A&A 531, A10 (2011)
DOI: 10.1051/0004-6361/201116680

Astronomy & Astrophysics

On Öpik's distance evaluation method in a cosmological context (Research Note)

P. Teerikorpi

Tuorla Observatory, Department of Physics and Astronomy, University of Turku, 21500 Piikkiö, Finland
e-mail: pekkatee@utu.fi

Received 9 February 2011 / Accepted 19 April 2011

ABSTRACT

Aims. Öpik derived the distance to M 31 using its rotation, flux and angular size. We describe his method and its reliance on the mass-to-luminosity ratio M/L in an instructive way and consider it formally within a family of dynamical cosmic distance indicators.
Methods. We consider ways of overcoming the M/d degeneracy in methods where together with the dynamical equation one uses an auxiliary relation $M \sim d^n$ to infer the distance d (with examples for $n = 0, 1, 2, 3$). As a possible deep space analog to Öpik's method we assess hypothetical Eddington radiators ($M/L = M/L_{\rm Edd}$).
Results. We describe ways of calibrating Öpik-related methods that do not use directly the angular size and consider their sensitivity to the used local H_0. We find, for instance, that when the size-luminosity relation is calibrated using distance-independent reverberation mapping to infer the BLR size, the derived mass $M \propto L^\alpha \Delta\lambda^2$ does not depend on H_0, whereas the Eddington ratio $L_{\rm bol}/L_{\rm Edd}$ does (as $\propto H_0^{-2}$). This is illustrated using very luminous yet quiescent radio quasars at redshifts 0.5–1.6.

Key words. galaxies: distances and redshifts – distance scale

1. Introduction

In the era of the Great Debate, Öpik (1922) made a remarkable determination of the distance to the Andromeda nebula, using its rotation (as reported already in 1918 in Moscow; e.g. Einasto 1994). From the data available, Öpik first calculated the distance of M 31 to be 785 kpc, a value that he proposed instead to be 450 kpc in the 1922 paper based on a different value of the mass-to-luminosity ratio of M 31.

It is useful to write the principle of Öpik's method in a transparent way. If the object's mass is distributed spherically up to the point where we take the rotation velocity (at the angle θ), we can write for the mass, using the the rotation of the Earth around the Sun as a meter stick and similarly for the luminosity in Solar units:

$$\frac{M}{M_\odot} = \frac{d}{1\ \mathrm{AU}}\theta\left(\frac{V_{\rm rot}}{30\ \mathrm{km\,s^{-1}}}\right)^2 \quad \text{and} \quad \frac{L}{L_\odot} = \left(\frac{d}{1\ \mathrm{AU}}\right)^2\frac{f}{f_\odot}, \tag{1}$$

where f is the flux within the wavelength band that the mass-to-luminosity ratio refers to. Denoting this ratio by γ (in the solar unit), we can express the unknown distance d

$$d = \frac{1}{\gamma}\theta\left(\frac{V_{\rm rot}}{30\ \mathrm{km\,s^{-1}}}\right)^2\frac{f_\odot}{f}\ \mathrm{AU}. \tag{2}$$

It is instructive to define first the M/L ratio $\gamma_{\rm M\,31}$ to be equal to unity, corresponding to an unrealistic system consisting entirely of Sun-like stars. A modern value of the flux ratio $f_\odot/f_{\rm M\,31}$ is about $10^{11.73}$ (based on the Galactic and internal extinction-corrected V magnitude difference 2.53 – (−26.8) = 29.33 from the HyperLeda Database) and the rotation velocity at the horizontal part of the rotation curve is about 225 km s^{-1} (Carignan et al. 2006), starting at $\theta \approx 2.5$ deg.

These values would give the distance $d_{\rm M\,31} \approx 13$ Mpc for $\gamma_{\rm M\,31} = 1$ and ≈4.3 Mpc for $\gamma_{\rm M\,31} = 3$, while the modern distance of 0.77 Mpc inferred from several other methods implies that $\gamma_{\rm M\,31} \approx 17$ within the 2.5 deg radius, which corresponds to a large amount of dark matter.

