Specific effects of large asteroids on the orbits of terrestrial planets and the ASETEP database[★]

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ABSTRACT

The necessity to take into account the perturbations caused by a large number of asteroids on the terrestrial planets is fundamental in the construction of modern numerical ephemeris on the solar system. Therefore about 300 of the largest asteroids were taken into account in recent ephemeris. Yet, the uncertainty on the mass values of the great majority of these asteroids constitutes a crucial and the main limit of accuracy of this ephemeris. Consequently, it is important to conduct a specific and detailed study of their individual effects especially on the terrestrial planets, which are far more affected than the giant planets. This was already done explicitly, but only for Mars and for only two orbital elements (a and λ). We aim both to confirm these previous results and to extend the study to all orbital elements and to the other three terrestrial planets (Mercury, Venus and the Earth), which are priori less affected by asteroid perturbations. Our methodology consists in several steps: we carried out precise computations of the orbital motions of the planets at short (100 y) and longer (1000 y) time scales with numerical integration. For that purpose we included the eight planets and also considered 43 of the most powerful asteroids. These were added to the numerical integrations once separately and once combined to determine their specific effects on the orbital elements of the Earth and the three other terrestrial planets. This procedure also allowed us to assess the spatial geocentric coordinates of the three terrestrial planets. We determined the signal that represents the effects by simple subtraction. Then we systematically analyzed this signal by FFT (fast Fourier transform), and finally we adjusted the signal with a set of sinusoidal components. We analyzed in detail the variations of the six orbital elements a , e , i , Ω , $\tilde{\omega}$ and λ of Mercury, Venus, the Earth-Moon barycenter (EMB) and Mars that are caused by the individual influences of the set of our 43 selected asteroids. We compared our results for Mars with the analytical ones on the semi-major axis and the longitude. The two studies agree very well. All our results, consisting of 1032 different curves (43 asteroids $\times$ 4 planets $\times$ 6 orbital elements) and the related tables that provide the fitted Fourier and Poisson components are gathered the ASETEP database (asteroid effect on the terrestrial planets). Moreover, we include in this database the influence of our sample of 43 asteroids on three fundamental parameters: the distance and the bi-dimensional orientation vector (α, δ) from the EMB to each of the other terrestrial planets. This database, which will be regularly updated by taking into account more asteroids with improved mass determinations, constitutes a precious tool for understanding specifically the influence of the large asteroids on the orbital motion of the terrestrial planets, and also for better understanding how modern ephemeris can be improved.

Key words. gravitation – minor planets, asteroids: general – ephemerides – astronomical databases: miscellaneous – astrometry – celestial mechanics

1. Introduction

There are two main ways of computing the ephemeris of celestial bodies. One is semi-analytical, which was used for the VSOP87 theory of the planets by [Bretagnon et al \(1987\)](#) and the ELP2000 solution for the orbital motion of the Moon ([Chapront et al. 1988](#)). In these successful and complex works, the coordinates of the moving bodies are given in the form of Fourier series of fundamental arguments as the mean longitudes of the planets or the Delaunay arguments. The interest of these analytical theories is that they enable us to understand the origin of the countless sinusoidal terms that originate in the perturbing function. But these theories can no longer reach the necessary accuracy to match numerical determinations of the motions that constitute the second way of computation.

Indeed, today the stability and rapidity of the numerical integrators allow an accurate construction of the ephemeris of the solar system's main bodies. As explained by [Standish \(2004\)](#), the accuracy of an ephemeris depends on three major factors. The first one is related to the equations of motion. These equations

might be imperfect for instance because they do not take all the gravitational effects into account or because the underlying analytical model contains some errors. The second factor concerns the integration of motion, which inevitably contains uncertainties that have to be estimated. The last factor is the exact determination of basic parameters such as the initial conditions (positions, velocities, masses etc.), which are given by the observations, and therefore contain errors that are conditioning the results of the integration. The two first factors are generally believed to be satisfactory, because the equations of motion look correct and complete, whereas the errors accompanying the integration algorithms have been proved to be negligible. The third factor is still subject to various investigations and numerous recent developments, in particular those coming from the recent missions to Mars ([Standish 2004](#); [Fienga et al. 2009](#)). The JPL ephemeris are undoubtedly used the most worldwide. Since the construction of an initial release DE200 in 1979, a lot of other versions have been constructed, such as DE405 in 1997, and DE409 in 2003. This last version in particular uses very recent VLBI observations of the MGS (Mars Global Surveyor) in 2001–2003 and Odyssey in 2002–2003, which leads to an accuracy for the determination of the position of Mars at the level

[★] Appendices A–C are available in electronic form at <http://www.aanda.org>

of a few milliarcseconds. The comparisons between the various versions of JPL ephemerides give a clear insight into the improvements. DE405 for instance, showed an improvement of an order of magnitude over DE200, especially for the inner planets, as explained by [Standish \(2004\)](#).

To evaluate the accuracy limits in the ephemeris of the four inner planets, [Standish et al. \(2002\)](#) considered the present-day knowledge of the asteroid masses determined by various methods, and modeled the distributions of those masses. Then they carried out a Monte-Carlo study, repeatedly adjusting the ephemeris to fit the observational data, by a suitable set of asteroid masses. The authors' conclusion is that by combining accumulated data for more than two decades, the orbits become distorted both in right ascension and in declination by as much as 5 km or more. At the level of present-time ephemerides accuracy, the influence of asteroids can no longer be neglected, as in the case of DE409. Many asteroids have enough mass to produce a significant gravitational effect on the inner planets. Nevertheless, because of the large uncertainty in the determination of asteroid masses, it is difficult or impossible to evaluate their effect with sufficient accuracy. Yet no specific study of the influence of each asteroid has been conducted to date. A pioneering work was carried out by [Williams \(1984\)](#), giving a list of the most influential asteroids on the orbit of Mars over a short period of time, roughly 50 years. Recently, [Mouret et al. \(2009\)](#) identified and provided a list of the main perturbers of Mars, which could cause significant disturbances over the next 50 years as well. Their analytical analysis is based on a harmonic decomposition of the perturbations on Mars as a function of the mean longitudes of the planet and the asteroid, to the first order in asteroid mass. Still, these two last studies ([Williams 1984](#); [Mouret et al. 2009](#)) can be considered as incomplete because they concentrate mainly on two orbital elements, the semi-major axis and longitude of Mars. Moreover, they only focus on this planet.

We aim to carry out an extensive study of the subject in the present paper. In [Sect. 2](#), after determining the asteroidal mass and its accuracy, we select a set of 43 asteroids. We explain in detail in [Sect. 3](#) the theoretical framework and our methodology with which we calculated the perturbing effect of any specific asteroid on a terrestrial planet. In [Sect. 4](#), we determine the effects of our 43 selected asteroids on the six Mars orbital elements, while the effects on the other telluric planets, the Earth (replaced in the following by the EMB (Earth-Moon barycenter), Mercury and Venus are presented in [Sect. 5](#). Thus we reconstruct the asteroidal perturbation as a combination of Fourier and Poisson components for each perturbing asteroid as well as each terrestrial planet and each orbital parameter. Moreover, as an important by-product of these computations, we calculate in [Sect. 6](#) the influences of each asteroid on two fundamental astrometric parameters namely the distance from the EMB to the barycenter of the other terrestrial planets (Mercury, Venus and Mars) and their two-dimensional positioning (α, δ) in the celestial sphere. Finally we describe in [Sect. 7](#) how we constructed our ASETEP (asteroidal effects on the terrestrial planets) database, which enables one to determine the influence of each asteroid on any of the four terrestrial planets precisely and in detail through their orbital elements and their geocentric positioning.

2. Mass determinations of the asteroids

Much information about the size, shape and pole direction of the leading asteroids has been obtained from analyses of light curves at different epochs, occultation, and/or images obtained through adaptive optics and speckle techniques. The mass is one of the

important characteristics, necessary for describing the perturbations on other bodies. There are two methods of estimating the mass: one is dynamical, the other is astrophysical.

The dynamical method is based on the analysis of perturbations of the studied asteroid on other celestial bodies. After a pioneering work by [Williams \(1984\)](#), studying how asteroid masses can be determined from their perturbations on Mars, [Standish et al. \(1989\)](#) showed that the measurements of Mars orbit perturbations can be conducted to estimate the masses of Ceres, Pallas and Vesta, with the intermediary of a least-squares adjustment of all ephemeris parameters, including the relevant asteroid masses. This adjustment was compiled on the basis of observational data encompassing the highly accurate Viking Lander ranging data. The accuracy of mass determination through this method highly depends on the precision of positional observations used and on the number of the observations and their distribution.

The astrophysical method is based on the data about the asteroid size and the taxonomic classification. This method assumes that the body has a spherical shape, a homogeneous structure, is single, etc. This method, in contrast to the dynamical one, has the advantage that it can be applied to many objects even if there is no close encounter with other bodies ([Kochetova 2004](#)), but is generally marred by a large uncertainty.

2.1. The mass of Ceres, Pallas and Vesta

The masses of the three largest asteroids, Ceres, Pallas and Vesta obtained by [Standish et al. \(1989\)](#) starting from the perturbations on Mars orbital motion were $(5.0 \pm 0.2) \times 10^{-10} M_S$, $(1.4 \pm 0.2) \times 10^{-10} M_S$ and $(1.5 \pm 0.3) \times 10^{-10} M_S$ respectively. They were updated by more recent studies, as we will show below. [Hilton \(1999\)](#) explained why the mass determination of Ceres is completely dependent on that of Pallas. The reason is the degeneracy between the two masses, which is in turn caused by the similarity of the semi-major axes of the two asteroids. In consequence, their mean motion is nearly the same, and their mutual synodic period is very long, around 2000 yr ([Williams 1989](#)). This amplifies the propagation of the errors on the mass of Pallas to the mass of Ceres, when the mass of the first asteroid is obtained starting from a series of close encounters with the second one. This explains why the mass estimations are highly dependent on old data with poor accuracy. Because of the high Pallas inclination and its high eccentricity, its close encounters with main-belt asteroids occur only rarely, and when they occur, the relative velocity is high so that mass determination is not very efficient. For Vesta, the mass was obtained starting from perturbations of both Ceres and 197 Arete. The values obtained by [Hilton \(1999\)](#) were $M_{\text{Ceres}} = 4.39(\pm 0.04) \times 10^{-10} M_S$, $M_{\text{Pallas}} = 1.59(\pm 0.05) \times 10^{-10} M_S$ and $M_{\text{Vesta}} = 1.69(\pm 0.11) \times 10^{-10} M_S$. Many new constraints for the masses of the three asteroids were obtained during one decade. For our computations, we have chosen the recent values determined by [Pitjeva \(2009\)](#) from data of the Martian landers and spacecraft by fitting the planet ephemeris to observations. Their values are significantly different from the estimations above, i.e. $4.712(\pm 0.006) \times 10^{-10} M_S$ for Ceres, $1.027(\pm 0.007) \times 10^{-10} M_S$ for Pallas, and $1.344(\pm 0.003) \times 10^{-10} M_S$ for Vesta.

2.2. The mass of the other large asteroids

We limit our study to the perturbations produced by asteroids with relatively large and well-constrained masses. According to [Mouret et al. \(2009\)](#), we can rely on 36 asteroid masses derived

from direct measurements with a precision better than 10 per cent, while only 54 reach the 50 per cent mark. These statistics come from [Mouret et al. \(2007\)](#), [Baer et al. \(2008a\)](#) and [Baer database](#)¹. All these masses were deduced from the gravitational pull exerted by the corresponding asteroid on other solar system objects, mainly smaller asteroids, and in special cases Mars or a spacecraft. In Table 1 we present the set of the 43 asteroids selected in this paper for which we will calculate the gravitational effects on the four terrestrial planets in the following, together with their mass determination and the relative uncertainty. Our choice was guided by two criteria: the asteroid must be big enough and its mass determination satisfactorily accurate. The data were taken from 12 different sources: [Pitjeva et al. \(2009\)](#), [Fienga et al. \(2009\)](#), [Baer et al. \(2008a\)](#), [Baer et al. \(2008b\)](#), [Bange \(1998\)](#), [Marchis et al. \(2008a\)](#), [Marchis et al. \(2008b\)](#), [Kochetova \(2004\)](#), [Ivantsov \(2008\)](#), [Aslan et al. \(2007\)](#), and [Yeomans et al. \(1997\)](#). Notice that in the near future, Gaia may significantly increase the number of favorable encounters to be analyzed for mass determination, as explained by [Mouret et al. \(2009\)](#).

3. Analytical expression of planet perturbations caused by an asteroid

After reviewing the classical theory of perturbations for a system of N planets orbiting the Sun, we show in this section how the individual effects of a given asteroid on one of these planets are determined analytically and explain our methodology of characterizing these effects with numerical integration.

