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Quantitative estimates of the constraints on solar-like models imposed by observables★

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ABSTRACT

Context. Seismic parameters such as the large Δ_0 and small δ_{02} frequency separations are now being measured in a very large number of stars and begin to be used to test the physics of stellar models.
Aims. We estimate the influence of different observed quantities (oscillation frequencies, interferometry, etc.) and the impact of their accuracy in constraining stellar model parameters.
Methods. To relate the errors in observed quantities to the precision of the theoretical model parameters, we analyse the behaviour of the χ^2 fitting function around its minimum using the singular value decomposition (SVD) formalism. A new indicator called "weighting" quantifies the relative importance of observational constraints on the determination of each physical parameter individually. These tools are applied to a grid of evolutionary sequences for solar-like stellar models with varying age and mass, and to a real case: HD 49933 – a typical case for which seismic observations are available from space using CoRoT.
Results. The mass $\mathcal{M}$ is always the best determined parameter. The new indicator "weighting" allows us to rank the importance of the different constraints: the mean large separation Δ_0, the radius $R/R_\odot$, the mean small separation δ_{02}, the luminosity $L/L_\odot$, the effective temperature $T_{\rm eff}$. If the metallicity and age parameters are known, for example in an open cluster, using either individual or mean frequency separations yields the same uncertainties for masses less than 1.1 $M_\odot$. For HD 49933 the combination of $\mathcal{M}$ and Y_0: $\mathcal{M}^2 Y_0$ is well determined because of their correlation. However, they are poorly constrained individually. The frequency difference δ_{01}, if known with an error of about 0.3%, can determine the size of the convective core overshooting with about 3% accuracy.

Key words. methods: numerical – stars: fundamental parameters – stars: interiors – stars: oscillations

1. Introduction

The most promising way to probe the internal structure of a star is asteroseismology. The Sun has shown the way with helioseismology: observations of p-modes in the Sun have greatly improved solar modelling. Presently, p-mode oscillations are being detected in solar-like stars. From the ground, the organization of coordinated campaigns has been successful (Bouchy & Carrier 2002; Carrier & Bourban 2003; Bedding et al. 2004; Kjeldsen et al. 2005; Fletcher et al. 2006; Bazot et al. 2007). From space, the results are now pouring in on many types of stars. MOST (Walker et al. 2003) and WIRE (Hacking et al. 1999) have detected oscillations in classical pulsators, while CoRoT (Baglin & The CoRoT Team 1998) and *Kepler* missions (Borucki et al. 1997) are observing the p-mode spectrum in many solar-like stars.

These quantities, independent of the distance of the star, have a very important diagnostic power. However, to be really efficient, seismic data have to be complemented as much as possible by classical measurements, such as surface abundances, effective temperature, and surface gravity, all of which are obtained by spectroscopy. For nearby stars, radii and luminosities are sometimes measurable by interferometry, photometry, and the astrometry. For some binary systems, especially for eclipsing binaries, masses and radii are available with high accuracy.

To be able to derive the greatest amount of information from the data of the observed frequencies, many authors have developed diagnostic tools for probing the stellar interior and constraining the model parameters (e.g., Christensen-Dalsgaard 1984; Gough 1987). These tools are the large frequency separation $\Delta_{n,\ell}$, and the small frequency separation $\delta_{n,\ell}$, defined as

$$\Delta_{n,\ell} \equiv \nu_{n,\ell} - \nu_{n-1,\ell}, \quad \delta_{n,\ell} \equiv \nu_{n,0} - \nu_{n-1,\ell+2}. \qquad (1)$$

These quantities were inferred from the low-degree modes asymptotic expression. The physical interpretation of these seismic parameters can be found in Tassoul (1980) and Gough & Novotny (1990). According to the asymptotic theory, the large separation is proportional to the characteristic frequency of a star Δ_0 (or $\Delta_{n,\ell=0}$): $\Delta_0 \propto (M/R^3)^{1/2}$. Thus, in a homogeneous sequence of stars, the different characteristic frequencies of the oscillation spectrum are scaled by Δ_0, which is closely related to the star's mean density $\bar{\rho}$ ($\bar{\rho} \propto M/R^3$). The small separation depends on the variation in the sound speed in the central part of the star and may provide information about the composition of the star in its nuclear region. Consequently, the large and small separations can be combined to give estimates of both the mass and the age of a field star (Ulrich 1986; Christensen-Dalsgaard 1984, 1988), and if other information about the star (for example

★ Appendices A and B are available in electronic form at http://www.aanda.org its luminosity L) is available, then something might be learned about the convective mixing length or the composition (Gough 1987).

Thanks to the space missions CoRoT and *Kepler*, we have moved from ambiguous detections to firm measurements of seismic data in solar-like stars. It is therefore of interest to estimate the influence of constraints from seismic data and those obtained by photometry, spectroscopy, and interferometry, and to study in detail the effect of their precision on the determination of the fundamental stellar structure parameters like mass, age, and chemical composition. The knowledge of these basic parameters of stellar structure for a suitable sample of stars has a direct bearing on our understanding of the age and chemical evolution of the Galaxy.

To investigate the potential utility of all of the observations in an optimal way for determining the stellar parameters, we use a formalism based on singular value decomposition (SVD). This formalism relates errors in observed quantities to those in model parameters. In this context, we here address the following questions:

- What is the importance of classical and seismic constraints on the determination of stellar parameters?
- What is the behaviour of the χ^2 function, defined by differences between observations and model, around its minimum in the parameter space?
- Are the parameters correlated?
- What precision can be obtained on each of these parameters for given errors on the observables?

The paper is organized as follows: Sect. 2 presents the mathematical background of the fitting procedure and the SVD analysis. A new indicator "weighting" is defined to address the question of the importance of the "individual" observables in constraining the parameters. Section 3 describes the physics of our models and the observables used in this work. In Sect. 4, the method is applied to to a grid of solar-like stellar models to study the influence of classical and seismic observables with respect to mass and evolution. Section 5 presents the case of HD 49933.

2. Methods

The first step in studying the behaviour of the χ^2 space consists of computing the best model that satisfies a set of observational constraints, called the reference model (RM).

2.1. The reference model (RM)

Given a set of n measurements $y_{\mathrm{obs},i}$ (e.g. effective temperature, luminosity, seismic constraints) with associated error (σ_i), we first determine the best-fit model, or the RM, depending on a set of m parameters x_j (e.g. age, mass, etc.). This RM minimizes the χ^2 fitting function defined as

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{y_{\mathrm{obs},i} - y_{\mathrm{the},i}(\boldsymbol{x})}{\sigma_i} \right)^2, \qquad (2)$$

where $\boldsymbol{x} = \{x_j\}$ ($j = 1, \ldots, m$) is the parameter vector and $y_{\mathrm{the},i}(\boldsymbol{x})$ ($i = 1, \ldots, n$) are the model functions. The model functions, which we call observables, are the theoretical predictions for the observed quantities $y_{\mathrm{obs},i}$ such as effective temperature T_{eff}, luminosity L, etc.

There are several local approaches available to find this minimum (Press et al. 1992), two notable ones are the Levenberg-Marquardt (LM) algorithm and the singular value decomposition (SVD) method, etc. To automatically minimize the χ^2 fitting function, we use the LM algorithm. This algorithm is more robust than the SVD method because it is designed for cases where the convergence is difficult. The LM method is a combination of the Newton and the steepest descent methods. This algorithm behaves like a Newton method when convergence is good and like the steepest descent method when convergence is difficult.

There are also algorithms that allow us to find the global minimum, for example the genetic algorithms as applied to white dwarf stars by Metcalfe & Charbonneau (2003). An advantage of our local approach is that the link between the different constraints and free parameters is naturally established through the evaluation of the derivatives, $(\partial y_{\mathrm{the},i}/\partial x_j)$, at each step of the iteration. Problems with this method can arise when there are several local minima in the same region of the HR diagram. Therefore, it is impossible to find the global minimum and convergence problems may occur. In this case, either one can use global methods or construct a stellar grid and analyse the behaviour of χ^2 function near the minima. Our study is restricted to well-constrained solar-like stars with useful precision of seismic (oscillation frequencies) and classical information (spectroscopy, etc.). We assume a fixed identification of the oscillation modes and that they are not very far away from the asymptotic regime. In this case, the local approach works very efficiently, particularly when one has a good idea beforehand about the χ^2 behaviour. It is then possible to restrict the parameter domain to a region where there is only one local minimum.

2.2. Error-uncertainty problem

To analyse the behaviour of the χ^2 around its minimum, we use the singular value decomposition SVD method. This method was first introduced in the context of asteroseismology by Brown et al. (1994) and was subsequently used by several authors (e.g. Creevey et al. 2007). We recall the basic lines because of its importance in this paper. We first take the first order Taylor expansion of the model function or observable $y_{\mathrm{the},i}(\boldsymbol{x})$ in the neighbourhood of a reference set of parameters x_{j0} that minimize the χ^2 fitting function:

$$y_{\mathrm{the},i} = y_{\mathrm{the},i0} + \sum_{j=1}^{m} \frac{\partial y_{\mathrm{the},i}}{\partial x_j} \delta x_j, \qquad (3)$$

where $y_{\mathrm{the},i0}$ is the set of observables resulting from the reference set of parameters x_{j0}, $\delta x_j = x_j - x_{j0}$ and the derivatives $\partial y_{\mathrm{the},i}/\partial x_j$ are evaluated at $x_j = x_{j0}$. To determine the precision obtained on the model parameters and to show which combinations of parameters are well or poorly determined, we analyse the behaviour of χ^2 around its minimum.

After substituting the expression (3) into Eq. (2) and some manipulation, the behaviour of χ^2 around its minimum can be expressed thus

$$\Delta\chi^2 = \chi^2 - \chi^2_{\mathrm{min}} = \|\mathbf{D}\delta\boldsymbol{x}\|^2, \qquad (4)$$

where χ_{min} is a minimum at $\boldsymbol{x}_0$ ($\boldsymbol{x}_0 = x_{j0}$) and $\mathbf{D}$ is called the design matrix (Brown et al. 1994):

$$D_{ij} = \frac{1}{\sigma_i} \frac{\partial y_{\mathrm{the},i}}{\partial x_j}. \qquad (5)$$

This matrix relates small changes in the parameters to corresponding changes in the observables. This error analysis for solar-like stars is carried out using the SVD method.