Why then could Öpik obtain such an accurate value for the distance without knowing about dark matter? Apart from Öpik having had good luck, he also had data that covered the innermost nebula (within 2.5 arcmin and not 2.5 degrees from the centre!), where the dark matter is not important.

We relate Öpik's method to dynamical distance indicators which use different kinds of additional information (such as M/L) (Sect. 2). We consider ways of deriving and calibrating the required size parameter (Sect. 3), and consider a possible, though not yet practical, deep space indicator consisting of radiators at the Eddington limit (Sect. 4).

2. Öpik's idea among dynamical distance indicators

One may define a distance indicator as a method where an object is placed in 3D space so that its properties as observed through space agree with what we know about these objects, their constituents and the propagation of light (Teerikorpi 1997). In particular, using Öpik's method one shifts a galaxy to the distance where its calculated M/L ratio is equal to the assumed value.

2.1. A family of dynamical distance indicators

It is interesting to compare Öpik's method to dynamical distance indicators in general. We write a relation between the system's mass and an observable velocity quantity for a test particle

$$M = arV^2 = a\theta dV^2. \tag{3}$$

This formula contains a size r within the object (seen at angle θ at distance d) and the relevant velocity V (or dispersion σ). For an orbit around the mass M, the constant of proportionality a is simply G^{-1}. Omitting projection factors, the observations of θ and V then give the quantity

$$M/d = a\theta V^2. \tag{4}$$

If one can express M/d in another form containing observable quantities and known constants, in addition to the unknown d, then one can bypass the M/d degeneracy and derive the distance d. For instance, if the mass M can be given as $\propto d^n$ where $n = 0, 2$ or 3, then one can solve for d, but if $M \propto d$, then any prior dependence on d disappears. Sometimes one may express θ or V in terms of d. We consider a few examples (not all of which are used in practice):

(i) $n = 0$,
which would correspond to a standard object with a constant mass M_0. The unknown distance would then be

$$d = \frac{M_0}{a} \times \frac{1}{\theta_0 V^2}\cdot \tag{5}$$

(ii) $n = 1$,
where, for example, one might measure the gravitational redshift of light from the surface of a system, $z \approx GM/c^2 r$ (i.e., $n = 1$). One then obtains the expression $z = (V/c)^2\theta^{-1}$, without the distance. For the Sun (θ = its angular radius), this indeed gives the observed gravitational redshift, if we assume that we know our orbital velocity V independent of the orbit radius. One can then overcome the M/d degeneracy by expressing V^2 in the right side of Eq. (3) so that it contains the distance ($V = 2\pi d/T$, and T is one year), thus obtaining an estimate of the Sun's distance.
Another $n = 1$ example is the gravitational lens system where the mass of the lensing galaxy is given by $M = \beta\theta_{\rm E}^2 d$ (where β depends on the relative distances from redshifts and $\theta_{\rm E}$ is Einstein's angle). This is not enough to derive the distance together with the velocity dispersion and the virial theorem; additional information from the time delay is needed (Paraficz & Hjorth 2009).

(iii) $n = 2$,
In a special case, one has measured for a particle orbiting a mass point at the angular distance θ its orbital speed V and in addition has been able to determine its centripetal acceleration $\dot{V}_{\rm r}$ ($M = r^2\dot{V}_{\rm r}/G$). The distance then becomes

$$d = \frac{V^2}{\theta \dot{V}_{\rm r}}\cdot \tag{6}$$

On the basis of these kinematics, Miyoshi et al. (1995) measured the distance to the spiral galaxy NGC 4258 from observations of megamasers close to its center.
If one expresses the mass M using the luminosity L and the mass-to-luminosity ratio γ, so that $M = \gamma L = \gamma f d^2$, then the distance can be solved as

$$d = \frac{a}{\gamma} \times \frac{\theta_0 V^2}{f}\cdot \tag{7}$$

This method is similar to those used to determine the dynamical parallaxes of binary stars. It is also Öpik's method. Its practical extension to distant galaxies is obviously impaired by various problems, e.g. radius- and type-dependent M/L arising from evolving stars and dark matter. In Sect. 4, we consider hypothetical Eddington radiators for which M/L has a known value from physics.

