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Thermodynamics and dark energy

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ABSTRACT

Significant observational effort has been directed to unveiling the nature of the so-called dark energy. However, given the large number of theoretical possibilities, it is possible that this a task cannot be based only on observational data. We discuss some thermodynamic properties of this energy component assuming a general time-dependent equation-of-state parameter $w(z) = w_0 + w_a f(z)$, where w_0 and w_a are constants and $f(z)$ may assume different forms. We show that very restrictive bounds can be placed on the $w_0 - w_a$ space when current observational data are combined with the thermodynamic constraints derived.

Key words. cosmological parameters – dark energy – cosmology: observations – cosmology: theory

1. Introduction

The observational evidence of an acceleration in the expansion of the Universe is now overwhelming, although the precise cause of this phenomenon is still unknown (see, e.g., Alcaniz 2006; Ratra & Vogeley 2008; Caldwell & Kamionkowski 2009; Silvestri & Trodden 2009; Sami 2009; Li et al. 2011, for some reviews). In this respect, and besides the need for more accurate estimates of cosmological parameters, the current state of affairs also brings to light some other important aspects regarding the physics of the mechanism behind cosmic acceleration.

One aspect is the thermodynamical behavior of a dark energy-dominated universe, and questions such as "what is the thermodynamic behavior of the dark energy in an expanding universe?" or, more precisely, "what is its temperature evolution law?" must be answered in the context of this new conceptual set-up. Another interesting aspect in this discussion is whether thermodynamics in the accelerating universe can place constraints on the time evolution of the dark energy and can also reveal some physical properties of this energy component.

The aim of this paper is twofold. First, we attempt to derive physical constraints on the dark energy from the second law of thermodynamics and to deduce the temperature evolution law for a dark component with a general equation-of-state (EoS) parameter $w(a)$. Second, we aim to perform a joint statistical analysis involving current observational data and the thermodynamic bounds on $w(a)$. To achieve both aims, we assume a generalized form of the time evolution of $w(a)$ (Barboza et al. 2009)

$$w(a) \equiv \frac{p_x}{\rho_x} = w_0 + w_a\frac{1-a^{\beta}}{\beta} = w_0 + w_a\frac{1-(1+z)^{-\beta}}{\beta}, \quad (1)$$

which recovers some well-known EoS parameterizations: P1 (Cooray & Huterer 1999; Astier 2001; Weller & Albrecht 2002), P2 (Efstathiou 1999), and P3 (Chevallier & Polarski 2001; Linder 2003) in the limits

$$w(z) = \begin{cases} w_0 + w_a\frac{(1-a)}{a} & (\text{P1}) \quad \beta \to -1, \\ w_0 - w_a \ln a & (\text{P2}) \quad \beta \to 0, \\ w_0 + w_a(1-a) & (\text{P3}) \quad \beta \to +1, \end{cases}$$

where p_x and ρ_x stand for the dark energy pressure and energy density, respectively (see also Wang 2001; Watson & Scherrer 2003; Corasaniti et al. 2004; Johri 2004; Wang & Tegmark 2004; Jassal et al. 2005; Barboza & Alcaniz 2008, for other EoS parameterizations). The analyses are performed using one the most recent type Ia supernovae (SNe Ia) observations, namely the nearby + SDSS + ESSENCE + SNLS + *Hubble* Space Telescope (HST) set of 288 SNe Ia discussed in Kessler et al. (2009) (which we refer to as the SDSS compilation). We consider two sub-samples of this latter compilation that use the SALT2 (Guy et al. 2007) and MLCS2k2 (Phillips 1993; Riess et al. 1993; Jha et al. 2007) SN Ia light-curve fitting method. Along with the SNe Ia data, and to help break the degeneracy between the dark energy parameters, we also use the baryonic acoustic oscillation (BAO) peak at $z_{\rm BAO} = 0.35$ (Eisenstein 2005) and the current estimate of the CMB shift parameter $\mathcal{R} = 1.71 \pm 0.019$ (Spergel et al. 2007). We work in units where $c = 1$. Throughout this paper, a subscript 0 stands for present-day quantities and a dot denotes a time derivative.

2. Thermodynamic analysis

Let us first consider a homogeneous, isotropic, spatially flat cosmologies described by the Friedmann-Robertson-Walker (FRW) flat line element, $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$, where $a(t) = 1/(1+z)$ is the cosmological scalar factor. The matter content is assumed to be composed of baryons, cold dark matter and a dark energy component.

