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Is there a spatial gradient in values of the fine-structure constant? A reanalysis of the results *(Research Note)*

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ABSTRACT

We statistically analyse a recent sample of data points measuring the fine-structure constant α (relative to the terrestrial value) in quasar absorption systems, where a spatial gradient in α was recently reported. We find agreement with previous authors that the dipole model is a robust, well-justified fit to the data. Using a simple analysis we find that the monopole term (the constant offset in $\Delta\alpha/\alpha$) may be caused by non-terrestrial magnesium isotope abundances in the absorbers. Finally we test the domain-wall model against the data.

Key words. cosmology: theory – methods: statistical – large-scale structure of Universe – cosmology: miscellaneous

Optical quasar absorption spectra provide a probe of variations in the fine-structure constant, $\alpha = e^2/\hbar c$, along a past-light cone centred on present-day telescopes. In this paper we analyse the results of a large sample of quasar absorption systems where values of $\Delta\alpha/\alpha$ have been measured using the many-multiplet method (Dzuba et al. 1999a,b). Here $\Delta\alpha/\alpha = (\alpha(\boldsymbol{r}) - \alpha_0)/\alpha_0$ is the relative variation in α at a particular position $\boldsymbol{r}$ in the Universe where the absorption occurs.

The data used in this paper are taken from Murphy et al. (2003, 2004, Keck telescope) and King et al. (2012, Very Large Telescope), kindly supplied by the King in a usable text format including the location of each absorber and its measured value of $\Delta\alpha/\alpha$. In total there are 293 points in our data file including 140 from Keck and 153 from VLT of which seven absorbers are seen in both the Keck and VLT samples. The data is described fully in King et al. (2012) (see Berengut et al. 2012 for a discussion pertinent to the current analysis). In Webb et al. (2011), the combined data sample is interpreted as providing evidence for variation in α throughout the Universe with an angular-dependence. This "dipole" model is found to be preferred to a monopole (constant offset) model of the variation at the 4.1σ level.

Note that both the Keck and VLT samples account for unknown sources of scatter in the data by including extra systematic errors, $\sigma_{\rm rand}$, that are added in quadrature with the underlying statistical error. The data we use includes a conservative (large) value which will tend to reduce the significance of the dipole relative to a monopole model. In practice the results of this paper are relatively insensitive to the exact $\sigma_{\rm rand}$ used.

The values of $\Delta\alpha/\alpha$ for each absorber (presented in Murphy et al. 2004; Webb et al. 2011) are modelled in Webb et al. (2011); King et al. (2012) using the formula

$$\frac{\Delta\alpha}{\alpha} = A + B(z)\cos\theta \qquad (1)$$

where θ is the angle between the direction of the measurement and the axis of the dipole, z is the redshift of the absorber, A is a constant (a "monopole" term) and $B(z)$ is the magnitude of the dipole term. In Webb et al. (2011) several variants were tested in which $B(z)$ was: constant, $B(z) = B$; $B(z) = B\,z^\beta$, where β is another fitting parameter; and $B(z) = B\,r(z)$, where $r = ct$ is the lookback time in giga-lightyears (Glyr). In this paper we test the dipole interpretation $B(z) \sim r$. In this form (1) represents a gradient in the value α throughout the Universe, and $r\cos\theta$ is the distance to a quasar absorption system along that gradient.

Of course, r itself is model dependent at large redshifts. In this work we use the standard $\Lambda_{\rm CDM}$ cosmology parametrized by WMAP5 (Hinshaw et al. 2009) to determine $r = ct$ (t is lookback time) from the redshift z:

$$r(z) = \frac{c}{H_0}\int_{1/(1+z)}^{1} \frac{1}{\sqrt{\Omega_{\rm m}a^{-3} + \Omega_\Lambda}}\frac{{\rm d}a}{a}. \qquad (2)$$

Again, this reflects the fact that we are taking measurements along a past light cone centred on present-day Earth. An alternative approach is to use the comoving distance:

$$d(z) = \frac{c}{H_0}\int_{1/(1+z)}^{1} \frac{1}{\sqrt{\Omega_{\rm m}a^{-3} + \Omega_\Lambda}}\frac{{\rm d}a}{a^2}. \qquad (3)$$

In this case the dipole model (1) is modified to use $B(z) = B\,d(z)$. This parameterisation makes physical sense if one imagines that any spatial α-variation is "fixed" to the CMB frame and follows the same scale factor, $a(t)$. The wall model uses this distance dependence.