Any $n \times m$ dimensional matrix $\boldsymbol{D}$ may be decomposed as

$$D_{n\times m} = U_{n\times m} W_{m\times m} V^{\mathrm{T}}_{m\times m}. \quad (6)$$

Inserting expression (6) into Eq. (4) and carrying out the matrix multiplications, one finds an m dimensional ellipsoidal equation, as shown in Brown et al. (1994)

$$\Delta\chi^2 = \frac{(\mathbf{V}^{(1)}\cdot\delta x)^2}{W_1^{-2}} + \frac{(\mathbf{V}^{(2)}\cdot\delta x)^2}{W_2^{-2}} + \ldots + \frac{(\mathbf{V}^{(m)}\cdot\delta x)^2}{W_m^{-2}}, \quad (7)$$

where $\mathbf{V}^{(1)}, \ldots, \mathbf{V}^{(m)}$ denote the columns of the matrix $\mathbf{V}$. In the SVD method, the columns of $\mathbf{V}$ are an orthonormal set of m vectors that are the principal axis of the error ellipsoid $\Delta\chi^2 = 1$, while the corresponding values of $\mathbf{W}^{-1}$ are the lengths of these axes.

The major advantage of the method is that it is more convenient to analyse the error ellipsoid in the parameter space, which gives information concerning the origin of the uncertainties on the obtained parameters. A vector V^j corresponding to a small singular value describes a direction in which χ^2 varies little. However, χ^2 increases very rapidly in the direction V^j, corresponding to a high singular value. If W_j is very high, the combination $V^{1,j}\delta x_1 + V^{2,j}\delta x_2 + \ldots$ is well-determined. However, if W_j is low, the combination $V^{1,j}\delta x_1 + V^{2,j}\delta x_2 + \ldots$ is very poorly determined.

The estimation of the variances-covariances matrix of the free parameters due to the measurement errors on the n observables is expressed by

$$Cov(\delta x_j, \delta x_k) = \sum_i^n \frac{V_{ji}V_{ki}}{W_i^2}. \quad (8)$$

The diagonal elements of Cov give the uncertainty in each of the parameters:

$$\epsilon(x_j) = \sqrt{Cov(\delta x_j^2)} = \sqrt{\sum_i^n \left(\frac{V_{ji}}{W_i}\right)^2}. \quad (9)$$

To rule out the ambiguity between the observable errors and the uncertainties in the parameters, we denote the former by σ and the latter by ϵ.

The variance-covariance matrix quantifies the correlation between the parameters j and k, given by

$$r_{jk} = \frac{Cov(\delta x_j, \delta x_k)}{\sqrt{Var(\delta x_j)Var(\delta x_k)}}. \quad (10)$$

If $|r_{jk}|$ is close to 1, then the parameters jk are highly correlated. This means χ^2 is partly degenerate and the parameters j and k are poorly constrained individually.

2.3. Weighting and significance

Many studies (e.g., Brown et al. 1994; Creevey et al. 2007; Ozel et al. 2010) have already asked the question of the role of the constraints on the results of the fitting. The usual approach consists of defining a quantity that estimates the importance of each observable on the *global* solution, called the "significance". This has been already done several times, in particular by Brown et al. (1994). Here we try to go a step further and evaluate the importance of the observables on the determination of *each* parameter of the solution.

Weighting of an observable: the approach used here allows us to quantify the importance of each observable on the determination of parameters. It evaluates the impact of a given observable to constrain *each individual parameter* on the given confidence limit. To measure the weighting of a given observable $y_{\mathrm{obs},i}$, we compare two situations, completing the SVD analysis with and without a given observable. If the uncertainty on the jth parameter $\epsilon(x_j)$ increases abruptly when we remove this constraint, we conclude that this observable has a large weight and is important in constraining this parameter. Otherwise, it has no effect in determining the parameter. Furthermore, this approach gives a quantitative error estimate that exactly shows how the uncertainties of the parameters change when one removes/includes one of the constraints.

Significance: the "significance" approach has been proposed by Brown et al. (1994). The significance points out the importance of each observable for *the full parameter solution*. It is defined by the following equation:

$$S_i = \left(\sum_j^M U_{ij}^2\right)^{1/2}. \quad (11)$$

In the remainder of the paper, the term "significance" represents this indicator. It can be understood as follows. Let us assume that for a set of observables $\boldsymbol{y}_{\mathrm{obs},0}$ we have found the minimum of χ^2 at $\boldsymbol{x}_0$. Then, we continuously modify the value of the ith observable $y_{\mathrm{obs},i}$ while keeping the other observables constant. At each new value of $y_{\mathrm{obs},i}$, we can associate a set of parameters $\boldsymbol{x}$ minimizing the χ^2 function. While doing this, we move from $\boldsymbol{x}_0$ to a point $\boldsymbol{x}_1$ such that $\Delta\chi^2 = \chi^2(\boldsymbol{x}_1) - \chi^2(\boldsymbol{x}_0) = 1$ (recall that the ellipsoid $\Delta\chi^2 < 1$ can be interpreted as a confidence region, see Press et al. 1992, Sect. 14.5: $\Delta\chi^2$ as a function of confidence level and degrees of freedom). We have

$$\boldsymbol{x}_1 - \boldsymbol{x}_0 = \mathbf{V}\mathbf{W}^{-1}\mathbf{U}^{\mathrm{T}}(0,\ldots,0,\delta y_{\mathrm{obs},i}/\sigma_i,0,\ldots,0)^{\mathrm{T}} \quad (12)$$

and thus,

$$\begin{aligned}\Delta\chi^2 &= 1 = \|D(\boldsymbol{x}_1 - \boldsymbol{x}_0)\|^2 \\ &= (\boldsymbol{x}_1 - \boldsymbol{x}_0)^{\mathrm{T}}\mathbf{V}\mathbf{W}^2\boldsymbol{V}^{\mathrm{T}}(\boldsymbol{x}_1 - x_0) \\ &= \|(0,..,0,\delta y_{\mathrm{obs},i}/\sigma_i,0,..,0)\mathbf{U}\|^2 = (\delta y_{\mathrm{obs},i}/\sigma_i)^2\|U_i\|^2 . \quad (13)\end{aligned}$$

Hence,

$$S_i = \left|\frac{\sigma_i}{\delta y_{\mathrm{obs},i}}\right|. \quad (14)$$

We can distinguish two cases.

1. $S_i \ll 1$. This implies $|\delta y_{\mathrm{obs},i}| \gg \sigma_i$ (Eq. (14)). It is very unlikely to encounter a measurement several standard deviations away from the mean value. A modification of the constraint $\delta y_{\mathrm{obs},i} \simeq \sigma_i$ would imply a negligible displacement of the minimum compared to the size of the $\Delta\chi^2 < 1$ domain, which means that this observable does not significantly affect the location of the minimum;
2. $S_i \sim 1$. Here, a variation $\delta y_{\mathrm{obs},i} \sim \sigma_i$ moves the minimum near the error ellipsoid edge ($\Delta\chi^2 = 1$), such an observable significantly affects the location of the minimum.

Table 1. Parameters of the models.

Parameters	Values
X_c	0.6, 0.4, 0.2
α	1.61
$\mathcal{M}$	0.9, 0.95, 1, 1.1, 1.25, 1.31, 1.4, 1.55 $M_\odot$
Y_0	0.2773
Z/X_0	0.0245

Table 2. Observables $y_{i,\mathrm{obs}}$ and expected errors σ_i.

Observables	Values ($y_{\mathrm{obs},i}$)	σ_i (%)
T_{eff} (K)	5250–7500	0.9
$L(L_\odot)$	0.4–7.5	2
$R(R_\odot)$	0.8–2	1
Δ_0 (μHz)	180–60	0.1
δ_{02} (μHz)	16.5–4.5	12.5
$Z/X_\star$	0.0245	23

Table 3. Increments δx_j for the RMs.

RM ($M_\odot$)	$\delta\tau$ (Myr)	$\delta\alpha$	$\delta\mathcal{M}$ ($M_\odot$)	δY_0	$\delta Z/X_0$
0.9	20	0.05	0.01	0.005	0.001
0.95	20	0.04	0.01	0.005	0.001
1	20	0.04	0.01	0.004	0.001
1.05	20	0.05	0.01	0.004	0.001
1.1	20	0.06	0.01	0.003	0.001
1.25	20	0.1	0.01	0.004	0.001
1.31	20	0.1	0.01	0.004	0.001
1.4	20	0.3	0.01	0.004	0.001
1.55	20	0.5	0.01	0.004	0.001

3. Stellar models and observables

3.1. Computation of stellar models and their seismic properties

A model of a solar-like star can be described with five free parameters: mass $\mathcal{M}$, age τ, mixing length parameter α to describe the outer convective zone, initial helium abundance Y_0, and initial ratio between heavy-element abundance and hydrogen Z/X_0. The reference values α and Y_0 are set to the values obtained by calibration of the Sun (see Table 1) and Z/X_0 corresponds to the solar abundances of Grevesse & Noels (1993).

Models are computed using CESAM2k (Morel & Lebreton 2008) for each parameter set ($\mathcal{M}, \tau, \alpha, Y_0, Z/X_0$). The code uses the OPAL equation of state and OPAL96 opacities (Iglesias & Rogers 1996), completed at low temperatures with the opacities of Alexander & Ferguson (1994). The physical description of the convective transport is the standard mixing-length theory (MLT, Böhm-Vitense 1958). Diffusion is not included. The oscillation frequencies are calculated with LOSC (Scuflaire et al. 2008) for modes of $n = 15$–25 and $\ell = 0$–3, which could correspond to the detectable ones in the space asteroseismology era. CESAM2k calculates the following observables: effective temperature T_{eff}, luminosity $L/L_\odot$, metallicity Z/X, radius $R/R_\odot$, and seismic quantities: mean and individual large and small frequency separations Δ_0, δ_{02}, $\Delta_{0,i}$, and $\delta_{02,i}$.