(iv) $n = 3$,
If the mass can be expressed as $\rho_0 r^3 = \rho_0(\theta_0 d)^3$, and assuming that the density ρ_0 is known, the distance is evaluated to be

$$d = \left(\frac{a}{\rho_0}\right)^{1/2} \times \frac{V}{\theta_0}\cdot \tag{8}$$

The zero-velocity radius $R_{\rm ZV}$ is often used to infer the mass of a galaxy system. The spherical model with $\Lambda = 0$ leads to the estimator $M_0 = (\pi^2/8G)t_0^{-2}R_{\rm ZV}^3$, where t_0 is the relevant timescale (Lynden-Bell 1981; Sandage 1986). The distance is then

$$d = \left(\frac{a}{bk^3}\right)^{1/2} \times \frac{\sigma}{\theta_0}, \tag{9}$$

where $b = (\pi^2/8G)t_0^{-2}$, θ_0 is the angular radius of the mass concentration within which the velocity dispersion σ has been measured, and $k = \theta_{\rm ZV}/\theta_0$, where $\theta_{\rm ZV}$ is the angle corresponding to $R_{\rm ZV}$.

The parameter n ($M \sim d^n$) need not be an integer. For example, there might be a radius–mass relation $M \propto R^\alpha$.

2.2. Angular size and luminosity distances

When distances are large, the formulae contain the angular size distance $d_{\rm ang}$ and/or the luminosity distance $d_{\rm lum}$. These are related as $d_{\rm lum} = (1+z)^2 d_{\rm ang}$. For Öpik's method, Eq. (7), in terms of the angular size distance, then becomes

$$d_{\rm ang} = \frac{a}{\gamma} \times \frac{\theta_0 V^2}{f} \times \frac{1}{(1+z)^4}\cdot \tag{10}$$

The angular size distance $d_{\rm ang}$ can be expressed via the redshift of the object and the parameters of the Friedmann model (Mattig's relation).

3. Öpik's method without angular size

Using the M/L ratio as in Eq. (7), one needs both the angle and the flux. For distant, small objects one may have to relate the size to measurements other than the angular size.

3.1. Light travel-time argument, independent of distance

One may infer a size from a light travel time-argument (Blandford & McKee 1982). For a fixed M/L ratio, one then derives the luminosity distance to be (where $1+z$ transforms the observed time interval Δt to the local frame of the object)

$$d_{\rm lum} = \left(\frac{ac}{\gamma}\right)^{1/2} \times V\left(\frac{\Delta t}{(1+z)f}\right)^{1/2}. \tag{11}$$

In this case, Eq. (3) does not contain the unknown distance d that appears in the auxiliary expression $M/d^2 \propto \gamma f$.

3.2. The size is derived from luminosity: calibration

One may try to infer the size of the dynamically relevant region from luminosity (Koratkar & Gaskell 1991). In practice, there could be a luminosity-size ($L - r$) relation, which must be calibrated at low redshifts (Kaspi et al. 2000).

With a $L - r$ power law, the dynamical equation given in Eq. (3) is replaced by the expression (where b is an empirical constant of proportionality)

$$M = bL^{\alpha}V^2 = bf^{\alpha}d^{\alpha}V^2, \quad (12)$$

and one can derive from observations the quantity $M/d^{2\alpha}$. Assuming that the M/L ratio is known and equal to γ (i.e. $M/d^2 \propto \gamma f$), one can evaluate the luminosity distance if $\alpha \neq 1$ to be

$$d^{2(1-\alpha)} = \frac{b}{\gamma} \times \frac{V^2}{f^{(1-\alpha)}}. \quad (13)$$

We note that for $\alpha = 0.5$, Eq. (13) transforms into the familiar "theoretical" Tully-Fisher relation $L \propto \gamma^{-2}V^4$, which can also be derived from Eq. (7), writing luminosity ∝ (radius)^2× surface brightness.

How does the way in which the $L-r$ relation is calibrated affect the derivation of the Hubble constant at high redshifts when the M/L ratio is fixed to an adopted value? This is an interesting question because depending on the adopted calibration procedure the calculated H_0 will be in different ways sensitive to the value of M/L. One may list three different assumptions:

(1) the $L - r$ relation does not depend on H_0;
(2) we can derive the size r independently of H_0 (reverbaration mapping), but L is tied to H_0;
(3) both the size (via angular size) and the luminosity (via flux) depend on an assumed Hubble constant.

