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Pick-up ion transport under conservation of particle invariants: how important are velocity diffusion and cooling processes?

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ABSTRACT

Context. The phase space transport of pick-up ions (PUIs) in the heliosphere has been studied for the cases that these particles are experiencing a second-order Fermi process, i.e. velocity diffusion, a convection with the solar wind, and adiabatic or "magnetic" cooling, i.e. cooling connected with the conservation of the magnetic moment.
Aims. The study aims at a quantification of the process of "magnetic cooling" that has recently been introduced as a modification of adiabatic cooling in the presence of frozen-in magnetic fields.
Methods. The isotropic PUI velocity distributions are obtained as numerical solutions of a Fokker-Planck phase space transport equation.
Results. It is demonstrated that this newly discussed process is, like adiabatic energy changes, not limited to cooling but can also, depending on the shape of the distribution function, result in heating. For pure cooling with negligible velocity diffusion a v^{-5} velocity power law is found for magnetic cooling, thus confirming earlier analytical results. For non-negligible second-order Fermi acceleration, the tails of the distribution functions exhibit different shapes, which in special cases are also close to the prominent v^{-5} behaviour, which is often found in observations.
Conclusions. The existence of an exact v^{-5} power law of PUI distribution functions can be confirmed for insignificant velocity diffusion and its approximate validity for specific choices of the velocity dependence of the diffusion coefficient.

Key words. acceleration of particles – solar wind – scattering – diffusion

1. Introduction

The transport theory of pick-up ions (PUIs) has been enriched recently by the consideration of two new processes. First, Fisk & Gloeckler (2008) have introduced a stochastic acceleration of these suprathermal particles by compressible fluctuations in order to explain the frequently (see Gloeckler 2003) (though not exclusively, e.g. see Dayeh et al. 2007) observed power laws in the form of v^{-5}-tails of the PUI velocity distributions. Second, Fahr (2007) has suggested that there is a need to modify the usual adiabatic cooling taking into account the anisotropic ion drifts in velocity space due to their co-motion with the heliospheric magnetic field which is frozen into the solar wind bulk flow. Interestingly, this latter modification as we shall show in this paper is related with an extension of the v^{-5}-power law shape of the PUI velocity distributions to speeds below the injection speed (i.e. at about $v \simeq U \simeq 450\ \mathrm{km\,s^{-1}}$).

In view of these extensions of the "traditional" theory (Vasyliunas & Siscoe 1976; Isenberg 1987; Chalov et al. 1995; Fichtner et al. 1996), it appears timely to revisit especially the phase space transport of PUIs, i.e. of suprathermal, middle-energetic ions. With an emphasis on the so-called magnetic cooling, in the next section the motivation for a modified transport is described along with its quantitative treatment. Section 3 contains a detailed presentation of the model and the solution procedure. In Sect. 4 various simulation results are given that (i) relate them to previous ones; (ii) shed light on the effect of magnetic cooling; and (iii) address the consequences for the interpretation of the often appearing v^{-5}-shape of the PUI velocity distributions. After a critical discussion of the findings in Sect. 5, the conclusions are summarized in Sect. 6.

2. PUI transport: the state-of-the-art

In order to relate the results of our study quantitatively to the above-mentioned recent developments we briefly summarize in this section the state-of-the-art regarding the modelling of the transport processes of suprathermal particles in the heliosphere.

2.1. Magnetic cooling

In the absence of stochastic processes like random collisions or wave-particle interactions ions moving along magnetic field-lines have to conserve their magnetic moment $\Gamma_\perp = (m/2)v_\perp^2/B$ (see, e.g., Baumjohann & Treumann 1996). It is interesting, but perhaps less familiar to the scientific community, that the quantity $\Gamma_\perp$ also plays the role of an invariant for ions co-convected with a plasma bulk with frozen-in magnetic field, if the magnetic field magnitude decreases in the direction of the bulk flow. The latter, for instance, is the case for the Archimedean spiral field in the inner heliosphere (Parker 1958; Forsyth et al. 2002), so that $\frac{v_\perp^2}{B}$ = const. holds while ions are convected outwards (Fahr 2007; Fahr & Siewert 2008, 2010). In addition, connected with differential motions of ions parallel to frozen-in fields, for bulk velocity gradients parallel to the field, another quantity appears that serves as a second particle invariant: $\Gamma_\parallel = v_\parallel B/\rho$ = const. (Fahr & Siewert 2008; Siewert & Fahr 2008). Now connected with changes of B and ρ in the direction of the wind flow, corresponding changes of the co-convected ion velocities occur which can be understood as due to ion velocity drifts $\dot{v}_\perp$ and $\dot{v}_\parallel$ in velocity space.

Starting from the Archimedean magnetic field one finds (Parker 1958)

$$\frac{1}{B}\frac{\partial B}{\partial r} = -\frac{2}{r}\left[1 - \frac{1}{2}\left(1 + ctg^2\varphi\right)^{-1}\right] \tag{1}$$

with φ denoting the angle between the directions of the local magnetic field and the solar wind velocity. With this and the above relation for the magnetic moment one obtains

$$\dot{v}_\perp = Uv_\perp \frac{1}{B}\frac{\partial B}{\partial r} = -\frac{2Uv_\perp}{r}\left[1 - \frac{1}{2}\sin^2\varphi\right] \tag{2}$$

and, furthermore, according to the second invariant

$$\dot{v}_\parallel = -\frac{Uv_\parallel}{r}\sin^2\varphi. \tag{3}$$

In the phase space transport considered in this paper we assume pitchangle-isotropic distributions. Since in that case only the drift in velocity magnitude is of interest, we proceed in the following manner:

With the representation $v_\parallel = v\cos\varphi$; $v_\perp = v\sin\varphi$; $v^2 = v_\parallel^2 + v_\perp^2$ we obtain

$$2v\dot{v} = 2v_\parallel\dot{v}_\parallel + 2v_\perp\dot{v}_\perp \tag{4}$$

and find

$$\dot{v} = \frac{1}{v}\left[-Uv^2\frac{1}{r}\cos^2\varphi\sin^2\varphi - v^2\frac{2U}{r}\sin^2\varphi\left[1 - \frac{1}{2}\sin^2\varphi\right]\right], \tag{5}$$

which finally can be represented by

$$\dot{v} = -\frac{Uv}{r}\left[\cos^2\varphi\sin^2\varphi + 2\sin^2\varphi\left(1 - \frac{1}{2}\sin^2\varphi\right)\right]. \tag{6}$$

As one can see, this asymptotically (i.e. for $r \to \infty; \varphi \to 90°$) delivers $\dot{v} = -\frac{Uv}{r}$, which is exactly what was found in Fahr & Siewert (2008). The term in the square brackets is, in the distance range $r \in [1, \infty)$ very close to unity, and therefore, can safely be approximated by this value. For results of a test simulation see Fig. 1 (right panel in middle row) and the corresponding discussion in Sect. 4.