3.1. Theoretical principles

Consider N planets $P_1, \dots, P_N$ orbiting around the Sun P_0 , with mass m_0 , according to Newton's law. The heliocentric position vector $\mathbf{r}_i$ of a given planet in the frame of this $N+1$ body problem is subject to the second order differential equation:

$$\frac{d^2 \mathbf{r}_i}{dt^2} = \text{grad}_{\mathbf{r}_i} \left(\frac{G(m_0 + m_i)}{r_i} + R_i \right), \quad (1)$$

where $i = 1, 2, \dots, N$ and the parameters m_i and G are the mass of the i th. planet and the gravitational constant. R_i , usually called the disturbing function on the planet P_i , is written as follows:

$$R_i = \sum_{j \neq i} Gm_j \left(\frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} - \frac{\mathbf{r}_i \cdot \mathbf{r}_j}{r_j^3} \right). \quad (2)$$

This disturbing function causes a perturbation of the planet's osculating orbital elements; in principle each of these perturbations can be calculated analytically according to the classical Lagrange equations for the osculating elements a, e, i, Ω, ϖ and λ (see for instance [Murray & Dermott 1999](#)) (for simplicity we did not take into account the index i in R_i)

$$\begin{aligned} \frac{da}{dt} &= \frac{2}{na} \frac{\partial R}{\partial \lambda} \\ \frac{de}{dt} &= -\frac{\sqrt{1-e^2}}{na^2 e} (1 - \sqrt{1-e^2}) \frac{\partial R}{\partial \lambda} - \frac{\sqrt{1-e^2}}{na^2 e} \frac{\partial R}{\partial \varpi} \end{aligned}$$

$$\begin{aligned} \frac{d\lambda}{dt} &= n - \frac{2}{na} \frac{\partial R}{\partial a} + \frac{\sqrt{1-e^2}(1 - \sqrt{1-e^2})}{na^2 e} \frac{\partial R}{\partial e} + \frac{\tan \frac{1}{2} I}{na^2 \sqrt{1-e^2}} \frac{\partial R}{\partial I} \\ \frac{d\Omega}{dt} &= \frac{1}{na^2 \sqrt{1-e^2} \sin I} \frac{\partial R}{\partial I} \\ \frac{d\varpi}{dt} &= \frac{\sqrt{1-e^2}}{na^2 e} \frac{\partial R}{\partial e} + \frac{\tan \frac{1}{2} I}{na^2 \sqrt{1-e^2}} \frac{\partial R}{\partial I} \\ \frac{dI}{dt} &= \frac{-\tan \frac{1}{2} I}{na^2 \sqrt{1-e^2}} \left(\frac{\partial R}{\partial \lambda} + \frac{\partial R}{\partial \varpi} \right) - \frac{1}{na^2 \sqrt{1-e^2} \sin I} \frac{\partial R}{\partial \Omega}. \quad (3) \end{aligned}$$

We focus on the perturbation caused on a planet P (one of the P_i) by an asteroid P' of mass m' added to the $N+1$ body system considered above. Let the position vector of the new body be denoted by $\mathbf{r}'$. Generally we define the perturbation ΔX of one orbital element X of the planet induced by the asteroid as the difference $\Delta X = \tilde{X} - X$, where $\tilde{X}$ stands for the perturbed element in the frame of the $N+2$ body problem (with the asteroid) and X for perturbed element in the frame of the $N+1$ body problem (without the asteroid). On an equivalent manner, the perturbing function ΔR can be written as

$$\Delta R = \tilde{R} - R = Gm' \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} - \frac{\mathbf{r} \cdot \mathbf{r}'}{r'^3} \right). \quad (4)$$

We can write the derivative of the perturbation induced by the extra body P' on the elements of the planet in the following form ([Kuchynka 2010](#)):

$$\begin{aligned} \frac{d\Delta a}{dt} &= \frac{2}{na} \frac{\partial \Delta R}{\partial \lambda} \\ \frac{d\Delta e}{dt} &= -\frac{\sqrt{1-e^2}}{na^2 e} (1 - \sqrt{1-e^2}) \frac{\partial \Delta R}{\partial \lambda} - \frac{\sqrt{1-e^2}}{na^2 e} \frac{\partial \Delta R}{\partial \varpi} \\ \frac{d\Delta \lambda}{dt} &= \Delta n - \frac{2}{na} \frac{\partial \Delta R}{\partial a} + \frac{\sqrt{1-e^2}(1 - \sqrt{1-e^2})}{na^2 e} \frac{\partial \Delta R}{\partial e} \\ &\quad + \frac{\tan \frac{1}{2} I}{na^2 \sqrt{1-e^2}} \frac{\partial \Delta R}{\partial I} \\ \frac{d\Delta \Omega}{dt} &= \frac{1}{na^2 \sqrt{1-e^2} \sin I} \frac{\partial \Delta R}{\partial I} \\ \frac{d\Delta \varpi}{dt} &= \frac{\sqrt{1-e^2}}{na^2 e} \frac{\partial \Delta R}{\partial e} + \frac{\tan \frac{1}{2} I}{na^2 \sqrt{1-e^2}} \frac{\partial \Delta R}{\partial I} \\ \frac{d\Delta I}{dt} &= \frac{-\tan \frac{1}{2} I}{na^2 \sqrt{1-e^2}} \left(\frac{\partial \Delta R}{\partial \lambda} + \frac{\partial \Delta R}{\partial \varpi} \right) - \frac{1}{na^2 \sqrt{1-e^2} \sin I} \frac{\partial \Delta R}{\partial \Omega}. \quad (5) \end{aligned}$$

The development of the perturbing function as a function of eccentricities and inclinations enables one to obtain the expressions of the perturbations as a function of the orbital elements both of the perturbed body (here the planet) and the perturbing body (the studied asteroid). This development was continued up to the fourth order by [Brouwer & Clemence \(1961\)](#). To develop the disturbing function ΔR given in Eq. (4), we distinguished between its direct part R_D and its indirect part R_E . According to classical celestial mechanics (see for instance [Laskar et al. 1995](#); or [Murray & Dermott 1999](#)) we have:

$$\Delta R = \frac{Gm'}{a'} (R_D + \alpha R_E), \quad (6)$$

¹ URL: <http://home.earthlink.net/~jimbaer1/astmass.htm>

With $\alpha = \frac{a'}{a}$, where a' stands for the semi-major axis of the asteroid, and:

$$R_D = \frac{a'}{|r - r'|} \quad (7)$$

$$+ \left(C_1^j + (e^2 + e'^2)C_2^j + (s^2 + s'^2)C_3^j \right) \cos[j\lambda - j\lambda']$$

$$+ ee' C_4^j \cos[j\lambda' - j\lambda + \varpi' - \varpi]$$

$$+ ss' C_6^j \cos[j\lambda' - j\lambda + \Omega' - \Omega]$$

$$+ eC_7^j \cos[j\lambda' + (1-j)\lambda - \varpi]$$

$$+ e' C_8^j \cos[j\lambda' + (1-j)\lambda - \varpi']$$

$$+ e^2 C_9^j \cos[j\lambda' + (2-j)\lambda - 2\varpi]$$

$$+ ee' C_1 0^j \cos[j\lambda' + (2-j)\lambda - \varpi' - \varpi]$$

$$+ e' C_1 1^j \cos[j\lambda' + (2-j)\lambda - 2\varpi']$$

$$+ e' C_1 2^j \cos[j\lambda' + (2-j)\lambda - 2\Omega]$$

$$+ ss' C_1 3^j \cos[j\lambda' + (2-j)\lambda - \Omega' - \Omega]$$

$$+ s'^2 C_1 4^j \cos[j\lambda' + (2-j)\lambda - 2\Omega']$$

$$R_E = -\frac{a'^2}{a} \frac{r \cdot r'}{r'^3} = -\frac{r}{a} \left(\frac{a'}{r'} \right)^2 \cos \psi \quad (8)$$

$$= -\left(-1 + \frac{1}{2}e^2 + \frac{1}{2}e'^2 + s^2 + s'^2 \right) \cos(\lambda' - \lambda)$$

$$- ee' \cos(2\lambda' - 2\lambda - \varpi' + \varpi) - \frac{1}{2}e \cos(\lambda' - 2\lambda + \varpi)$$

$$- 2ss' \cos(\lambda' - \lambda - \Omega' + \Omega) + \frac{3}{2}e \cos(\lambda' - \varpi)$$

$$- 2e' \cos(2\lambda' - \lambda - \varpi') - \frac{3}{8}e^2 \cos(\lambda' - 3\lambda + 2\varpi)$$

$$- \frac{1}{8}e^2 \cos(\lambda' + \lambda - 2\varpi) + 3ee' \cos(2\lambda - \varpi' - \varpi)$$

$$- \frac{1}{8}e'^2 \cos(\lambda' + \lambda - 2\varpi') - s'^2 \cos(\lambda' + \lambda - 2\Omega')$$

$$- s^2 \cos(\lambda' + \lambda - 2\Omega) + 2ss' \cos(\lambda' + \lambda - \Omega' - \Omega)$$

$$- \frac{27}{8}e'^2 \cos(3\lambda' - \lambda - 2\varpi'),$$

where e' , λ' , ϖ' , Ω' stand for the orbital elements of the asteroid. ψ is the angle between the position vectors r and r' , $s = \sin \frac{1}{2}i$, $s' = \sin \frac{1}{2}i'$. The coefficients C_i^j depend on the Laplace coefficients

$$b_s^j(\alpha) = \frac{1}{\pi} \int_0^{2\pi} \frac{\cos j\rho \, d\rho}{(1 - 2\alpha \cos \rho + \alpha^2)^s}.$$

The developments in (7) and (8) show that after integration of the variational Eqs. (5) we can expect that the variations of orbital elements be expressed as a series of linear combinations of λ and λ' , taking into account that the parameters $\tilde{\omega}$, $\tilde{\omega}'$, $\tilde{\Omega}$ and $\tilde{\Omega}'$ have a very low frequency with respect both to λ and λ' . Therefore, they can be considered as nearly constant during the interval of time considered (100 y or 1000 y). This explains why both Williams (1984) and Mouret et al. (2009) constructed their developments of a_{Mars} and λ_{Mars} only with the help of sinusoidal terms with high frequency, function of the arguments of the form $j\lambda + j'\lambda'$. Nevertheless this study is incomplete, because it does not take into account two long-periodic terms that can come from two different origins: first the resonant terms in (7) and (8) with very

Table 1. Mass estimation of the 43 asteroids selected in this paper, with their relative uncertainty.

Name	$M [M_\odot]$	$\Delta M [\%]$	Authors
1 Ceres	0.472×10^{-09}	0.127	Pitjeva et al. (2009)
2 Pallas	0.103×10^{-09}	0.682	Pitjeva et al. (2009)
3 Juno	0.750×10^{-11}	20.000	Fienga et al. (2009)
4 Vesta	0.134×10^{-09}	0.223	Pitjeva et al. (2009)
6 Hebe	0.759×10^{-11}	18.709	Baer et al. (2008a)
7 Iris	0.500×10^{-11}	8.621	Fienga et al. (2009)
8 Flora	0.426×10^{-11}	10.563	Baer et al. (2008b)
9 Metis	0.103×10^{-10}	23.301	Baer et al. (2008a)
10 Hygiea	0.454×10^{-10}	3.304	Baer et al. (2008a)
11 Parthenope	0.316×10^{-11}	1.899	Baer et al. (2008a)
13 Egeria	0.820×10^{-11}	19.512	Baer et al. (2008b)
14 Irene	0.413×10^{-11}	17.676	Baer et al. (2008b)
15 Eunomia	0.168×10^{-10}	4.762	Baer et al. (2008a)
16 Psyche	0.129×10^{-10}	13.178	Baer et al. (2008a)
17 Thetis	0.617×10^{-12}	10.373	Baer et al. (2008a)
18 Melpomene	0.151×10^{-11}	33.775	Baer et al. (2008b)
19 Fortuna	0.541×10^{-11}	14.048	Baer et al. (2008a)
20 Massalia	0.242×10^{-11}	16.942	Bange (1998)
21 Lutetia	0.129×10^{-11}	9.302	Baer et al. (2008b)
22 Kalliope	0.407×10^{-11}	2.457	Marchis et al. (2008b)
24 Themis	0.567×10^{-11}	37.919	Baer et al. (2008b)
28 Bellona	0.700×10^{-11}	28.571	Ivantsov (2008)
29 Amphitrite	0.100×10^{-10}	35.000	Baer et al. (2008a)
31 Euphrosyne	0.313×10^{-11}	18.850	Baer et al. (2008b)
45 Eugenia	0.900×10^{-11}	33.333	Ivantsov. (2008)
46 Hestia	0.109×10^{-10}	62.385	Bange (1998)
47 Aglaja	0.109×10^{-11}	39.450	Baer et al. (2008b)
48 Doris	0.610×10^{-11}	49.180	Kochetova (2004)
49 Pales	0.135×10^{-11}	18.519	Baer et al. (2008a)
52 Europa	0.976×10^{-11}	22.643	Baer et al. (2008a)
65 Cybele	0.759×10^{-11}	23.979	Baer et al. (2008a)
87 Sylvia	0.743×10^{-11}	0.404	Baer et al. (2008a)
88 Thisbe	0.572×10^{-11}	29.196	Baer et al. (2008a)
90 Antiope	0.414×10^{-12}	1.208	Baer et al. (2008a)
107 Camilla	0.563×10^{-11}	2.664	Marchis et al. (2008b)
111 Ate	0.840×10^{-10}	22.619	Ivantsov (2008)
121 Hermione	0.272×10^{-11}	5.515	Marchis et al. (2005)
130 Elektra	0.332×10^{-11}	6.024	Marchis et al. (2008a)
165 Loreley	0.160×10^{-10}	62.500	Aslan et al. (2007)
189 Phthia	0.187×10^{-13}	34.225	Baer et al. (2008a)
243 Ida	0.192×10^{-13}	4.688	Baer et al. (2008a)
253 Mathilde	0.519×10^{-13}	4.239	Yeomans et al. (1997)
283 Emma	0.694×10^{-12}	2.162	Marchis et al. (2008a)

low frequency; and second the long periodic terms in the disturbing function with arguments as a linear combination of the sole $\tilde{\omega}$, $\tilde{\omega}'$, $\tilde{\Omega}$ and $\tilde{\Omega}'$ not shown in (7) and (8).

The planetary orbital perturbations due to the asteroids were also discussed by Williams (1984) and Simon (1987), focusing exclusively on the planet Mars. Simon (1987) describes the analytical construction of programs to calculate the first and second derivatives of the Lagrange equations, and to express the theoretical perturbations due to an asteroid on the six ecliptical elements of a planet $\Delta\sigma_k$, where the variables $\sigma_{k=1,\dots,6}$ are: a , λ , the mean longitude (written as $\lambda = \lambda_0 + nt$, n being the mean motion), $k = e \cos \varpi$, $h = e \sin \varpi$, $q = \sin \frac{1}{2} \cos \Omega$, $p = \sin \frac{1}{2} \sin \Omega$. The method is based on a harmonic decomposition with respect to the mean longitudes of the planet $\bar{\lambda}$ and the asteroid $\bar{\lambda}'$. Finally the perturbations can be written as

$$\Delta\sigma_k = \sigma_k^{\text{sec}} t + \sum_{i,j} a_{ij}^k \cos(i\bar{\lambda} + j\bar{\lambda}') + b_{ij}^k \sin(i\bar{\lambda} + j\bar{\lambda}'), \quad (9)$$

as an extension of this work, Mouret et al. (2009) presented in a detailed manner the application of this decomposition for each perturbation of some selected asteroids on the Mars semi-major axis and longitude which were also the only two orbital elements studied by Williams (1984).