In this framework, the thermodynamic states of a relativistic fluid are characterized by an energy momentum tensor (perfect-type fluid)

$$T^{\mu\nu} = \rho_x u^\mu u^\nu - p_x h^{\mu\nu}, \quad (2)$$

a particle current

$$N^\mu = n u^\mu, \quad (3)$$

and an entropy current

$$S^\mu = n\sigma u^\mu, \quad (4)$$

where $h^{\mu\nu} := g^{\mu\nu} - u^\mu u^\nu$ is the usual projector onto the local rest space of u^μ and n and σ are the particle number density and the specific entropy (per particle), respectively (Landau & Lifshitz 1959). The conservation laws for energy and particle number densities read

$$u_\nu T^{\mu\nu};_\mu = \dot{\rho}_x + (\rho_x + p_x)\theta = 0, \quad (5)$$
$$N^\mu;_\mu = \dot{n} + n\theta = 0, \quad (6)$$

where semi-colons mean a covariant derivative, $\theta = 3\dot{a}/a$ is the scalar of expansion and the quantities p_x, ρ_x, and n and σ are related to the temperature T by means of the Gibbs law $nT\mathrm{d}\sigma = \mathrm{d}\rho_x - \frac{\rho_x + p_x}{n}\mathrm{d}n$. From the energy conservation equation above, it follows that the energy density for a general $w(a)$ component can be written as

$$\rho_x \propto \exp\left[-3\int \frac{1+w(a)}{a}\mathrm{d}a\right], \quad (7)$$

where $w(a) = w_0 + w_a f(a)$ and $f(a) = (1 - a^\beta)/\beta$. Following standard lines (see, e.g., Weinberg 1971; Lima & Germano 1992; Silva et al. 2002), it is possible to show that the temperature evolution law is given by

$$\frac{\dot{T}}{T} = \left(\frac{\partial p_0}{\partial \rho_x}\right)_n \frac{\dot{n}}{n} + \left(\frac{\partial \Pi}{\partial \rho_x}\right)_n \frac{\dot{n}}{n}, \quad (8)$$

where we have separated the dark energy pressure into the components[1]

$$p_x \equiv p_0 + \Pi = w_0\rho_x + w_a f(a)\rho_x. \quad (9)$$

By combining the above equations, we also find that

$$T \propto \exp\left[-3\int \frac{w(a)}{a}\mathrm{d}a\right], \quad (10)$$

where we have assumed that $n \propto a^{-3}$, as given by the conservation of particle number density (Eq. (6)). For parameterization P_β, shown in Eq. (1), Eq. (10) can be rewritten as

$$T^x = T_0^x a^{-3(w_0 + \frac{w_a}{\beta})} \exp\left[-\frac{3w_a}{\beta}\left(\frac{1-a^\beta}{\beta}\right)\right], \quad (11)$$

which reduces to the generalized Stefan-Boltzmann law for $w_a = 0$ (Lima & Alcaniz 2004; Lima & Santos 1995). From the above expressions, we confirm the results of Lima & Alcaniz (2004) (for a constant EoS parameter) and find that dark energy becomes hotter in the course of the cosmological expansion since the its EoS parameter must be a negative quantity. A possible physical explanation of this behavior is that thermodynamic work is being done on the system (see, e.g., Fig. 1 of Lima & Alcaniz 2004). In particular, for the vacuum state ($w = -1$) we find that $T \propto a^3$.

[1] In doing so, Eq. (5) can be rewritten as $\dot{\rho}_x + (\rho_x + p_0)\theta = -\Pi\theta$, where the time-dependent part of the dark energy pressure can be identified as a source term.