It is perhaps interesting to compare the above result with a velocity magnitude drift that prevails in a radially symmetric solar wind without magnetization (i.e. no frozen-in field). In that case one only obtains a differential bulk velocity drift without magnetic control. For an ion with a speed v and an angle α of $\boldsymbol{v}$ with respect to the bulk flow $\boldsymbol{U}$ one then finds

$$\dot{v}(\alpha) = (\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{U} = \frac{v}{r}\sin\alpha\cdot U,$$

which under pitch-angle isotropic conditions simply leads to

$$\dot{v} = \int_0^\pi \dot{v}(\alpha)\sin\alpha\,\mathrm{d}\alpha = -\frac{1}{2}\frac{v}{r}U.$$

With some interest one may recognize here that this cooling drift is only half of what it is for a magnetized flow which of course is due to non-conservation of ion magnetic moments under the absence of magnetic fields.

2.2. Comparison with adiabatic cooling

So far in most cases, in the literature a type of phase space transport equation including adiabatic ion cooling has been applied that was originally developed for the transport of cosmic rays (Parker 1965; Gleeson & Axford 1967) and is a result of a diffusion approximation. A few studies, like that by le Roux & Webb (2009), have used the more general focused transport equation. Because this approach, with respect to the action of adiabatic cooling, gives results very similar to those obtained with the diffusion approximation we conduct the following discussion within the realm of the latter.

The Parker equation, though well approved for high-energy cosmic rays, may, however, become questionable for low- or middle-energetic particles like SW ions or PUIs with KeV energies. This we demonstrate in the following.

The adiabatic cooling term appearing in the cosmic ray (CR) transport equation up to now is as well used in the PUI transport equation (Vasyliunas & Siscoe 1976; Isenberg 1987; Chalov et al. 1995; Fichtner et al. 1996; Mall 2000). This term appears to be justified, if the pressure of the ions does work at the volume expansion. Connected with the expansion of a spherically diverging solar wind flow a loss of internal PUI energy Ξ (i.e. enthalpy) must be expected. In the wind frame it is given by

$$\frac{\mathrm{d}\Xi}{\mathrm{d}t} = nV\frac{\mathrm{d}\chi}{\mathrm{d}t} = -P\frac{\mathrm{d}V}{\mathrm{d}t}$$

where $\chi = \Xi/nV$ denotes the enthalpy per particle. This effect is expected from thermodynamic principles under conditions where the gas reacts adiabatically in the expanding flow, i.e. under subsonic expansion rates and a quasi-isentropic gas reaction.

These conditions are, however, not fulfilled in the region of the supersonically expanding solar wind, because the thermodynamic equilibrium is perturbed (like a gas with a piston expanding supersonically). Thus, at great heliospheric distances, i.e. beyond 1 AU, conditions for low-energy particles are invalidating the above-mentioned adiabatic relation. The ion gas cannot react adiabatically, if the typical volume scale growth speed $c_V \simeq (2/3)U$ is greater than the thermal ion velocities $c_{\rm th}$. This is not critical for high-energy CR particles with velocities $v_{\rm CR}$ or sound speeds $c_{\rm CR} = \sqrt{\mathrm{d}P_{\rm CR}/\mathrm{d}\rho_{\rm CR}}$ much greater than the solar wind velocity U. These particles, just like galactic and anomalous cosmic rays, undergoing scattering at statistically distributed, magnetic inhomogeneities quasi-frozen into the supersonically expanding solar wind, thus in fact react isentropically and adiabatically (see also Toptygin 1985). This is different for low-energy, "subsonic" ions. Both exospheric solar wind theories (see Griffel & Davis 1969; Lemaire & Scherer 1971; Marsch & Livi 1985) and in situ plasma observations (Marsch et al. 1981) clearly show, that solar wind ions at their co-motion with the wind behave non-adiabatically and non-isentropically.

Nevertheless, we re-investigate the adiabatic approach here again for the purpose of comparing adiabatic with magnetic cooling. For a spherically symmetric solar wind with constant bulk velocity magnitude U the change of the co-moving proper volume V with time is given by

$$\frac{\mathrm{d}V}{\mathrm{d}t} = (\nabla\cdot\boldsymbol{U})\,V = 2\frac{U}{r}V \tag{7}$$

and thus with the relation further above delivers the following equation

$$n\frac{\mathrm{d}\chi}{\mathrm{d}t} = -P\frac{1}{V}\frac{\mathrm{d}V}{\mathrm{d}t} = -2P\frac{U}{r}. \tag{8}$$

