fit to the observations, where the Si XIIId/Si XIII ratios are much more enhanced with respect to the Maxwellian spectra (Dzifčáková et al. 2008; Kulinová et al. 2011).

The combination of the DL-distribution with the Maxwellian distribution results in a strong increase of the Si XIV/Si XIII ratio with the increasing portion of the Maxwellian distribution (Fig. 4). Moreover, the increase in the Si XIIId/Si XIII ratios with

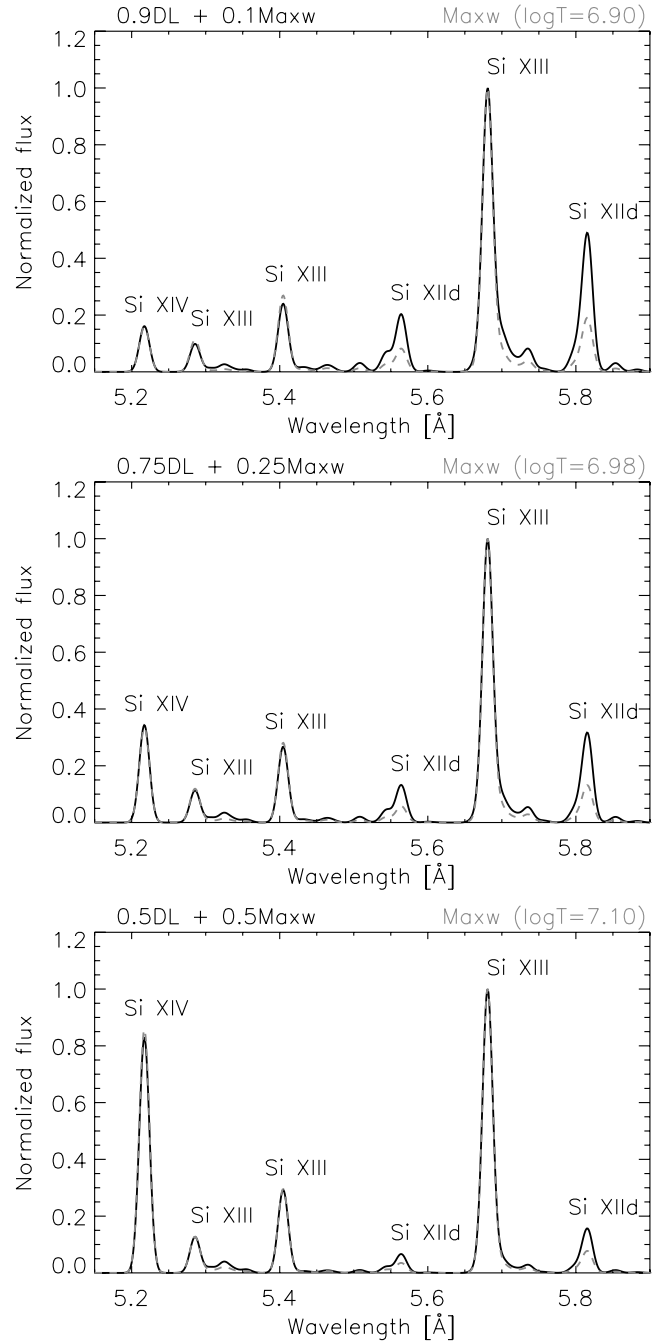


Fig. 4. Synthetic spectra (full black lines) for the combination of the DL-distribution with the Maxwellian distribution (Fig. 2 *bottom*), both having $\log(\tau) = 7.28$. The relative weights of each distribution are indicated in each figure. Dashed grey lines show Maxwellian spectra having the same Si XIV/Si XIII ratio. The parameters of each distribution are indicated in each figure.

respect to the corresponding purely Maxwellian spectra (dashed lines in Fig. 4) are significant for all considered weights. This increase is about a factor of 2 or more. Both these effects are in good agreement with the characteristics of the spectra observed by the RESIK instrument (cf. Fig. 1), exhibiting significant increase of the Si XIIId/Si XIII ratios, together with the presence of a strong Si XIV resonance line during the impulsive phase of several flares (Dzifčáková et al. 2008; Kulinová et al. 2011). From the distributions considered here, we conclude that the combination of the DL-distribution with the Maxwellian

distribution can reproduce all features of the spectra observed by the RESIK instrument. This provides strong support for the presence of DLs in solar flares.

4. Summary and discussion

We investigated the influence of various distribution functions on the resulting X-ray synthetic spectra of Si emitted at 5–6 Å under the conditions corresponding to the impulsive phase of the solar flares. The considered distributions included the parametric n -distributions, the analytical moving Maxwellian distribution, and the distribution formed in plasma DL. While the ratios of dielectronic-to-allowed Si XIIId/Si XIII lines do not depend on the relative ion abundances, the ratio of the allowed Si XIV/Si XIII is influenced by the ionization equilibrium. The DL-distribution has the narrowest peak of all the considered distributions, resulting in highest Si XIIId/Si XIII ratios, but very small values of the Si XIV/Si XIII ratio. Distributions with a wider peak show a decrease in the relative intensities of the Si XIIId lines, accompanied by an increase in the relative intensity of the allowed Si XIV transition.

We also analyzed the more realistic situation where the observed line emission originates in both DL and thermal background plasma having the same mean energy. The addition of a thermal source leads to an increase in the Si XIV/Si XIII ratio, while keeping the Si XIIId/Si XIII ratios several times higher than the purely Maxwellian spectrum with the same Si XIV/Si XIII ratio. These results are in agreement with the characteristics of the solar flare spectra observed by the RESIK instrument, showing both a strong Si XIV line and significantly enhanced Si XIIId lines. We conclude that these results provide evidence for the presence of DL in the impulsive phase of solar flares, i.e., at times when particle acceleration can be inferred directly from RHESSI and radio observations. We note that the presence of DLs in the impulsive flare phase is further supported by direct observations of the DL by the FAST satellite in the terrestrial aurora, where the physical conditions are similar (Andersson et al. 2008).

The presence of a high-energy non-Maxwellian tail has only a small influence on the Si XIIId/Si XIII ratios (Dzifčáková et al. 2011). For the typical relative number of particles present in the high-energy tail, as inferred from the RHESSI observations (e.g., Warmuth et al. 2009; Veronig et al. 2010; Guo et al. 2011; Lin 2011), the effect of the tail is negligible. Therefore, the increase in Si XIIId excitation is indicative of the departures of the bulk of the distribution from the Maxwellian distribution (Dzifčáková et al. 2011). However, the presence of the high-energy tail can influence the Si XIV/Si XIII ratio through the ionization equilibrium. A complete diagnostic of all effects would require simultaneous observations of the high-energy tail, together with several He-like ions with the corresponding X-ray dielectronic satellite lines, to constrain the distribution function at multiple energies. This remains a challenge for future instrumentation.

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RAL (UK), MSSL (UK), the Universities of Florence (Italy) and Cambridge (UK), and George Mason University (USA). CHIANTI is a great spectroscopic database and software, and the authors are very grateful for its existence and availability.

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