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On the physical meaning of *n*-distributions in solar flares

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ABSTRACT

Aims. We investigate the physical meaning of the n-distributions detected in solar flares.

Methods. We consider a Maxwellian velocity distribution with a velocity drift. This distribution is analytically integrated to obtain the energy distribution, and its stability is investigated numerically using a fully electromagnetic particle-in-cell code.

Results. It is shown that the derived moving Maxwellian energy distribution is very similar to the n-distribution, especially in their high-energy parts. Both these distributions are mutually fitted and a relation between their parameters found. Contrary to the n-distribution, the moving Maxwellian distribution has a simple physical meaning, e.g., the electron component of the return current in the beam-plasma system. However, for high drift velocities of such a component, the moving Maxwellian distribution is unstable. Therefore to keep the form of this distribution similar to the n-distribution, some stabilization processes are necessary. If so, then the high intensities of the Si XIId 5.56 Å and 5.82 Å satellite lines and their evolution in solar flares can be explained by moving Maxwellian distributions instead of the n-distributions. Thus, our previous results connected with the n-distributions can be understood in a new, physically profound way.

Key words. Sun: flares – Sun: X-rays, gamma rays

1. Introduction

It is well known that during solar flares, especially during their impulsive phases, electrons are strongly accelerated (e.g., Priest 1982; Tandberg-Hanssen & Emslie 1998; Somov 2007). The suprathermal electrons propagate as electron beams out of their acceleration regions, and their distributions evolve in both space and time thanks to the ballistic mode and various wave-particle and particle-particle interactions (Tsytovich 1970; Melrose 1980). Thus, electron distributions in some regions of solar flares are non-Maxwellian.

There are many observations which confirm the presence of the non-Maxwellian electron distributions in solar flares. For example, the radio bursts observed during solar flares are radio signatures of these distributions. In their models in decimetric wavelength range, where the plasma emission processes are dominant, the most appropriate electron distributions are the bump-on-tail or loss-cone distributions (Krüger 1979; Karlický 1997; Aschwanden 2002; Chernov 2006). On the other hand, in the microwave range, where the gyro-synchrotron or synchrotron emission mechanisms dominate, the power-law distributions are used (Dulk 1985). Furthermore, in the hard X-ray range the observed spectra (e.g. by RHESSI) are interpreted using the power-law, double power-law or even κ-distributions (Brown 1971; Holman et al. 2003; Kontar et al. 2004; Brown et al. 2008; Krucker & Lin 2008; Krucker et al. 2008; Kašparová & Karlický 2009; Asai et al. 2009; Warmuth et al. 2009; Veronig et al. 2010; Kurt et al. 2010; Zharkova et al. 2010; Guo et al. 2011). Acceleration and propagation of particles with energies of tens of keV or higher have been extensively studied (e.g., Zharkova & Gordovskyy 2005, 2006; Petkaki & MacKinnon 2007, 2011; Siversky & Zharkova 2009; Godrovskyy et al. 2010a,b; Browning et al. 2010). A review is provided by Zharkova et al. (2011).

Apart from the existence of high-energy tails during flares, the flare line spectra of He-like ions and associated dielectronic satellites show considerable departures of the particle distribution from the Maxwellian one at energies of few keV (Seely et al. 1987). Dzifčáková et al. (2008) and Kulinová et al. (2011) concluded that the observed Si XIId/Si XIII line ratio can be explained by the presence of particle distribution decreasing much more steeply in the range of few keV than the Maxwellian distribution. These authors were able to explain the observations using an analytical n-distribution of the form

$$f_n(E)\mathrm{d}E = B_n \frac{2E^{1/2}}{(k_\mathrm{B}T)^{3/2}\sqrt{\pi}}\left(\frac{E}{k_\mathrm{B}T}\right)^{\frac{n-1}{2}} \exp\left[-\frac{E}{k_\mathrm{B}T}\right]\mathrm{d}E, \quad (1)$$

$$B_n = \frac{\sqrt{\pi}}{2\Gamma(n/2+1)}, \quad (2)$$

where E is the electron energy, k_B the Boltzmann constant, and $n \in \langle 1, \infty)$ the parameter of the distribution. For $n = 1$, the distribution is Maxwellian. The mean energy of the n-distribution depends on n

$$\langle E \rangle = \frac{3}{2}k_\mathrm{B}\tau = \left(\frac{n}{2}+1\right)k_\mathrm{B}T, \quad (3)$$

where τ is the pseudo-temperature and T is just a parameter of the distribution. This type of distribution was also considered in the laboratory experiment of laser-irradiated targets (Hares et al. 1979).