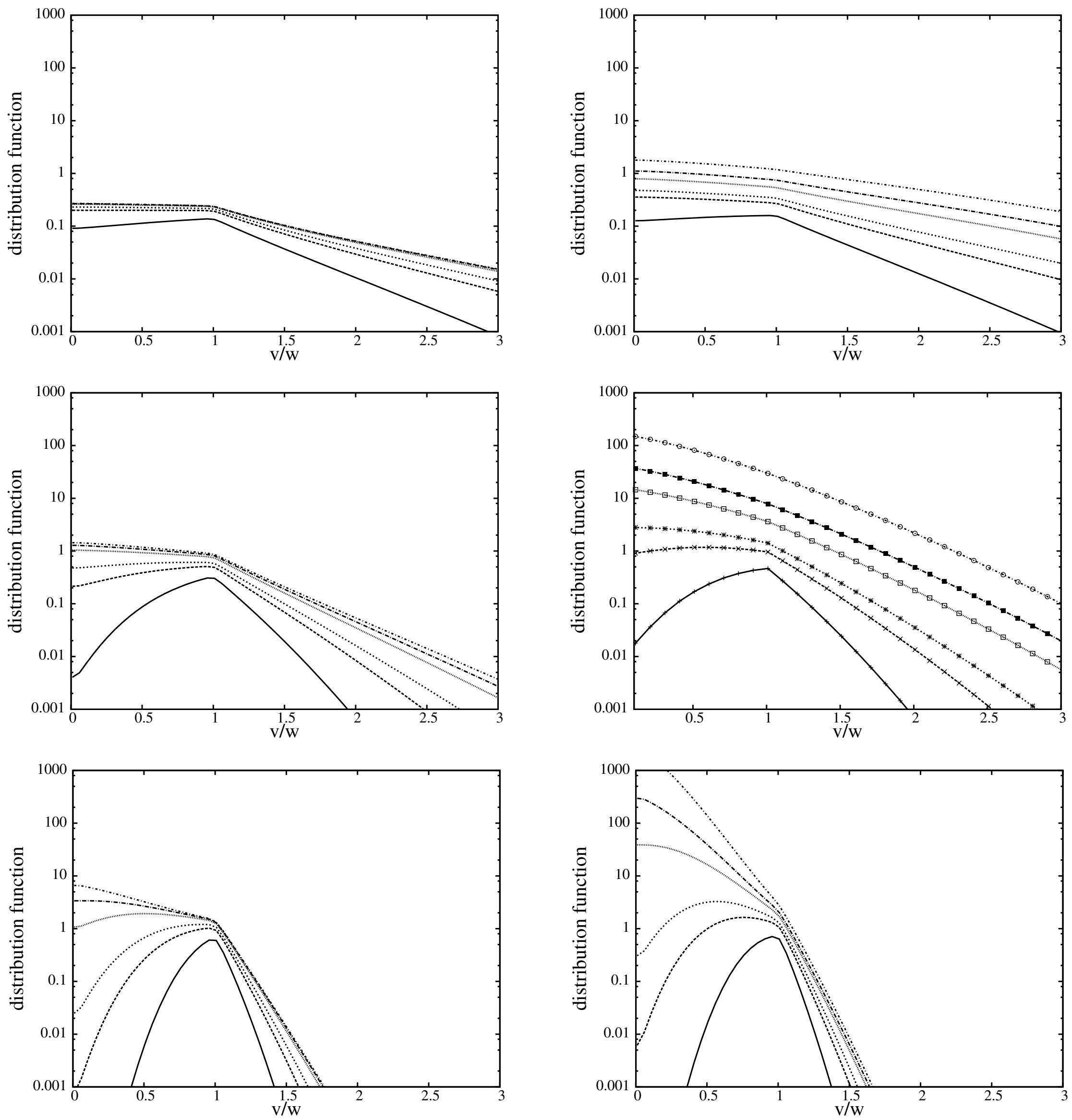

Fig. 1. PUI transport with adiabatic (*left-hand panels*) and magnetic (*right-hand panels*) energy changes: the normalized PUI velocity distribution $f2U^2r/3Q$ as a function of normalized PUI speed (with $w = U$) and heliocentric distance (and, thus, of time): from bottom to top the curves are for $r/r_0 = 1.2$, 1.6, 2.0, 4.0, 8.0 and 29 (approximating the asympotic shape). The three rows are the diffusion coefficients $D = 5\ D_F(r, v)$, $D = D_F(r, v)$, and $D = 0.2\ D_F(r, v)$. The symbols in the right middle panel illustrate that the very weak distance dependence of the square bracket term in Eq. (6) has no effect on the solution.

Usually, but without a sound physical basis, one replaces macroscopic thermodynamic by corresponding kinetic quantities by taking the enthalpy per particle and the pressure in the form

$$\chi = \frac{\gamma}{\gamma - 1}\left\langle\frac{p^2}{2m}\right\rangle \tag{9}$$

with

$$P = nkT \simeq n\frac{2}{3}\left\langle\frac{p^2}{2m}\right\rangle\cdot \tag{10}$$

This yields the relation

$$n\frac{1}{P}\frac{\mathrm{d}\chi}{\mathrm{d}t} \simeq \frac{3}{2}\frac{\gamma}{\gamma-1}\frac{\frac{\mathrm{d}}{\mathrm{d}t}\left\langle p^2\right\rangle}{\left\langle p^2\right\rangle} = -2\frac{U}{r}\cdot \tag{11}$$

Transcribing now this macroscopic relation to properties of individual particles, not caring for probability weights given by the distribution function, – a potentially problematic procedure –, will then finally yield

$$\frac{3}{2}\frac{\gamma}{\gamma-1}\frac{2p\frac{\mathrm{d}}{\mathrm{d}t}p}{p^2} = -2\frac{U}{r} \tag{12}$$

leading to the well-known adiabatic momentum change – or to be better comparable with the above derivations – to the adiabatic particle drift given by

$$\dot{v}_{\mathrm{ad}} = -\frac{2}{3}\frac{\gamma-1}{\gamma}\frac{Uv}{r}\cdot \tag{13}$$

The usual values $(4/3) \leq \gamma \leq (5/3)$ would then suggest adiabatic drifts $\dot{v}_{\mathrm{ad}}$ between $(1/6) \leq \dot{v}_{\mathrm{ad}}r/(Uv) \leq (1/4)$, i.e. substantially smaller than magnetic drifts.

In the following we study the influence of magnetic or adiabatic drifts on the ion distribution functions found with corresponding ion phase space transport equations.

2.3. The adiabatic and magnetic cooling terms compared

According to Fahr (2007) and Fahr & Siewert (2008) the above considerations result in the requirement to modify the "classical" adiabatic cooling term as used since Vasyliunas & Siscoe (1976)

$$L_{\rm ac} = \frac{v}{3}(\nabla \cdot \boldsymbol{U})\ \frac{\partial f}{\partial v} = \frac{2vU}{3r}\frac{\partial f}{\partial v} \tag{14}$$

with the PUI speed v, the radial solar wind velocity $\boldsymbol{U} = U\boldsymbol{e}_r$ and the isotropic PUI phase space distribution $f(\boldsymbol{r}, v, t)$ at the location $\boldsymbol{r}$ at time t. On the basis of two particle invariants valid for collision-free motions of ions co-moving with magnetized plasma flows, one instead derives for the new expression

$$L_{\rm mc} = \frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2\frac{Uv}{r}f\right) = \frac{Uv}{r}\frac{\partial f}{\partial v} + \frac{3U}{r}f. \tag{15}$$

Obviously, $L_{\rm mc}$ is only negative for distribution functions steeper than v^{-3}, it vanishes for $f \sim v^{-3}$ and is positive otherwise. So, like $L_{\rm ac}$, which is a cooling term only for $\partial f/\partial v < 0$, also $L_{\rm mc}$ can lead to a heating. In general, one should refer to "adiabatic" and "magnetically induced" energy changes.

The last equality in Eq. (15) reveals that such magnetically induced energy changes can be characterized by an energy change analogous to adiabatic ones and an energy gain, as indicated by the first and second term on the right-hand side, respectively.

In the following we study the transport of PUIs in the expanding solar wind for the case that adiabatic cooling is replaced by magnetic cooling.