3.2. Methodology of studying asteroidal effects

For the studies mentioned above, the corresponding analytical developments of the perturbing function ΔR expressed in (6)–(8) from the orbital elements of the two bodies considered (the planet and the perturbing asteroid) are particularly cumbersome and complex, all the more so if we aim to include very long-periodic components, which are difficult to determine analytically, and could be treated as polynomial expansions. But it is possible to evaluate the perturbations in a straightforward manner from numerical integration. Therefore we computed the classical nine-body problem (the Sun and the eight planets), and then carried out the same integration with the sole addition of the perturbing asteroid (Ceres, Pallas, etc.). A simple subtraction of the two corresponding signals of one orbital parameter of the planet gives a clear and direct determination of the effect of the asteroid on this parameter.

As mentioned above, the objective of this work is to try to assess the specific influence of each of the 43 big selected asteroids on the motion of the terrestrial planets. For that goal we focused our efforts in the following complementary steps:

- we evaluated and then removed the quadratic variation of the differential signal, to isolate the periodic components;
- we performed the frequency analysis of the signal, using fast Fourier transform to determine the leading frequencies;
- we carried out a nonlinear regression in which the differential signal is modeled by the least-square method following an expression combining Fourier-type and Poisson-type components in the form

$$F(t) = \sum_{i=1}^N F_i(t) = \sum_{i=1}^N A_i \sin f_i t + B_i \cos f_i t + C_i t \sin f_i t + D_i t \cos f_i t. \quad (10)$$

We evaluated the individual influence of the gravitational action of each asteroid listed in Table 1 on the six orbital elements of each of the four terrestrial planets. Because of the number of asteroids considered (43), of orbital elements (6) and of the terrestrial planets (4), we obtained total of $43 \times 6 \times 4 = 1032$ curves, which are gathered together with the related tables of components A_i , B_i , C_i , D_i in a database called ASETEP (asteroid effects on the terrestrial planets), as detailed in Sect. 7. In addition, we also calculated and included in this data base the influences of each asteroid on the distance from the EMB to a given terrestrial planet (Mercury, Venus or Mars) and its orientation vector with respect to the EMB. This is of particular interest for the specific study of geocentric observations of the planets.

Because of the very huge number of results above, our purpose in the next section is only to present a few characteristic examples, and some generalizing tables, which aim to understand both the quality of our determinations and the impact of the gravitational effects of each asteroid on the orbital parameters of the terrestrial planets. Although these effects are included in the construction of modern ephemeris of planets, the individual signature of each asteroid is generally not explicitly given, and the effect of mass uncertainty is not fully characterized. One

Fig. 1. Effects of Ceres on a_{Mars} for a 100 y time interval from JJ 2009.81; raw signal (black), fitting curve (red) and residuals (in green).

of the interests of our study is to give complete empirical tables of the asteroidal effects on the orbital elements of a given planet, which could be easily re-adjusted (by simple proportionality) after revision of the asteroidal masses. Then we can classify the asteroidal effects by the range of amplitude or/and frequency.

4. Perturbation on the Mars orbital parameters

By using the methodology detailed in the previous section, we computed at first the effects of the asteroids on Mars orbital parameters with the help of a Runge-Kutta twelfth order integration, with a 0.01-day stepsize. The rectangular coordinates obtained with respect to the ecliptic and equinox of J2000.0 are directly converted into orbital parameters with respect to this reference plane and point.

4.1. Perturbation on a_{Mars} and λ_{Mars}

As a basic and typical example of our computations, and to compare them with other authors, such as Mouret et al. (2009), we show in Figs. 1 and 2 the influence of the gravitational action of Ceres on the Mars semi-major axis and longitude. The time interval here is 1000 y starting from 2009 Sept. 26, i.e. between JD 2009.81 and JD 3009.81. For the clarity we plotted the figures for a 100 y time span. According to (10), we fitted the curves as a sum of Fourier (with coefficients A_i and B_i) and Poisson components (C_i and D_i).

In both curves, with peak-to-peak amplitudes of 400 m and 0.006'' for a_{Mars} and λ_{Mars} , respectively, we observe the good quality of the fit resulting from the least-squares-analysis: the fitting curve can hardly be distinguished from the original one. The flat curve represents the residuals obtained after subtracting a combination of 27 sinusoids for the semi-major axis from the original signal, and 10 sinusoids for the longitude. These sinusoids are listed in Tables 2 and 3 in the typical ASETEP database format. Notice that similar tables have been generated automatically for the 1032 cases mentioned above.

t is given in Julian year: its value is $t = 2009.81$ at the origin and not $t = 0$, so that the value of the Poisson coefficients contribution can be quite high all along the interval of time explored. Nevertheless, this high value is fully compensated by the high value of some Fourier components. In other words, to best fit the signal according to (10) requires introducing two contributions that can be very high individually compared with the signal itself but which give the best fit when mixed together. Therefore

Table 2. Coefficients of Fourier and Poisson series for the effects of Ceres on a_{Mars} .

Period (Day)	Period (Year)	SIN (m)	COS (m)	(m)	T SIN $\times 10^{-4}$ (m)	T COS $\times 10^{-4}$ (m)	$\times 10^{-3}$ (m)	Amplitud $\times 10^1$ (m)	Asteroid
27960.94	76.55	2.644	2.158	3.413	0.291	-3.054	0.432	0.563	1_Ceres
16191.36	44.33	-54.282	-53.165	75.981	67.182	62.329	8.815	5.757	1_Ceres
4886.42	13.38	-1.227	9.966	10.042	2.164	-13.844	1.958	0.691	1_Ceres
3754.72	10.28	15.277	57.810	59.794	-9.105	-0.557	0.079	5.954	1_Ceres
3047.01	8.34	-16.680	-0.495	16.687	7.171	0.221	0.031	1.542	1_Ceres
1877.01	5.14	24.877	16.008	29.582	-20.336	-10.365	1.466	2.495	1_Ceres
1251.31	3.43	8.691	-0.233	8.694	-10.205	0.628	0.089	0.671	1_Ceres
1161.51	3.18	6.924	-12.634	14.407	6.635	2.299	0.325	1.484	1_Ceres
887.06	2.43	3.562	-39.634	39.793	14.293	19.194	2.714	3.622	1_Ceres
717.51	1.96	-5.645	-11.862	13.136	9.587	5.867	0.830	1.149	1_Ceres
687.06	1.88	-3.017	-0.612	3.078	1.168	-1.459	0.206	0.304	1_Ceres
602.38	1.65	-4.163	-2.346	4.779	6.308	0.588	0.083	0.316	1_Ceres
580.76	1.59	-43.229	24.966	49.921	-13.036	-34.487	4.877	4.935	1_Ceres
502.95	1.38	-11.922	15.858	19.840	-7.375	-15.356	2.172	1.844	1_Ceres
443.53	1.21	-0.836	8.157	8.199	-5.500	-8.805	1.245	0.669	1_Ceres
408.23	1.12	11.695	-37.093	38.893	-56.454	184.861	26.143	1.956	1_Ceres
396.66	1.09	1.263	2.717	2.996	-3.399	-2.282	0.323	0.215	1_Ceres
387.17	1.06	19.798	0.132	19.798	-3.570	24.294	3.436	2.008	1_Ceres
350.98	0.96	9.686	-4.009	10.483	-0.729	12.473	1.764	0.962	1_Ceres
320.97	0.88	3.050	-3.512	4.652	0.594	5.911	0.836	0.402	1_Ceres
314.71	0.86	4.284	2.959	5.207	-5.783	3.139	0.444	0.473	1_Ceres
290.37	0.79	-7.373	-1.951	7.626	5.054	-12.518	1.770	0.819	1_Ceres
272.16	0.75	13.490	11.265	17.575	-66.768	-54.343	7.685	0.852	1_Ceres
AMPLITUDE		Before = 409.294276			After = 57.641405				

Notes. Bias: 0.137559×10^{-2} ; linear: -0.377681×10^{-2} ; T^2 : 0.333290×10^{-5} . The next-to-last column indicates the peak-to-peak amplitude of the total mixed component for the 1000 y time interval considered.

Table 3. Coefficients of Fourier and Poisson series for the effects of Ceres on λ_{Mars} .

period (Day)	period (Year)	SIN $\times 10^{-5}$ (")	COS $\times 10^{-5}$ (")	$\times 10^{-4}$ (")	T SIN $\times 10^{-8}$ (")	T COS $\times 10^{-8}$ (")	$\times 10^{-8}$ (")	Amplitud $\times 10^{-2}$ (")	Asteroid
27961.03	76.55	-10.158	13.521	1.691	0.841	0.666	0.942	0.021	1_Ceres
16190.53	44.33	149.608	-165.221	22.289	-14.986	19.171	27.112	0.177	1_Ceres
15442.23	42.28	-8.026	-1.417	0.815	2.600	0.650	0.919	0.001	1_Ceres
4886.18	13.38	-11.271	-0.854	1.130	1.674	0.200	0.282	0.007	1_Ceres
3754.68	10.28	-56.393	15.559	5.850	0.449	-0.867	1.227	0.057	1_Ceres
3046.73	8.34	-0.633	-5.797	0.583	-0.101	0.045	0.064	0.006	1_Ceres
1877.01	5.14	-7.474	12.606	1.465	0.362	-1.003	1.418	0.013	1_Ceres
1161.51	3.18	7.066	3.973	0.811	-0.223	0.274	0.387	0.008	1_Ceres
887.06	2.43	11.749	1.247	1.182	-0.559	0.397	0.562	0.011	1_Ceres
580.76	1.59	-5.492	-9.687	1.114	0.781	-0.210	0.297	0.011	1_Ceres
AMPLITUDE		Before = 0.005746			After = 0.000604				

Notes. Bias: 0.221687; linear: -0.550293×10^{-4} ; T^2 : -0.278178×10^{-7} . The next-to-last column indicates the peak-to-peak amplitude of the total mixed component for the 1000 y time interval considered.

we should not refer directly to the values of A_i , B_i , C_i and D_i to evaluate the amplitude of a component at a given frequency f_i in (10) but to the peak-to-peak amplitude of $F_i(t)$ during the interval of time considered. This peak-to-peak amplitude is given in the next-to-last columns of Tables 2 and 3, where each component has been ranged in decreasing order of period. For information and to avoid too many data in the paper, the results of the effects of Ceres on e_{Mars} , i_{Mars} , Ω_{Mars} and ϖ_{Mars} are given in Figs. A.1 to A.4 and Tables A.1 to A.4.

For comparison we also show in Figs. 3 and 4 the effects of Pallas and Vesta on the Mars semi-major axis for the same time interval as in Fig. 1. The peak-to-peak amplitudes reach roughly 200 m and 400 m, this last value is roughly the same as for Ceres, but the nature of the three signals is quite different

from each other. For the sake of clarity the tables characterizing the adjustment are presented in Tables A.5 and A.6.

4.2. Comparison with the analytical results in Mouret et al. (2009)

It is interesting and challenging to compare our numerical results with the analytical ones found by Mouret et al. (2009) when using developments as in (7) and (8). For that purpose it is necessary to choose the same time interval as these authors, that is to say 1000 y, because the developments in series are highly dependent on this time interval. In Tables 4 and 5 we show this comparison for the variation of the Mars semi-major axis and mean longitude due to the gravitational influence of Ceres, Pallas and

Fig. 2. Effects of Ceres on λ_{Mars} for a 100 y time interval from JJ 2009.81; raw signal (in black), fitting curve (in red) and residuals (in green).

Fig. 3. Effects of Pallas on a_{Mars} for a 100 y time interval from JJ 2009.81; raw signal (in black), fitting curve (in red) and residuals (in green).

Fig. 4. Effects of Vesta on a_{Mars} for a 100 y time interval from JJ 2009.81; raw signal (in black), fitting curve (in red) and residuals (in green).

Vesta. We can notice the remarkable general agreement between our numerical determination of the periods and amplitudes of the sinusoidal components that characterize the signal, and the analytical terms developed by Mouret et al. (2009), whose the arguments are a linear combination of the mean longitudes of Mars and the asteroid.

For the effect of Ceres on a_{Mars} (Table 4) a small but significant discrepancy arises only for the leading term with period 44.33 y and amplitude 57.57 m, which seems to correspond to the component with the period 45.77 y and 60.59 m amplitude in the above paper. This discrepancy probably arises because we used a quadratic regression and removed it from the signal to isolate the periodic parts, which should affect the determination of low-frequency terms. We also notice components in our calculations that are not mentioned in the paper above, with significant amplitudes 19.56 m, 8.52 m, 5.63 m and 4.78 m and periods 1.12 y, 0.75 y, 76.55 y and 0.56 y. the reason for this could be that analytical developments, even at a low order of masses and eccentricities, are cumbersome with a mixing of many sinusoidal contributions involving the mean longitudes. Therefore some components may be missing.

For the effect of Ceres on λ_{Mars} , the comparison between our results and Mouret et al. (2009) still agrees very well for the majority of components both in amplitudes and periods. Differences similar to those noted for a_{Mars} concern the leading term with period 44.33 y instead of 45.77 y and amplitude 1.77 mas (milliarcsecond).

The agreement is almost perfect for the influences of Pallas and Vesta, if we exclude large amplitudes without counterparts in the analytical developments, such as the component with amplitude 19.02 m and period 1.12 y for a_{Mars} and the second-largest component with amplitude 0.333 mas and period 59.79 y for λ_{Mars} .

4.3. Perturbations on the other orbital parameters e , i , Ω , $\tilde{\omega}$

The gravitational effects of Ceres and other leading asteroids on Mars orbital parameters e_{Mars} , i_{Mars} , Ω_{Mars} , $\tilde{\omega}_{\text{Mars}}$ have not been explicitly given either by Williams (1984) or by Mouret et al. (2009) although their relative amplitude is of the same order (and sometimes even larger) than the corresponding effects on a_{Mars} and λ_{Mars} described above.

Two important remarks about the curves plotted for Ceres, Pallas and Vesta in Figs. 5 to 8 need to be made. First of all, in contrast to what happens for the semi-major axes (see Fig. 1), long periodic effects that can be characterized by linear or quadratic expressions appear clearly, in particular for i_{Mars} (Fig. 5) and Ω_{Mars} (Fig. 7), for which they dominate the periodic oscillations, even during the short time span considered here (100 y). Second, we can observe that although its mass is considerably smaller than that of Ceres (with a ratio roughly 1/3), the effect of Vesta on the Mars orbital elements can be significantly stronger, as is the case for e_{Mars} (Fig. 6) and $\tilde{\omega}_{\text{Mars}}$ (Fig. 8). For the three asteroids considered here, the peak-to-peak amplitudes are on the order of 10^{-8} for e_{Mars} , 1 mas (milliarcsecond) for i_{Mars} , 0'05 for Ω_{Mars} and 0'01 for $\tilde{\omega}_{\text{Mars}}$, in the 100 y time interval considered.