The model parameters are given in Table 1. The observables and their expected standard errors that define the reference models are given in Table 2. Depending on the evolutionary state of the model, the values of the observables $y_{i,\mathrm{obs}}$ vary between the intervals shown in the second column. The expected errors of the reference models σ_i are given in percent.

Each derivative of the matrix D is computed from differences of δx centred on the RM values $\boldsymbol{x}_{j0}$ given in Table 1, i.e., $x_{j0} \pm \delta x$. The interval δx has to be sufficiently small so that the linear approximation is still valid, but also large enough to guarantee sufficient numerical accuracy. The increments δx_j for the grid of RM are reported in Table 3.

3.2. Observables and their errors

Spectroscopy provides effective temperature T_{eff}, surface gravity $\log g$, and surface abundances. The intrinsic luminosity $L_\star$ is derived from the apparent magnitude, the bolometric correction, and the distance. If the distance is not known, one has to rely on photometric calibrations. If the star is an eclipsing binary, mass M and radius R can be determined at a high accuracy. If it is sufficiently close, the limb-darkened angular diameter θ_{ld} can be estimated, and combined with the parallax, can give a direct estimate of the radius. The oscillation frequencies are obtained using photometric and/or spectroscopic observations over a long time series.

The relative errors of the observables in Table 2 are chosen as the limit of the present instrumental techniques. The oscillation frequencies can be determined with an accuracy of about 0.1 μHz. This is representative of current CoRoT observations lasting for about 120 days. The number of large individual separations used as constraints is 23, and that of individual small separations is 6.

To investigate how seismic data contribute to determining physical stellar parameters, we consider three cases:

1. using the mean large Δ_0 and small δ_{02} frequency separations as seismic constraints;
2. including the individual large and small separations ($\Delta_{0,i}, \delta_{i,02}$);
3. reducing the errors on some of the observables by considering a future situation.

We decided not to use the individual oscillation frequencies for several reasons.

- First, for a given absolute error on the frequencies $\delta\nu$, the relative error $\delta\nu/\nu$ (used in our analysis) decreases as the frequency ν increases, which means that more weight would be given to higher frequencies. By high frequency we mean a frequency typically higher than the frequency of maximum power $\nu_{\max}$.
- There are systematic errors in the estimation of theoretical frequencies. These errors can be caused by the adiabatic approximation or by not taking into account the dynamical effects of convection in the treatment of the physics of the outermost layers (Christensen-Dalsgaard & Thompson 1997). These systematic errors are more dominant at high frequency. Therefore, high frequencies should not be given a large weight.
- These systematic errors can be partly removed by using frequency separations.

It is accepted at present that the effective temperature T_{eff} is determined with a typical error of 100 K by photometry, and of about 50 K by spectroscopy (Bruntt 2009). σ[Fe/H], determined by a detailed spectroscopic analysis, may be of the order of 0.05 dex if the star is bright enough, does not rotate very rapidly ($v \sin i < 25$ km s^{-1}), and has a spectrum with sufficient quality (Bruntt 2009). We choose σ[Fe/H] $= 0.1$ dex. The metallicity

(Z/X) of the star is derived as $[\mathrm{Fe/H}] = \log \frac{(Z/X)_\star}{(Z/X)_\odot}$. Consequently, the relative error on $(Z/X)_\star$ is 23%. The accuracy of the surface gravity determinations is of the order of 0.2 dex (Kupka & Bruntt 2001).

The error on luminosity depends on the errors on the apparent magnitude (m_V), bolometric correction (BC), and parallax π. We can consider the relative error value on the parallax of 1% for bright stars, which translates to a relative error of 2% in luminosity.

The radius of a few single bright stars can be measured with an accuracy of about 1% (North et al. 2007) with interferometric methods or for eclipsing binaries. The error on radius depends on the errors on the parallax π and the angular size θ: $D(D_\odot) = 107.47\theta(\mathrm{mas})/\pi(\mathrm{mas})$. Stellar masses are measured directly for the eclipsing binary systems, with a relative error of 1%. However, they should be in general considered as unknown.

Table 2 lists the observables, including their relative errors in percent for the reference models. Scales and units used for errors on the observables and uncertainties on the parameters are expressed in logarithmic scale (relative error) so that errors are independent of the value of the measured observables and to facilitate comparison between errors in the different parameters. Using the logarithmic scale is not mandatory. However, it is the more appropriate choice if one does not want the results to depend on the units, the aim instead is to consider a broad part of the HR diagram, and the relative errors, not the absolute errors, are not expected to change much from one model to the other.

4. Results

We present two approaches for studying the parameter uncertainties. The first is a global analysis using all of the five free parameters and the second is restricted to well-chosen subparameter spaces.

The global analysis corresponds to real cases where all global parameters are unknown. However, it often appears in practical cases that the number of free parameters is larger than the number of observables. The SVD method is not designed for these underdetermined ($n < m$) and ill-posed problems. On the contrary, it is particularly powerful for overdetermined ($n > m$) systems. In our global analysis, there are five free parameters and six constraints. However, these constraints are not independent because the luminosity L, effective temperature T_{eff}, and radius R are related to each other by the Stefan-Boltzmann law. Thus, there are as many independent observables as there are free parameters. This means that the system is not overdetermined in the global analysis and it is not possible to use the "weighting" indicator.

Thus, for a detailed analysis of the relation between observables and parameters, we later restrict the parameter space to three parameters (for an analysis with two free parameters, see Ozel et al. 2010). This approach allows us to determine the "weighting" of each observable. We also note that some parameters are well known in some particular situations. For example, if the star is a member of a cluster, the age or the chemical composition could be well determined. Along the same lines as discussed in Brown et al. (1994), where the concept of pseudo-observable is introduced, the number of free parameters could be reduced in this situation. Furthermore, if one of the parameters is uncorrelated with all other parameters and has a fairly direct connection with one observable, then this parameter and the corresponding constraint can be safely ignored. A typical example is the metallicity, as we show below.

Table 4. Predicted uncertainties on the parameters $\epsilon(\boldsymbol{x})$.

X_c	$\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$	$\epsilon(Z/X_0)$	$\epsilon(\tau)$
			%		
			0.9 $M_\odot$		
0.6	2.73	13.51	12.39	23.00	27.30
0.4	2.74	11.20	8.86	23.00	15.01
0.2	2.77	10.20	6.94	23.00	14.63
			0.95 $M_\odot$		
0.6	2.76	16.29	14.34	23.00	30.37
0.4	2.74	10.62	8.56	23.00	15.55
0.2	2.75	10.47	6.95	23.00	14.90
			1 $M_\odot$		
0.6	2.71	11.17	10.67	23.00	23.80
0.4	2.73	10.57	8.72	23.00	19.72
0.2	2.85	17.58	5.58	23.00	47.64
			1.05 $M_\odot$		
0.6	2.70	11.34	10.74	23.00	26.79
0.4	2.70	10.54	9.03	23.00	23.19
0.2	2.71	10.44	8.00	23.00	20.78
			1.1 $M_\odot$		
0.6	2.68	10.91	10.69	23.00	27.32
0.4	2.68	10.56	9.56	23.00	27.02
0.2	2.70	10.68	8.68	23.00	21.35
			1.25 $M_\odot$		
0.6	2.70	10.89	14.67	23.00	26.63
0.4	2.74	10.50	13.59	23.00	15.56
0.2	2.84	10.25	15.04	23.00	13.17
			1.31 $M_\odot$		
0.6	3.30	9.93	17.42	23.00	25.99
0.4	3.26	10.87	18.71	23.00	17.47
0.2	2.77	9.76	18.50	23.00	4.63
			1.4 $M_\odot$		
0.6	4.83	11.32	27.28	23.00	29.35
0.4	5.28	13.98	28.74	23.00	18.52
0.2	3.14	11.71	24.30	23.00	13.86
			1.55 $M_\odot$		
0.6	5.61	17.92	279.36	23.00	3.74
0.4	9.23	22.95	139.61	23.00	17.72
0.2	12.52	31.96	76.34	23.00	26.52

Notes. The predicted uncertainties on the stellar parameters $\epsilon(\boldsymbol{x})$ are given in percent. X_c corresponds to the central hydrogen abundance. The set of observables and parameters is $\boldsymbol{Y}_{\mathrm{obs}} = (T_{\mathrm{eff}}, L/L_\odot, Z/X_\star, \Delta_0, \delta_{02}, R/R_\odot)$ and $\boldsymbol{P} = (\mathcal{M}, Y_0, \alpha, Z/X_0, \tau)$, respectively. The three best-constrained parameters are shown in black. The uncertainties on the metallicity parameter $\epsilon(Z/X_0)$, which have the same precision for all reference models, are shown in grey and those of the least determined ones are shown in red.

The difficulty related to the small number of observables compared to the number of parameters was partly overcome thanks to the new seismic observables. However, the situation here is quite critical. It can also be overcome by reducing the number of parameters and trying to reach a less complex situation.

Finally, reducing the number of parameters facilitates the interpretation of the results. To understand the connection between different parameters and observables, we take the three best constrained parameters, shown in black in Table 4, assuming that the other parameters are precisely known, as in open clusters. This is the approach adopted in this paper.

4.1. Global analysis

The relative uncertainties on the parameters $\epsilon(\boldsymbol{x})$ resulting from the SVD analysis for the global analysis are shown in Table 4.

In this table we give the results for the models with the central hydrogen abundance X_c = 0.6, 0.4, and 02, corresponding to the beginning, the middle, and the end of the main sequence. The three best-determined parameters are shown in black. The precisions on the metallicity parameter are shown in grey; they are the same for all reference models. The least determined parameter is shown in red. These results are interpreted as follows.

4.1.1. Mass ($\mathcal{M}$)

In all analyses, $\mathcal{M}$ is the best-determined parameter. For a relative error of 1% on the radius, we can achieve a precision of less than 3% on $\mathcal{M}$ for models less than 1.3 $M_\odot$. For models grater than 1.3 $M_\odot$, the precision decreases towards more massive solar-like stars. This increase of $\epsilon(\mathcal{M})$ comes from the correlation between $\mathcal{M}$ and other parameters, particularly mixing length parameter α and age τ. For example, the correlations between $\mathcal{M}$ and α ($|r_{\mathcal{M}\alpha}| = 0.18$), and $\mathcal{M}$ and τ ($|r_{\mathcal{M}\tau}| = 0.33$) for a 1.25 $M_\odot$ star increase to $|r_{\mathcal{M}\alpha}| = 0.61$, and $|r_{\mathcal{M}\tau}| = 0.69$ for a 1.31 $M_\odot$ star.