Dzifčáková et al. (2008) have studied the M4.9 flare on 2003 January 7/8 and found an increase of the parameter n during the impulsive phase of this flare. Furthermore, analyzing the RESIK and RHESSI spectra of three solar flares, Kulinová et al. (2011) have recognized a presence of the n-distributions with the parameter n up to $n = 11$. These n-distributions were detected at times of observation of type III radio bursts suggesting possible connection between the appearance of the n-distributions and the presence of electron beams.

Dzifčáková & Karlický (2008) studied the influence of the electron beam and return current on the distribution of particle energies. They were able to show that the resulting energy distribution has a narrower peak than the Maxwellian one and could thus explain the observed intensities of the satellite lines Si XIId 5.56 Å and 5.82 Å.

However, there still remains a question what is the physical meaning of these n-distributions and how these distributions are formed in solar flares. Answering this question about the detection of the n-distributions can give us further information about processes in solar flares. In the paper by Dzifčáková & Karlický (2008), where a simple model of the return current in the beam-plasma system was presented, it was proposed that the n-distribution corresponds to the Maxwellian distribution with a velocity drift (for short: moving Maxwellian distribution). We analyze this idea in detail.

2. Energy distribution of electrons with moving Maxwellian distribution

We begin by taking the moving Maxwellian distribution function in the velocity space as

$$f(v_x, v_y, v_z) = C \exp\left[-\frac{(v_x - v_0)^2 + v_y^2 + v_z^2}{v_T^2}\right], \qquad (4)$$

where v_x, v_y, and v_z are the electron velocities in the x-, y-, and z-coordinates, v_0 is the velocity shift, v_T the thermal velocity, and C the normalization constant. We integrate this function along the surface with the constant value of $v_x^2 + v_y^2 + v_z^2 \sim E$ using the relation (Fichtengolc 1969)

$$\iint_S f(v_x, v_y, v_z)\, dS = \iint_M f(v_x, v_y, \varphi(v_x, v_y)) \times \sqrt{1 + \left(\frac{\partial \varphi}{\partial v_x}\right)^2 + \left(\frac{\partial \varphi}{\partial v_y}\right)^2}\, dv_x dv_y,$$

where S is, in our case, the surface of the sphere having its centrum in the coordinate null point of the velocity space and radius $\sim\sqrt{E}$, and M is the area of the projection of this sphere in the $v_x - v_y$ plane.

Then the energy distribution function $f(E, v_0)$, where E is the electron energy, can be written as

$$f(E, v_0)dE = C_1 \frac{\sinh\left[2\frac{v_0}{v_T}\sqrt{\frac{E}{k_B T}}\right]}{\frac{v_0}{v_T}} \exp\left[-\frac{E}{k_B T}\right] dE, \qquad (5)$$

$$C_1 = \frac{1}{\sqrt{\pi} k_B T \exp\left[(\frac{v_0}{v_T})^2\right]}, \qquad (6)$$

where T is the electron temperature. For $v_0 = 0$ this distribution is Maxwellian. As can be seen, this distribution is symmetric for v_0 and $-v_0$. The mean energy of this distribution can be expressed as

$$\langle E \rangle = \frac{3}{2} k_B T + \frac{1}{2} m_e v_0^2 = \frac{3}{2} k_B \tau, \qquad (7)$$

where m_e is the electron mass, T the plasma temperature in the frame moving with the velocity v_0, and τ the pseudo-temperature corresponding to the total energy.