In Figs. A.1–A.4 we show the effects of Ceres after the quadratic regression is removed. For each of the parameters, leading sinusoidal signatures appear clearly and the same fitting method as previously can be applied. Results of the decomposition are shown in Tables A.1–A.4.

4.4. Amplitude of the perturbations for each parameter and each asteroid

As we already mentioned in Sect. 3.2, we extended our study to the 1032 cases (43 asteroids $\times$ 6 planets $\times$ 6 orbital elements). The output of our computations is presented in Table 6: we

Table 4. Comparisons of the amplitude of the sinusoids found in this paper with the analytical ones of Mouret et al. (2009) of the effects of Ceres, Pallas and Vesta on a_{Mars} .

Our analysis		Mouret et al. (2009)			
Period (y)	Amplitude (m)	Period (y)	Amplitude (m)	Argument $i\lambda_{\text{M}} + j\lambda_{\text{c}}$ i j	
Ceres					
10.280	59.5370	10.25	59.390	1	-2
44.330	57.5675	45.77	60.587		
1.590	49.3516	1.59	49.517	2	-2
2.429	36.2161	2.43	36.352	2	-3
5.139	24.9480	5.12	25.132	2	-4
1.118	19.5619				
1.060	20.0837	1.06	19.897	3	-3
1.377	18.4410	1.38	18.550	3	-4
8.342	15.4159	8.37	15.708	1	-3
3.180	14.8437	3.18	14.900	1	-1
1.964	11.4949	1.96	11.369	3	-5
0.961	9.6248	0.96	9.694	4	-5
0.745	8.5178				
0.795	8.1945	0.79	7.944	4	-4
13.378	6.9088	13.20	7.465	3	-7
3.426	6.7110	3.42	6.672	3	-6
1.214	6.6896	1.21	6.627	4	-6
76.553	5.6321				
0.738	4.8935	0.74	4.952	5	-6
0.862	4.7347	0.86	4.682	3	-2
0.559	4.7800				
0.879	4.0163	0.88	3.934	5	-7
1.649	3.1624	1.65	3.665	4	-7
0.678	3.4828	0.68	3.366	4	-3
1.881	3.0392	1.88	2.992	1	0
Pallas					
49.058	73.5324	48.65	74.350	2	-5
1.118	19.0229				
12.855	14.7998	12.89	14.750	3	-7
24.518	14.1215	24.32	13.883	4	-10
5.093	13.0090	5.10	13.224	2	-4
2.421	9.6624	2.42	9.619	2	-3
17.424	6.2069	17.54	7.121	5	-12
1.588	7.1841	1.59	6.537	2	-2
10.185	6.8003	10.19	6.508	1	-2
8.433	6.9895	8.43	6.358	1	-3
3.395	5.7901	3.40	5.789	3	-6
7.196	5.5975	7.18	5.505	3	-8
1.956	4.6407	1.96	4.638	3	-5
5.683	4.8822	5.69	4.593	4	-9
27.009	4.4647	27.44	3.964	7	-17
4.217	3.8491	4.21	3.934	2	-6
1.374	3.6767	1.37	3.590	3	-4
16.358	2.8954	16.22	3.366	6	-15
2.546	2.9761	2.55	2.857	4	-8
2.546	2.9761	1.64	2.423	4	-7
1.058	2.2976				
1.210	2.1458	1.21	2.035	4	-6
Vesta					
25.855	38.9394	25.93	38.746	2	-4
1.952	30.6734	1.95	30.368	2	-2
4.223	25.5473	4.22	24.983	2	-3
1.301	15.7973	1.30	15.708	3	-3
2.029	13.9884	2.03	13.254	3	-4
3.904	11.5442	3.90	11.325	1	-1
4.598	10.5150	4.59	9.829	3	-5
17.238	9.9542	17.29	9.140	3	-6
0.976	8.5505	0.98	8.198	4	-4
1.335	8.3356	1.33	8.078	4	-5

Notes. Coefficients are ranged in decreasing order of amplitude.

Table 5. Comparisons of the sinusoids found in this paper with those in Mouret et al. (2009) of the effects of Ceres, Pallas and Vesta on λ_{Mars} .

Our analysis		Mouret et al. (2009)			
Period (y)	Amplitude $\times 10^{-3}''$	Period (y)	Amplitude $\times 10^{-3}''$	Argument $i\lambda_M + j\lambda_c$ i j	
Ceres					
10.280	0.573	10.25	0.571	1	-2
44.330	1.768	45.77	1.914	2	-5
76.553	0.205				
1.590	0.108	1.59	0.127	2	-4
2.429	0.108	2.43	0.109	2	-2
5.139	0.126	5.12	0.108	2	-3
8.342	0.058	8.37	0.060	1	-3
3.180	0.080	3.18	0.039	3	-4
13.378	0.069	13.20	0.025	3	-6
Pallas					
49.058	2.521	48.65	2.537	2	-5
59.790	0.333				
24.517	0.209	24.32	0.227	4	-10
12.855	0.148	12.89	0.153	3	-7
17.424	0.088	17.54	0.099	5	-12
Vesta					
51.657	4.372				
25.854	0.679	25.93	0.668	2	-4
4.223	0.112	4.22	0.110	2	-3
40.162	0.107				
17.237	0.104	17.29	0.103	3	-6
1.118	0.096				
15.779	0.083				
1.952	0.078	1.95	0.078	2	-2
32.816	0.078				
2.029	0.036	2.03	0.035	3	-4
1.301	0.032	1.30	0.031	3	-3

Notes. Coefficients are ranged in decreasing order of amplitude.

Fig. 5. Effects of Ceres, Pallas and Vesta on i_{Mars} , for a 100 y time interval, starting from JD 2009.81.

provide the peak-to-peak amplitudes of the effects of each of the 43 asteroids listed in Table 1 on each Mars orbital element. This is done for a 1000 y time interval, beginning in 2009. The amplitudes stand for the total signal including secular trends and periodic components. Moreover, to compare the amplitudes of the effects on e_{Mars} to those on a_{Mars} with the same scale, we multiplied these effects by the nominal value a_0 of the Mars semi-major axis. Thus we can remark that throughout, $a_0 \times \Delta e$ is significantly higher than Δa with a ratio typically on the order of 2 or 3, but sometimes considerably more, as for Pallas and Vesta. The relatively large amplitude of the effects due to the asteroid

Fig. 6. Effects of Ceres, Pallas and Vesta on e_{Mars} , for a 100 y time interval starting from JD 2009.81.

Fig. 7. Effects of Ceres, Pallas and Vesta on Ω_{Mars} , for a 100 y time interval, starting from JD 2009.81.

Fig. 8. Effects of Ceres, Pallas and Vesta on $\tilde{\omega}_{\text{Mars}}$, for a 100 y time interval, starting from JD 2009.81.

111 Ate compared to those due to these two last asteroid is remarkable. This is because the mass of Ate is comparable to that of these two big objects. For Ceres, during the 1000 y interval of time considered, the peak-to-peak amplitudes are 416 m for Δa , 1300 m for $a_0 \times \Delta e$, 6.8 mas (milliarcsec) for Δi , 0'.410 for $\Delta \Omega$ and 0'.197 for $\Delta \lambda$. These effects of the biggest asteroid are surpassed by those of Vesta for Δa (546.7 m) and $a_0 \times \Delta e$ (4666 m), by Pallas and 111 Ate for $a_0 \times \Delta e$ (2025 m and 426 m).

As a direct by-product of our results, we present in Table 7 the uncertainty of each Mars orbital parameter due to the mass uncertainty of each selected asteroid. Indeed, according to (6), this uncertainty is directly dependent on the asteroid mass, it is

Fig. 9. Effects of Ceres on a_{EMB} , for a 100 y time interval, from JD 2009.81. Residuals (in green) after removing the sinusoidal components from the least-squares analysis.

sufficient for the purpose to multiply each number of Table 6 by the relative mass uncertainty $\Delta M/M$, whose value has been already given for each object in Table 1. Because of its relatively large mass (17.7% that of Ceres) and its very high mass uncertainty (22.6%), the asteroid 111 Ate is by far the most problematic one, causing uncertainties higher than 93 times that of Ceres for a_{Mars} , 197 times for $a_0 \Delta e_{\text{Mars}}$, 44 times for i_{Mars} , 23 times for Ω_{Mars} , 58 times for ϖ_{Mars} and 295 times for λ_{Mars} . Uncertainties on the same order can be observed for 6 Hebe (18.7% mass uncertainty), 7 Iris (8.6%), 9 Metis (23.3%) 19 Fortuna (14.0%), 20 Massalia (16.9%) and 46 Hestia (63%). We think that a table such as Table 7 shows in a very straightforward manner for which asteroid observational campaigns should be launched to minimize errors in the Mars ephemeris.

5. Perturbation on the EMB, Mercury and Venus

In this section we carry out the same kind of study as in the previous section for the terrestrial planets other than Mars. Instead of the Earth isolated, we consider the EMB. This facilitates our calculations noticeably. A specific study of the asteroidal effects on the Earth and the Moon taken individually is scheduled for a future paper.

5.1. Effects on EMB

As an example we show in Figs. 9–11 the effects of the gravitational action of Ceres, Pallas and Vesta on the EMB semi-major axis. They are characterized by very stable and regular leading oscillations with peak-to-peak amplitudes about 25 m for Ceres, 9 m for Pallas and 15 m for Vesta. The flat curves in each figure represent the residuals after subtracting the fit with the help of Fourier and Poisson components. We were able to reconstruct the curve of Ceres with as many as 50 frequencies. The leading components are shown in Table 8. We note that the amplitudes and periods of the leading oscillations are significantly shorter for the EMB than for Mars.

In Table 9 we present the peak-to-peak amplitudes of the effects of each asteroid on the orbital parameters of the EMB for a 1000 y time interval. They are generally, but no systematically smaller than the corresponding effect on Mars. This is because the mass of Mars is much smaller than the mass of our planet ($M_{\text{Mars}}/M_{\text{Earth}} \approx 0.107$) and its orbit is much closer to the asteroid main belt.

Table 6. Peak-to-peak amplitudes of the variations of the six Mars orbital elements for a 1000 y time interval starting from JD 2009.81.

	asteroid	Δa m	$\Delta e \times a_{\text{Mars}}$ m	Δi $\times 10^{-6}''$	$\Delta \Omega$ $\times 10^{-6}''$	$\Delta \varpi$ $\times 10^{-6}''$	$\Delta \lambda$ $\times 10^{-6}''$
1	Ceres	415.976	1300.018	6764.552	409910.423	101767.790	197860.644
2	Pallas	212.853	2025.012	4517.624	110490.432	21951.167	143118.437
3	Juno	38.999	166.213	381.803	8282.676	1641.266	24833.521
4	Vesta	546.665	4665.960	5665.561	98770.967	50554.943	623437.866
6	Hebe	144.013	607.501	606.177	2122.292	6015.502	40524.178
7	Iris	67.551	589.408	77.461	11921.764	1707.940	3249.652
8	Flora	28.615	351.759	264.020	1527.953	1472.282	23161.386
9	Metis	68.308	608.089	96.367	10037.872	5999.291	6453.236
10	Hygiea	53.587	547.873	225.513	13260.778	5733.690	6918.631
11	Parthenope	8.210	65.473	76.711	197.072	688.702	2268.556
13	Egeria	14.479	57.479	77.631	15907.661	1666.410	6619.356
14	Irene	14.634	79.531	122.264	5390.858	1888.597	11809.004
15	Eunomia	41.963	416.526	593.602	16346.103	1348.323	48827.323
16	Psyche	10.560	135.172	101.183	2213.723	824.135	6357.709
17	Thetis	2.740	23.640	20.280	38.826	173.695	1944.757
18	Melpomene	6.828	57.461	138.647	1381.789	195.621	497.143
19	Fortuna	48.578	308.680	30.604	3799.559	1832.499	46311.253
20	Massalia	80.479	238.095	9.043	2019.272	2971.102	47763.291
21	Lutetia	6.698	48.954	10.871	207.539	516.701	7525.555
22	Kalliope	2.682	36.025	21.367	3708.021	401.749	1934.839
24	Themis	14.250	50.778	1.832	527.693	1237.710	16186.184
28	Bellona	16.140	74.403	214.068	1144.324	2820.137	7919.977
29	Amphitrite	17.267	165.764	224.757	2016.979	2826.971	12679.011
31	Euphrosyne	13.346	56.216	54.024	3639.611	482.373	6169.238
45	Eugenia	8.092	107.311	202.524	1652.153	1867.647	2507.753
46	Hestia	20.516	229.351	112.954	5941.046	1115.982	11354.487
47	Aglaja	0.743	4.292	10.767	84.131	29.633	59.064
48	Doris	4.005	28.678	58.280	2075.386	802.772	969.684
49	Pales	1.635	27.457	5.568	440.772	120.879	210.067
52	Europa	9.826	68.303	131.673	607.090	1828.279	13298.494
65	Cybele	3.061	33.727	37.098	520.461	303.235	2904.304
87	Sylvia	1.974	7.763	16.837	2297.924	229.907	419.793
88	Thisbe	7.671	48.380	54.682	3342.237	256.270	6600.855
90	Antiope	0.866	4.080	0.385	4.311	34.948	628.948
107	Camilla	3.072	6.013	58.238	1078.727	503.130	278.376
111	Ate	218.942	1425.543	1683.832	53568.836	33425.288	328096.755
121	Hermione	1.001	11.230	4.441	587.147	95.005	1318.865
130	Elektra	4.738	23.884	110.486	393.899	589.786	3224.094
165	Loreley	12.197	52.039	256.157	5936.699	937.680	901.986
189	Phthia	0.132	0.395	0.255	21.056	7.059	152.358
243	Ida	0.108	0.116	0.049	3.560	3.224	28.511
253	Mathilde	0.193	0.395	1.320	42.175	12.877	224.965
283	Emma	0.486	4.920	9.306	230.447	22.563	56.147

5.2. Effects on Mercury and Venus

The effects of the asteroids on the orbital parameters of the two internal planets Mercury and Venus are presented in Tables 10 and 11 also for a 1000 y time span. The peak-to-peak amplitudes for a_{Mercury} and a_{Venus} do not exceed 1 m and 10 m, i.e. roughly 400 times and 40 times less than for a_{Mars} . Nevertheless, the ratio is significantly lower for other parameters, also because of the predominance of long-term components, which can be characterized by quadratic or polynomial expressions in the interval of time considered.