4.1.2. Mixing length parameter (α)

The relative uncertainty on this parameter $\epsilon(\alpha)$ varies between 14 and 6% for all masses. It also decreases during the evolution for a 1.1 $M_\odot$ star owing to the well constrained radius. For stars larger than 1.1 $M_\odot$, $\epsilon(\alpha)$ increases. For a 1.55 $M_\odot$ star, the uncertainty on this parameter is very large. This is because for more massive stars, $M/M_\odot > 1.55$, the change of α does not affect the general characteristics of the star (i.e., the radius R, the effective temperature $T_{\rm eff}$), because the stellar structure becomes insensitive to the value of α. Thus, the behaviour of χ^2 in the parameter space does not change in the direction corresponding to this parameter. It is therefore poorly determined.

4.1.3. Initial helium abundance (Y_0)

The uncertainty on Y_0, $\epsilon(Y_0)$, lies between 10 and 13% for stars less than 1.4 $M_\odot$ and is more or less constant for all considered evolutionary states. $\epsilon(Y_0)$ increases for a 1.55 $M_\odot$ star as the star evolves.

In fact, the uncertainty of this parameter as a function of the mass does not show a very clear trend because the relationship between helium abundance and internal structure is indeed complicated. If we increase the abundance of hydrogen (and thus decrease Y_0),

- the stellar structure changes in the deeper layers because the nuclear reaction rate increases;
- the number of free electrons increases, which then increases the opacity (κ);
- the mean molecular weight (μ) decreases, therefore, the equation of state changes.

This parameter is strongly correlated with the other parameters. The uncertainties on the other parameters (e.g., $\epsilon(\alpha)$ increases with the mass) will therefore affect the uncertainty of Y_0. Thus, it is difficult to interpret the behaviour of $\epsilon(Y_0)$.

4.1.4. Initial metallicity (Z/X_0)

Table 5 shows the coefficients of the correlation between Z/X_0 and other parameters for a 0.9 $M_\odot$ star. Z/X_0 is not correlated with the best determined parameter, the mass $\mathcal{M}$. Thus, any improvement on the accuracy of the observables other than the metallicity ($Z/X_\star$) allows us to better constrain the mass parameter without affecting the precision on the metallicity parameter Z/X_0.

Table 5. Coefficients of the parameters correlation.

X_c	$\lvert r_{Z/X_0,\mathcal{M}}\rvert$	$\lvert r_{Z/X_0,Y_0}\rvert$	$\lvert r_{Z/X_0,\alpha}\rvert$	$\lvert r_{Z/X_0,\tau}\rvert$
0.6	0.02	0.52	0.32	0.06
0.4	0.02	0.61	0.44	0.11
0.2	0.03	0.68	0.56	0.22

Notes. The coefficients of the correlation between the parameters are given for a 0.9 $M_\odot$ star. The set of observables and parameters is the same as that for Table 4. The relative error on the initial metallicity parameter is $\sigma(Z/X_*)/(Z/X_*) = 23\%$.

Table 6. Predicted uncertainties on the parameters $\epsilon(\boldsymbol{x})$.

X_c	$\epsilon(\mathcal{M})$	$\epsilon(\alpha)$	$\epsilon(Z/X_0)$ %	$\epsilon(\tau)$
0.6	2.53	9.65	19.63	18.49
0.4	2.36	8.16	18.09	11.78
0.2	2.31	6.87	16.77	11.82

Notes. $\epsilon(\boldsymbol{x})$ are given for a 0.9 $M_\odot$ star. The set of observables is the same as that for Table 4. The combination of the parameters is $\boldsymbol{P} = (\mathcal{M}, \alpha, Z/X_0, \tau)$. The relative error on the initial metallicity parameter is $\sigma(Z/X_*)/(Z/X_*) = 23\%$. The uncertainties on the metallicity parameter $\epsilon(Z/X_0)$ are shown in red.

It is normal that for a relative error on the constraint $Z/X_\star$, $\sigma(Z/X_\star)/(Z/X_\star) = 23\%$, we recover the same precision $\epsilon(Z/X_0) = 23\%$ for all reference models, which are shown in grey in Table 4. As just mentioned, we deal in reality with as many independent constraints as free parameters. The constraint of the metallicity $Z/X_\star$ determines the metallicity parameter Z/X_0, and independently (no-correlation) the other constraints determine the other parameters. This is the case for all considered values of $\sigma(Z/X_\star)$.

To have an overdetermined system, we remove one parameter in the analysis. For example, Table 6 shows the parameter uncertainties when we remove the initial helium abundance Y_0 parameter. Obviously, here the uncertainty on the metallicity parameter is smaller than the error on the constraint. Thus, one needs more independent observables than parameters to improve the precision on this parameter.

4.1.5. Age (τ)

The uncertainty on the age is expressed as the absolute error normalized by the lifetime of the star on the main sequence ($\delta\tau/\tau_{\rm MS}$). The precision on this parameter increases during the evolution particularly from a 1.1 $M_\odot$ star.

It is well known that the small separation $\boldsymbol{\delta}_{02}$ is sensitive to the evolutionary state of the star and consequently its age (Christensen-Dalsgaard 1984). For a more detailed analysis, we reduce the relative error on this constraint by a factor of 2 ($\sigma(\delta_{02})/\delta_{02} = 12.5 \rightarrow 6.25\%$) for a reference model of 0.9 $M_\odot$. The results of the SVD analysis are shown in Table 7. A comparison with the results from Table 4 shows that improving the accuracy of δ_{02} mainly gives a better determination of the age parameter.

Table 7. Predicted uncertainties on the parameters $\epsilon(\boldsymbol{x})$.

X_c	$\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$ %	$\epsilon(Z/X_0)$	$\epsilon(\tau)$
0.6	2.72	10.48	8.42	23.00	13.79
0.4	2.73	9.85	6.76	23.00	7.68
0.2	2.77	9.54	5.63	23.00	7.99

Notes. $\epsilon(\boldsymbol{x})$ are given for a 0.9 $M_\odot$ star. The set of observables and parameters is the same as that for Table 4. The relative error on δ_{02} is $\sigma(\delta_{02})/\delta_{02}$ = 6.25%. The three best determined parameters are shown in red.

4.2. Sub-space of the three best-constrained parameters and relative importance of each observable

To quantify the "weighting" of each observable, we give the uncertainties of the sub-spaces of the parameters for a reference model of 0.9 $M_\odot$ in Table 8 by including or excluding a given constraint. In Table 8, for example, the combinations of the varying parameters during the evolution of this model are $\mathcal{M}$, Y_0, and α, assuming that Z/X_0 and τ are fixed (i.e., precisely known). The results for each reference model are shown in Tables A.1–A.3 in the on-line material. In section Ⓢ$_A$ in these tables we present the uncertainties when we include all observables $\boldsymbol{Y}_{\rm obs}$ for each reference model in the SVD analysis. In section Ⓢ$_B$ we include the individual small and large separations $(\Delta_{0,i}, \delta_{02,i})$ instead of their mean values (Δ_0, δ_{02}). In section Ⓢ$_C$ we remove each constraint one by one in the SVD analysis to determine its weighting.

The major results are the following:

1. *Mean large separation* Δ_0: it has the largest weight on the determination of the mass parameter $\mathcal{M}$. Indeed, removing this constraint strongly increases the uncertainty of the mass $\epsilon(\mathcal{M})$. For example, for a 0.9 $M_\odot$ star in Table 8, $\epsilon(\mathcal{M})$ increases from 2.70% (general case: using all observables $Y_{\rm obs}$ in the SVD analysis) to 759.77% (removed case: removing each constraint one by one in the SVD analysis).
   In addition, there is a strong correlation between $\mathcal{M}$ and other parameters ($|r_{\mathcal{M},Y_0}| \simeq |r_{\mathcal{M},\alpha}| \simeq |r_{\mathcal{M},\tau}| \sim 1$). Therefore, removing this constraint not only reduces the precision of $\mathcal{M}$ but also the precision of the other parameters because of the correlation.
2. *Radius* $R/R_\odot$: with a relative error $\sigma(R/R_\odot)/(R/R_\odot)$ = 1%, it is one of the most important constraints after the mean large separation Δ_0. The error on the radius has an important weight on the determination of $\mathcal{M}$, α, and Y_0 as the uncertainties of these parameters increase by a factor of about 2 when this constraint is removed. Its impact on the determination of $\mathcal{M}$ is larger when it is combined with Δ_0 because $M \propto \Delta_0^2 R^3$.
3. *Mean small separation* δ_{02}: with a relative error of about 13%, the exclusion of this constraint has no influence on the determination of the parameters. However, with a good precision of about 7% on δ_{02} , as illustrated in Table 7, it has an important weight on the age τ.
4. *Luminosity* $L/L_\odot$ *and effective temperature* $T_{\rm eff}$: with a relative error $\sigma(L/L_\odot)/L/L_\odot \sim 2\%$ and $\sigma(T_{\rm eff})/T_{\rm eff} \sim 0.9\%$, $L/L_\odot$ and $T_{\rm eff}$ have much less weight than Δ_0 and $R/R_\odot$ on the determination of the parameters.
5. *Individual* $(\Delta_{0,i}, \delta_{02,i})$ *instead of mean* (Δ_0, δ_{02}): both give approximately the same results on the parameter uncertainties for models less than 1.1 $M_\odot$. For example, for a 1.1 $M_\odot$ star and X_c = 0.6 in Table A.2, $\epsilon(\mathcal{M})$ = 2.65% and 2.25%; $\epsilon(Y_0)$ = 6.26% and 5.30%; $\epsilon(\alpha)$ = 3.28% and 2.96% in section Ⓢ$_A$ and in section Ⓢ$_B$, respectively. This is because we are close to the asymptotic regime where the individual large separations are close to the mean values and the errors on the small separations are too large to give a significant additional constraint.
   For models greater than 1.1 $M_\odot$, the precision on the parameters is better with individual frequency separations because we are farther from the asymptotic regime. For example, for a 1.25 $M_\odot$ star and X_c = 0.6 in Table A.2, $\epsilon(\mathcal{M})$ = 2.53% and 1.29%; $\epsilon(Y_0)$ = 6.11% and 2.95%; $\epsilon(\alpha)$ = 3.14% and 2.43% in section Ⓢ$_A$ and in section Ⓢ$_B$, respectively.