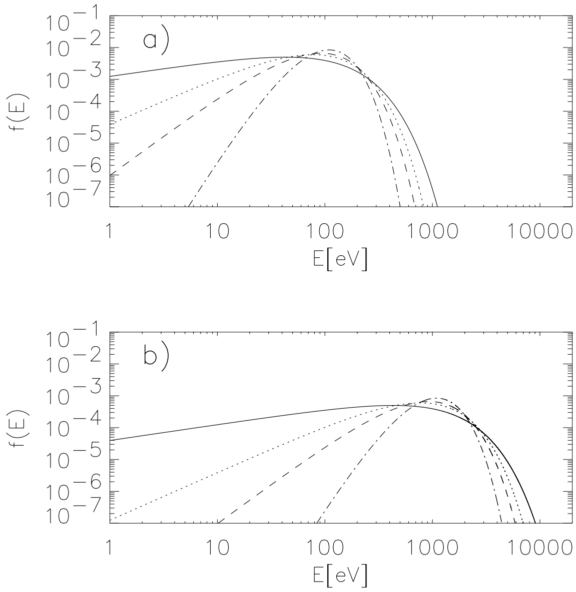

Fig. 1. The n-distribution functions for $\tau = 10^6$ K **a)** and $\tau = 10^7$ K **b)**. The full line means the n-distribution for $n = 1$ (Maxwellian), dotted line for $n = 3$, dashed line for $n = 5$, and dash-dotted line for $n = 11$.

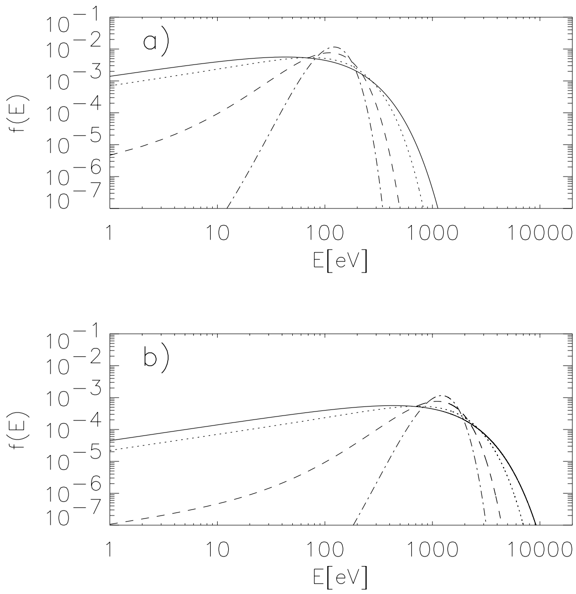

Fig. 2. Moving Maxwellian distributions for $\tau = 10^6$ K **a)** and $\tau = 10^7$ K **b)**. The full line means the distribution for $v_0/v_T = 0$ (Maxwellian with no velocity shift), dotted line for $v_0/v_T = 1.4$, dashed line for $v_0/v_T = 3$, and dash-dotted line for $v_0/v_T = 5$.

3. Comparison of the *n*-distribution with moving Maxwellian one

In Fig. 1 we present the n-distributions with two different energies, expressed by $\tau = 10^6$ K (a) and $\tau = 10^7$ K (b), for the n parameter $n = 1$ (Maxwellian), $n = 3$, $n = 5$, and $n = 11$. For comparison Fig. 2 shows the moving Maxwellian distributions for two different total energies (thermal plus kinetic energies) corresponding to $\tau = 10^6$ K (a) and $\tau = 10^7$ K (b). As can be seen here, both moving Maxwellian and n-distributions are similar. More detailed comparison of both distributions is presented in Fig. 3. The upper part (a) shows a comparison of the moving Maxwellian distribution ($T = 10^6$ K and $v_0/v_T = 1.4$) with the Maxwellian (without the velocity shift), but with the