Depending on how the calibration is performed, the factor b in Eq. (13) will depend on the assumed Hubble parameter h as follows for the three cases mentioned above: (1) $b \propto h^0$; (2) $b \propto h^{2\alpha}$; (3) $b \propto h^{2\alpha-1}$. The calculated $M/L = \gamma$ for a high-redshift object will then depend on h as follows for each of the above assumptions respectively:

(1) $M \propto h^{-2\alpha}$, $\quad \gamma \propto h^{-2(\alpha-1)}$;
(2) $M \propto h^0$, $\quad \gamma \propto h^{+2}$;
(3) $M \propto h^{-1}$, $\quad \gamma \propto h^{+1}$.

In case (2), we find that as the size does not depend on the distance scale, a change in H_0 causes the $L - r$ relation from the calibrator sample to simply shift along the luminosity axis by a factor $(H/H_0)^{-2}$. As the inferred luminosity of a higher-z object is also changed by this same factor, a change in H_0 to H does not change the mass at all, but alters the M/L ratio by a factor of $(H/H_0)^2$.

3.3. On the calibration case 2

In practice, the calibration case (2) occurs when the masses of the compact nuclei in galaxies and quasars are derived using the known link between the size of the broad line region $R_{\rm BLR}$ and the optical luminosity at a continuum wavelength (see Vestergaard 2004), where the velocity is given by the line width $\Delta\lambda$ of some emission line (Hβ, CIV, or MgII)

$$M_{\rm BH} \propto \left[\lambda L_{\lambda}(\lambda)/10^{44}\,{\rm erg\,s^{-1}}\right]^{\alpha} \Delta\lambda^2. \quad (14)$$

A relation between size $R_{\rm BLR}$ and luminosity L can been obtained at low redshifts ($\lesssim$0.2) using a value of H_0 (Kaspi et al. 2000). The size can be inferred from reverberation mapping, and, as we have explained, a change in H_0 does not affect the derived masses, but in contrast can significantly affect the Eddington ratios $L/L_{\rm Edd}$

$$M_{\rm BH} \propto h^0, \qquad L/L_{\rm Edd} \propto 4\pi r_{\rm lum}^2 f_{\rm bol}/M_{\rm BH} \propto h^{-2}. \quad (15)$$

Table 1. Data ($M_{\rm min} < -25.5$ mag, $0.5 < z < 1.6$).

RA	δ	z	$M_{\rm min}$	$\log L_{\rm bol}$	$\log M_{\rm BH}$	Ref.
0024	+2225	1.118	–26.0	47.40	9.45	2
0405	–1219	0.574	–26.0	47.69	9.58	1
0414	–0601	0.781	–25.7	46.97	9.52	2
0454	+0356	1.345	–26.7	47.60	9.39	2
0637	–7513	0.651	–26.3	47.46	9.54	1
0952	+1757	1.472	–25.9	47.41	9.23	2
1458	+7152	0.905	–25.7	47.28	9.14	1
1954	–3853	0.626	–25.8	46.61	8.75	1
2216	–0350	0.901	–26.0	47.52	9.40	1
2255	–2814	0.926	–26.1	47.32	9.32	1
2344	+0914	0.677	–25.8	47.38	9.44	1

References. 1. Woo & Urry (2002); 2. Shields et al. (2003).

4. An illustration of the calibration case 2: Eddington radiators

In active galactic nuclei, the energy generation may be either linked to or limited by the Eddington luminosity. In its simplest form, this maximum value of luminosity that can be powered by spherical accretion depends only on the mass of the radiating object and on physical constants: $L_{\rm Edd} = 1.26 \times 10^{38}(M/M_{\odot})$ erg^{-1}. The corresponding M/L ratio is small, $\gamma_{\rm Edd} = 3 \times 10^{-5}$. These "Eddington radiators" would be among the most luminous stable sources within a class of a fixed mass.

