The correlation between C/O ratio, metallicity, and the initial WD mass for SNe Ia[★]

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Received 15 December 2010 / Accepted 12 May 2011

ABSTRACT

Context. When type Ia supernovae (SNe Ia) were chosen as distance indicators to measure cosmological parameters, the Phillips relation was applied. However, the origin of the scatter in the maximum luminosity of SNe Ia (or the variation in the production of ⁵⁶Ni) remains unclear. Both the metallicity in general and the carbon abundance of a white dwarf (WD) before a supernova explosion are important parameters, but neither has the ability to interpret the scatter in the maximum luminosity of SNe Ia.

Aims. We attempt to check whether or not the carbon abundance can be affected by initial metallicity.

Methods. We calculated a series of stellar evolution models with various masses and metallicities.

Results. We found that when $Z \leq 0.02$, the carbon abundance is almost independent of metallicity if it is plotted against the initial WD mass. However, when $Z > 0.02$, the carbon abundance is not only a function of the initial WD mass, but also metallicity, i.e. for a given initial WD mass, the higher the metallicity, the lower the carbon abundance. On the basis of some previous studies, i.e. which find that both a high metallicity and a low carbon abundance lead to a lower production of ⁵⁶Ni being formed during a SN Ia explosion, the effects of the carbon abundance and the metallicity on the amount of ⁵⁶Ni are enhanced by each other, which may account, at least qualitatively, for the variation in the maximum luminosity of SNe Ia.

Conclusions. Since the central density of WD before a supernova explosion may also play a role in the production of ⁵⁶Ni and the parameters (the carbon abundance, the metallicity, and the central density) are all determined by the initial parameters of the progenitor system, i.e. the initial WD mass, metallicity, orbital period and secondary mass, the amount of ⁵⁶Ni might be a function of the initial parameters. Then, our results might construct a bridge linking the progenitor model to the explosion model of SNe Ia.

Key words. white dwarfs – supernovae: general

1. Introduction

As principal distance indicators, type Ia supernovae (SNe Ia) can provide important constraints in determining cosmological parameters, which resulted in the discovery of the accelerating expansion of the universe (Riess et al. 1998; Perlmutter et al. 1999). The result was exciting and suggested the presence of dark energy. At present, SNe Ia are regarded as critical cosmological probes for testing both the evolution of the dark energy equation of state with time and the evolutionary history of the universe (Riess et al. 2007; Kuznetsova et al. 2008; Howell et al. 2009a).

When SNe Ia are applied as a distance indicator, the Phillips relation is adopted, which is a linear relation between the absolute magnitude of SNe Ia at maximum light and the magnitude drop in the B light curve during the first 15 days following the maximum (Phillips 1993). This relation was motivated by the observations of two peculiar events, i.e. SN 1991bg and SN 1991T, and implies that the brightness of SNe Ia is mainly determined by one parameter. There is a consensus that a SN Ia is produced by a thermonuclear explosion of a carbon-oxygen white dwarf (CO WD) and the amount of ⁵⁶Ni formed during the supernova explosion dominates the maximum luminosity of SNe Ia (Arnett 1982). However, the origin of the variation in the amount

of ⁵⁶Ni for different SNe Ia is still unclear (Podsiadlowski et al. 2008). Many efforts have been made to resolve this problem. Some multi-dimensional numerical simulations have shown that the ignition intensity (the number of ignition points) at the center of WDs or the transition density from deflagration to detonation are wonderful parameters for interpreting the Phillips relation (Hillebrandt & Niemeyer 2000; Höflich et al. 2006, 2010; Kasen et al. 2010). In addition, the ratio of nuclear-statistical-equilibrium (NES) to intermediate-mass elements (IME) in the explosion ejecta is likely to be the key parameter for the width of SN Ia light curve and its peak luminosity (Pinto & Eastman 2001; Mazzali et al. 2001, 2007). For the simulations above, a reasonable assumption is that these parameters are determined by one or some properties of SNe Ia progenitors. Since these parameters are allowed to vary in the numerical simulations of SNe Ia explosions, the question has become which property or properties of progenitor system determine these parameters, one that remains unclear. Lesaffre et al. (2006) carried out a systematic study of the sensitivity of ignition conditions for H-rich Chandrasekhar-mass single degenerate exploders to various properties of the progenitors, and suggested that the central density of the WD at ignition may be the origin of the Phillips relation (see also Podsiadlowski et al. 2008). This suggestion was upheld by detailed multi-dimensional numerical simulations of explosion (Krueger et al. 2010). However, in the models of Höflich et al. (2010), the central density is only a second parameter, and the cooling time of the WDs before mass transfer in Lesaffre et al. (2006) and