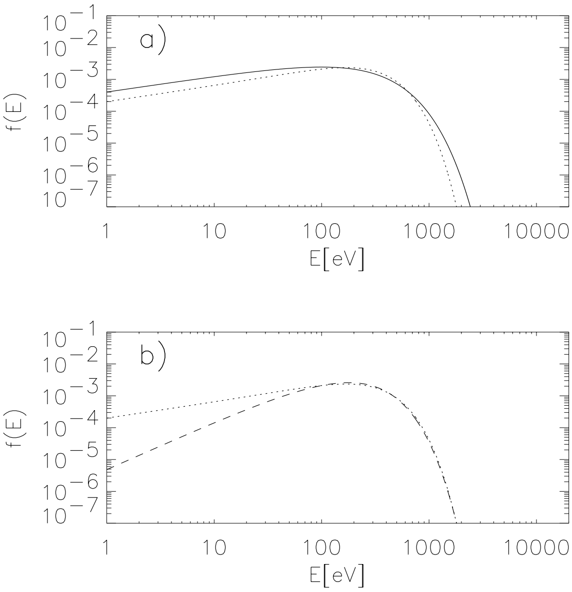

Fig. 3. **a)** Moving Maxwellian distribution for $T = 10^6$ K and for $v_0/v_\mathrm{T} = 1.4$ (dotted line) and the Maxwellian distribution for $v_0/v_\mathrm{T} = 0$ with the same energy (full line). **b)** Moving Maxwellian distribution for $T = 10^6$ K and for $v_0/v_\mathrm{T} = 1.4$ (dotted line) and the n-distribution with the same energy, for $n = 3$.

same energy. On the other hand, part (b) shows the moving Maxwellian distribution – the same as in the part (a) with the n-distribution with the same energy and for $n = 3$. The n-distribution and the moving Maxwellian one are nearly the same in the energy range above their maxima, and they differ in their low-energy parts. Therefore, in the high-energy part of distributions, moving Maxwellian distribution can fully replace the n-distribution and be used for spectroscopic purposes instead of the n-distribution.

We numerically fitted the n-distributions in their high-energy parts by moving Maxwellian ones for the parameters $n = 3$–27. For the same energy of both the distributions we find that the parameters of fitted distributions n and v_0/v_T are related, as shown in Fig. 4. Analysis of this relation shows that a form of the high-energy parts of both these distributions (relations 1 and 5) is mainly given by the function $\exp(-E/k_\mathrm{B}T)$. However, for the distributions with the same energy, the variables $k_\mathrm{B}T$ in both the distributions are related as

$$(k_\mathrm{B}T)_n = C_2(k_\mathrm{B}T)_\mathrm{M}, \quad (8)$$

$$C_2 = \frac{1 + \frac{2}{3}(\frac{v_0}{v_\mathrm{T}})^2}{\frac{n}{2} + 1}, \quad (9)$$

where $(k_\mathrm{B}T)_\mathrm{M}$ and $(k_\mathrm{B}T)_n$ are for moving Maxwellian and n-distributions, respectively. The distributions (1 and 5) become similar in their high-energy parts, if $(k_\mathrm{B}T)_\mathrm{M} = (k_\mathrm{B}T)_n = k_\mathrm{B}T$, i.e. $C_2 \sim 1$, which gives a relation between v_0/v_T and n as

$$\frac{v_0}{v_\mathrm{T}} = \sqrt{\frac{3}{4}n}. \quad (10)$$

This relation (Fig. 4) is in good agreement with the one obtained numerically. According to these relations, the n-distribution in its high-energy part can be replaced by a moving Maxwellian one.

We now focus on the role of the of moving Maxwellian distributions in flare spectroscopy. Figure 5 shows moving

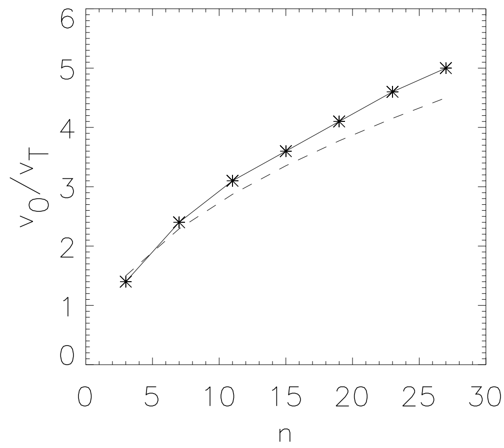

Fig. 4. The relation between parameters n and v_0/v_T derived numerically (asterisks) and according to the relation (10) (dashed line).