6. Influences of asteroid on the EMB-Mars distance and Mars orientation

In view of the ground-based observations of Mars, the fundamental parameters are both the distance d_{Mars} from the Earth geocenter (or the EMB in an alternative case considered here) to the planet geocenter, and the geocentric direction of the planet

generally given through the set of its geocentric coordinates (α_{Mars} , δ_{Mars}). Thus the gravitational effect of a given asteroid on these parameters comes directly from the combination of the influences of the asteroid on the orbital elements of the EMB and the planet, which we studied above. As for these influences, the uncertainty of the asteroid masses directly affects the accuracy of the determination of d_{Mars} , α_{Mars} and δ_{Mars} . Moreover, the analytical characterization of the asteroidal effects on these three geocentric parameters is still much more complicated than for the heliocentric orbital parameters, because the determination of the vector from the EMB to planet, even assuming that the two bodies follow a Keplerian trajectory, is a much more complex function that involves Legendre polynomials, than for the heliocentric vector of the planet.

We recall that to facilitate the computations, we replaced the Moon and the Earth by the EMB, i.e. a single point whose mass is the sum of the masses of the two bodies, and which is located at their barycenter. We will calculate in a future paper the differential effects of our asteroids on the Earth to Moon distance,

Table 7. Uncertainty on the values of the six Mars orbital elements, for a 1000 y, because of the lack of accuracy in the measurements of the asteroid mass.

	Asteroid	Δa [m]	$\Delta e \times a_{\text{Mars}}$	Δi $\times 10^{-6}''$	$\Delta \Omega$ $\times 10^{-6}''$	$\Delta \varpi$ $\times 10^{-6}''$	$\Delta \lambda$ $\times 10^{-6}''$
1	Ceres	0.529	1.653	8.599	521.073	129.366	251.518
2	Pallas	1.451	13.802	30.792	753.100	149.618	975.491
3	Juno	7.800	33.243	76.361	1656.535	328.253	4966.704
4	Vesta	1.220	10.415	12.646	220.471	112.846	1391.602
6	Hebe	26.943	113.656	113.409	397.056	1125.430	7581.599
7	Iris	13.510	117.882	15.492	2384.353	341.588	649.930
8	Flora	3.023	37.158	27.890	161.404	155.523	2446.625
9	Metis	15.917	141.691	22.455	2338.922	1397.893	1503.667
10	Hygiea	1.771	18.102	7.451	438.131	189.439	228.589
11	Parthenope	0.156	1.243	1.457	3.742	13.077	43.074
13	Egeria	2.825	11.215	15.148	3103.934	325.153	1291.582
14	Irene	2.587	14.058	21.611	952.864	333.820	2087.306
15	Eunomia	1.998	19.835	28.267	778.386	64.206	2325.111
16	Psyche	1.392	17.813	13.334	291.731	108.607	837.838
17	Thetis	0.284	2.452	2.104	4.027	18.017	201.725
18	Melpomene	2.306	19.408	46.828	466.697	66.071	167.909
19	Fortuna	6.824	43.364	4.299	533.764	257.431	6505.832
20	Massalia	13.635	40.338	1.532	342.108	503.369	8092.128
21	Lutetia	0.623	4.554	1.011	19.306	48.065	700.052
22	Kalliope	0.066	0.885	0.525	91.106	9.871	47.539
24	Themis	5.404	19.255	0.695	200.095	469.326	6137.619
28	Bellona	4.612	21.258	61.162	326.950	805.754	2262.851
29	Amphitrite	6.044	58.018	78.665	705.943	989.440	4437.654
31	Euphrosyne	2.516	10.597	10.184	686.061	90.927	1162.892
45	Eugenia	2.698	35.770	67.508	550.718	622.549	835.918
46	Hestia	12.799	143.081	70.467	3706.341	696.209	7083.534
47	Aglaja	0.293	1.693	4.248	33.190	11.690	23.301
48	Doris	1.970	14.104	28.663	1020.682	394.806	476.894
49	Pales	0.303	5.085	1.031	81.624	22.385	38.901
52	Europa	2.225	15.466	29.815	137.466	413.986	3011.237
65	Cybele	0.734	8.087	8.896	124.801	72.713	696.421
87	Sylvia	0.008	0.031	0.068	9.277	0.928	1.695
88	Thisbe	2.240	14.125	15.965	975.793	74.820	1927.173
90	Antiope	0.010	0.049	0.005	0.052	0.422	7.596
107	Camilla	0.082	0.160	1.552	28.741	13.405	7.417
111	Ate	49.523	322.444	380.867	12116.761	7560.482	74212.364
121	Hermione	0.055	0.619	0.245	32.379	5.239	72.732
130	Elektra	0.285	1.439	6.656	23.729	35.529	194.223
165	Loreley	7.623	32.525	160.098	3710.437	586.050	563.742
189	Phthia	0.045	0.135	0.087	7.206	2.416	52.144
243	Ida	0.005	0.005	0.002	0.167	0.151	1.336
253	Mathilde	0.008	0.017	0.056	1.788	0.546	9.536
283	Emma	0.011	0.106	0.201	4.982	0.488	1.214

which likely has some interesting implications on lunar laser ranging (LLR) measurements. We calculated the geocentric coordinates of Mars by simply subtracting the heliocentric vector of the EMB from that of Mars. The effects of the leading asteroids Ceres, Pallas and Vesta on the EMB-Mars distance are plotted in Figs. 12–14. In any case the signal is dominated by a single wave with increasing amplitude and a period that slightly varies at each oscillation, which makes the adjustment through a least-squares analysis uneasy, indeed even inadequate. As expected, the period is clearly close to the synodic period of the planet, which changes slightly at each synodic cycle. The increase of amplitudes with respect to the time are undoubtedly caused by the secular effects of the asteroid on the EMB and Mars orbital parameters, characterized by quadratic trends as studied in Sect. 4.3. These secular effects gradually enhance the effects on the EMB-Mars vector. The peak-to-peak amplitudes of the effects of Ceres, Pallas and Vesta on the EMB-to-Mars distance are, 34 km, 31 km and 116 km respectively during the 100 y time interval considered.

In Figs. 15 and 16, we present the influence of Ceres on Mars geocentric right ascension and declination. Their peak-to-peak amplitudes reach 0.08 and 0.05 arcseconds during the 100 y time interval. The peaks correspond to Mars oppositions, for which the relatively close distance of the planet from the Earth enhances the angular effects of the perturbations. In parallel we can observe the asymmetrical nature of the two curves originating from the fact that the effects are proportional to the distance from the Earth to Mars, which varies roughly from 0.5 to 2.5 AU.

7. ASETEP database

7.1. Construction of the ASETEP database.

As a result of all our computations, we constructed the ASETEP (asteroid effects on the terrestrial planets) database, which contains at its present extent the curves showing the individual effects of each of the 43 asteroids listed in Table 1 on each of the 6 orbital elements for any of the 4 terrestrial planets (Mercury,

Table 8. Coefficients of Fourier and Poisson series for the effects of Ceres on a_{EMB} .

Period (Day)	Period (Year)	SIN $\times 10^{-2}$	COS $\times 10^{-2}$	$\times 10^{-1}$	T SIN $\times 10^{-5}$	T COS $\times 10^{-5}$	$\times 10^{-5}$	Amplitud $\times 10^0$	Asteroid
4264.15	11.67	-5.833	-4.725	0.751	4.218	4.478	6.332	0.089	1_Ceres
2785.82	7.63	14.760	-17.093	2.258	-9.642	14.800	20.930	0.164	1_Ceres
1048.02	2.87	14.503	-23.187	2.735	-13.280	16.173	22.872	0.199	1_Ceres
645.80	1.77	23.772	1.851	2.384	4.417	7.592	10.736	0.389	1_Ceres
583.74	1.60	-4.586	80.143	8.027	2.322	-39.840	56.342	0.048	1_Ceres
466.62	1.28	-107.748	26.526	11.096	-3.096	-9.959	14.085	1.127	1_Ceres
399.51	1.09	-1.046	24.761	2.478	-0.533	-9.255	13.089	0.106	1_Ceres
365.29	1.00	2.167	4.551	0.504	1.556	-0.782	1.106	0.131	1_Ceres
337.45	0.92	0.631	7.730	0.776	-0.767	-3.066	4.336	0.080	1_Ceres
322.86	0.88	8.798	48.792	4.958	1.456	-5.255	7.431	0.358	1_Ceres
312.83	0.86	7.583	-2.635	0.803	-4.257	0.966	1.366	0.216	1_Ceres
299.95	0.82	-6.889	11.029	1.300	5.119	-5.229	7.395	0.294	1_Ceres
291.95	0.80	24.131	76.178	7.991	-12.129	-38.203	54.027	0.144	1_Ceres
281.05	0.77	9.514	-6.059	1.128	-2.537	2.629	3.718	0.399	1_Ceres
273.99	0.75	-51.020	-30.936	5.967	25.466	14.530	20.548	0.519	1_Ceres
AMPLITUDE		Before= 22.943527			After= 0.353778				

Notes. Bias: 0.561802×10^{-3} ; linear: -0.549138 ; T^2 : 0.133527×10^{-3} . The next-to-last column indicates the peak-to-peak amplitude of the total mixed component for the 100 y time interval.

Fig. 10. Effects of Pallas on a_{EMB} , for a 100 y time interval, from JD 2009.81. Residuals (in green) after removing the sinusoidal components from the least-squares analysis. **Fig. 11.** Effects of Vesta on a_{EMB} , for a 100 y time interval, from JD 2009.81. Residuals (in green) after removing the sinusoidal components from the least-squares analysis.

Venus, the EMB, Mars) both for a 100 y time interval and a 1000 y time interval from JD 2009,7. In parallel we deliver for each of the curves the corresponding tables giving the Fourier and Poisson components for fitting these curves. This ASETEP database will be subject to regular and continuous updates with an increasing number of new asteroids taken into account, if their mass is determined with a satisfactory accuracy, i.e. better than 65% uncertainty, as for the 43 asteroids listed in Table 1. One of the advantages of the ASETEP database is that the figures and tables already produced for a given asteroid can be updated immediately each time a new and more accurate determination of the mass of this asteroid is reached, by simple proportionality.

7.2. Individual and combined asteroidal effects

In all computations we considered the effect of each asteroid taken individually, which neglects the effects from the combined gravitational attraction of the asteroids themselves. A priori these effects which result principally from the reciprocal gravitational interactions of the asteroids affecting their positions, are of second order because of the smallness of asteroid masses. They can be neglected at first approximation as long as

the interval of time of the study Δt is short, as for $\Delta t = 100$ y. To obtain some information about the error caused by neglecting the effects due to the combination of asteroids, we carried out a test with numerical integration: it consists of studying the difference between two numerical computations: first we summed up the individual effects of the 43 asteroids considered here on the Mars orbital parameters, each of which was obtained in the frame of an n -body problem with $n = 10$ (Sun + 8 planets + asteroid); second we included all asteroids in the frame of an n -body problem with $n = 52$ (Sun + 8 planets + 43 asteroids).

In Fig. 17, we plot the combined effect of the 43 asteroids taken together on the Mars semi-major axis. The residuals of the two computation methods presented above do not exceed 2.5 m peak-to-peak, as can be seen in Fig. 18. This is by two orders less than the signal itself (650 m peak-to-peak), and the error caused by treating individual effects separately can be neglected in first approximation. This is still true for the other Mars parameters (see Appendix B), for which the errors are below 1.2×10^{-11} for e (Figs. B.1 and B.2), 1.8 mas for i (Figs. B.3 and B.4), 10^{-2} mas for Ω (Figs. B.5 and B.6), 5×10^{-3} mas for $\dot{\omega}$ (Figs. B.7 and B.8).

Table 9. Peak-to-peak amplitudes of the variations of the six EMB orbital elements due to each given asteroid for a 1000 y time interval starting from JD 2009.81.

	Asteroid	Δa m	$\Delta e \times a_{\text{Earth}}$ m	Δi $\times 10^{-6}''$	$\Delta \Omega$ $\times 10^{-6}''$	$\Delta \varpi$ $\times 10^{-6}''$	$\Delta \lambda$ $\times 10^{-6}''$
1	Ceres	25.443	543.236	4929.530	464926.356	6311.044	2882.002
2	Pallas	10.608	147.959	219.655	1138450.592	2759.495	26618.466
3	Juno	1.347	29.962	29.840	55190.496	1976.844	5736.100
4	Vesta	19.679	221.168	1729.805	303243.082	55222.264	40243.056
6	Hebe	1.575	44.829	128.567	66754.335	1169.070	2754.659
7	Iris	1.916	49.911	67.099	5392.495	2470.755	2850.537
8	Flora	1.141	36.128	61.188	13731.489	989.258	194.963
9	Metis	1.324	23.418	97.963	18171.585	2828.013	301.391
10	Hygiea	2.154	28.291	122.134	18072.496	6670.653	4666.845
11	Parthenope	0.402	8.018	19.419	6613.715	1071.967	1323.398
13	Egeria	0.647	6.346	114.239	52651.902	504.693	173.308
14	Irene	0.624	17.746	52.094	3431.527	414.283	1124.330
15	Eunomia	2.074	60.980	236.253	49917.699	1191.803	1711.442
16	Psyche	1.673	37.652	18.645	13866.105	2789.469	7791.016
17	Thetis	0.119	0.879	4.588	1577.995	306.011	204.078
18	Melpomene	0.631	16.064	17.729	13773.197	234.733	1303.507
19	Fortuna	1.107	30.447	8.395	5701.767	689.533	2320.502
20	Massalia	0.688	1.965	1.549	1315.434	922.438	953.087
21	Lutetia	0.235	6.677	7.069	685.747	598.547	132.662
22	Kalliope	0.381	3.543	41.715	8466.995	239.465	775.256
24	Themis	0.456	7.424	1.398	908.453	142.142	794.665
28	Bellona	0.675	10.883	40.815	25052.303	1082.842	801.938
29	Amphitrite	0.773	14.573	6.314	36621.458	577.161	632.069
31	Euphrosyne	0.318	2.090	23.015	18359.137	122.700	67.111
45	Eugenia	0.618	10.152	35.986	25097.892	1631.268	501.838
46	Hestia	3.892	78.350	3.219	18091.046	4244.686	10698.904
47	Aglaja	0.140	1.865	0.227	2219.030	223.737	84.944
48	Doris	0.412	1.419	1.141	12170.155	138.494	549.257
49	Pales	0.156	3.498	3.626	498.753	155.474	501.291
52	Europa	0.569	4.259	41.151	14775.024	553.168	2008.514
65	Cybele	0.349	1.907	8.621	5582.948	883.675	375.725
87	Sylvia	0.350	3.944	33.215	6532.738	334.762	847.921
88	Thisbe	0.704	11.842	32.942	2892.127	1809.126	1924.267
90	Antiope	0.090	0.626	0.569	128.930	74.697	95.295
107	Camilla	0.503	1.431	6.525	11211.884	73.744	476.956
111	Ate	9.223	76.054	449.106	151257.961	12292.524	28037.019
121	Hermione	0.192	2.596	9.092	1821.472	60.102	613.565
130	Elektra	0.277	4.675	29.562	17655.726	232.881	840.610
165	Loreley	1.280	5.862	102.851	29936.733	1867.874	974.511
189	Phthia	0.056	0.036	0.066	62.766	3.047	179.217
243	Ida	0.076	0.131	0.016	8.814	11.517	233.687
253	Mathilde	0.069	0.386	0.016	231.658	24.807	92.696
283	Emma	0.094	1.347	3.668	1093.080	54.125	181.180

Fig. 12. Effects of Ceres on the distance from the EMB to Mars for a 100 y time interval from JD 2009.81. The curve in green corresponds to the post-fit residuals. **Fig. 13.** Effects of Pallas on the distance from the EMB to Mars for a 100 y time interval from JD 2009.81. The curve in green corresponds to the post-fit residuals.