5. Application to a real case: HD 49933

HD 49933 is a typical example of a single field star, in which strong classical constraints do not exist, but for which the seismic data are now available from the CoRoT satellite.

5.1. Observables and their errors

Mosser et al. (2005) have detected solar-like oscillations in this star during a ten-night observational run with the HARPS spectrometer. The first space-based high-quality photometric results obtained by CoRoT were presented by Appourchaux et al. (2008). The data consist of 42 individual frequencies of degree ℓ = 0, 1 and 2, lying between 1.2 and 2.4 μHz. The average large separation Δ_0 was found to be 85.9 ± 0.15 μHz derived from the degree l = 0 and l = 2 modes. We estimated the arithmetic mean frequency difference $\delta_{01} = \nu_{n,0} - (\nu_{n,1} + \nu_{n-1,1})/2 = -0.48 \pm 0.32$, by using the set of frequencies reported by Appourchaux et al. (2008). We do not consider here the small separation δ_{02} because the associated error bars are too large.

HD 49933 has a measured iron abundance of [Fe/H] = −0.37 dex (Solano et al. 2005), lower than the Sun and Procyon. An effective temperature of 6780 ± 130 K was determined by Bruntt et al. (2008), who also found [Fe/H] = −0.46 ± 0.08 dex. More recently, Gillon & Magain (2006) found a quite similar value, which we adopted here, [Fe/H] = −0.37 ± 0.03 dex. With the solar value $Z/X_\odot$ = 0.0245 given by Grevesse & Noels (1993), it gives the metallicity $Z/X_\star$ and the error $\sigma_{Z/X_\star}$ given in Table 10. Bruntt (2009) reanalysed the star with high-quality spectra and determined more accurate atmospheric constraints ($T_{\rm eff}$ = 6570 ± 60 K, $\log g$ = 4.28 ± 0.06 dex, [Fe/H] = −0.44 ± 0.03 dex).

The absolute visual magnitude derived by Appourchaux et al. (2008) is based on the revised Hipparcos catalogue by van Leeuwen (2007). Using the bolometric correction from Bessell et al. (1998), they estimated the luminosity to be $L/L_\odot$ = 0.53 ± 0.01. Mosser et al. (2005) proposed a mass around 1.15 $M_\odot$ by matching theoretical evolution tracks to the observed $L - T_{\rm eff}$ error box in the colour–magnitude diagram.

The reference model of HD 49933 was calculated by Goupil et al. (2011) using the full-spectrum-of-turbulence treatment (FST Canuto & Mazzitelli 1992). The model indicates that core overshooting is necessary. The overshooting parameter ($d_{\rm ov}$ = 4.02) has been included in the study.

The other input physics are identical to those adopted for the reference models of solar-like stars. The adiabatic oscillation frequencies are calculated for the modes of degrees ℓ = 0−2, n = 13−27. In Table 9 we show the derivative (or design) matrix of the computed reference models of HD 49933. The complete design matrix is given in Table B.1 in the on-line material.

Table 8. Predicted uncertainties on the parameters $\epsilon(\boldsymbol{x})$.

$M = 0.9\ M_\odot$	$X_c = 0.6$				$X_c = 0.4$				$X_c = 0.2$			
	$\epsilon(\mathcal{M})$	$\epsilon(Y_0)$ %	$\epsilon(\alpha)$		$\epsilon(\mathcal{M})$	$\epsilon(Y_0)$ %	$\epsilon(\alpha)$		$\epsilon(\mathcal{M})$	$\epsilon(Y_0)$ %	$\epsilon(\alpha)$	
Ⓢ$_A$ General Case $\boldsymbol{Y}_{obs}$	2.70	6.54	5.79		2.73	6.28	4.41		2.77	5.89	3.38	
Ⓢ$_B$ Included Obs. in $\boldsymbol{Y}_{obs}$ $(\Delta_{0,i}, \delta_{02,i})$	2.48	6.01	5.38		2.64	6.07	4.31		2.72	5.77	3.39	
Ⓢ$_C$ Removed Obs. in $\boldsymbol{Y}_{obs}$	*Weighting*			S_i	*Weighting*			S_i	*Weighting*			S_i
Δ_0	759.77	1928.91	3019.79	0.99	50.87	122.47	156.54	0.99	36.97	81.35	84.58	0.99
δ_{02}	2.71	6.55	5.80	0.05	2.73	6.28	4.41	0.04	2.77	5.90	3.38	0.12
$R/R_\odot$	5.96	14.33	12.56	0.89	6.04	13.84	9.33	0.89	6.13	12.99	6.57	0.89
$L/L_\odot$	3.03	6.78	7.41	0.90	3.06	6.59	6.43	0.90	3.11	6.26	5.88	0.89
T_{eff}	3.02	7.53	6.21	0.63	3.05	7.20	4.58	0.63	3.09	6.69	3.39	0.62

Notes. $\epsilon(\boldsymbol{x})$ are given for a 0.9 $M_\odot$ star. The set of observables is $\boldsymbol{Y}_{obs} = (T_{eff}, L/L_\odot, \Delta_0, \delta_{02}, R/R_\odot)$. The combination of parameters is $\boldsymbol{P} = (\mathcal{M}, Y_0, \alpha)$, assuming that Z/X_0 and τ are known. Section Ⓢ$_A$ corresponds to the uncertainties if we include all observables $\boldsymbol{Y}_{obs}$ for each reference model in the SVD analysis. In section Ⓢ$_B$ we include the individual small and large separations $(\Delta_{0,i}, \delta_{02,i})$ instead of their mean values (Δ_0, δ_{02}). In the first part of section Ⓢ$_C$ we give the uncertainties obtained by removing each constraint one by one in the SVD analysis, which we define as the weighting. The second part of Ⓢ$_C$ shows the significance S_i of each constraint of the set of $\boldsymbol{Y}_{obs}$ in the SVD analysis.

Table 9. Derivative matrix.

Constraints	Parameters $\ln\tau$	$\ln\alpha$	$\ln\mathcal{M}$	$\ln Y_0$	$\ln Z/X_0$	$\ln d_{ov}$
$\ln T_{eff}$	–6.57	4.61	–5.96	–1.62	–2.41	1.53
$\ln L/L_\odot$	35.67	–0.51	332.56	147.39	–34.42	–2.77
$\ln\Delta_0$	–463.07	158.08	–2534.54	–1210.27	208.50	65.19
$\ln\delta_{01}$	7.17	–13.33	74.67	34.40	–8.97	–15.73
$\ln Z/X_\star$	0.0	0.0	0.0	0.0	14.42	0.0
$\ln\Delta_0(17)$	–145.03	33.61	–698.05	–333.54	52.14	22.19
$\ln\Delta_0(20)$	–47.95	21.74	–291.07	–140.00	26.05	5.94
$\ln\Delta_1(19)$	–84.56	45.03	–568.25	–271.55	54.20	11.05
$\ln\Delta_1(27)$	–31.40	12.95	–187.74	–89.64	16.63	4.27
$\ln\delta_{01}(19)$	12.89	–6.05	57.70	29.26	–5.34	–9.86
$\ln\delta_{01}(27)$	1.69	0.45	4.22	2.65	–0.43	–0.15

Notes. Logarithmic derivatives of various properties of the computed reference model of HD 49933 with respect to the model parameters. The mean and individual separations were defined (Δ_0, δ_{01}), as were some of the individual frequency separations $(\Delta_\ell(n), \delta_\ell(n))$.

5.2. Results

Table 10 summarizes the observational constraints and the characteristics of the reference model of HD 49933; the first column gives the values of the constraints and the corresponding measurement errors used as input for the determination of the reference model by the Levenberg-Marquardt algorithm; the second column gives the theoretical values of these constraints and the six model parameters $(\mathcal{M}, Y_0, \alpha, Z/X_0, \tau, d_{ov})$ for the reference model.

Given the uncertainties with the identification of the modes, we discuss three cases:

– Case 1: including the individual large separations$(\Delta_{0,i})$;
– Case 2: using all of the individual large seperations $(\Delta_{0,i})$ and the small frequency differences $(\delta_{01,i})$;
– Case 3: reducing the errors on the seismic data considered in Case 2, by factors of 5 and 10. This accuracy can be achievable by the missions such as *Kepler* (Borucki et al. 1997), SONG (Grundahl et al. 2008).

Table 11 presents the parameter matrix **V** of the SVD solution for case 1 (left panel) and case 3 (right panel) where the errors on the seismic data are reduced by a factor of 10 in case 3. The columns are ordered from the highest to the lowest singular value in Table 11. Table 12 shows the uncertainties on the parameters for HD 49933 for a global analysis in cases 1, 2, and 3.

– *The best-determined parameters $\mathcal{M}$ and Y_0.* The combination of parameters corresponding to the first column of **V** (Table 11) is best determined. This vector mainly lies in the $\mathcal{M}$-Y_0 plane both in case 1 and 3. This shows that the combination $0.88\ln\mathcal{M} + 0.42\ln Y_0$, or equivalently $\mathcal{M}^{2.1}Y_0$ is determined with highest precision. The relative uncertainty on this combination is given by $\delta(\mathcal{M}^{2.1}Y_0)/(\mathcal{M}^{2.1}Y_0) = 1/(0.42W_1) = 0.038\%$ for case 1 and 0.0036% for case 3. However, the individual uncertainties on these parameters are much larger, particularly in case 1 (9.48 and 22.48%, Table 12), because these two parameters also have significant components in the poorly determined vectors (e.g., V^5 for case 1, see Table 11). In addition, these two parameters are very highly correlated in case 1 : $r_{MY_0} = -0.99$.
These two effects are smaller in case 3, as shown by the significant decrease of their uncertainties from case 1 to case 3 in Table 12.