In preparation for this work, we have statistically reanalysed the astronomical data using different techniques to the original authors (Berengut et al. 2012). We have tested the assumption of normalcy of the residuals using a quantile-quantile plot, and concluded that χ^2 is a reasonable statistic for the data. We have found general agreement with previous authors that a dipole model is a well-justified fit to the data. Unfortunately the data is too limited to distinguish between a distance-dependence using lookback time from comoving distance.

In this paper we show that approximately 40% of the data must be removed in a heavily biased way to reduce the significance of the α-dipole to 1σ; therefore, the dipole result is not due to a small, errant portion of the data sample. The monopole term may be an artefact due to non-terrestrial magnesium isotope ratios in the absorbers. We find that a small change (approximately 5%) in the ratio of ^{26}Mg to ^{24}Mg could account for the monopole, leaving a purely spatial gradient in values of α across the Universe. Finally we develop a genetic algorithm to find the best fit parameters for the domain-wall model of α-variation suggested by Olive et al. (2011). We find that the best fit direction of the wall is not consistent with the dipole direction. This is because χ^2 in the best-fit region is anomalously low with respect to its local neighbourhood of parameter space. On the other hand if we force the domain wall to be perpendicular to the best-fit dipole direction, the resulting χ^2 are significantly larger than those of the dipole model.

Robustness of the dipole model: we wish to test whether the result that the dipole model is statistically preferred to a simpler monopole model is due to a small proportion of the data sample. To do so we perform biased and unbiased clippings of the data. We start with a biased iterative clipping method, where at each increment we remove a point that lends support to the dipole model. One option is to remove the absorber producing the smallest residual value $\xi_i = (x_i - \bar{x}_i)/\sigma_i$, where the $\bar{x}_i$ is the dipole model value for that absorber. However, the data points with small residuals may be absorbers lying near the plane orthogonal to the fitted dipole axis, and these would not contribute to the dipole effect. Therefore we introduce an angular weighting that further biases the clipping towards removal of points that lie on the dipole axis, $\xi_i/\cos\theta_i$ where θ_i is the angle from the dipole axis.

After each clipping of the absorber with smallest weighted residual, we calculate the significance of the dipole using the modified Akaike information criterion (AIC$_c$; Sugiura 1978), F-test, and error-ellipsoid method (EEM; similar to the method of Cooke & Lynden-Bell 2010). We describe these methods, along with our modifications to the EEM to allow for various distance-dependent dipoles, in Berengut et al. (2012). The result of our clipping tests are presented in Fig. 1. The solid black line gives the significance of preference for the dipole+monopole model over the pure monopole model as calculated via the F-test; it indicates that the dipole model is preferred over the monopole (at $>1\sigma$ confidence) until approximately 120 (40%) of the most significant points are removed. The significance as calculated using EEM is given by the dashed line of Fig. 1: we see that the significance assessed using F-tests and EEM are consistent until $\sim$120 points have been removed and the significance is smaller than 1σ. The vertical dashed line indicates the point at which the dipole+monopole model is no longer preferred over the monopole model as determined by the Jeffreys (1961) criterion, viz. that the difference in the AIC$_c$ of the two models is <5. Again, this occurs after $\sim$120 points have been removed.

Based on our heavily biased clipping we expect that if we randomly remove 120 data points from the sample and re-fit our data, the dipole+monopole model would still be preferred over the monopole model. In Fig. 2 we present the probability density function for this unbiased clipping performed 10 000 times. Each time, the dipole significance is calculated using an F-test. We find that, as expected, the significance is at least 1σ in almost all of our subsets. Taken together with our biased clipping, this shows that the dipole result is not due to a small number of outliers in the sample.

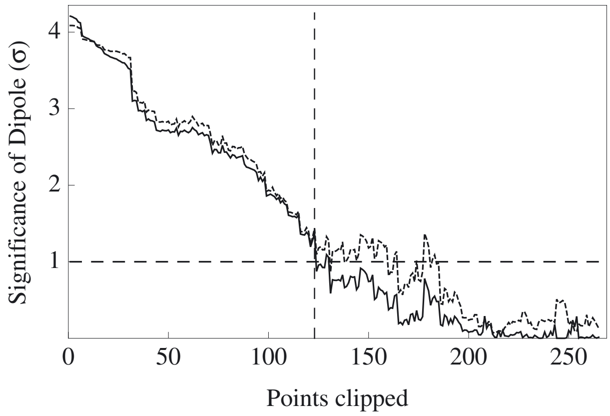

Fig. 1. Significance of the preference for a dipole+monopole model over the monopole model versus number of data points clipped. At each step we have removed the absorber with smallest weighted residual $\xi/\cos\theta$, where θ is the angle between the location of the absorber and the dipole axis. Solid line: significance assessed using F-test; dashed line: significance assessed using EEM. The vertical dashed line indicates the point at which the dipole model is not preferred according to the AIC$_c$.