Table 10. Peak-to-peak amplitudes of the variations of the six Mercury orbital elements due to each given asteroid for a 1000 y time interval starting from JD 2009.81.

	Asteroid	Δa m	$\Delta e \times a_{\text{Mercury}}$ m	Δi $\times 10^{-6}''$	$\Delta \Omega$ $\times 10^{-6}''$	$\Delta \varpi$ $\times 10^{-6}''$	$\Delta \lambda$ $\times 10^{-6}''$
1	Ceres	1.027	17.040	506.486	1525.642	5274.314	1031.921
2	Pallas	0.236	21.632	546.181	3524.193	680.519	2027.054
3	Juno	0.031	0.226	23.731	194.155	53.392	1045.685
4	Vesta	0.444	1.697	312.317	1284.460	2605.141	9801.069
6	Hebe	0.035	1.712	38.624	149.068	85.198	742.517
7	Iris	0.050	0.311	3.638	190.978	91.108	1633.283
8	Flora	0.036	0.610	10.833	69.548	81.990	1386.485
9	Metis	0.032	0.097	6.998	53.990	161.183	1166.247
10	Hygiea	0.054	1.107	12.770	512.880	391.011	1769.981
11	Parthenope	0.020	0.202	5.388	50.541	51.114	965.908
13	Egeria	0.033	0.866	7.069	161.930	104.715	559.951
14	Irene	0.043	0.472	6.181	2.022	61.844	2266.552
15	Eunomia	0.046	1.757	38.713	391.935	218.771	398.350
16	Psyche	0.037	0.389	9.404	146.207	111.991	868.428
17	Thetis	0.021	0.015	1.276	9.532	10.570	1164.296
18	Melpomene	0.024	0.282	6.566	45.892	22.050	776.751
19	Fortuna	0.028	0.299	2.340	119.341	79.661	99.164
20	Massalia	0.021	0.275	0.760	48.928	36.882	163.080
21	Lutetia	0.023	0.194	0.909	16.061	22.953	380.720
22	Kalliope	0.020	0.055	2.041	34.978	37.148	904.473
24	Themis	0.035	0.325	0.393	43.222	44.341	2114.352
28	Bellona	0.031	0.613	15.834	93.611	61.361	745.187
29	Amphitrite	0.032	0.383	13.422	69.474	135.370	275.212
31	Euphrosyne	0.015	0.351	5.117	60.569	13.988	107.380
45	Eugenia	0.032	0.463	16.170	134.059	102.246	1300.468
46	Hestia	0.056	0.989	8.991	226.888	156.690	2138.406
47	Aglaja	0.019	0.084	0.771	6.255	11.698	333.500
48	Doris	0.024	0.331	5.801	81.630	39.975	1628.596
49	Pales	0.026	0.041	0.345	16.042	9.940	566.625
52	Europa	0.032	0.298	11.911	65.363	66.695	1956.630
65	Cybele	0.020	0.069	4.196	52.779	46.187	221.947
87	Sylvia	0.026	0.165	1.920	18.840	41.512	869.579
88	Thisbe	0.029	0.288	3.084	107.780	73.901	517.634
90	Antiope	0.029	0.026	0.075	2.631	3.275	544.525
107	Camilla	0.018	0.242	6.226	53.845	24.304	708.428
111	Ate	0.195	4.328	93.162	1380.844	1113.267	1479.395
121	Hermione	0.015	0.088	0.568	0.221	15.036	342.469
130	Elektra	0.017	0.498	11.278	34.221	10.621	593.566
165	Loreley	0.042	0.823	22.573	186.959	138.388	1484.663
189	Phthia	0.019	0.017	0.024	0.565	0.252	792.788
243	Ida	0.023	0.012	0.005	0.243	0.353	1287.098
253	Mathilde	0.011	0.011	0.099	1.381	0.617	456.920
283	Emma	0.012	0.043	0.732	8.355	5.916	591.482

Fig. 14. Effects of Vesta on the distance from the EMB to Mars for a 100 y time interval from JD 2009.81. The curve in green corresponds to the post-fit residuals.

7.3. Use of ASETEP for ephemerides study

Most of these asteroids have a very large mass uncertainty, sometimes by far exceeding their nominal values, which is why it appears to be ill-advised to include them individually in the ephemerides computations. But they can be assigned a reasonable distribution of masses according to some models such as the statistical asteroid model (Tedesco et al. 2005). The problem of mass undetermination of small asteroids was there partly solved by replacing all asteroids except for the 303 most perturbing ones by a ring. Through Monte Carlo experiments, where asteroids are assigned random but reasonable masses, it is possible to calculate the corresponding perturbations from the selected set of asteroids (Fienga et al. 2009). An estimate of the mass of the ring by Kuchynka et al. (2008) was $M_{\text{ring}} = 1 \pm 0.3 \times 10^{-10} M_{\text{Sun}}$ for a ring at 3.14 AU. Yet compared this statistical evaluation, the uncertainty concerning the largest asteroids looks like a severe limitation for the ephemeris accuracy. For instance, the mass

Table 11. Peak-to-peak amplitudes of the variations of the six Venus orbital elements due to the action of each asteroid for a 1000 y time interval starting from JD 2009.81.

	Asteroid	Δa m	$\Delta e \times a_{\text{Venus}}$ m	Δi $\times 10^{-6}''$	$\Delta \Omega$ $\times 10^{-6}''$	$\Delta \varpi$ $\times 10^{-6}''$	$\Delta \lambda$ $\times 10^{-6}''$
1	Ceres	8.873	104.443	48.038	31824.583	36061.152	70697.133
2	Pallas	3.622	42.975	1518.868	4481.157	2990.961	13040.604
3	Juno	0.520	12.100	67.545	237.562	844.492	749.814
4	Vesta	5.436	113.260	411.520	6658.590	35311.328	16328.430
6	Hebe	0.363	12.581	87.162	486.578	2490.318	662.926
7	Iris	0.647	17.653	1.688	902.456	2010.286	3453.414
8	Flora	0.225	10.603	18.079	143.247	567.063	1067.780
9	Metis	0.292	10.441	8.033	336.941	2196.530	174.656
10	Hygiea	0.655	14.605	16.991	2080.629	4350.355	1846.343
11	Parthenope	0.129	0.871	9.425	12.093	1109.277	555.197
13	Egeria	0.239	0.504	53.465	939.036	928.800	2454.464
14	Irene	0.179	4.529	2.946	292.024	649.127	210.247
15	Eunomia	0.435	17.318	69.851	2515.550	1056.317	2316.562
16	Psyche	0.681	8.146	18.972	245.119	1104.041	647.785
17	Thetis	0.034	0.526	2.198	4.257	214.398	83.850
18	Melpomene	0.191	4.177	16.546	5.985	669.946	832.947
19	Fortuna	0.195	8.576	6.152	369.171	512.983	1946.517
20	Massalia	0.176	1.709	1.578	146.463	923.534	881.744
21	Lutetia	0.085	0.941	0.244	7.793	692.551	295.897
22	Kalliope	0.113	1.642	6.392	315.607	171.884	98.980
24	Themis	0.135	1.459	0.592	110.140	457.939	793.104
28	Bellona	0.267	0.896	34.004	52.256	1521.361	1362.879
29	Amphitrite	0.216	5.020	43.100	337.808	252.462	1806.797
31	Euphrosyne	0.101	0.899	21.759	238.441	141.318	516.297
45	Eugenia	0.169	4.213	33.420	75.524	850.141	667.179
46	Hestia	0.652	13.456	21.361	588.879	3725.682	168.093
47	Aglaja	0.085	0.211	2.609	23.395	250.509	277.676
48	Doris	0.134	0.905	15.407	180.786	166.177	1191.363
49	Pales	0.083	1.194	0.484	65.283	55.044	315.793
52	Europa	0.153	0.666	21.136	112.815	824.430	911.623
65	Cybele	0.119	0.888	7.796	66.585	786.489	359.006
87	Sylvia	0.131	1.011	4.817	230.323	445.904	256.907
88	Thisbe	0.261	0.760	3.815	476.506	1825.573	703.813
90	Antiope	0.061	0.103	0.056	3.868	94.980	676.628
107	Camilla	0.154	0.265	14.943	65.902	140.830	170.971
111	Ate	2.328	15.164	198.173	6787.420	15143.916	10854.568
121	Hermione	0.064	0.781	1.315	48.973	144.406	429.713
130	Elektra	0.098	1.419	24.887	146.941	390.856	201.809
165	Loreley	0.361	1.349	41.093	1179.523	1587.762	431.086
189	Phthia	0.055	0.035	0.069	1.701	6.693	268.361
243	Ida	0.086	0.105	0.014	0.812	29.902	879.399
253	Mathilde	0.040	0.123	0.267	2.917	32.359	300.357
283	Emma	0.063	0.319	1.421	46.358	89.405	437.905

evaluation of asteroid 111 Ate is $0.84 \times 10^{-10} M_{\text{Sun}}$. This is very close to the evaluation of the mass of the ring above, and is accompanied by a 22% mass uncertainty (see Table 1). This will cause a severe degradation of the accuracy in the Mars positioning in particular. To clearly demonstrate this degradation, we show in Fig. 19 the difference of the EMB-to-Mars distance from two types of planetary ephemeris DE405 (Standish 1998) and INPOP08 (Fienga et al. 2008). In the same figure we represent also the difference in the EMB-to-Mars distance due to the uncertainty on the sole mass of 111 Ate.

We point out the astonishing correspondence between the two curves in phase, frequency and amplitude. This suggests that the differences in the values of masses for this asteroid between the two ephemerides are highly responsible for their mutual discrepancy. For comparison we show in Figs. 20 to 22 the effects on the EMB-Mars distance of the mass uncertainty of 46 Hestia, 19 Fortuna, 1 Ceres, 2 Pallas, and 4 Vesta (Appendix C). We note that for these three last asteroids, which are the biggest ones, the

effect is negligible, because of the very good mass accuracy. In contrast, for the first two the effect becomes clearly significant after a few years with an amplitude of roughly 1/8th and 1/16th that of 111 Ate (see Fig. 19) for a 100 y time interval, that is to say, 2 km and 1 km instead of 16 km.

In this context, our ASETEP database is an interesting and useful tool for understanding in a direct manner the limitations of modern ephemerides of terrestrial planets caused by the uncertainty on asteroid masses. It enables one to clearly indicate which asteroid would have to be observed preferentially in order to limitate its degradation on ephemeris accuracy. We limited our study to a small sample of 43 asteroids for which the mass is known with relatively good accuracy, i.e. with less than 65% uncertainty, as we explained in Sect. 2. But we can easily extend this study to a much larger set of asteroids. To some extent our work is similar to that of Fienga et al. (2009), who constructed the INPOP08 planetary ephemeris: they also analyzed the individual effects of very many asteroids one by one,

Fig. 15. Effects of Ceres on α_{Mars} for a 100y time interval from JD 2009.81. The curve in green corresponds to the post-fit residuals.

Fig. 16. Effects of Ceres on δ_{Mars} for a 100y time interval from JD 2009.81. The curve in green corresponds to a post-fit residuals.

Fig. 17. Peak-to-peak amplitude of the combined effects of the 43 asteroids on α_{Mars} . The curve in green represents the difference between two ways of computation: adding individual perturbations, or making a combined integration.

but without carrying out a Fourier and Poisson series decomposition of each signal as we did, and without comparing this decomposition with specific studies (Williams 1984). Indeed, the great advantage of such a decomposition is to understand, at a given period (or frequency) range, which asteroid is dominant both in terms of amplitude (related to the mass M) and amplitude uncertainty (related to $\delta M/M$).

Fig. 18. Zoom of the green curve of Fig. 17, showing the difference between two ways of computation.

Fig. 19. Uncertainty of the EMB-to-Mars distance due to the lack of accuracy in Ate's mass (red curve) and difference in the EMB-to-Mars distance between DE405 and INPOP08 (black curve).

Fig. 20. Uncertainty of the EMB-to-Mars distance due to the lack of accuracy in Hestia's mass (red curve) and difference in the EMB-to-Mars distance between DE405 and INPOP08 (black curve).

Another use of ASETEP is the straightforward listing of asteroidal effects. In Table 12, we show the effects of the asteroid mass uncertainty on fundamental parameters such as the determination of the distance from the EMB to Mars, Venus and Mercury. For each planet, the value of the uncertainty on the EMP-to-planet distance is given in decreasing order of amplitude. The most problematic asteroids for Mars are 111 Ate, with a dramatic 139.0 km uncertainty, followed by 46 Hestia

Fig. 21. Uncertainty of the EMB-to-Mars distance due to the lack of accuracy in Fortuna's mass (red curve) and difference in the EMB-to-Mars distance between DE405 and INPOP08 (black curve).