Table 10. Observations of HD 49933 and the properties of the RM.

Observations		σ	Model observables	Model parameters	
$T_{\rm eff}$(K)	6780	130	6669	τ (Gyr)	3.72
$L/L_\odot$	0.53	0.01	0.54	α	1.03
Δ_0 (μHz)	85.9	0.15	85.63	Y_0	0.282
δ_{01} (μHz)	–0.48	0.32	0.41	Z/X_0	0.011
$Z/X_\star$	0.01024	0.00071		$\mathcal{M}$	1.15
				$d_{\rm ov}$	0.402

Table 11. Parameter matrix **V** for case 1 and case 3 of HD 49933.

$\mathcal{M}$	0.88	–0.14	0.20	–0.13	0.36	0.09		0.88	–0.03	0.10	–0.32	0.30	–0.12
Y_0	0.42	0.02	–0.24	0.00	–0.84	–0.24		0.42	0.07	0.05	0.57	–0.44	0.54
α	–0.05	0.27	0.92	0.06	–0.25	–0.10		–0.06	0.47	–0.22	0.32	0.74	0.26
Z/X_0	–0.07	0.36	–0.05	–0.93	0.00	–0.02		–0.07	0.22	–0.24	–0.67	–0.15	0.64
τ	0.17	0.87	–0.21	0.33	0.10	0.24	,	0.16	0.43	–0.71	0.04	–0.33	–0.43
$d_{\rm ov}$	–0.02	–0.16	0.07	–0.09	–0.30	0.93		–0.04	0.73	0.62	–0.10	–0.19	–0.17
	↓	↓	↓	↓	↓	↓		↓	↓	↓	↓	↓	↓
$W_{i,...,6}$	2618	64	29	14	4	2		27 820	1660	534	84	54	25

Notes. The parameter matrix **V** of the SVD solution for case 3 is calculated for a factor of 10. The last lines show the corresponding singular values obtained by the SVD analysis.

Table 12. Predicted uncertainties on the parameters $\epsilon(\boldsymbol{x})$.

	Case 1	Case 2	Case 3	
ϵ (P)%	$\Delta_{0,i}$	$\Delta_{0,i}, \delta_{01,i}$	$\Delta_{0,i}, \delta_{01,i}(\sigma_{\Delta_{0,i},\delta_{01,i}} \searrow 5)$	$\Delta_{0,i}, \delta_{01,i}(\sigma_{\Delta_{0,i},\delta_{01,i}} \searrow 10)$
$\mathcal{M}$	9.48	5.34	1.47	0.81
Y_0	22.48	13.12	4.23	2.37
α	8.00	4.98	2.58	1.75
Z/X_0	6.87	6.71	4.41	2.67
τ	10.16	4.79	2.80	1.79
$d_{\rm ov}$	37.81	2.96	1.22	0.77

Notes. $\epsilon(\boldsymbol{x})$ are given for HD 49933 for a global analysis in case 1, 2, and 3. The set of parameters is $\boldsymbol{P} = (\mathcal{M}, Y_0, \alpha, Z/X_0, \tau, d_{\rm ov})$. The set of classical observables $(T_{\rm eff}, L/L_\odot, Z/X_\star)$ is included in all cases. Case 1 corresponds to using Δ_i, while case 2 corresponds the situation where all seismic data are available. Case 3 illustrates the importance of increasing the accuracy on the seismic parameters. In Case 3 the first and second columns show the results by reducing the errors on the seismic data by a factor of 5 and 10, respectively.

– *The α and τ parameters.* Note that the α parameter for HD 49933 is described by the full-spectrum-of-turbulence treatment (FST Canuto & Mazzitelli 1992). In case 1 in Table 11 these parameters (α, τ) appear as dominant components in V^2 and V^3, which explains their relatively small uncertainties. However they do not appear as a dominant component in any vector V^i in case 3. This means that the error ellipsoid is oblique with respect to this direction. As a consequence, these parameters are not well constrained individually.
– *The initial metallicity Z/X_0.* The results are similar to those of the grid of reference models in Sect. 4.1.4. With the current precision in case 1, the $Z/X_\star$ observable determines Z/X_0 and the other observables constrain independently the other parameters. The comparison of case 1 and case 3 shows a slight decrease of the uncertainty on Z/X_0, indicating that if seismic data become very precise, they could help improving the precision on the metallicity parameter.
– *The overshooting parameter $d_{\rm ov}$.* The uncertainty on $d_{\rm ov}$ significantly decreases from case 1 to case 2 and then to case 3 ($\epsilon(d_{\rm ov})$ = 37.8, 2.96, and 0.77%, respectively, see Table 12). The huge improvement from case 1 to case 2 reflects the importance of the frequency difference information $\delta_{01,i}$ as an indicator of the extension of the convective core. This tendency is well known, e.g., Goupil et al. (2011) for HD 49933, Miglio & Montalbán (2005) for α Cen system, but we quantify it here.

6. Conclusions

Using classical methods (χ^2 minimisation and SVD analysis), we have studied the characteristics of the χ^2 surface close to its minimum. The analysis of the axes of the error ellipsoid $\Delta\chi^2 = 1$ helps to understand the relation between observable errors and uncertainty of the parameters. In addition to a classical global evaluation using the "significance", we also use a new indicator, the "weighting", which is able to quantify the relative importance of the observational constraints on the determination of *each physical parameters individually*.

In the mass and age range we examined, the "weighting" analysis ranks the relative importance of the different constraint: mean large separation Δ_0, radius $R/R_\odot$, mean small separation δ_{02}, luminosity $L/L_\odot$, and finally effective temperature $T_{\rm eff}$. However, the degree of their importance is different and varies depending on the mass, the evolutionary stage, and the considered combination of parameters. For models greater than 1.1 $M_\odot$, with an accuracy of 0.1 μHz, the importance of Δ_0 decreases because of the deviation of the frequencies from the asymptotic regime. With a relative error of 1%, the radius R has an important weight on the determination of $\mathcal{M}$, α, and Y_0 for models less than 1.3 $M_\odot$.

- The mean large separation Δ_0 plays an important role in constraining the mass parameter $\mathcal{M}$. By combining the two constraints Δ_0 and R, the relative precision on $\mathcal{M}$ is significantly improved because $M \propto \Delta_0^2 R^3$.
- With a relative error of about 13% on the mean small separation (δ_{02}), this constraint has no significant influence on the determination of the parameters. However, for a relative error of about 7%, the precision obtained on the parameters is significantly improved, particularly for the age.
- With a relative error on the effective temperature $\sigma(T_{\rm eff})/T_{\rm eff} \sim 0.9\%$ and the luminosity $\sigma(L/L_\odot)/L/L_\odot \sim 2\%$, these constraints have no significant weight in determining the parameters when R, Δ_0, and δ_{02} are available. However, we must not conclude from this that the non-seismic constraints $T_{\rm eff}$ and $L/L_\odot$ are no longer required in the asteroseismology era. When the mode identification remains uncertain and/or few frequencies are detected with large error bars, the non-seismic constraints such as $T_{\rm eff}$, $L/L_\odot$, and $\log g$ remain crucial for the analysis, particularly for distinguishing between multiple solutions.
- As for most stars of CoRoT and *Kepler*, radius and mass constraints are not available. Such a "real case" represented by HD 49933 shows that the new seismic ($\Delta_{0,i}, \delta_{01,i}$) constraints, associated with the classical ones ($T_{\rm eff}, L/L_\odot, Z/X_\star$), allow us to reach a precision of 6% on the mass, 5% on the age, 5% on the mixing length parameter α, 3% on the overshooting parameter, and about 13% on the initial helium abundance Y_0.
- The combination of $\mathcal{M}$ and Y_0: $\mathcal{M}^2 Y_0$ is well determined because of their correlation *for all the considered cases of HD 49933*. However, they are poorly constrained individually.
- As already known, in stars with a convective core the major indicator of the size of the overshooting region is the seismic quantity δ_{01}. This allows the uncertainty of the overshooting parameter $d_{\rm ov}$ to be determined better than 3% if we have a precision of factors of 5 or 10 better than the actual precision on δ_{01}.
- The seismic constraints with current precision cannot constrain the metallicity better than the spectroscopic value.

The type of the analysis described here can also be used to select the best targets for future seismic missions and to define the most relevant follow up observations from the ground.

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Appendix A: Results of $\epsilon(x)$ to sub-spaces of RMs

Table A.1. Predicted uncertainties on the parameters $\epsilon(\boldsymbol{x})$.

$M = 0.9\ M_\odot$	$X_c = 0.6$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$		$X_c = 0.4$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$		$X_c = 0.2$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$	
Ⓢ$_A$ General Case $\boldsymbol{Y}_{obs}$	2.70	6.54	5.79		2.73	6.28	4.41		2.77	5.89	3.38	
Ⓢ$_B$ Included Obs. in $\boldsymbol{Y}_{obs}$ $(\Delta_{0,i}, \delta_{02,i})$	2.48	6.01	5.38		2.64	6.07	4.31		2.72	5.77	3.39	
Ⓢ$_C$ Removed Obs. in $\boldsymbol{Y}_{obs}$	*Weighting*			S_i	*Weighting*			S_i	*Weighting*			S_i
Δ_0	759.77	1928.91	3019.79	0.99	50.87	122.47	156.54	0.99	36.97	81.35	84.58	0.99
δ_{02}	2.71	6.55	5.80	0.05	2.73	6.28	4.41	0.04	2.77	5.90	3.38	0.12
$R/R_\odot$	5.96	14.33	12.56	0.89	6.04	13.84	9.33	0.89	6.13	12.99	6.57	0.89
$L/L_\odot$	3.03	6.78	7.41	0.90	3.06	6.59	6.43	0.90	3.11	6.26	5.88	0.89
T_{eff}	3.02	7.53	6.21	0.63	3.05	7.20	4.58	0.63	3.09	6.69	3.39	0.62