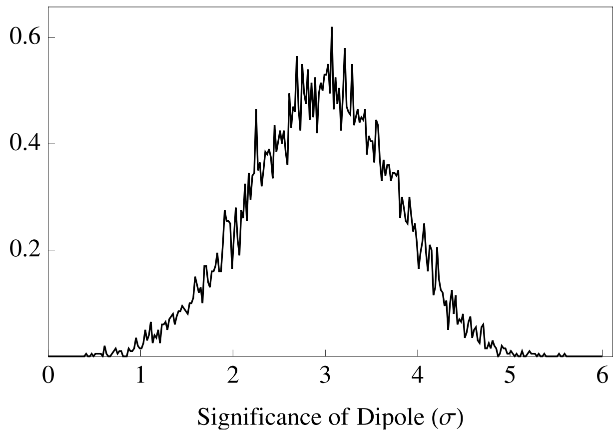

Fig. 2. Probability density function for the significance of the dipole after random removal of 40% of data, assessed using the F-test. We see that in almost all cases the dipole+monopole model is preferred to the pure monopole model at more than 1σ significance.

It is useful to contrast our biased iterative clipping to that presented in Webb et al. (2011). In that work, the point with *largest* residual was removed at each step of the procedure. This is useful for showing that the dipole is not caused by a few outlying data points, but it can also remove absorbers that do not support the dipole model. For example, in Webb et al. (2011) there is no weighting by $\cos\theta$, and deviant points that lie in the perpendicular plane may be removed. Therefore it is unsurprising that the significance of the dipole increases at first (Fig. 4 in Webb et al. 2011). However, there is a trade-off: the significance eventually falls as more points are removed because there are simply fewer data. Webb et al. (2011) found that approximately 60% of data with the largest residuals had to be discarded before the significance dropped below 3σ.

Magnesium and the Monopole: it was seen in King et al. (2012) (see also Berengut et al. 2012) that that there is some justification for including the monopole term of (1) as well as the dipole term. One natural interpretation of such a term is time-variation of the fine-structure constant, of the kind that was indicated in previous Keck studies (Murphy et al. 2003). One

Table 1. Best fit parameters and values of χ^2 and χ^2/ν for different model fits to the data when all absorbers with Mg II have been removed.

Model		Parameter values				χ^2	χ^2/ν
		A ($\times 10^{-5}$)	B ($\times 10^{-5}$ Glyr^{-1})	RA (h)	Dec		
Subset with Mg II *data removed:*							
Null	$\Delta\alpha/\alpha = 0$	–	–	–	–	126.565	1.1200
Monopole	$\Delta\alpha/\alpha = A$	0.16 (15)	–	–	–	125.421	1.1198
Dipole	$\Delta\alpha/\alpha = B\,r\cos\theta$	–	0.105 (35)	16.0 (1.8)	−63 (12)°	117.491	1.0681
Dipole + monopole	$\Delta\alpha/\alpha = A + B\,r\cos\theta$	0.12 (15)	0.103 (35)	15.8 (1.8)	−61 (12)°	116.863	1.0721
All data:							
Dipole + monopole	$\Delta\alpha/\alpha = A + B_0\,r\cos\theta$	−0.19 (8)	0.106 (22)	17.4 (1.0)	−62 (10)°	279.66	0.9677

Notes. In models that include a dipole, the direction of the dipole axis is specified in equatorial coordinates.

would then expect the monopole term to have an explicit time-dependence, $A = A(z)$, although the form may not be resolved by the data.

Another possible explanation for the monopole could be chemical evolution of the Universe, which changes the isotope abundance ratios. Since terrestrial abundances are assumed, any deviation from these ratios in the absorber will shift the centroid of the line profile, and this might mimic a change in α. Accounting for a systematic such as this is difficult, especially since the isotope shift is unknown for many of the lines used in the analysis. The transition with the largest known shift used in the analysis are the $\lambda\lambda$2796 and 2803 lines in Mg II, and calculations suggest that the unknown isotope shifts are smaller and less important (Kozlov et al. 2004).

We have applied a simple test to see whether the isotope abundance ratios of magnesium could cause the monopole. We have simply removed all data points taken from absorbers where the Mg II lines are present and used in the analysis. 113 absorbers remain from our initial sample of 293: our statistical significance is hugely reduced. The removal of points with Mg II is by no means a random sampling: these Mg II lines are seen in low redshift systems since at $z \gtrsim 2$ they are redshifted outside the range of optical telescopes.