2.4. An analogy to results from the relaxation theory of suprathermal particle distributions

The concept of a Fermi-2 type velocity diffusion due to scattering between counterflowing wave trains has often been used in the past to describe a kind of diffusive ion drift in velocity space taking care of transporting newly injected PUIs from 1 keV to higher (and lower) energies (see Isenberg 1987; Bogdan et al. 1991; Chalov et al. 1995; Fichtner et al. 1996) This velocity diffusion is generally ascribed to pitch-angle scattering and resonant transfer processes at counterflowing Alfvén waves. The efficiency of this process, however, very much depends on properties of the turbulence power spectrum of the Alfvén waves, see, e.g. Isenberg (2005). In his study of the turbulent heating of the solar wind, that has recently been generalized by Isenberg et al. (2010), he used the more fundamental Fokker-Planck approach allowing for anisotropic distributions. He found not only that the energization of the PUIs is weak, with details critically depending on the turbulence properties, but also that dispersive effects have essentially no influence on the energization. This way the dispersionless analysis used in Isenberg (1987) and the later studies cited above are demonstrated to be justified. In addition, in a more consistent description, unfortunately, at least at larger solar distances, the effect of velocity diffusion turns out to be essentially negligible (see Chalov et al. 2003; Fahr et al. 2007; Fisk & Gloeckler 2008).

Therefore it may be worthwhile to look into alternative processes taking care of a redistribution of ions in velocity space.

In this respect it is perhaps interesting to notice that in the literature there is the following form of a Boltzmann-Vlasow equation that describes the relaxation of an ion distribution function towards a quasi-equilibrium (Hinton 1983; Karney 1986; Shizgal 2004), (e.g. usually describing the relaxation of a hot ion population towards a cool background population)

$$\frac{\partial f}{\partial t} = \frac{A}{v^2}\frac{\partial}{\partial v}\left[D_1(v)\left(1 + \frac{v_{\rm qe}^2}{v}\frac{\partial}{\partial v}\right)\right]f.$$

While in the mentioned references the relaxation process is envisioned as operating on the basis of Coulomb collisions of a light gas with a heavy background gas, one can as well understand this process as expressing the relaxation of hot gas atoms with a quasi-equilibrium background population of its own gas atoms where the background for example is defined as quasi-equilibrium between ions and and a nonlinear background MHD wave turbulence with energy density of $\varepsilon_{\rm qe} \simeq n\cdot(mv_{\rm qe}^2)$ (see, e.g., Treumann 2001). For the stationary quasi-equilibrium distribution that is approached under these conditions one, thus, also has to solve a type of the above equation, which when written in a more explicit form, attains the form

$$0 = \frac{A}{v^2}\left[\frac{\partial}{\partial v}\left[D_1(v)f_{\rm qe}\right] + \left(D_1(v)\frac{v_{\rm qe}^2}{v}\right)\frac{\partial^2 f_{\rm qe}}{\partial v} + \frac{\partial}{\partial v}\left[D_1(v)\frac{v_{\rm qe}^2}{v}\right]\frac{\partial f_{\rm qe}}{\partial v}\right]. \tag{16}$$

It is interesting to notice that the first term appearing in the above equation is exactly of the form that we find for the cooling drift term, namely

$$\frac{\partial f}{\partial t}_{\rm c} = \frac{1}{v^2}\left[\frac{\partial}{\partial v}\left[v^2\left(-\frac{Uv}{r}\right)f\right] = \frac{A}{v^2}\left[\frac{\partial}{\partial v}\left[D_1(v)f\right]\right.\right.$$

for $A = 1$ and $D_1 = -\frac{Uv^3}{r}$, i.e. one part of the relaxation can be described as operating similar to our cooling drift term.

The second interesting point is that taking the quantity $D_1(v)$ as derived by Hinton (1983) (i.e. for Coulomb relaxation) one finds that the above requirement for the stationary function $f_{\rm qe}$ is fulfilled at high velocities $v \gg v_{\rm qe}$ only by a velocity power law with $\gamma = -2$, i.e. by the form $f_{\rm qe} \sim (v/v_{\rm qe})^{-2}$.

Furthermore, it has been shown that the above relaxation equation can be generalized including spatial and velocity diffusion terms. First, allowing only for a velocity diffusion process leads to the more general Fokker-Planck type equation

$$\frac{\partial f}{\partial t} = \frac{A}{v^2}\frac{\partial}{\partial v}\left[D_1(v)\left(1 + \frac{v_{\rm qe}^2}{v}\frac{\partial}{\partial v}\right)\right]f + \frac{B}{v^2}\frac{\partial}{\partial v}\left[v^2 D_2(v)\frac{\partial f}{\partial v}\right] \tag{17}$$

where it is again interesting to see that one part of the relaxation term acts identical to the diffusion term and allows us to write the above equation in the form

$$\frac{\partial f}{\partial t} = \frac{A}{v^2}\frac{\partial}{\partial v}\left[D_1(v)f\right] + \frac{B}{v^2}\frac{\partial}{\partial v}\left[v^2\left[D_2(v) + \frac{A}{B}\frac{v_{\rm qe}^2}{v^3}D_1(v)\right]\frac{\partial f}{\partial v}\right]. \tag{18}$$

Using the earlier identification $D_1 = -\frac{Uv^3}{r}$ one eventually finds

$$\frac{\partial f}{\partial t} = \frac{A}{v^2}\frac{\partial}{\partial v}\left[-\frac{Uv^3}{r}f\right] + \frac{B}{v^2}\frac{\partial}{\partial v}\left[v^2\left[D_2(v) - \frac{A}{B}v_{\rm qe}^2\frac{U}{r}\right]\frac{\partial f}{\partial v}\right]. \tag{19}$$

This shows that the relaxation process also includes a velocity diffusion, however, operating with the opposite sign compared to the usual term, i.e. diffusion fluxes are oriented in the opposite direction of the velocity gradient of the distribution function f.

In view of the above Fokker-Planck equation it is tempting to study solutions of this equation, because as a hint it has already been found by Shizgal (2007) that this equation naturally leads to so-called κ-distributions, which become pure power laws at high velocities, if the velocity diffusion coefficient varies like $D_2 \sim v^{-1}$.

2.5. Velocity diffusion in an idealistic equilibrium state

Relaxation always means relaxation towards an attracting high-entropy equilibrium state. In a plasma without collisions, however, it seems hard to imagine what stochastic processes may be responsible to establish such a state, and, even harder, how this state might look like eventually. Treumann (2001) has given some principle ideas how the approach towards a turbulent quasi-equilibrium state may be realized for ion plasmas with wave-ion interactions. For the purpose of obtaining a more quantitative and analytic result, however, we look here at a more idealistic equilibrium state in which Alfvénic wave turbulence is permanently fed by wave growth driven by ions newly injected into unstable modes, while these turbulences by nonlinear wave-ion interactions act back on the ion distributions. These considerations may indicate towards what quasi-equilibrium state ions would relaxate whatever the actual non-equilibrium state of the plasma might be.