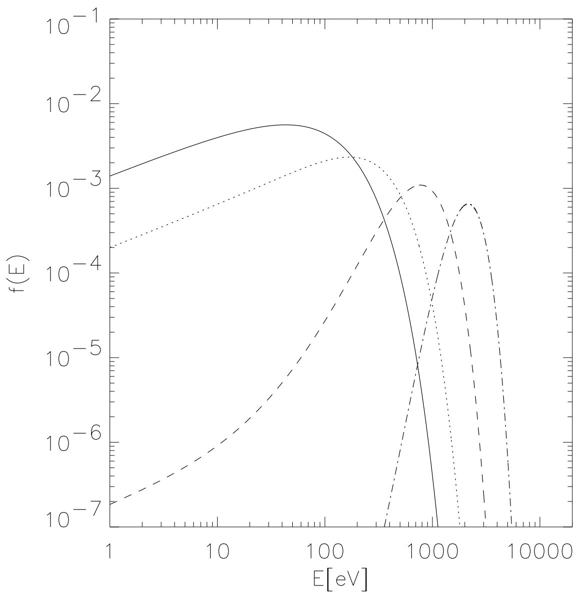

Fig. 5. Moving Maxwellian distributions for $T = 10^6$ K. The full line means the distribution for $v_0/v_\mathrm{T} = 0$, dotted line for $v_0/v_\mathrm{T} = 1.4$, dashed line for $v_0/v_\mathrm{T} = 3$, and dash-dotted line for $v_0/v_\mathrm{T} = 5$.

Maxwellian distributions with the same temperature $T = 10^6$ K, but with the different v_0/v_T. As seen here, increasing the v_0/v_T makes distribution narrower, corresponding to an increase in the n parameter in the equivalent n-distribution. However, both the maximum and steep high-energy part of this distribution move to higher energies. If we want to generate the distribution with an increasing steepness of its high-energy part in some specific energy interval, as requested for specific spectral lines (in our case in 1–4 keV for the Si XIId 5.56 Å and 5.82 Å satellite lines), then we need to increase the velocity shift v_0/v_T, but simultaneously decrease the plasma temperature, as shown in Fig. 6.

4. Stability of moving Maxwellian distribution

It is well known that the the Maxwellian distribution of electrons shifted in velocity space (moving Maxwellian), together with the background plasma (the electron beam – plasma system) or with static protons (current or return current), is unstable due to the two-stream (bump-on-tail) or Buneman instabilities (Mikhailovskii 1975). Although the role of slow electron beams cannot be excluded, the most relevant model of generation of

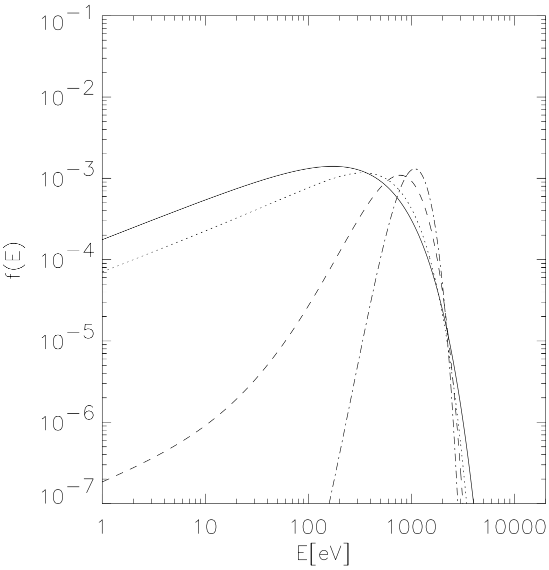

Fig. 6. Moving Maxwellian distributions showing a change of their derivatives in the interval in 1–4 keV. The full line means the distribution for the temperature $T = 4 \times 10^6$ K and $v_0/v_T = 0$, dotted line for $T = 2 \times 10^6$ K and $v_0/v_T = 1.4$, dashed line for $T = 10^6$ K and $v_0/v_T = 3$, and dash-dotted line for $T = 0.5 \times 10^6$ K and $v_0/v_T = 5$.