[★] Appendix is available in electronic form at
<http://www.aanda.org>

Krueger et al. (2010) is shorter than 1 Gyr. However, there are SNe Ia as old as 10 Gyr. The WDs with such a long cooling time may become more degenerate before the onset of accretion phase. Some other processes such as C and O separation or crystallization may occur, and dominate the properties of the CO WD (Fontaine et al. 2001). How the extremely degenerate conditions affect the properties of SNe Ia remains unclear. The suggestion of Lesaffre et al. (2006) should then be checked carefully under extremely degenerate conditions (Bravo et al. 2011). Some numerical and synthetical studies have shown that metallicity has an effect on the final amount of ^{56}Ni , and thus the maximum luminosity (Timmes et al. 2003; Travaglio et al. 2005; Podsiadlowski et al. 2006; Bravo et al. 2010) and that some observational evidence does exist of the correlation between the properties of SNe Ia and metallicity (Branch & Bergh 1993; Hamuy et al. 1996; Wang et al. 1997; Cappellaro et al. 1997; Shanks et al. 2002). However, the metallicity dose not seem to have the ability to interpret the whole scatter in the maximum luminosity of SNe Ia (Timmes et al. 2003; Gallagher et al. 2008; Howell et al. 2009b).

Nomoto et al. (1999, 2003) suggested that the average¹ ratio of carbon to oxygen (C/O) of a white dwarf at the moment of explosion is the dominant parameter of the Phillips relation (see also Umeda et al. 1999b). The higher the C/O, the larger the amount of nickel-56, and then the higher the maximum luminosity of SNe Ia. By comparing theory and observations, the results of Meng et al. (2009) and Meng & Yang (2010a) upheld this suggestion. Nomoto et al. (1999, 2003) used a total ^{12}C mass fraction included in the convective core of mass $M = 1.14 M_{\odot}$ just before the SN Ia explosion, $X(c)$, to represent the C/O ratio. Their suggestion is based on the dependence of $X(c)$ on metallicity being small when it is plotted against the initial mass of WDs when $Z \leq 0.02$. When $Z \leq 0.02$, the effect of metallicity on the amount of ^{56}Ni can be neglected. Only when $Z > 0.02$, does metallicity have a significant influence on the production of ^{56}Ni in SNe Ia explosions (Timmes et al. 2003). Here, we wish to verify whether the C/O ratio remains unaffected by metallicity when $Z > 0.02$.

In Sect. 2, we simply describe our method, and present the calculation results in Sect. 3. In Sect. 4, we show discussions and our main conclusions.

2. Method

Our work presented here is based on the single degenerate scenario, i.e. that a SN Ia originates from a CO WD in a binary system and that its companion is a main sequence or a slightly evolved star (WD+MS), a red giant star (WD+RG) or a helium star (WD + He star) (Whelan & Iben 1973; Nomoto et al. 1984). This scenario has been widely studied by many groups (Li & van den Heuvel 1997; Hachisu et al. 1999a; Langer et al. 2000; Han & Podsiadlowski 2004; Chen & Li 2007; Hachisu et al. 2008; Meng et al. 2009; Lü et al. 2009; Wang et al. 2009a,b; Meng & Yang 2010b,c; Wang et al. 2010). We assume that an initial CO WD is derived from a main sequence (MS) star in a primordial binary system (primary). The CO WD accretes hydrogen-rich material from its companion via a Roche lobe overflow or wind, where the companion is a normal star. The accreted hydrogen-rich material is burned into helium, and then the helium is converted to both carbon and oxygen. The CO WD increases its mass until the mass reaches $1.378 M_{\odot}$ (close to the

Chandrasekhar mass limit, Nomoto et al. 1984) where it explodes in a thermonuclear supernova. A binary system with the same primordial primary but different orbital period may produce a CO WD of different mass. For simplicity, we assume that if the MS mass of the primary in a primordial binary system is the same, the initial mass of the CO WD from the binary system is the same and equal to the core mass derived from the envelope-ejection model in Han et al. (1994) and Meng et al. (2008) (see below in details). Because the primary in a binary system may lose its hydrogen envelope because of the influence of secondary before it fulfills the criterion for the envelope ejection, the initial mass of CO WD in this paper should be taken as an upper limit to the real case. This assumption cannot affect our results because we only wish to find a relation between the C/O ratio, the WD mass, and metallicity. The method used here is similar to that in Umeda et al. (1999a) and Höflich et al. (2010).