Quasars exhibit a wide range of Eddington ratios $\epsilon = L/L_{\rm Edd}$. However, in a sample of the SDSS quasars within $0.5 \lesssim z \lesssim 2$ "the Eddington luminosity is still a relevant physical limit to the accretion rate of luminous quasars" (McLure & Dunlop 2004) and from the AGES survey Kollmeier et al. (2006) conclude that the black holes gain most of their mass while radiating near $L_{\rm Edd}$. These and similar works based on Eq. (14) usually assume a standard cosmology with $h \approx 0.7$.

We are presently unable to use either the spectrum or some other property that can be measured without knowing the distance, to identify quasars shining close to $L_{\rm Edd}$. This may change in the future. For instance, Wilhite et al. (2008) and Bauer et al. (2009) suggest that optical activity correlates inversely with the Eddington ratio (small variability at $L_{\rm Edd}$). Dong et al. (2009) conclude that the equivalent width of the MgII λ2800 emission line is governed by the Eddington ratio (small width, high ratio).

We can illustrate the result encapsulated in Eq. (15) without insisting that $\epsilon = 1$, though it is instructive to consider possible $\epsilon \approx 1$ candidates. Thus, we take from our previous work "AI" quasars, around $M_{\rm min} \approx -26.0 + 5\log h_{100}$ (a minimum brightness V mag). About 30 potential AI objects in the redshift range 0.5–1.7 are found in a list of radio quasars with UBV photometry (Teerikorpi 2000). They are very luminous ($M_{\rm min} < -25.6$), but of lower activity than fainter quasars on the basis of their optical variability and polarization (other properties in Teerikorpi 2001, 2003).

Among these objects, 11 (Table 1) appear in the lists of $M_{\rm BH}$ (Woo & Urry 2002; Shields et al. 2003) where $\alpha = 0.7$ and 0.5, respectively, were used in Eq. (14)[1]. Adjusted to the cosmology $(\Omega_{\rm m}, \Omega_{\Lambda}) = (0.3, 0.7)$, the $L_{\rm bol}$ versus $M_{\rm BH}$ graph (Fig. 1) shows the effect of varying H_0. With $h_{100} = 0.45$ (0.80), the AIs radiate above (below) the Eddington value. There is just a vertical shift by the factor $2\log 80/45$ and the masses $M_{\rm BH}$

[1] Two quasars would lie far below the trend followed by the others in Fig. 1: 0414-0601 and optically active 1954-3853 ($P_{\rm pol} = 11\%$, var $\geq 0.8^{\rm m}$).

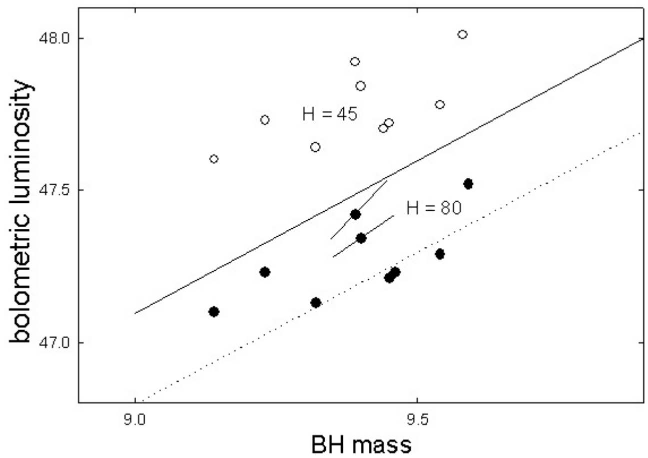

Fig. 1. $L_{\rm bol}$ vs. $M_{\rm BH}$ (in logarithmic scales) for very luminous yet quiescent quasars ($0.6 < z < 1.5$) for two values of H_0. The upper line is $L_{\rm Edd}$, and the lower one is $0.5L_{\rm Edd}$. Varying H_0 causes only a vertical shift (Eq. (15)). Two quasars show the effect of $\Delta\Omega_\Lambda = \pm 0.15$ ($\Omega = 1$); the steeper slope corresponds to $\alpha = 0.5$, the shallower one to 0.7.

remain the same. Normalized to approximate integral values of the linewidth-to-velocity factor f and the bolometric correction k, the Hubble parameter from this small sample may be written as $h \sim 0.65\epsilon^{-1/2}(k/10)^{1/2}/(f/1)$.