The results of our fitting are shown in Table 1. For models that include a dipole we use the light-travel time as a measure of distance $B(z) = B\,r(z)$. We see that the best-fit dipole parameters for the Mg II-removed data are consistent with those of the complete data set, but we no longer have a statistically significant monopole. Using an F-test we find that the dipole+monopole model is preferred over the monopole model at 1.9σ significance for the Mg II-removed set.

We may try to estimate the magnitude of the change in Mg II abundances that could mimic the observed monopole term in the original data set. The two Mg II lines (at ∼35 000 cm^{-1}) are often observed simultaneously with Fe II lines (∼40 000 cm^{-1}) which in these systems will provide most of the sensitivity to α-variation ($q \sim 1500$ for positive shifters in Fe II). To obtain our order-of-magnitude estimate we assume here that only Mg II and positive-shifting Fe II lines are present in the system.

The procedure of Webb et al. (2011) is to simultaneously fit the redshift z and $\Delta\alpha/\alpha$ (along with column densities, Doppler widths, etc.) for the entire quasar spectrum. The monopole term in the full data set, $\Delta\alpha/\alpha = -0.19 \times 10^{-5}$, corresponds to a shift in the Fe II lines of $\Delta\omega/\omega = 2q/\omega \cdot \Delta\alpha/\alpha \approx -1.4 \times 10^{-7}$. This shift manifests as an effective change in the redshift of the quasar absorption system as measured by the Fe II lines, but not as measured by the Mg II lines. It is possible to obtain consistency in measured z if the relative isotope abundances of Mg II are assumed to vary. The isotope shift of the Mg II lines is $\Delta\omega_{IS} = \omega^{26} - \omega^{24} = 0.102$ cm^{-1} (Drullinger et al. 1980). In order to compensate the monopole term of the observed α-variation, the relative abundance of $x = {}^{26}\mathrm{Mg}/{}^{24}\mathrm{Mg}$ would have to change by $x\,\Delta\omega_{IS}/\omega = 1.4 \times 10^{-7}$, implying $x \approx 0.05$. That is, a roughly 5% increase in the relative abundance of ^{26}Mg could remove the observed monopole in $\Delta\alpha/\alpha$. Alternatively, a ∼10% increase in the ^{25}Mg abundance relative to ^{24}Mg, would also work – or any combination of the two (e.g. the absorber may have the terrestrial ^{25}Mg : ^{26}Mg ratio, and a reduction in ^{24}Mg relative to the heavier ^{25,26}Mg isotopes).

Of course our assumptions are quite rough. However a previous study of the effect of isotope abundance in the Keck data came to much the same conclusion (Murphy et al. 2004). In their method, the quasar absorption spectra were refitted using a different value of magnesium heavy-isotope abundance and the $\Delta\alpha/\alpha$ extracted were averaged assuming a monopole model. This found a linear relationship between their assumed $^{25,26}\mathrm{Mg}/^{24}\mathrm{Mg}$ ratio and the extracted $\Delta\alpha/\alpha$. A change in $\Delta\alpha/\alpha$ of 0.2×10^{-5} required an increase in the abundance of heavy Mg isotopes relative to ^{24}Mg of around 10% (see Fig. 6 of Murphy et al. 2004). A similar analysis applied to the $z < 1.6$ subsample of the Keck+VLT data also found that the monopole could be removed by a change in the $^{25,26}\mathrm{Mg}/^{24}\mathrm{Mg}$ abundance ratio from the terrestrial value of 0.21 to a heavy-isotope enhanced value of 0.32 ± 0.03 (King et al. 2012).

Generally other species are present in our quasar absorption spectra, and this could remove the degeneracy between variation of magnesium isotope abundance and variation of α. A more complete analysis should allow Mg isotope abundances to vary in the fitting procedure, which may be possible if this is restricted to only one additional parameter in the entire sample of ∼300 absorption systems.

Domain wall model: recently a different model for spatial variation of α has been proposed by Olive et al. (2011) where the Universe is divided into two domains, each with a different value of α. On the Earth-side of the domain wall α takes the terrestrial value, while on the other side it takes a different value. We can parametrize this model by the equation

$$\frac{\Delta\alpha}{\alpha} = \begin{cases} A, & d\cos\theta - d_{\mathrm{wall}} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where d_{wall} is the shortest distance to the wall (we use comoving distance, Eq. (3), in this section) and θ is the angle between the direction of the shortest distance to the wall and a quasar. The wall itself is parametrized by A, d_{wall}, and the direction to the wall in equatorial coordinates; these can be described by a single four-dimensional vector in the parameter space, $\boldsymbol{p}$.