We thus study here an equilibrium state of a moving magnetized plasma with a constant flow velocity U into which PUIs are permanently injected with a constant rate β_i. These freshly injected PUIs are incorporated into the bulk plasma flow in unstable velocity-space modes and drive turbulence power. These turbulent modes diffuse in wave vector space to both sides of the injection wave length $k_{\rm inj} = \Omega/U$, and a part of their power is re-absorbed both by background protons and PUIs (see Fahr & Chashei 2002; Chalov et al. 2004). This enforces energy diffusion of the PUIs starting from the injection point to lower and higher velocities and tends to establish a stationary equilibrium state, if it is guaranteed that the number of injected ions equals the number of ions lost at the lower border v_0 to a cold background of co-moving solar wind ions.

This situation is consistently formulated by the following two coupled differential equations for the distribution function $f(v)$ and the wave power function $W_k(k)$ (see, e.g. Chalov et al. 2006)

$$\frac{\partial f}{\partial t} = 0 = \frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2 D_{vv}\frac{\partial f}{\partial v}\right) + S\,(v) \qquad (20)$$

and

$$\frac{\partial W_k}{\partial t} = 0 = \frac{\partial}{\partial k}\left(D_{kk}\frac{\partial W_k}{\partial k}\right) + \gamma_k W_k + Q_k \qquad (21)$$

where D_{vv} and D_{kk} denote velocity-space and wave vector-space diffusion coefficients, γ_k is the wave growth coefficient, Q_k is the power injection due to wave generation by freshly injected PUIs, and $S\,(v)$ is the PUI injection rate.

Assuming a power law for the suprathermal ions in the form $f \sim v^{-\gamma}$ one finds the solution for the ion velocity distribution in the form

$$f(v) = f\,(v_0)\left(1 - \gamma\int_1^y \frac{{\rm d}y}{y^2 D_{yy}}\right) \qquad (22)$$

where $y = v/v_0$ and $D_{yy} = D_{vv}/D^0_{vv}$ has been introduced. Adopting the velocity diffusion according to Miller & Roberts (1995) leads to

$$D_{yy} = \frac{D_{vv}}{D^0_{vv}} = \frac{v_0}{v}\frac{\int_{k_v}^{\infty}\left(1-\frac{k_v}{k}\right)\frac{W_k}{k}{\rm d}k}{\int_{k_0}^{\infty}\left(1-\frac{k_0}{k}\right)\frac{W_k}{k}{\rm d}k} = y^{\alpha-1} \qquad (23)$$

where the lower resonance points are defined by $k_v = \Omega/v$ and $k_0 = \Omega/v_0$.

It can be shown that the stationary equilibrium condition is fulfilled, if f and W_k attain power laws in the forms $f(v) \sim (v/v_0)^{-\gamma}$ and $W_k(k) \sim (k/k_0)^{-\alpha}$ and if the condition $\alpha = \gamma = 3$ holds.

This then tells us that in such an idealistic equilibrium state the velocity diffusion driven by PUI-generated waves is characterized by the following velocity-dependence (see Eq. (23))

$$D_{yy} = \frac{D_{vv}}{D^0_{vv}} = y^{\alpha-1} = (v/v_0)^2\,. \qquad (24)$$

3. The PUI phase space transport equation

The transport of PUIs in the heliosphere is governed by three physical processes, namely convection with the solar wind, adiabatic or magnetic cooling, and diffusion in velocity space in one of the forms we have discussed in the sections before. For the case that magnetic cooling is considered, the equation describing the time evolution of the isotropic part of the pick-up ion phase space distribution (compare with, e.g., Isenberg 1987; Chalov et al. 1995; Fichtner et al. 1996, who studied adiabatic cooling) then reads

$$\frac{\partial f}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\left(v^2 D_{vv}\frac{\partial f}{\partial v}\right) + \frac{1}{v^2}\frac{\partial}{\partial v}\left(\frac{v^3 U}{r}\,f\right) - U\,\frac{\partial f}{\partial r} + S\,(r,v,t) \qquad (25)$$

where radial symmetry has been assumed, D_{vv} denotes a velocity diffusion coefficient, and $S\,(r,v,t)$ is the ion injection function. For our purpose the use of the diffusion approximation and, thus, an isotropic PUI distribution function is sufficient. This has been demonstrated by le Roux & Webb (2009), who used the more detailed focused transport approach that allows for anisotropic distributions (see also Isenberg 1997). They obtained for no pre-acceleration an asymptotic PUI distribution very close to that calculated by Vasyliunas & Siscoe (1976) and this way confirmed the validity of the diffusion approximation used for the analysis in the present paper. We generalize their approach in two respects: first, we include the PUI pre-acceleration and, second, we study the effect of a magnetically modified adiabatic cooling. While the pre-acceleration can in principle be studied with the focused transport as well, this is not true for the magnetic cooling because the CGL approximation, on which its derivation is based, cannot be derived from the focused transport equation.

We have solved the partial differential Eq. (25) with the well-tested VLUGR3 code developed by Blom & Verwer (1994) by employing the boundary conditions $(\partial f/\partial v)_{v=0} = 0$, $f(v_{\rm max}) = 0$, $(\partial f/\partial r)_{r=1\ \rm AU} = 0$, and an open (extrapolating) one at $r_{\rm max}$. In the considered domain of phase space the computed solutions turned out to be insensitive to the boundary conditions at $v_{\rm max} = 10$ U or 100 U and $r_{\rm max} = 29$ AU.

4. Simulations

4.1. Adiabatic cooling and diffusion: the reference case

Isenberg (1987) studied the PUI transport assuming adiabatic cooling and velocity diffusion due to resonant wave-particle interactions (Fermi II process) with low-frequency Alfvénic fluctuations propagating along the direction of the heliospheric magnetic field. Considering Alfvén waves and assuming power laws