Fig. 22. Uncertainty of the EMB-to-Mars distance due to the lack of accuracy in Ceres' mass (red curve) and difference in the EMB-to-Mars distance between DE405 and INPOP08 (black curve).

(22.0 km), 20 Massilia (14.0 km), 19 Fortuna (12.1 km), and 3 Juno (10.5 km). In the case of Venus, 46 Hestia is dominant with a 7.2 km amplitude uncertainty, followed by 111 Ate (4.2 km), 3 Juno (1.4 km), 7 Iris (1.3 km) and 16 Psyche (1.2 km). The leading uncertainties for Mercury come from 111 Ate (4.1 km), 46 Hestia (3.8 km) and 165 Loreley (0.9 km). Additional information of ASETEP is the ratio of the amplitude of the uncertainty of Mars to that of Venus and Mercury, respectively. Because of the proximity of Mars to the asteroids this ratio would a priori be significantly higher by one or two orders. This is indeed what we can notice except for a few cases. The minimum and maximum values of the ratio are 0.15 (47 Aglaja) and 186.27 (19 Fortuna) respectively for Venus; they are 0.32 (47 Aglaja) and 224.54 (31 Euphrosyne) for Mercury.

In Fig. 23 we represent as a function of the mass of each of the 43 asteroids both the amplitude of its effect on the EMB-to-Mars distance and the amplitude of the uncertainty of determination of this effect. Clearly, if we exclude 111 Ate which offers by far the largest uncertainty, the following asteroids in decreasing amplitude of effects are not the largest ones but constitute a cluster with mass ranging between $3 \times 10^{-12} M_S$ and $3 \times 10^{-11} M_S$.

8. Conclusion and prospects

We have carried out detailed studies of the individual effects on the orbital parameters and geocentric positions of the terrestrial planets of a set of 43 of the most interesting asteroids,

Fig. 23. Peak-to-peak effects (black circles) and respective uncertainty (red circles) of the EMB-to-Mars distance for each asteroid mass.

selected for their large mass and the relatively good accuracy of their mass determination. This continues previous research of Williams (1984) and Mouret et al. (2009) in a more systematic and extended way. These authors had restricted their results solely on the planet Mars and only two orbital parameters, i.e. a_{Mars} and λ_{Mars} .

1. In contrast to these authors we performed a purely numerical study, based on the numerical integration of the n-body problem. To validate our method, we compared our results for the effects of Ceres, Pallas, Vesta and other leading asteroids on a_{Mars} and λ_{Mars} with Mouret et al. (2009). We characterized the signals by a set of Fourier and Poisson series, whose arguments and amplitudes fit the analytical tables of Mouret et al. (2009) very well, which were constructed as a Fourier series as a function of the mean longitudes of Mars and the asteroids. This both validates our methodology and confirms the analytical tables of Williams (1984) and Mouret et al. (2009).
2. We extended our investigations to the other parameters e_{Mars} , i_{Mars} , Ω_{Mars} , ω_{Mars} , and we summarized in a table the amplitude of the effect of each asteroid, and the uncertainty on the modelisation of this effect directly proportional to the relative mass uncertainty $\Delta M_{\text{ast.}}/M_{\text{ast.}}$.
3. Moreover, we studied the other terrestrial planets Mercury, Venus and the Earth (the Earth-Moon barycenter) in the same way, gathering our results in recapitulative tables.
4. We also evaluated the influences of each asteroid on the equatorial coordinates α , δ and the geocentric distance d of the three terrestrial planets Mercury, Venus and Mars, as well as the uncertainty on these parameters that are directly linked to the mass uncertainty of each asteroid. Our results will be compared, with those obtained recently by Kuchynka et al. (2010), in the case of common asteroids.
5. All results of our study are gathered in the ASETEP database (asteroid effects on the terrestrial planets), i.e. the tables of the decomposition of the signals with the leading components of the fit as a function of Fourier and Poisson series, and also the curves showing the asteroid effects and the residuals after subtraction of the fit. These tables are available only for a 1000 y time span. The ASETEP database can be accessed at <http://hpiers.obspm.fr/icrs-pc/>. As a first step of construction of ASETEP we restricted our study on

Table 12. Uncertainty of the distances Earth-Mercury, Earth-Venus, and Earth-Mars, due to the asteroids mass unaccuracy for a 1000 y time interval.

Mars		Venus			Mercury		
Asteroid	EMB-Mars	Asteroid	EMB-Venus	Ratio	Asteroid	EMB-Mercury	Ratio
111_ Ate	139044.10	46_ Hestia	7184.52	3.07	111_ Ate	4073.74	34.13
46_ Hestia	22055.13	111_ Ate	4188.84	33.19	46_ Hestia	3782.45	5.83
20_ Massalia	14016.59	3_ Juno	1399.89	7.55	165_ Loreley	916.20	0.86
19_ Fortuna	12090.17	7_ Iris	1334.75	0.85	3_ Juno	824.34	12.82
3_ Juno	10566.12	16_ Psyche	1181.23	2.55	16_ Psyche	718.58	4.19
24_ Themis	10531.30	165_ Loreley	931.12	0.84	48_ Doris	429.47	1.30
6_ Hebe	9948.96	88_ Thisbe	806.58	5.23	88_ Thisbe	428.74	9.85
29_ Amphitrite	7805.64	18_ Melpomene	770.59	1.14	24_ Themis	409.79	25.70
52_ Europa	5856.20	28_ Bellona	662.53	6.81	18_ Melpomene	391.27	2.24
28_ Bellona	4510.37	13_ Egeria	468.42	5.07	6_ Hebe	283.14	35.14
8_ Flora	4421.21	29_ Amphitrite	436.53	17.88	7_ Iris	249.36	4.53
88_ Thisbe	4221.34	6_ Hebe	432.03	23.03	19_ Fortuna	220.82	54.75
15_ Eunomia	4199.23	48_ Doris	335.68	1.66	189_ Phthia	214.23	0.83
14_ Irene	3522.35	20_ Massalia	333.67	42.01	45_ Eugenia	213.72	6.66
16_ Psyche	3008.18	2_ Pallas	288.46	4.93	29_ Amphitrite	193.07	40.43
9_ Metis	2772.05	52_ Europa	270.33	21.66	9_ Metis	182.47	15.19
4_ Vesta	2382.66	10_ Hygiea	235.19	1.22	14_ Irene	180.04	19.56
13_ Egeria	2376.39	14_ Irene	197.95	17.79	2_ Pallas	114.65	12.39
31_ Euphrosyne	1995.00	65_ Cybele	184.85	7.53	20_ Massalia	104.96	133.55
45_ Eugenia	1423.89	49_ Pales	156.28	0.65	52_ Europa	100.20	58.45
2_ Pallas	1420.99	4_ Vesta	138.08	17.26	8_ Flora	98.61	44.84
65_ Cybele	1391.56	47_ Aglaja	133.58	0.15	65_ Cybele	85.12	16.35
21_ Lutetia	1211.49	45_ Eugenia	129.20	11.02	28_ Bellona	81.86	55.10
7_ Iris	1128.66	8_ Flora	103.45	42.74	17_ Thetis	72.16	4.60
18_ Melpomene	875.10	1_ Ceres	98.58	4.94	10_ Hygiea	70.72	4.05
165_ Loreley	785.95	31_ Euphrosyne	95.83	20.82	13_ Egeria	63.56	37.39
48_ Doris	557.76	189_ Phthia	74.72	2.38	47_ Aglaja	61.06	0.32
1_ Ceres	486.84	19_ Fortuna	64.90	186.27	15_ Eunomia	55.59	75.54
130_ Elektra	402.76	24_ Themis	59.06	178.32	4_ Vesta	46.54	51.19
17_ Thetis	332.10	9_ Metis	56.95	48.67	49_ Pales	46.45	2.19
10_ Hygiea	286.44	243_ Ida	54.77	0.29	243_ Ida	46.23	0.34
189_ Phthia	177.82	130_ Elektra	44.75	9.00	130_ Elektra	16.80	23.98
121_ Hermione	174.91	15_ Eunomia	36.12	116.25	21_ Lutetia	14.81	81.82
11_ Parthenope	111.49	21_ Lutetia	25.10	48.27	121_ Hermione	14.04	12.45
22_ Kalliope	107.91	22_ Kalliope	19.96	5.41	253_ Mathilde	12.39	1.14
49_ Pales	101.64	11_ Parthenope	16.64	6.70	283_ Emma	10.60	0.49
47_ Aglaja	19.69	253_ Mathilde	15.27	0.93	31_ Euphrosyne	8.88	224.54
243_ Ida	15.80	17_ Thetis	15.20	21.85	11_ Parthenope	7.98	13.98
253_ Mathilde	14.17	283_ Emma	14.15	0.37	107_ Camilla	7.17	1.93
107_ Camilla	13.81	107_ Camilla	13.15	1.05	22_ Kalliope	6.90	15.64
90_ Antiope	13.21	121_ Hermione	11.46	15.26	90_ Antiope	2.88	4.58
87_ Sylvia	7.56	90_ Antiope	7.92	1.67	1_ Ceres	2.43	200.25
283_ Emma	5.21	87_ Sylvia	4.36	1.73	87_ Sylvia	1.40	5.38

Notes. The third column for Mercury and Venus indicates the ratio of the effect with respect to the corresponding effect on Mars.

a limited set of 43 asteroids. But one of the advantages of this database is that it is generated automatically and can be updated each time a new and more accurate determination of the masses of selected asteroids is made or when a new asteroid is included. The ASETEP database enables one to characterize the influence of each asteroid on the orbital motion of the terrestrial planets in an accurate and detailed manner, and will guide the efforts toward improvements of the ephemeris. The case of 111 Ate is particularly significant, because we were able to show that the differences given by two types of ephemerides (DE405 and IPOP08) on the EMB-Mars distance is remarkably similar to the effect caused by the mass uncertainty on this asteroid, which suggests the choice of two different masses for 111 Ate in the ephemeris given above.

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Appendix A: Effects on the Mars orbital motion

Fig. A.1. Effects of Ceres on e_{Mars} for a 100 y time interval from JJ 2009.81; raw signal (in black), fitting curve (in red) and residuals (in green).

Fig. A.2. Effects of Ceres on i_{Mars} for a 100 y time interval from JJ 2009.81; raw signal (in black), fitting curve (in red) and residuals (in green).

Fig. A.3. Effects of Ceres on Ω_{Mars} for a 100 y time interval from JJ 2009.81; raw signal (in black), fitting curve (in red) and residuals (in green).

Fig. A.4. Effects of Ceres on ω_{Mars} for a 100 y time interval from JJ 2009.81; raw signal (in black), fitting curve (in red) and residuals (in green).

Table A.1. Coefficients of Fourier and Poisson series for the effects of Ceres on e_{Mars} .

period (Day)	period (Year)	SIN $\times 10^{-10}$	COS $\times 10^{-10}$	$\times 10^{-10}$	T SIN $\times 10^{-14}$	T COS $\times 10^{-14}$	$\times 10^{-14}$	Amplitud $\times 10^{-8}$	Asteroid
16800.43	46.00	71.042	-57.060	91.120	-336.820	219.769	310.801	0.145	1_ Ceres
4862.41	13.31	0.650	4.085	4.137	1.708	-18.545	26.227	0.064	1_ Ceres
3752.63	10.27	-5.746	-12.352	13.623	-14.963	26.378	37.305	0.127	1_ Ceres
3046.97	8.34	2.753	-1.422	3.099	8.535	4.919	6.957	0.046	1_ Ceres
1875.99	5.14	-4.305	-4.983	6.585	11.563	32.195	45.530	0.028	1_ Ceres
1682.02	4.61	-4.362	-1.963	4.783	26.924	7.169	10.138	0.013	1_ Ceres
1250.69	3.42	-1.101	7.132	7.217	6.247	-31.438	44.460	0.009	1_ Ceres
1083.67	2.97	1.155	-0.487	1.254	-7.228	0.406	0.574	0.007	1_ Ceres
886.91	2.43	-3.654	0.482	3.686	22.982	4.080	5.769	0.018	1_ Ceres
840.95	2.30	-0.706	-0.207	0.736	0.831	-0.538	0.761	0.007	1_ Ceres
717.34	1.96	1.234	1.552	1.983	-2.558	-8.717	12.328	0.009	1_ Ceres
687.04	1.88	0.063	-0.479	0.483	-3.217	2.391	3.381	0.006	1_ Ceres
502.91	1.38	1.721	-0.327	1.752	-8.277	-1.340	1.894	0.006	1_ Ceres
DISPERSION		Before = 0.437036×10^{-8}			After = 0.449648×10^{-9}				

Notes. Bias: -0.147732×10^{-5} ; linear: 0.767680×10^{-9} ; T^2 : $-0.246106 \times 10^{-13}$. The next-to-last column indicates the peak-to-peak amplitude of the total mixed component for the 100 y time interval.

Table A.2. Coefficients of Fourier and Poisson series for the effects of Ceres on i_{Mars} .