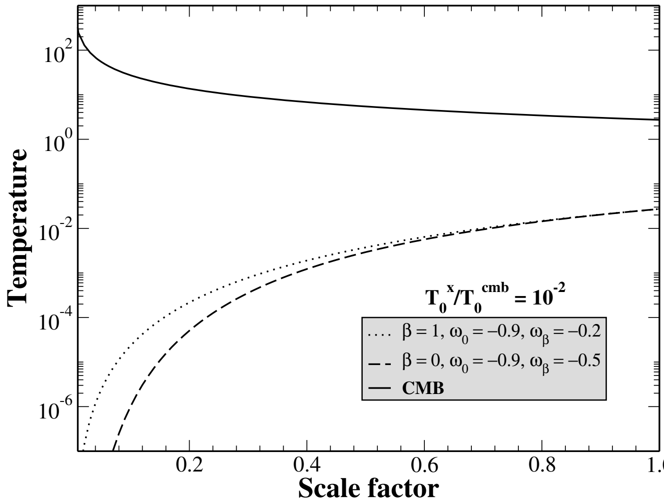

Fig. 1. Temperatute evolution law for parameterizations P2 ($\beta = 0$) and P3 ($\beta = 1$) assuming some arbitrary values of w_0, w_a, and $T_0^x = 10^{-2}T_0^{\mathrm{CMB}}$, where $T_0^{\mathrm{CMB}} = 2.73$ K. For comparison, we also show the CMB temperature curve (solid line).

To illustrate this behavior, Fig. 1 shows the dark energy temperature as a function of the scale factor for $\beta = 0$ (P2) and $\beta = 1$ (P3)[2] by assuming arbitrary values of w_0, w_a, and $T_0^x = 10^{-2}T_0^{\mathrm{CMB}}$, where $T_0^{\mathrm{CMB}} = 2.73$ K. From this analysis, it is clear that an important point for the thermodynamic fate of the universe is to know how long the dark energy temperature will take to become the dominant temperature of the universe. A basic difficulty in estimating such a time interval, however, is that the present-day dark energy temperature has not been measured, being completely unknown.

Assuming that the chemical potential for this $w(a)$-fluid is null (as occurs for $w = 1/3$), the Euler's relation defines its specific entropy, i.e.,

$$\sigma \equiv \frac{S_x}{N} = \frac{\rho_x + p_x}{nT}. \quad (12)$$

Now, by combining the above equations, it is straightforward to show that the product $\rho_x a^3/T \equiv$ const., so that

$$S_x \propto (1 + w). \quad (13)$$

For a constant EoS parameter, the above expression recovers some of the results of Lima & Alcaniz (2004). We also note that the vacuum entropy is zero ($w = -1$), whereas for phantom dark energy ($w < 1$), which violates all the energy conditions (Caldwell 2002; Carroll et al. 2003; Alcaniz 2004), the entropy is negative and, therefore, physically meaningless. For a discussion of the behavior of a phantom fluid with a nonzero chemical potential, we refer to Silva, Alcaniz & Lima (2007); Lima & Pereira (2008); Pereira & Lima (2008). We also refer to Gonzalez-Diaz & Siguenza (2004) for an alternative explanation in which the temperature of the phantom component takes negative values and Izquierdo & Pavon (2006), Bilic (2008);

[2] As is well-known, P1 has the drawback of becoming rather unphysical for $w_0 > 0$, with the dark energy density ρ_x increasing exponentially as $e^{3w_0 z}$ at high-z. We, therefore, do not consider this parameterization in our analyses.

Myung (2009), and Saridakis et al. (2009) for other thermodynamic analyses of dark energy.

Two cases of interest arise directly from Eq. (13). The case in which S_x = constant implies necessarily that $w_a = 0$ for all the above parameterizations[3]. The second case is even more interesting, with the $w(a)$-fluid mimicking a fluid with bulk viscosity where the viscosity term is identified with the varying part of the dark energy pressure, i.e., $\Pi = w_a f(a)\rho_x$. For this latter case, we note that the positiveness of S_x implies that

$$1 + w(a) \geq 0. \tag{14}$$

Therefore, using the generalized formula for the time evolution of $w(a)$ (Eq. (1)), we obtain

$$w_a \geq -\frac{1 + w_0}{f(a)}, \tag{15}$$

which clearly is not defined at $a = 1$, where $w = w_0$.

By taking the divergence of entropy current and combining with the conservation of the particle number density, we have

$$S^{\nu}{}_{;\nu} = n\dot{\sigma}. \tag{16}$$

Finally, when combined with Eq. (12) the above equation provides

$$S^{\nu}{}_{;\nu} \propto \frac{dw(a)}{dt} \geq 0 \tag{17}$$

or, equivalently, $w_a \leq 0$.