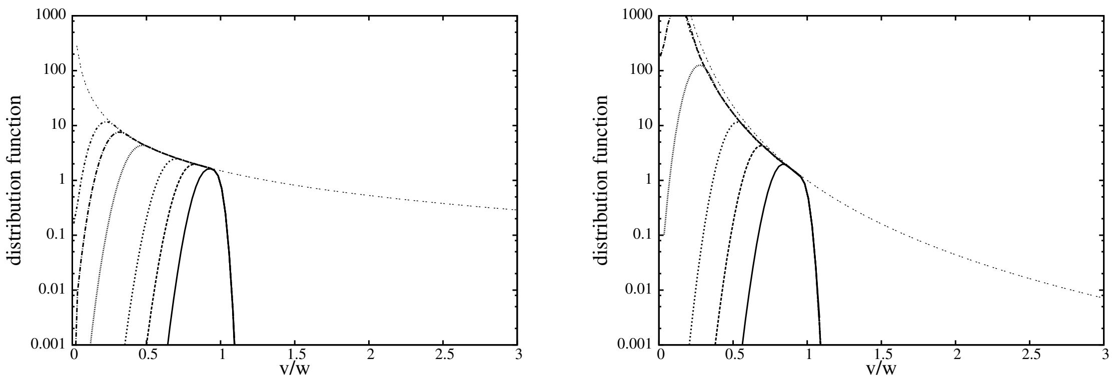

Fig. 2. The evolution of the PUI distribution with increasing heliocentric distance for negligible velocity diffusion. As in Fig. 1 the normalized distribution $\mathcal{F}$ is shown at the given heliocentric distances vs. normalized speed with $w = U$. With increasing heliocentric distance the initial (delta-)distribution is more strongly broadening towards lower velocities for magnetic cooling (*right panel*) as compared to adiabatic cooling (*left panel*).

for both the heliospheric magnetic field strength in radial direction $B(r)$ and the wave power $A(r)$ in the form

$$B(r) = B_0\left(\frac{r_0}{r}\right) \tag{26}$$

$$A(r) = A_0\left(\frac{r_0}{r}\right)^{\alpha} \tag{27}$$

the diffusion coefficent can be put into the form

$$D_F(r, v) = D_0\left(\frac{v}{U}\right)^{\gamma-1}\left(\frac{r}{r_0}\right)^{\gamma-\alpha} \tag{28}$$

$$D_0 = \frac{\pi v_{A0}^2 A_0 U^{\gamma-3}}{\gamma(\gamma+2)B_0^{\gamma}}\left(\frac{q}{mc}\right)^{2-\gamma}, \quad \gamma = \frac{5}{3} \tag{29}$$

with q and m the PUI's charge and mass, respectively, and c the speed of light. Assuming the same parameter values as in Isenberg (1987), namely

$$B_0 = 5\times10^{-5}\ \mathrm{G} \tag{30}$$

$$A_0 = 3.5\times10^{18}\ \mathrm{G}^2\,\mathrm{cm}^2 \tag{31}$$

$$\alpha = 8/3 \tag{32}$$

$$U = 400\ \mathrm{km\,s}^{-1} \tag{33}$$

$$v_{\mathrm{A}_0} = 50\ \mathrm{km\,s}^{-1} \tag{34}$$

and the source function as

$$S(r, v, t) = \frac{Q}{2\pi U^2}\,H(t-t_0)\;\delta(v-U) \tag{35}$$

the numerical solution of the transport equation with adiabatic cooling results in the evolutions of the pick-up proton velocity distribution shown in the left column of Fig. 1, which correspond to those determined with the semi-analytical procedure employed by Isenberg (1987) and given in Fig. 1 therein.

This successfully reproduced test serves in the following as a reference case.

4.2. Magnetic cooling and diffusion

The corresponding solutions of the transport Eq. (25) for magnetic cooling are displayed in the right column of Fig. 1.

The differences to the adiabatic case are obvious. For intermediate (middle panel) and strong diffusion (top panel) the distribution function is in the region $v < 1$ flatter than v^{-3}, which results in an actual heating, see the discussion in Sect. 3.4. Consequently, the distribution is "lifted" to higher values in these cases. That this effect is strongest for the intermediate case shown in the middle panel results from facts that both the gradient and the source term (on the right-hand side of Eq. (15)) are larger than in the case of strong diffusion. Clearly, in these cases the distribution function needs much longer to attain an asymptotic shape than in the corresponding Isenberg reference cases displayed in the left-hand panels.

For weak diffusion (bottom panel) the distribution function quickly develops strong negative velocity gradients significantly steeper than v^{-3}. Thus, there is actually magnetic *cooling*, and it is more efficient than the adiabatic cooling, so that the net acceleration of the PUIs due to the Fermi II acceleration process is less efficient. Consequently, the velocity spectra are slightly softer (i.e. steeper) for magnetic cooling and exhibit significantly higher intensities at lower speeds. As one must expect this becomes much clearer with less efficient diffusive acceleration, i.e. with a decreasing diffusion coefficient.

We also tested the effect of the square bracket term in Eq. (6) for the intermediate case. The results shown in the right panel in the middle row of Fig. 1 demonstrate that, as expected, it has no effect on the solution due to its very weak variation in the considered distance range.

Finally, note that for both adiabatic and magnetically induced energy changes the slopes of the velocity distributions are generally very similar as long as velocity diffusion is effective.

4.3. Transport without diffusion

In order to further compare the effects of the two alternative cooling terms, we have computed the two cases without velocity diffusion. This is shown in Fig. 2.

Now no PUIs are accelerated at all, but all are only cooled after injection into the solar wind bulk. Consequently, the newly, according to the source function $S(r, v, t)$ (see Eq. (35)), injected PUIs continuously decelerate and, thus, the velocity intervall below the injection velocity is gradually filled. The higher efficiency hereby of the magnetic cooling is very clear from the rapid evolution of the distribution and also manifests in a different asymptotic shape of the PUI distribution with higher values at lower velocities.

In both cases the asymptotic shape is a power law, which is indicated with the solid smooth lines in Fig. 2. While for the adiabatic cooling case one finds $f \sim v^{-3/2}$, a result originally already found by Vasyliunas & Siscoe (1976), for the magnetic cooling case one finds $f \sim v^{-5}$, a result already found analytically by Fahr (2007) showing that transport of PUIs solely under magnetic cooling leads to a v^{-5}-power law for ion energies below the PUI injection speed, see also Appendix B.

4.4. Alternative transport with different diffusion coefficients

In view of the various alternatives for the velocity diffusion coefficient as discussed in the introduction, a comparison of the corresponding asymptotic distributions is interesting. Therefore, in addition to the well-known Isenberg case with $D \sim v^{\gamma-1} = v^{2/3}$ (see Eq. (28)), we alternatively consider the cases

1. $D \sim 1/v$, i.e. a v-dependence that is consistent with κ-distributions (compare with Sect. 2.4); and
2. $D \sim v^2$, the v-dependence resulting for an idealistic equilibrium state treated in Sect. 2.5;
3. $D \sim v^4$, which for $U = 0$ or no radial divergence would result in an equilibrium spectrum of $f \sim v^{-5}$.

The resulting distributions are shown in Fig. 3 for the adiabatic (upper and middle panels) and the magnetically induced energy changes (lower panels), respectively.