Again, values for d_{wall} and the direction of the wall are chosen to minimise χ^2, however in this case χ^2 is discontinuous with respect to changes in the parameters: any given quasar must either be on this side of the wall (model value $\Delta\alpha/\alpha = 0$) or the other (model value $\Delta\alpha/\alpha = A$). This necessitates a genetic algorithm with simulated annealing to perform the fit.

In our genetic algorithm, sets of model parameters $\boldsymbol{p}_i$ are produced in "generations" with the 10 sets producing the best fits (in our case the smallest χ^2) in a particular generation retained to "breed", producing the next brood of parameters. The 500 offspring have genes taken to be the average of the parent values $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ with an additional random "mutation" term: $\boldsymbol{p}_{\rm child} = (\boldsymbol{p}_i + \boldsymbol{p}_j)/2 + \boldsymbol{\xi} \cdot (\boldsymbol{p}_i - \boldsymbol{p}_j)$ where the four terms in $\boldsymbol{\xi}$ are drawn randomly from a Gaussian distribution with standard deviation $\sigma_{\rm extra}$. The "mutations" are necessary to produce the quasi-random variations needed to cover the parameter space. The magnitude of such "mutations" are incrementally reduced in a process of simulated annealing so that the mutations become finer as parameters converge to their best fit values: $\sigma_{\rm extra} = \sigma_{\rm max}(500-g)/500$, where g is an index of the generation (500 generations per annealing).

As is usual when using genetic algorithms to determine best fits, several annealings are required. Unusually, in this case due to the discontinuous nature of the parameter space, we not only require several annealing steps but also several cycles of sets of annealing with new "isolated" (random) populations of parameters in order to avoid a situation where the entire population inhabits local minima in the parameter space. See Berengut et al. (2012) for full details.

The best fit model for the entire data sample was found to have $A = -1.055 \times 10^{-5}$, $d_{\rm wall} = 5.513$ Gly, RA = 20.1 h, Dec = 68°. The model has 4-parameters with $\chi^2 = 281.76$; this can be compared with the 4-parameter dipole+monopole model with comoving distance, for which $\chi^2 = 279.85$. However it is worth noting that this minimum χ^2 is obtained for an extremely narrow range of parameters, and even a small deviation in any of them increases it dramatically. For example, in Fig. 3 we present χ^2 in the direction of best-fit (solid line) as a function of $d_{\rm wall}$.

The best-fit wall is located at an angle 47° to the dipole axis (towards negative values of $\Delta\alpha/\alpha$, i.e. 180° from the directions presented in Table 1). This is not consistent: if the Universe really does have a domain wall in values of α one would expect a dipole fit to the data to have the same direction. If we calculate χ^2 as a function of $d_{\rm wall}$ along the direction that is given by the best-fit dipole we obtain the dashed line of Fig. 3. It is seen that the behaviour is much as would be expected if the dipole model is correct: as $d_{\rm wall}$ increases, fewer absorbers are found on the other side of the wall, the model gives $\Delta\alpha/\alpha = 0$ for more absorbers, and χ^2 increases.

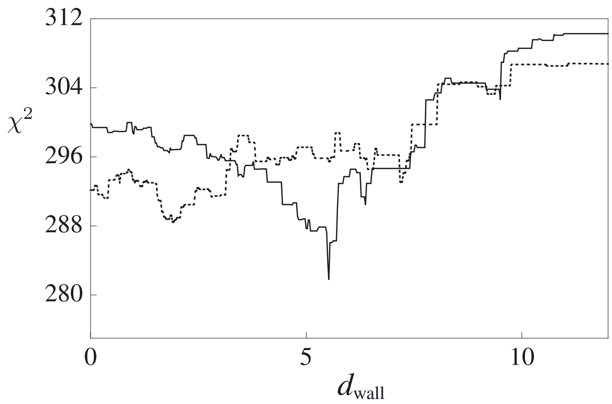

Fig. 3. χ^2 for the wall model as a function of comoving distance to the wall, $d_{\rm wall}$. Solid line: in the direction of the best fit for the wall (the best fit occurs at $d_{\rm wall} = 5.513$ Gly); dashed line: in the direction of the dipole axis (decreasing α direction). By comparison, χ^2 for the spatial gradient (dipole) model is 279.66.

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