3. Constraints on the $w_0 - w_a$ plane

We now combine the above physical constraints in Eqs. (15) and (17) with current observational data to impose bounds on the dark energy parameters. We use one of the most recent SNe Ia data sets available, namely, the SDSS compilation discussed in Kessler et al. (2009). This compilation comprises 288 SNe Ia and uses both SALT2 (Guy et al. 2007) and MLCS2k2 (Phillips 1993; Riess et al. 1993; Jha et al. 2007) light-curve fitters (see also Frieman 2008 for a discussion on these light-curve fitters) and is distributed across a redshift interval $0.02 \leq z \leq 1.55$. Along with the SNe Ia data, and to help break the degeneracy between the dark energy parameters w_0 and w_a, we use the BAO (Eisenstein 2005) and shift parameters (Spergel et al. 2007)

$$\mathcal{A} = D_V \frac{\sqrt{\Omega_{\rm m} H_0^2}}{z_{\rm BAO}} = 0.469 \pm 0.017, \tag{18}$$

$$\mathcal{R} = \Omega_{\rm m}^{1/2} r(z_{\rm CMB}) = 1.71 \pm 0.019, \tag{19}$$

where $D_V = [r^2(z_{\rm BAO}) z_{\rm BAO}/H(z_{\rm BAO})]^{1/3}$ is the so-called dilation scale, defined in terms of the dimensionless comoving distance r, $z_{\rm BAO} = 0.35$ and $z_{\rm CMB} = 1089$. In our analyses, we minimize the function $\chi^2 = \chi^2_{\rm SNe} + \chi^2_{\rm BAO} + \chi^2_{\rm CMB}$, which takes into account all the data sets mentioned above and marginalize over the present values of the matter density $\Omega_{\rm m}$ and Hubble parameters H_0.

Figures 2 and 3 show the main results of our joint analyses. We plot contours of $\Delta\chi^2 = 6.17$ in the parametric space w_0–w_a for P2 and P3, respectively. The light gray region displayed in the plots stands for the physical constraint given in Eq. (15).

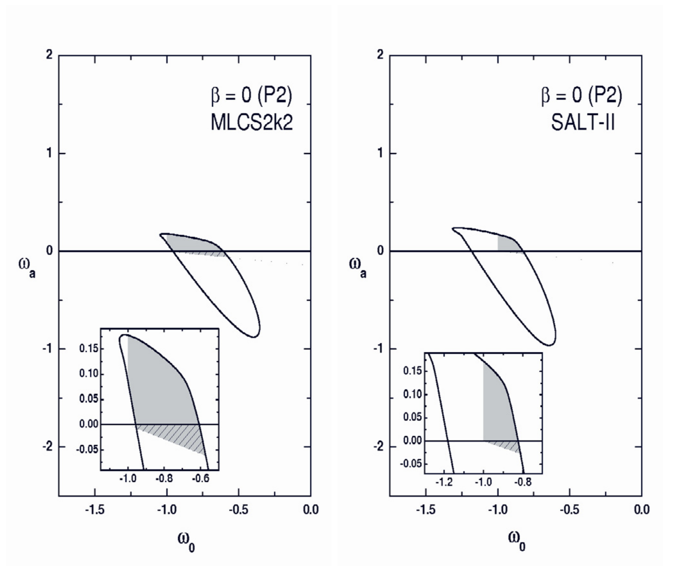

Fig. 2. Contours of $\Delta\chi^2 = 6.17$ in the parametric plane $w_0 - w_a$ for P2 ($\beta = 0$). The light gray area represents the thermodynamic constraint of Eq. (15), whereas the small dashed area is the resulting parametric space for which the constraint from the second law of thermodynamics (Eq. (17)) is added to the analysis.