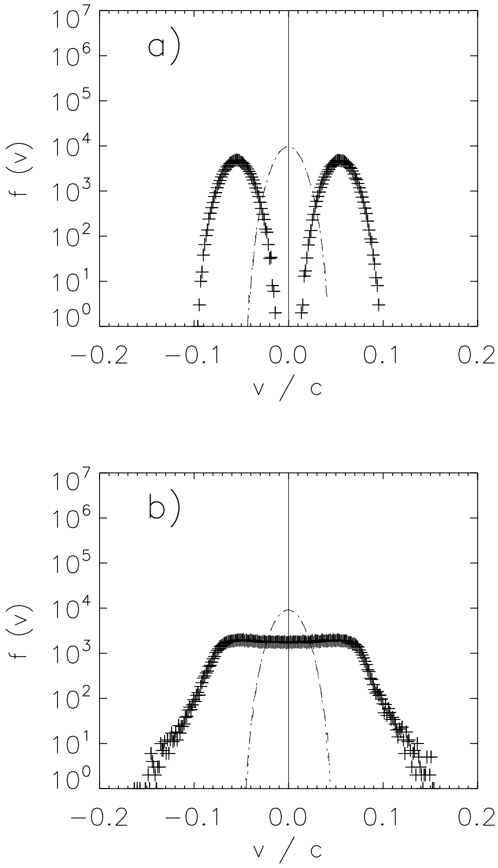

Fig. 7. a) The velocity distributions in the initial state of the PIC simulation ($f(v_x)$ crosses, $f(v_y)$ and $f(v_z)$ dashed-dot line), c means the speed of light. **b)** The velocity distributions after a saturation of the Buneman instability.

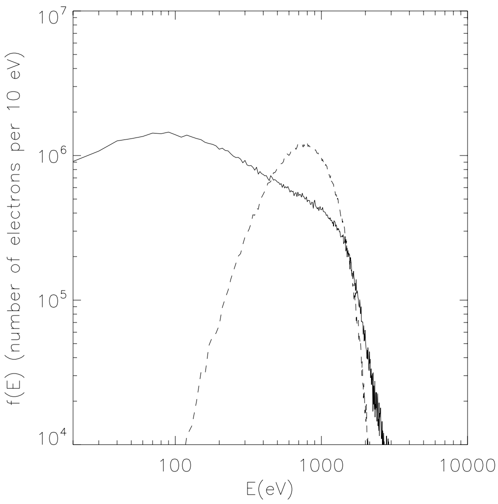

Fig. 8. The energy distributions corresponding to the velocity distributions in Fig. 7; in the initial state (dashed line) and after a saturation of the Buneman instability (full line).

the Si XIId 5.56 Å and 5.82 Å satellite lines is the one based on the n-distribution formed by the return current (Dzifčáková & Karlický 2008). The electric current or return current is unstable if $v_0/v_T > 1$ (Mikhailovskii 1975). Thus, moving Maxwellian distributions forming the return current with the parameters analyzed above are in the range of the Buneman instability.

Now a question arises about how moving Maxwellian distribution, which is a part of the return current, will be changed after the saturation of the Buneman instability. We used a 3D fully electromagnetic particle-in-cell model (e.g., Karlický 2009) with two counterstreaming beams having the same density and the same temperature $T = 10^6$ K. Each of these symmetric beams can be considered as the electron beam generating the return current (second beam in this model). Such a symmetric configuration was chosen to have zero net current and zero magnetic field in the initial state. In the coordinate system of protons (having the Maxwellian distribution with the temperature $T = 10^6$ K), these two beams have the velocities $v_0/v_T = \pm 3$. Their initial velocity distributions are shown in Fig. 7a. The corresponding energy distribution is the Maxwellian one shifted in the velocity space (moving Maxwelian, Fig. 8). Because of the Buneman instability, plasma waves are generated, which on the other hand modify the distributions. When the Buneman instability is saturated, the distributions have the forms shown in Figs. 7b and 8. In our case the saturation of the Buneman instability occurs at about $t = 100 t_p$, where t_p is the electron plasma period.

This computation illustrates how, in very short times, moving Maxwellian distribution is modified by plasma processes. While a change in its high-energy part is small, changes in its low-energy part are essential. Because the distribution after the saturation of the Buneman instability (Fig. 7b) still deviates from the Maxwellian distribution with no velocity shift, more instabilities and modifications of the distribution can be expected.