Evidently, most distributions do not exhibit a v^{-5}-power law, because the shape of the distributions is, of course, codetermined by the diffusion process, i.e. the velocity dependence of the diffusion coefficient. Out of the alternatives discussed in Sect. 2, the third case with $D \sim v^4$ is, as expected, closest to a v^{-5} behaviour.

As already observed for the results displayed in Fig. 1, again the slopes of the distributions obtained for adiabatic and for magnetically induced energy changes are very similar, so that our findings for the power law tails are also true for the latter case.

5. Discussion of further improvements

While the theoretical basis of the effective cooling in the magnetized solar wind, as it appears to us, is safe and well described in this paper, the question of effectiveness and velocity dependence of the energy diffusion process is still not satisfactorily solved and perhaps needs further input from theoretical and observational studies. Therefore, also an alternative view on the basis of the velocity diffusion process should be investigated, different from Fermi-2 acceleration due to nonlinear scattering at counterflowing Alfven waves, which basically was the subject in this paper.

Another form of ion acceleration, for instance, is due to ion interactions with compressive MHD fluctuations like recently treated by Zhang (2010) or Fahr & Siewert (2011). Zhang (2010) describes the acceleration of suprathermal particles by compressional plasma wave trains represented by an infinite chain of identical, consecutive bulk velocity jumps characterizing traveling rarefaction and compression regions. Then solving a typical phase space transport equation including spatial diffusion and ion injection, he could calculate asymptotic forms of the ion distribution functions resulting after passage of an infinite number of such bulk velocity structures. In his derivation, however, for mathematical reasons, he assumes that PUI injection only takes place at the steps of the traveling structures and that cooling can be separated from spatial diffusion, which is treated by a constant, velocity-independent spatial diffusion coefficient. Depending then on the effectiveness of cooling he obtains asymptotic power law spectra with spectral indices $s \le -3$.

Fahr & Siewert (2011) show that a chain of bulk velocity jumps passing over a co-moving ion population under favourite quasi-equilibrium conditions leads to power laws. In order to validate this, two particle invariants should be conserved at bulk velocity jumps and during the interim periods between the passage of consecutive bulk velocity jumps the jump-induced anisotropic distribution should isotropize by pitch-angle scattering. If these conditions are fulfilled, a power law distribution with a spectral velocity power index of $s = -3$ is found. Spectral indices of $s = s_{\rm ob} \simeq -5$ as often found by plasma analysers in heliospheric space plasmas (see, e.g., Gloeckler 2003) in contrast are obtained, if only incomplete isotropization of the ions takes place. This may allow the conclusion that bulk velocity fluctuations lead to power laws, but only to those with actually observed power indices, if the effects of ion cooling and incomplete isotropization of distribution functions are properly taken into account.

Conservation of particle invariants as used by Fahr & Siewert (2011) in addition can only be valid for ions with subcritical velocities $v \le v_{\rm c} = 3L_U^2 \Delta U / \Lambda \Delta L$. If, to the contrast, passage times of jump structures and diffusion times are becoming comparable, i.e. $\tau_t/\tau_{\rm dif} = v\Lambda\Delta L/3L_U^2\Delta U \simeq 1$, then ions effectively escape from the jump structure not conserving the applied invariants, suggesting an upper validity border of $v_{\rm c} = 3U \dfrac{\Delta U}{U}\dfrac{L_U}{\Delta L}\dfrac{L_U}{\Lambda}$ where ΔL is the extent of the transition region from one to the next bulk velocity, i.e. from U_1 to U_2. $\Delta U = U_1 - U_2$ denotes the bulk velocity jump, L_U is the scale of a mono-bulk domain, v is the ion velocity, and Λ is the ion mean free path with respect to wave scattering.

With values $\Delta U/U = 50/400 = 1/8$, $L_U/\Delta L = 10$ AU/10^{-2} AU $= 10^3$, and $L_U/\Lambda = 10$ AU/1 AU $= 10$, this then evaluates to $v_{\rm c} \simeq 3.75 \times 10^3 U$, and indicates that their theory does not apply to ion velocities with $v \ge v_{\rm c}$. At these supercritical energies rather the theoretical approach presented by Fisk & Gloeckler (2006), Fisk & Gloeckler (2007), Fisk & Gloeckler (2008), and Fisk et al. (2010) will apply.

6. Summary and conclusions

We have studied the phase space transport of pick-up ions (PUIs) in the heliosphere for the cases that the particles are experiencing a second-order Fermi process, i.e. velocity diffusion, a convection with the solar wind, and adiabatic or the recently suggested magnetic cooling. The last process has not been studied before in detail within the framework of other transport codes. The reason for this is that the new process is not inherently included in the standard transport theory but needs an independent consideration based on the CGL approximation.

We have demonstrated that the magnetic cooling is, like adiabatic energy changes, not actually limited to cooling but can also, depending on the shape of the distribution function, result in heating if second order Fermi acceleration is not negligible. For those cases we found the slopes of the distribution functions obtained for adiabatic and for magnetically induced energy changes to be very similar. This similarity of the slopes does not hold for pure cooling with negligible velocity diffusion. While for adiabatic cooling the well-known $v^{-3/2}$-behaviour occurs, for magnetic cooling a v^{-5} velocity power law is found, thus confirming earlier analytical results by Fahr (2007).

For non-negligible second-order Fermi acceleration, the tails of the distribution functions are depending on the velocity diffusion coefficient. In the special case that the latter is proportional to v^4, a behaviour close to the v^{-5} power law is found, even at velocities higher than the injection velocity U.