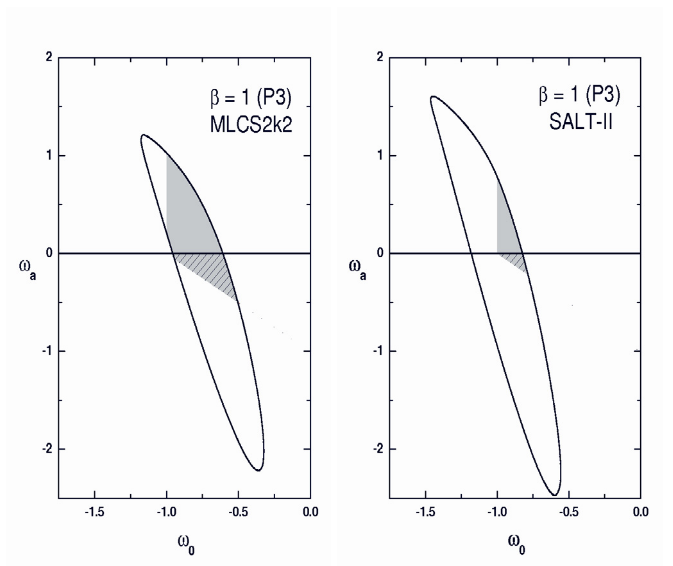

Fig. 3. The same as in Fig. 2 for parameterization P3 ($\beta = 1$).

Since this inequality is a function of time, the region is plotted by assuring its validity from $a = 10^{-4}$ up to today at $a = 1$. The resulting parametric space, in which all the observational data discussed above are combined with the constraints given in Eqs. (15) and (17), corresponds to the small dashed area right below the $w_a = 0$ line.

These results clearly illustrate the effect that the thermodynamic bounds discussed in the previous section may have on the determination of the dark energy EoS parameters. In particular, we note that the resulting allowed regions are even tighter for the logarithmic parameterization P2 than for P3 (CPL). Since the SALT2 compilation allows for more negative values of w_0, the joint constraints involving this SNe Ia sub-sample are also more restrictive (Figs. 2b and 3b). For completeness, we display in Table I the changes in the 2σ estimates of w_0 and w_a due to the thermodynamic bounds in Eqs. (15) and (17).

[3] This result implies that time-dependent EoS parameterizations without the constant term w_0 are imcompatible with the case S_x = const.

Table 1. Constraints on ω_0 and ω_a.

Test		w_0	w_a
SNe Ia (MLCS2k2)[a]	P2	$-0.78^{+0.33}_{-0.21}$	$0.02^{+0.14}_{-0.66}$
SNe Ia (MLCS2k2)[a] + T[b]	P2	$-0.77^{+0.16}_{-0.14}$	$0.00^{+0.00}_{-0.05}$
SNe Ia (SALT2)[a]........................	P2	$-1.05^{+0.34}_{-0.20}$	$0.13^{+0.09}_{-0.80}$
SNe Ia (SALT2)[a] + T[b]..............	P2	$-0.99^{+0.14}_{-0.01}$	$0.00^{+0.00}_{-0.02}$
SNe Ia (MLCS2k2)[a]....................	P3	$-0.81^{+0.37}_{-0.29}$	$0.18^{+0.99}_{-1.81}$
SNe Ia (MLCS2k2)[a] + T[b]..........	P3	$-0.77^{+0.22}_{-0.14}$	$0.00^{+0.00}_{-0.45}$
SNe Ia (SALT2)[a]........................	P3	$-1.09^{+0.41}_{-0.30}$	$0.52^{+0.96}_{-2.26}$
SNe Ia (SALT2)[a] + T[b]..............	P3	$-0.99^{+0.17}_{-0.01}$	$0.00^{+0.00}_{-0.18}$

Notes. [(a)] + CMB + BAO [(b)] Thermodynamic constraints (Eqs. (15) and (17)).

4. Conclusions

In spite of its fundamental importance to a clear understanding of the evolution of the universe, the relevant physical properties of the dominant dark energy component remain completely unknown. In this paper, we have investigated some thermodynamic aspects of this energy component assuming that its constituents are massless quanta with a general time-dependent EoS parameter $w(a)$. We have discussed its temperature evolution law and derived constraints from the second law of thermodynamics on the values of w_0 and w_a for a family of $w(a)$ parameterizations given by Eq. (1). When combined with current data from SNe Ia, BAO, and CMB observations, we have shown that these constraints provide very restrictive limits on the parametric space $w_0 - w_a$ (see Figs. 2 and 3).

Finally, we note that in the present analysis we have assumed that the chemical potential μ for the $w(a)$-fluid representing the dark energy is null. Under these conditions, we have shown that a phantom dark energy component is ruled out by thermodynamic considerations. However, it is worth mentioning that this is not the case when a more general analysis relaxing this condition ($\mu \neq 0$) is considered. This analysis is currently in progress and will appear in an upcoming paper.

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