5. Discussion

It was shown that the n-distributions and moving Maxwellian ones are very similar, and their high-energy parts are nearly the same for appropriate parameters (presented in Fig. 4 and expressed by the relation 10). Thus for the spectroscopic studies where these high-energy parts are important, e.g., in the range of a few keV, where the Si XIId 5.56 Å and 5.82 Å satellite lines are formed, these distributions can be replaced.

The moving Maxwellial distribution is not a unique equivalent of the n-distribution. Distributions in velocity space generally carry more information than those in energy space. But contrary to the n-distribution, which is a parametric distribution with unclear physical meaning, the moving Maxwellian one has a simple physical meaning – the electron beam with a specific velocity and temperature. In our present case it is the electron component of the return current.

The observed values of the parameter n of the n-distribution give the values of the parameter v_0/v_T of moving Maxwellian distribution that are greater than 1. This means that this distribution, together with the charge neutralizing protons (forming current or return current), is unstable, mainly because of the Buneman instability. Using PIC simulation we showed that the Buneman instability modifies strongly the low-energy part of moving Maxwellian distribution. On the other hand, changes in its high-energy part, which are probably caused by additional wave-wave and wave-particle processes (e.g., Melrose 1980), are much smaller. It is interesting to see that the exponential high-energy end of the distribution in its initial state changes into the power-law one. Such a power-law high-energy end of the distribution would result in a decrease in the Si XIId/Si XIII ratio, since its gradient in the energy range of a few keV decreases with respect to the moving Maxwellian distribution.

However, there are further instabilities, e.g. those due to non-isotropic temperatures (Mikhailovskii 1975), which can destroy this distribution further. Therefore, if the moving Maxwellian distribution exists in the form according to the relation (5), then in solar flares there have to be some processes to stabilize this distribution, at least in some localized regions.

If we accept that the n-distributions are formed as the electron component of strong return currents in the beam-plasma systems (Dzifčáková & Karlický 2008), then variations in the n parameter in solar flares can be explained by variations in not only the parameter v_0/v_T, but also by variations in v_T. Namely, these variations require a change in the steepness of the electron distribution in a specific energy range as shown in Fig. 6. For the parameter n derived from variations of the Si XIId 5.56 Å and 5.82 Å satellite lines, it is in the 1–4 keV range. Furthermore, Kulinová et al. (2011) find that the n parameter of the n-distribution in solar flares increases with an increase in the X-ray emission, which is given by an increase in the electron beam flux. With this increase, the beam current, as well as return current, also increases. It furthermore means that the return-current electrons are shifted to higher energies (at locations of the flaring atmosphere with the same plasma density) or that they have the same energies, but for the return current in denser layers of the flaring atmosphere. Because Si XIId satellite lines are generated by a resonance process at specific energies, the region where these lines are formed have to be shifted to denser parts of the flaring atmosphere. Assuming now that these denser layers have lower temperatures (for the pressure equilibrium this is fulfilled), then the high-energy part of moving Maxwellian distribution will be steeper (see Fig. 6); i.e., the parameter n of the corresponding n-distribution will be higher. For a decrease in the electron beam flux, this effect will be the opposite, in agreement with observations.

6. Conclusions

We found that the moving Maxwellian distribution is very similar to the n-distribution. Both these distributions are nearly the same in their high-energy parts. Based on their mutual fitting in these parts we found a simple relationship between parameters of these distributions. We found that observed values of the parameter n of the n-distribution correspond to high $v_0/v_T > 1$ values of the parameter v_0/v_T of the moving Maxwellian distribution. For such high values, the moving Maxwellian distribution is unstable mainly because of the Buneman instability. Therefore to keep the form of this distribution similar to the n-distribution, at least in some regions of the flaring atmosphere, some stabilization processes are necessary.

The possibility of substituting the moving Maxwellian distribution for the n-distribution provides much deeper understanding of the previous results connected with the n-distribution. In accordance with our previous results, we assume that the moving Maxwellian distribution corresponds to the electron component of the return current in the beam-plasma system. This approach enables us to explain variations in the n and v_0/v_T parameters during the impulsive phase of solar flares.

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