Period (Day)	Period (Year)	SIN $\times 10^{-5}$	COS $\times 10^{-5}$	$\times 10^{-5}$	T SIN $\times 10^{-9}$	T COS $\times 10^{-9}$	$\times 10^{-9}$	Amplitud $\times 10^{-4}$	Asteroid
16704.15	45.73	-30.404	-41.976	51.830	128.455	201.120	284.427	0.354	1_ Ceres
4871.57	13.34	-3.481	-0.198	3.486	16.174	2.520	3.564	0.081	1_ Ceres
3745.29	10.25	2.366	2.832	3.691	-6.410	-15.360	21.722	0.143	1_ Ceres
3043.72	8.33	0.387	0.930	1.007	-1.532	-9.749	13.788	0.111	1_ Ceres
1878.41	5.14	0.518	0.763	0.922	-1.876	-5.321	7.525	0.049	1_ Ceres
1683.67	4.61	1.111	0.683	1.304	-7.355	-4.239	5.995	0.052	1_ Ceres
1251.11	3.43	-1.761	0.037	1.762	8.127	0.333	0.471	0.026	1_ Ceres
1161.41	3.18	1.597	0.156	1.604	-5.001	-0.533	0.754	0.067	1_ Ceres
1083.47	2.97	0.253	0.348	0.430	-2.531	-1.831	2.590	0.027	1_ Ceres
887.20	2.43	0.772	-1.393	1.593	-2.531	7.513	10.625	0.043	1_ Ceres
840.94	2.30	-0.626	0.521	0.814	0.989	-3.728	5.273	0.050	1_ Ceres
687.10	1.88	-0.399	0.843	0.932	1.609	-5.043	7.132	0.024	1_ Ceres
580.74	1.59	-0.469	-0.210	0.514	0.789	0.457	0.647	0.038	1_ Ceres
560.56	1.53	-0.124	-0.154	0.198	1.263	1.183	1.673	0.017	1_ Ceres
503.00	1.38	-0.861	0.430	0.963	3.812	-2.672	3.779	0.020	1_ Ceres
487.77	1.34	0.452	-0.048	0.454	-1.446	1.033	1.462	0.023	1_ Ceres
387.16	1.06	-0.661	-0.235	0.701	3.708	1.657	2.344	0.017	1_ Ceres
AMPLITUDE		Before = 0.000024			After = 0.237359×10^{-4}				

Notes. Bias: 0.258949×10^{-1} ; linear: -0.251401×10^{-4} ; T^2 : 0.600592×10^{-8} . The next-to-last column indicates the peak-to-peak amplitude of the total mixed component for the 100 y time interval.

Table A.3. Coefficients of Fourier and Poisson series for the effects of Ceres on Ω_{Mars} .

Period (Day)	Period (Year)	SIN $\times 10^{-4}$	COS $\times 10^{-4}$	$\times 10^{-4}$	T SIN $\times 10^{-7}$	T COS $\times 10^{-7}$	$\times 10^{-7}$	Amplitud $\times 10^{-2}$	Asteroid
17237.30	47.19	-254.868	163.129	302.603	120.532	-84.520	119.529	0.126	1_Ceres
10945.54	29.97	3.333	13.201	13.616	-1.550	-6.445	9.114	0.116	1_Ceres
7586.87	20.77	-3.312	-4.891	5.907	1.250	2.352	3.326	0.054	1_Ceres
4921.78	13.48	7.651	-0.338	7.658	-3.232	0.853	1.207	0.036	1_Ceres
3754.45	10.28	1.482	-1.680	2.241	2.192	0.170	0.241	0.078	1_Ceres
3045.26	8.34	0.149	5.993	5.995	-2.205	-4.246	6.005	0.056	1_Ceres
1877.72	5.14	-6.977	1.328	7.102	4.562	-0.518	0.733	0.030	1_Ceres
1681.32	4.60	-0.203	-3.210	3.216	-1.143	1.428	2.019	0.029	1_Ceres
1521.01	4.16	3.786	-0.861	3.882	-1.957	0.150	0.212	0.006	1_Ceres
1252.35	3.43	-0.665	0.300	0.729	0.284	0.253	0.357	0.014	1_Ceres
1161.48	3.18	-2.464	1.778	3.038	0.017	-1.536	2.172	0.031	1_Ceres
1083.21	2.97	-1.423	-0.137	1.429	1.120	0.322	0.456	0.010	1_Ceres
887.02	2.43	5.332	-2.557	5.913	-3.223	1.151	1.628	0.018	1_Ceres
840.68	2.30	-0.150	-0.303	0.338	0.904	0.412	0.582	0.019	1_Ceres
717.62	1.96	3.271	1.048	3.435	-1.827	-0.739	1.045	0.009	1_Ceres
686.93	1.88	0.479	-5.096	5.119	0.187	2.734	3.866	0.012	1_Ceres
580.74	1.59	-3.886	-1.628	4.214	2.118	1.406	1.989	0.016	1_Ceres
560.52	1.53	0.764	1.457	1.645	-0.390	-0.971	1.374	0.006	1_Ceres
502.94	1.38	-1.693	-1.005	1.969	1.027	0.679	0.960	0.007	1_Ceres
487.69	1.34	-2.055	1.376	2.473	0.739	-0.930	1.316	0.008	1_Ceres
431.64	1.18	2.216	3.012	3.739	-1.231	-1.633	2.309	0.006	1_Ceres
387.16	1.06	0.068	3.096	3.097	-0.005	-1.781	2.519	0.006	1_Ceres
343.47	0.94	-1.495	-3.287	3.611	0.833	1.882	2.661	0.008	1_Ceres
AMPLITUDE		Before = 0.005407			After = 0.000974				

Notes. Bias: -0.811114 ; linear: 0.427426×10^{-3} ; T^2 : -0.121763×10^{-7} . The next-to-last column indicates the peak-to-peak amplitude of the total mixed component for the 100 y time interval.

Table A.4. Coefficients of Fourier and Poisson series for the effects of Ceres on $\tilde{\omega}_{\text{Mars}}$.

Period (Day)	Period (Year)	SIN $\times 10^{-4}$	COS $\times 10^{-4}$	$\times 10^{-4}$	T SIN $\times 10^{-7}$	T COS $\times 10^{-7}$	$\times 10^{-7}$	Amplitud $\times 10^{-2}$	Asteroid
17112.22	46.85	-156.276	611.164	630.828	70.223	-290.043	410.183	0.311	1_Ceres
10842.14	29.68	276.449	3.355	276.470	-134.342	-2.937	4.153	0.378	1_Ceres
4924.23	13.48	0.168	-17.541	17.542	0.160	9.637	13.629	0.115	1_Ceres
3755.58	10.28	25.200	-2.573	25.331	-0.378	2.019	2.855	0.273	1_Ceres
3046.85	8.34	-7.000	-5.383	8.831	2.778	-2.097	2.965	0.099	1_Ceres
1877.78	5.14	-2.572	-2.221	3.398	3.756	1.278	1.808	0.060	1_Ceres
1681.96	4.60	-2.301	12.463	12.674	0.668	-7.302	10.327	0.031	1_Ceres
1251.92	3.43	-5.802	-7.270	9.301	3.454	3.746	5.298	0.014	1_Ceres
1083.69	2.97	6.023	-8.188	10.165	-3.391	4.274	6.044	0.012	1_Ceres
887.10	2.43	6.554	1.009	6.632	-4.692	-1.180	1.669	0.039	1_Ceres
840.86	2.30	0.293	0.606	0.673	-0.137	0.308	0.435	0.012	1_Ceres
717.65	1.96	3.002	2.539	3.931	-1.857	-1.823	2.578	0.017	1_Ceres
687.08	1.88	4.640	3.120	5.591	-2.526	-1.043	1.474	0.012	1_Ceres
502.96	1.38	0.513	-1.741	1.815	0.007	1.204	1.703	0.010	1_Ceres
AMPLITUDE		Before = 0.009966			After = 0.000944				

Notes. Bias: $-0.605169 \times 10^{+2}$; linear: -0.121834×10^{-1} ; T^2 : 0.156335×10^{-5} . The next-to-last column indicates the peak-to-peak amplitude of the total mixed component for the 100 y time interval.

Table A.5. Coefficients of Fourier and Poisson series for the effects of Pallas on a_{Mars} .

Period (Day)	Period (Year)	SIN $\times 10^0$	COS $\times 10^0$	$\times 10^0$	T SIN $\times 10^{-3}$	T COS $\times 10^{-3}$	$\times 10^{-3}$	Amplitud $\times 10^1$	Asteroid
21839.28	59.79	-10.566	1.282	10.644	3.470	-0.149	0.210	1.031	2_ Pallas
17918.54	49.06	-107.297	3.679	107.360	16.781	-2.048	2.896	7.353	2_ Pallas
9865.09	27.01	-9.001	4.336	9.991	2.672	-0.949	1.342	0.446	2_ Pallas
8955.04	24.52	12.925	19.794	23.640	-3.318	-3.703	5.236	1.412	2_ Pallas
6364.17	17.42	5.432	9.561	10.996	-0.971	-1.935	2.736	0.621	2_ Pallas
5974.81	16.36	4.910	-2.340	5.439	-0.825	1.045	1.478	0.290	2_ Pallas
4695.27	12.85	13.748	-17.279	22.081	-2.510	2.891	4.088	1.480	2_ Pallas
3720.02	10.18	-3.842	-4.707	6.075	-0.432	0.013	0.018	0.680	2_ Pallas
3080.06	8.43	-2.749	-5.035	5.736	0.396	-0.555	0.785	0.699	2_ Pallas
2628.38	7.20	-8.639	-1.328	8.740	1.655	0.137	0.194	0.560	2_ Pallas
2075.66	5.68	1.660	7.042	7.236	-0.231	-1.369	1.937	0.488	2_ Pallas
1860.14	5.09	15.833	-9.731	18.584	-2.247	1.721	2.434	1.301	2_ Pallas
1540.23	4.22	3.956	-4.351	5.880	-0.578	0.830	1.173	0.385	2_ Pallas
1240.07	3.40	0.060	9.006	9.006	-0.144	-1.609	2.275	0.579	2_ Pallas
930.06	2.55	-3.330	-2.179	3.980	0.558	0.278	0.393	0.298	2_ Pallas
884.18	2.42	-8.246	9.304	12.432	0.721	-1.233	1.744	0.966	2_ Pallas
714.39	1.96	-2.652	-6.204	6.747	0.530	0.901	1.274	0.464	2_ Pallas
579.91	1.59	1.465	-5.380	5.576	0.679	-0.289	0.408	0.718	2_ Pallas
501.71	1.37	3.219	3.088	4.461	-0.590	-0.005	0.008	0.368	2_ Pallas
442.08	1.21	-2.850	0.973	3.011	0.416	-0.174	0.246	0.215	2_ Pallas
408.23	1.12	-10.042	36.276	37.640	4.874	-18.183	25.715	1.902	2_ Pallas
386.59	1.06	-0.348	0.504	0.613	-0.644	-0.204	0.288	0.230	2_ Pallas
272.16	0.75	-15.594	-2.638	15.815	7.718	1.194	1.688	0.770	2_ Pallas
AMPLITUDE		Before = 214.335797			After = 72.316238				

Notes. Bias: $0.398536 \times 10^{+4}$; linear: $-0.326098 \times 10^{+1}$; T^2 : 0.651096×10^{-3} . The next-to-last column indicates the peak-to-peak amplitude of the total mixed component for the 1000 y time interval considered.

Table A.6. Coefficients of Fourier and Poisson series for the effects of Vesta on a_{mars} .

Period (Day)	Period (Year)	SIN $\times 10^0$	COS $\times 10^0$	$\times 10^0$	T SIN $\times 10^{-3}$	T COS $\times 10^{-3}$	$\times 10^{-3}$	Amplitud $\times 10^2$	Asteroid
18867.98	51.66	83.834	-107.509	136.331	-3.213	4.449	6.291	1.267	4_ Vesta
9443.39	25.85	17.064	-41.235	44.626	-0.487	2.848	4.027	0.389	4_ Vesta
6296.23	17.24	3.447	-4.872	5.968	1.009	-0.857	1.211	0.100	4_ Vesta
1679.56	4.60	0.997	12.655	12.695	0.599	-1.360	1.923	0.105	4_ Vesta
1542.42	4.22	0.669	28.454	28.462	-0.482	-1.519	2.149	0.255	4_ Vesta
1425.93	3.90	-1.302	10.131	10.215	-0.384	0.443	0.627	0.115	4_ Vesta
740.95	2.03	-0.607	13.687	13.700	-0.711	-0.238	0.337	0.140	4_ Vesta
712.97	1.95	-8.158	30.122	31.208	-1.160	-0.688	0.972	0.307	4_ Vesta
487.59	1.33	-1.036	10.484	10.535	-1.058	-1.360	1.924	0.083	4_ Vesta
475.32	1.30	-5.351	15.041	15.964	-1.000	-0.548	0.775	0.158	4_ Vesta
408.23	1.12	30.965	-107.157	111.541	-14.906	53.474	75.624	0.555	4_ Vesta
356.49	0.98	-2.836	7.467	7.988	-1.067	-0.451	0.638	0.086	4_ Vesta
272.16	0.75	40.158	30.629	50.506	-19.865	-14.742	20.848	0.240	4_ Vesta
AMPLITUDE		Before = 545.913598			After = 146.660553				

Notes. Bias: $0.152498 \times 10^{+3}$; linear: -0.209840×10^{-1} ; T^2 : 0.445801×10^{-5} . The next-to-last column indicates the peak-to-peak amplitude of the total mixed component for the 1000 y time interval considered.

Appendix B: individual and combined asteroidal effects

Fig. B.1. Combined effects of the 43 asteroids on e_{Mars} . The curve in green represents the difference between two ways of computation: adding individual perturbations, or making a combined integration.

Fig. B.2. Zoom of the green curve of Fig. B.1, showing the difference between two ways of computation.

Fig. B.3. Combined effects of the 43 asteroids on i_{Mars} . The curve in green represents the difference between two ways of computation: adding individual perturbations, or making a combined integration.

Fig. B.4. Zoom of the green curve of Fig. B.3, showing the difference between two ways of computation.

Fig. B.5. Combined effects of the 43 asteroids on Ω_{Mars} . The curve in green represents the difference between two ways of computation: adding individual perturbations, or making a combined integration.

Fig. B.6. Zoom of the green curve of Fig. B.5, showing the difference between two ways of computation.

Fig. B.7. Combined effects of the 43 asteroids on $\tilde{\omega}_{\text{Mars}}$. The curve in green represents the difference between two ways of computation: adding individual perturbations, or making a combined integration.

Fig. B.8. Zoom of the green curve of Fig. B.7, showing the difference between two ways of computation.

Fig. B.9. Combined effects of the 43 asteroids on λ_{Mars} . The curve in green represents the difference between two ways of computation: adding individual perturbations, or making a combined integration.

Fig. B.10. Zoom of the green curve of Fig. B.9, showing the difference between two ways of computation.

Appendix C: Use of ASTEP for ephemerides study

Fig. C.1. Uncertainty of the EMB-to-Mars distance due to the lack of accuracy in Pallas' mass (red curve) and difference in the EMB-to-Mars distance between DE405 and INPOP08 (black curve).

Fig. C.2. Uncertainty of the EMB-to-Mars distance due to the lack of accuracy in Vesta's mass (red curve) and difference in the EMB-to-Mars distance between DE405 and INPOP08 (black curve).