5. Concluding remarks

Grouping dynamical distance indicators according to how the M/d degeneracy in Eq. (3) is overcome by assuming an auxiliary relation $M \sim d^n$ (Sect. 2) seems to be interesting in bringing some order to the diverse indicators. It may even inspire one to probe new possibilities, such as the hypothetical Eddington radiators. It is often useful to consider how a cosmic phenomenon is related to the distance, even when its practical value as a distance indicator is not high.

Thus, if one assumes that a sample of quasars radiate at $L \approx L_{\rm Edd}$, one may infer which value of H_0 leads to this efficiency, though only if the systematic errors arising from the various steps of calculation are under control[2]. The objects may actually radiate below $L_{\rm Edd}$, so the inferred H_0 is a lower limit. Of course, one usually assumes H_0 to be equal to a standard value and then studies $L/L_{\rm Edd}$. Our results mean that the massive quasars here used as test objects do not radiate very far from $L_{\rm Edd}$.

It should be noted that the dynamical relation (in Eq. (3)) used above does not contain any local dark energy term, which would in all cases have a minor effect on systems smaller than galaxy groups (Chernin et al. 2009).

Acknowledgements. The author thanks the referee for his/her useful comments.

References

Bauer, A., Baltay, C., Coppi, P., et al. 2009, ApJ, 696, 1241
Blandford, R. D., & McKee, C. F. 1992, ApJ, 255, 419
Carignan, C., Chemin, L., Huchtmeier, W. K., & Lockman, F. J. 2006, ApJ, 641, L109
Chernin, A. D., Teerikorpi, P., Valtonen, M. J., et al. 2009, A&A, 507, 1271
Dong, X.-B., Wang, T.-G., Wang, J.-G., et al. 2009, ApJ, 703, L1
Einasto, J. 1994, Mitt. Astron. Gesellschaft, 77, 19, also http://www.aai.ee/muuseum/Reprints/HTML/ernstopikcentenary.htm
Elvis, M., Wilkes, B. J., McDowell, J. C., et al. 1994, ApJS, 95, 1
Kaspi, S., Smith, P. S., Netzer, H., et al. 2000, ApJ, 533, 631
Kollmeier, J. A., Onken, C. A., Kochanek, C. S., et al. 2006, ApJ, 648, 128
Koratkar, A. P., & Gaskell, C. M. 1991, ApJ, 370, L61
Lynden-Bell, D. 1981, The Observatory, 101, 111
McLure, R. J., & Dunlop, J. S. 2004, MNRAS, 352, 1390
Miyoshi, M., Moran, J., Herrnstein, J., et al. 1995, Nature, 373, 127
Paraficz, D., & Hjorth, J. 2009, A&A, 507, L49
Sandage, A. 1986, ApJ, 307, 1
Shields, G. A., Gebhardt, K., Salviander, S., et al. 2003, ApJ, 583, 124
Teerikorpi, P. 1997, ARA&A, 35, 101
Teerikorpi, P. 2000, A&A, 353, 77
Teerikorpi, P. 2001, A&A, 375, 752
Teerikorpi, P. 2003, A&A, 399, 829
Vestergaard, M. 2004, ApJ, 601, 676
Wilhite, B. C., Brunner, R. J., Grier, C. J., Schneider, D. P., & Vanden Berk, D. E. 2008, MNRAS, 383, 1232
Woo, J.-H., & Urry, C. M. 2002, ApJ, 579, 530
Öpik, E. 1922, ApJ, 55, 406

[2] The error due to the cosmological Ω parameters is rather small (Fig. 1). Changing $(\Omega_{\rm m}, \Omega_\Lambda)$, comoving distances change, depending on z, and one must correct each L to $(r_2/r_1)^2L$. This affects $L/L_{\rm Edd}$ only as $(r_2/r_1)^{2(1-\alpha)}$ (for $\alpha = 0.6$, $(r_2/r_1)^{0.8}$).