$M = 0.95\ M_\odot$	$X_c = 0.6$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$		$X_c = 0.4$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$		$X_c = 0.2$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$	
Ⓢ$_A$ General Case $\boldsymbol{Y}_{obs}$	2.70	6.53	4.80		2.73	6.18	3.72		2.74	5.98	2.84	
Ⓢ$_B$ Included Obs. in $\boldsymbol{Y}_{obs}$ $(\Delta_{0,i}, \delta_{02,i})$	2.53	6.11	4.57		2.63	5.96	3.65		2.68	5.82	2.84	
Ⓢ$_C$ Removed Obs. in $\boldsymbol{Y}_{obs}$	*Weighting*			S_i	*Weighting*			S_i	*Weighting*			S_i
Δ_0	2032.09	5118.59	6793.08	0.99	47.62	111.92	125.07	0.99	32.05	72.13	62.59	0.99
δ_{02}	2.71	6.54	4.81	0.05	2.73	6.19	3.72	0.04	2.75	5.99	2.84	0.11
$R/R_\odot$	5.98	14.33	10.35	0.89	6.05	13.65	7.70	0.89	6.05	13.17	5.11	0.89
$L/L_\odot$	3.03	6.77	6.46	0.90	3.07	6.48	5.80	0.90	3.09	6.34	5.39	0.89
T_{eff}	3.02	7.51	5.09	0.63	3.05	7.09	3.81	0.63	3.06	6.81	2.84	0.63

$M = 1M_\odot$	$X_c = 0.6$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$		$X_c = 0.4$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$		$X_c = 0.2$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$	
Ⓢ$_A$ General Case $\boldsymbol{Y}_{obs}$	2.70	6.44	4.08		2.72	6.17	3.29		2.72	5.93	2.65	
Ⓢ$_B$ Included Obs. in $\boldsymbol{Y}_{obs}$ $(\Delta_{0,i}, \delta_{02,i})$	2.51	6.00	3.88		2.60	5.90	3.22		2.62	5.72	2.63	
Ⓢ$_C$ Removed Obs. in $\boldsymbol{Y}_{obs}$	*Weighting*			S_i	*Weighting*			S_i	*Weighting*			S_i
Δ_0	274.35	680.79	795.75	0.99	48.48	113.60	113.72	0.99	35.53	79.51	64.07	0.99
δ_{02}	2.70	6.45	4.08	0.05	2.72	6.17	3.29	0.04	2.72	5.94	2.65	0.09
$R/R_\odot$	5.95	14.14	8.66	0.89	6.03	13.62	6.61	0.89	5.99	13.03	4.49	0.89
$L/L_\odot$	3.03	6.69	5.82	0.90	3.06	6.46	5.47	0.90	3.07	6.28	5.28	0.89
T_{eff}	3.01	7.42	4.26	0.63	3.03	7.07	3.33	0.63	3.04	6.77	2.67	0.63

Notes. $\epsilon(\boldsymbol{x})$ are given for 0.9, 0.95 and 1 $M_\odot$ stars. The set of observables is $\boldsymbol{Y}_{obs} = (T_{eff}, L/L_\odot, \Delta_0, \delta_{02}, R/R_\odot)$. Section Ⓢ$_A$ corresponds to the uncertainties when we include all observables $\boldsymbol{Y}_{obs}$ for each reference model in the SVD analysis. In section Ⓢ$_B$, we include the individual small and large separations $(\Delta_{0,i}, \delta_{02,i})$ instead of their mean values (Δ_0, δ_{02}). In the first part of the section Ⓢ$_C$, we remove each constraint one by one in the SVD analysis to determine its weighting. The second part of Ⓢ$_C$ shows the significance S_i of each constraint of the set of $\boldsymbol{Y}_{obs}$ in the SVD analysis.

Table A.2. Predicted uncertainties on the parameters $\epsilon(\boldsymbol{x})$.

$M = 1.05\ M_\odot$	$X_c = 0.6$				$X_c = 0.4$				$X_c = 0.2$			
	$\epsilon(\mathcal{M})$	$\epsilon(Y_0)$ %	$\epsilon(\alpha)$		$\epsilon(\mathcal{M})$	$\epsilon(Y_0)$ %	$\epsilon(\alpha)$		$\epsilon(\mathcal{M})$	$\epsilon(Y_0)$ %	$\epsilon(\alpha)$	
Ⓢ$_{\rm A}$ General Case $\boldsymbol{Y}_{\rm obs}$	2.68	6.44	3.54		2.69	6.14	2.93		2.70	5.93	2.52	
Ⓢ$_{\rm B}$ Included Obs. in $\boldsymbol{Y}_{\rm obs}$ $(\Delta_{0,i}, \delta_{02,i})$	2.42	5.80	3.32		2.48	5.65	2.82		2.58	5.65	2.49	
Ⓢ$_{\rm C}$ Removed Obs. in $\boldsymbol{Y}_{\rm obs}$	*Weighting*			S_i	*Weighting*			S_i	*Weighting*			S_i
Δ_0	118.24	293.56	306.84	0.99	42.32	99.33	90.71	0.99	32.98	74.37	55.79	0.99
δ_{02}	2.68	6.44	3.55	0.04	2.69	6.14	2.93	0.03	2.70	5.94	2.52	0.07
$R/R_\odot$	5.93	14.17	7.39	0.89	5.97	13.54	5.61	0.89	5.99	13.09	3.94	0.89
$L/L_\odot$	3.02	6.68	5.41	0.90	3.04	6.43	5.24	0.90	3.05	6.26	5.26	0.89
$T_{\rm eff}$	2.99	7.41	3.64	0.63	3.01	7.04	2.93	0.63	3.02	6.77	2.57	0.63

$M = 1.1\ M_\odot$	$X_c = 0.6$				$X_c = 0.4$				$X_c = 0.2$			
	$\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$		$\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$		$\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$	
Ⓢ$_{\rm A}$ General Case $\boldsymbol{Y}_{\rm obs}$	2.65	6.26	3.28		2.66	6.02	2.79		2.68	5.90	2.50	
Ⓢ$_{\rm B}$ Included Obs. in $\boldsymbol{Y}_{\rm obs}$ $(\Delta_{0,i}, \delta_{02,i})$	2.25	5.30	2.96		2.25	5.09	2.58		2.40	5.29	2.43	
Ⓢ$_{\rm C}$ Removed Obs. in $\boldsymbol{Y}_{\rm obs}$	*Weighting*			S_i	*Weighting*			S_i	*Weighting*			S_i
Δ_0	92.73	226.03	229.42	0.99	40.50	94.13	84.19	0.99	31.13	70.65	51.75	0.99
δ_{02}	2.65	6.27	3.28	0.04	2.66	6.02	2.79	0.03	2.68	5.90	2.51	0.06
$R/R_\odot$	5.84	13.74	6.66	0.89	5.87	13.25	5.09	0.89	5.95	13.08	3.63	0.89
$L/L_\odot$	3.00	6.51	5.31	0.90	3.01	6.29	5.26	0.90	3.02	6.19	5.37	0.89
$T_{\rm eff}$	2.96	7.21	3.33	0.63	2.96	6.91	2.79	0.63	2.98	6.74	2.60	0.63

$M = 1.25\ M_\odot$	$X_c = 0.6$				$X_c = 0.4$				$X_c = 0.2$			
	$\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$		$\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\alpha)$		$\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\tau)$	
Ⓢ$_{\rm A}$ General Case $\boldsymbol{Y}_{\rm obs}$	2.53	6.11	3.14		2.63	6.14	2.78		2.45	6.20	3.72	
Ⓢ$_{\rm B}$ Included Obs. in $\boldsymbol{Y}_{\rm obs}$ $(\Delta_{0,i}, \delta_{02,i})$	1.29	2.95	2.43		1.76	4.19	2.60		1.87	4.36	2.11	
Ⓢ$_{\rm C}$ Removed Obs. in $\boldsymbol{Y}_{\rm obs}$	*Weighting*			S_i	*Weighting*			S_i	*Weighting*			S_i
Δ_0	5.69	86.33	23.34	0.99	6.42	12.98	11.75	0.99	23.83	60.79	16.00	0.99
δ_{02}	2.53	6.11	3.14	0.02	2.63	6.14	2.79	0.05	2.47	6.30	3.84	0.26
$R/R_\odot$	5.59	13.47	5.92	0.89	5.83	13.54	3.28	0.89	5.27	13.32	6.56	0.89
$L/L_\odot$	2.89	6.36	5.71	0.90	2.98	6.38	6.21	0.90	3.01	6.52	4.76	0.88
$T_{\rm eff}$	2.81	7.03	3.14	0.63	2.93	7.06	3.05	0.63	2.63	6.98	4.62	0.60

Notes. $\epsilon(\boldsymbol{x})$ are given for 1.05, 1.1, and 1.25 $M_\odot$ stars. The set of observables $\boldsymbol{Y}_{\rm obs}$ and the sections Ⓢ$_{\rm A}$, Ⓢ$_{\rm B}$, and Ⓢ$_{\rm C}$ are the same as the ones in Table A.1.

Table A.3. Predicted uncertainties on the parameters $\epsilon(\boldsymbol{x})$.