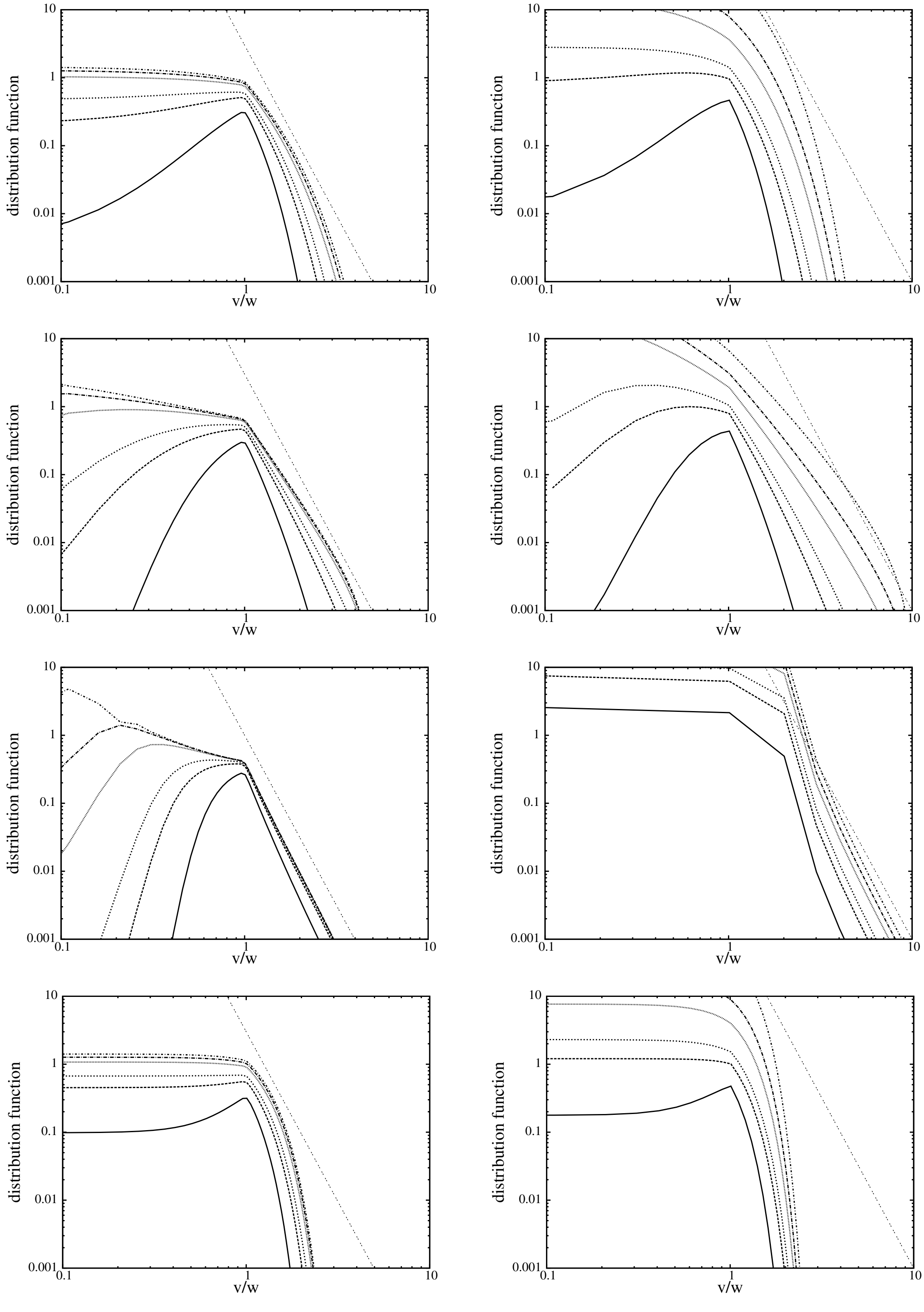

Fig. 3. The normalized distributions vs. normalized speed (as in Fig. 1) for four different diffusion coefficients. From the top to the bottom row: $D \sim v^{\gamma-1}$, $D \sim v^2$, $D \sim v^4$, and $D \sim 1/v$. The *left column* is for adiabatic cooling, the *right* for magnetically induced energy changes. In each panel the dashed straight line indicates the slope of v^{-5}.

The existence of an exact v^{-5} power law of PUI distribution functions can thus be confirmed for insignificant velocity diffusion and magnetic cooling. For non-negligible second-order Fermi acceleration this power law may be approximately valid given a specific velocity dependence of the diffusion coefficient.

In a forthcoming paper we shall present results obtained on the basis of the PUI transport equation, which we have solved here – then, however, applied to a newly derived diffusion process due to the stochastic action of consecutive solar wind bulk velocity jumps (see Sect. 5) on the ion distribution function.

Appendix A: Particle number conservation for magnetic cooling

Starting from Eq. (25) without the source term $S(r, v, t)$ one obtains after multiplication with v^2

$$\frac{\partial\left(fv^2\right)}{\partial t} = \frac{\partial}{\partial v}\left(v^2 D\frac{\partial f}{\partial v}\right) - U\,\frac{\partial\left(fv^2\right)}{\partial r} + \frac{\partial}{\partial v}\left(\frac{vU}{r}fv^2\right)$$

yielding with $n = \int_0^\infty fv^2 \mathrm{d}v$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \frac{\partial n}{\partial t} + U\frac{\partial n}{\partial r} = \left[v^2 D\frac{\partial f}{\partial v}\right]_0^\infty + \left[\frac{v^3 U}{r}f\right]_0^\infty = 0.$$

The second term on the right-hand side is vanishing because at the lower boundary $v = 0$ and at the upper one $f(\infty) = 0$. The first term vanishes as well because also $\partial f/\partial v|_\infty = 0$. Consequently, particle number (density) is conserved.

Appendix B: Analytic solution for magnetic cooling

For vanishing energy diffusion and constant H-atom density the solution for the pick-up ion distribution reads (Fahr 2007)

$$f(r, v) = \frac{1}{4\pi}\frac{r\beta\left(\frac{vr}{U}\right)}{Uv^3}. \tag{B.1}$$

Inserting this into Eq. (25), yields

$$\frac{1}{Uv}\frac{\partial}{\partial t}\left[r\beta\left(\frac{vr}{U}\right)\right] = \frac{\partial}{\partial v}\left[\beta\left(\frac{vr}{U}\right)\right] + 2\beta(r)\delta(v-U). \tag{B.2}$$

This can be transformed into ($r_v = vr/U$)

$$\frac{1}{v}\left[U\beta\left(\frac{vr}{U}\right) + r\frac{\partial\beta}{\partial r_v}v\right] - U\frac{\partial\beta}{\partial r_v}\frac{r}{U} = 2U\beta(r)\delta(v-U) \tag{B.3}$$

yielding

$$\frac{U}{v}\beta\left(\frac{vr}{U}\right) = 2U\beta(r)\delta(v-U), \tag{B.4}$$

which evidently can be rearranged into the form

$$1 = 2\,v\,\frac{\beta(r)}{\beta\left(\frac{vr}{U}\right)}\,\delta(v-U). \tag{B.5}$$

Integration with respect to the velocity v

$$\int_0^U \mathrm{d}v = 2\,\beta(r)\int_0^U \frac{v\,\delta(v-U)}{\beta\left(\frac{vr}{U}\right)}\,\mathrm{d}v \tag{B.6}$$

leads to

$$U = 2\,U\,\beta(r)\int_0^1 \frac{x\delta(x-1)}{\beta(xr)}\,\mathrm{d}x, \tag{B.7}$$

demonstrating the validity of the solution given in Eq. (B.1). Thus, at distances where H-atom densities can be taken as constant, the distribution function obeys a v^{-5}-power law.

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