$M = 1.31\ M_{\odot}$	$X_c = 0.6$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$ %	$\epsilon(\alpha)$		$X_c = 0.4$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$ %	$\epsilon(\tau)$		$X_c = 0.2$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$ %	$\epsilon(\tau)$	
Ⓢ$_A$ General Case $\boldsymbol{Y}_{obs}$	2.29	5.56	3.02		2.64	6.12	3.03		2.21	5.51	1.16	
Ⓢ$_B$ Included Obs. in $\boldsymbol{Y}_{obs}$ $(\Delta_{0,i}, \delta_{02,i})$	0.96	2.13	2.43		2.34	5.05	2.13		1.10	2.47	0.66	
Ⓢ$_C$ Removed Obs. in $\boldsymbol{Y}_{obs}$		*Weighting*		S_i		*Weighting*		S_i		*Weighting*		S_i
Δ_0	25.14	63.23	76.66	0.99	7.94	17.36	12.32	0.99	11.08	28.13	1.59	0.99
δ_{02}	2.29	5.56	3.02	0.02	2.65	6.13	3.06	0.14	2.21	5.51	1.17	0.11
$R/R_{\odot}$	5.09	12.28	5.30	0.89	5.76	13.36	3.66	0.89	5.22	12.98	1.33	0.91
$L/L_{\odot}$	2.67	5.79	5.90	0.90	3.20	6.34	6.53	0.89	2.67	5.70	2.30	0.89
T_{eff}	2.54	6.39	3.03	0.63	2.87	7.04	3.29	0.63	2.38	6.21	1.43	0.61
$M = 1.4\ M_{\odot}$	$X_c = 0.6$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(Z/X_0)$		$X_c = 0.4$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\tau)$		$X_c = 0.2$ $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\tau)$	
Ⓢ$_A$ General Case $\boldsymbol{Y}_{obs}$	2.39	9.10	12.25		2.62	5.99	3.06		2.69	6.41	2.33	
Ⓢ$_B$ Included Obs. in $\boldsymbol{Y}_{obs}$ $(\Delta_{0,i}, \delta_{02,i})$	2.15	8.49	12.18		2.19	4.77	2.37		2.35	5.21	1.30	
Ⓢ$_C$ Removed Obs. in $\boldsymbol{Y}_{obs}$		*Weighting*		S_i		*Weighting*		S_i		*Weighting*		S_i
Δ_0	2.81	13.35	22.83	0.99	9.34	21.07	14.96	0.99	13.25	31.88	7.53	0.99
δ_{02}	2.40	9.10	12.26	0.21	2.63	6.01	3.08	0.11	2.69	6.41	2.34	0.10
$R/R_{\odot}$	3.98	14.86	17.59	0.81	5.66	13.14	5.12	0.89	5.90	14.01	2.34	0.89
$L/L_{\odot}$	2.60	10.36	17.34	0.82	3.41	6.57	6.06	0.89	3.20	6.64	5.08	0.89
T_{eff}	2.58	9.62	12.26	0.63	2.79	6.74	3.08	0.63	2.96	7.40	2.72	0.63
$M = 1.55\ M_{\odot}$	X_c=0.6 $\epsilon(\mathcal{M})$	$\epsilon(Y_0)$	$\epsilon(\tau)$		$X_c = 0.4$ $\epsilon(\mathcal{M})$	$\epsilon(Z/X_0)$	$\epsilon(\tau)$		$X_c = 0.2$ $\epsilon(\mathcal{M})$	$\epsilon(Z/X_0)$	$\epsilon(\tau)$	
Ⓢ$_A$ General Case $\boldsymbol{Y}_{obs}$	1.07	2.84	0.41		2.15	13.94	2.97		2.18	14.42	2.82	
Ⓢ$_B$ Included Obs. in $\boldsymbol{Y}_{obs}$ $(\Delta_{0,i}, \delta_{02,i})$	0.34	1.42	0.00		2.00	12.50	2.78		1.14	7.89	1.21	
Ⓢ$_C$ Removed Obs. in $\boldsymbol{Y}_{obs}$		*Weighting*		S_i		*Weighting*		S_i		*Weighting*		S_i
Δ_0	3.11	6.84	6.32	0.99	3.28	21.01	6.17	0.99	3.32	22.76	5.21	0.99
δ_{02}	1.08	2.86	0.42	0.12	2.17	14.06	2.99	0.06	2.18	14.43	2.82	0.10
$R/R_{\odot}$	2.33	5.89	0.88	0.89	3.01	19.47	4.10	0.89	2.98	19.94	3.46	0.88
$L/L_{\odot}$	1.23	2.87	0.53	0.89	2.58	14.34	3.90	0.89	2.76	15.53	5.15	0.89
T_{eff}	1.19	3.43	0.44	0.63	2.24	15.14	3.04	0.63	2.25	15.42	2.84	0.64

Notes. $\epsilon(\boldsymbol{x})$ are given for 1.31, 1.4, and 1.55 $M_{\odot}$ stars. The set of observables $\boldsymbol{Y}_{obs}$ and the sections Ⓢ$_A$, Ⓢ$_B$, and Ⓢ$_C$ are the same as the ones in Table A.1.

Appendix B: Derivative matrix for HD 49933

Table B.1. Derivative matrix

Constraints	Parameters					
	$\ln \tau$	$\ln \alpha$	$\ln \mathcal{M}$	$\ln Y_0$	$\ln Z/X_0$	$\ln d_{ov}$
$\ln T_{\rm eff}$	–6.57	4.61	–5.96	–1.62	–2.41	1.53
$\ln L/L_\odot$	35.67	–0.51	332.56	147.39	–34.42	–2.77
$\ln \Delta_0$	–463.07	158.08	–2534.54	–1210.27	208.50	65.19
$\ln \delta_{01}$	7.17	–13.33	74.67	34.40	–8.97	–15.73
$\ln Z/X_\star$	0.0	0.0	0.0	0.0	14.42	0.0
$\ln \Delta_0(15)$	–83.57	27.36	–444.42	–211.62	35.40	11.16
$\ln \Delta_0(16)$	–118.91	30.68	–592.85	–283.15	44.88	17.41
$\ln \Delta_0(17)$	–145.03	33.61	–698.05	–333.54	52.14	22.19
$\ln \Delta_0(18)$	–217.07	45.70	–1013.67	–486.53	70.42	31.06
$\ln \Delta_0(19)$	–78.81	37.32	–509.87	–243.52	47.44	10.59
$\ln \Delta_0(20)$	–47.95	21.74	–291.07	–140.00	26.05	5.94
$\ln \Delta_0(21)$	–37.96	16.65	–228.98	–110.12	20.22	4.89
$\ln \Delta_0(22)$	–23.94	10.37	–144.96	–69.18	13.09	3.44
$\ln \Delta_0(23)$	–28.44	8.03	–141.67	–67.69	10.73	4.04
$\ln \Delta_0(24)$	–49.36	14.60	–259.59	–123.46	20.58	7.61
$\ln \Delta_0(25)$	–87.62	25.13	–450.42	–214.64	35.63	13.43
$\ln \Delta_0(26)$	–105.98	29.51	–530.19	–254.34	40.02	15.27
$\ln \Delta_0(27)$	–55.40	21.96	–329.12	–157.42	28.95	8.54
$\ln \Delta_1(15)$	–31.08	8.14	–153.13	–73.15	11.22	4.49
$\ln \Delta_1(16)$	–44.22	12.12	–225.93	–107.65	17.72	6.85
$\ln \Delta_1(17)$	–73.60	14.50	–335.01	–160.39	22.91	11.16
$\ln \Delta_1(18)$	–80.55	25.79	–438.57	–209.65	35.19	11.37
$\ln \Delta_1(19)$	–84.56	45.03	–568.25	–271.55	54.20	11.05
$\ln \Delta_1(20)$	–52.70	22.33	–308.09	–148.26	26.37	6.49
$\ln \Delta_1(21)$	–48.31	23.36	–309.54	–147.80	28.53	6.54
$\ln \Delta_1(22)$	–55.56	19.42	–302.12	–143.94	24.88	7.72
$\ln \Delta_1(23)$	–50.74	13.91	–254.42	–121.11	18.99	7.15
$\ln \Delta_1(24)$	–49.28	15.31	–266.17	–126.20	21.69	7.42
$\ln \Delta_1(25)$	–70.73	18.65	–347.23	–165.91	25.81	9.70
$\ln \Delta_1(26)$	–42.79	14.49	–236.48	–113.15	19.30	5.92
$\ln \Delta_1(27)$	–31.40	12.95	–187.74	–89.64	16.63	4.27
$\ln \Delta_2(14)$	–25.16	8.91	–138.01	–65.57	11.25	3.36
$\ln \Delta_2(15)$	–38.56	9.99	–191.95	–91.64	14.39	5.65
$\ln \Delta_2(16)$	–53.30	12.94	–260.44	–124.31	19.68	8.20
$\ln \Delta_2(17)$	–132.65	27.38	–614.31	–294.68	42.11	19.12
$\ln \Delta_2(18)$	–51.35	23.84	–329.16	–157.09	30.24	6.93
$\ln \Delta_2(19)$	–21.35	10.06	–132.17	–63.48	11.94	2.64
$\ln \Delta_2(20)$	–20.67	9.11	–124.57	–59.88	10.93	2.61
$\ln \Delta_2(21)$	–19.99	8.96	–123.16	–58.72	11.19	2.82
$\ln \Delta_2(22)$	–23.82	6.89	–119.60	–57.09	9.10	3.31
$\ln \Delta_2(23)$	–26.86	7.92	–140.96	–66.99	11.08	4.02
$\ln \Delta_2(24)$	–28.62	8.32	–148.37	–70.61	11.77	4.29
$\ln \Delta_2(25)$	–41.26	11.37	–205.63	–98.56	15.40	5.73
$\ln \Delta_2(26)$	–19.77	7.79	–117.41	–56.12	10.27	2.92
$\ln \delta_{01}(15)$	2.45	–0.45	9.09	5.01	–1.13	–1.67
$\ln \delta_{01}(16)$	0.94	–0.31	3.87	2.08	–0.48	–0.70
$\ln \delta_{01}(17)$	1.27	–0.44	5.30	2.82	–0.54	–0.94
$\ln \delta_{01}(18)$	1.19	–3.77	22.42	11.28	–3.81	–3.33
$\ln \delta_{01}(19)$	12.89	–6.05	57.70	29.26	–5.34	–9.86
$\ln \delta_{01}(20)$	4.49	–3.52	28.93	14.07	–3.08	–4.93
$\ln \delta_{01}(21)$	105.94	–129.88	893.65	413.04	–105.15	–159.17
$\ln \delta_{01}(22)$	–2.43	1.53	–11.88	–5.39	0.75	2.44
$\ln \delta_{01}(23)$	1.09	7.32	–32.05	–12.97	4.23	7.09
$\ln \delta_{01}(24)$	0.77	2.88	–8.54	–2.85	0.97	2.49
$\ln \delta_{01}(25)$	10.23	11.10	–6.05	3.64	–0.20	7.12
$\ln \delta_{01}(26)$	2.00	0.84	1.78	1.48	0.11	0.15
$\ln \delta_{01}(27)$	1.69	0.45	4.22	2.65	–0.43	–0.15

Notes. Logarithmic derivatives of various properties of the computed reference model of HD 49933 with respect to the model parameters. The mean and individual separations have been defined (Δ_0, δ_{01}) and ($\Delta_\ell(n), \delta_\ell(n)$), respectively.