The C/O ratio before the SN Ia explosion is a result of both stellar and binary evolution, i.e. during central helium burning and thin shell burning during the stellar evolution and the accretion to close to the Chandrasekhar mass limit ($1.378 M_{\odot}$, Nomoto et al. 1984). After the central helium burning phase, the C/O ratio is low, i.e. 0.25–0.5 depending on the initial mass and metallicity of a main sequence (MS) star (Umeda et al. 1999a). The C/O ratio obtained from the burning shell is ≈ 1 , because the helium in the shell has a lower density and a higher temperature than helium burning in the core (Höflich et al. 2010).

In the paper, we calculated a series of stellar evolution models with the primordial MS mass from $1 M_{\odot}$ to $6.5 M_{\odot}$ until the stars evolve to the asymptotic giant branch (AGB) stage. When a star evolves to the stage, its envelope may be lost if the binding energy (BE) of the envelope transforms from a negative phase to a positive one (Paczynski & Ziolkowski 1968; see also Fig. 2 in Meng et al. 2008). We calculated the BE of the envelope to be

$$\Delta W = \int_{M_c}^{M_s} \left(-\frac{Gm}{r} + U \right) dm, \quad (1)$$

where M_c is the core mass, M_s is the surface value of the mass coordinate m , and U is the internal energy of thermodynamics where those due to ionization of H and dissociation of H_2 , as well as the basic $\frac{3}{2} \mathcal{R}T/\mu$ for a perfect gas are all included. Here, we assume that a star will lose its envelope when the BE of the star's envelope increases to the point of $\Delta W = 0$ and the core mass at the point is the final WD mass. The method here is robust and its virtue is significant because we need not consider the specific mechanism of mass loss since the mass loss rate is very uncertainty (see Meng et al. 2008, in details about this method). We assume that after envelope ejection the remnant is a CO WD if carbon and oxygen have not been ignited at the moment of envelope ejection. Following shell burning, we assume that the C/O ratio is 1² until $M_{\text{WD}} = 1.378 M_{\odot}$ as did in Umeda et al. (1999a) and Höflich et al. (2010).

We use the stellar evolution code of Eggleton (1971, 1972, 1973), which has been updated with the latest input physics over the past three decades (Han et al. 1994; Pols et al. 1995, 1998).

¹ If there is no special statement, the C/O ratio in this paper is the average value over the whole WD structure just before the SN explosion.

² For the core chemical profile produced during the AGB phase, the steady increase in the carbon abundance in the region adjacent to the inner flat profile is synthesized by the shell in the early AGB, whereas the following abrupt peak is left by the thermally pulsing phase. The slope of the profile produced during the early AGB depends on both the initial mass and metallicity, and the C/O = 1 is not always a good approximation. However, to compare with previous results, we still simply make a constant-ratio assumption.

The chemistry of a WD is mainly determined by the competition between two major nuclear reactions powering the He burning, i.e. 3α and $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$. As discussed by Imbriani et al. (2001) and Prada Moroni & Straniero (2002), the final C/O after the central helium exhaustion not only depends on the rate of these two reactions, but also is significantly influenced by the efficiency of the convective mixing operating at the central helium burning phase. We set the ratio of mixing length to local pressure scale height, $\alpha = l/H_p$, to 2.0, and set the convective overshooting parameter, δ_{ov} , to 0.12 (Pols et al. 1997; Schröder et al. 1997), which roughly corresponds to an overshooting length of $0.25H_p$. The two parameters are adopted during the whole evolution of star. For the occurrence of convective instability near the He exhaustion in the central core, some breathing pulses are expected. (Castellani 1985, 1989). This phenomenon may also occur naturally in our code. The reaction rates are from Caughlan & Fowler (1988), except for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction, which is taken from Caughlan et al. (1985). The range of metallicity is from 0.0001 to 0.1, i.e. 0.0001, 0.0003, 0.001, 0.004, 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.08, and 0.1. The opacity tables for these metallicities are compiled by Chen & Tout (2007) from Iglesias & Rogers (1996) and Alexander & Ferguson (1994). For a given Z , the initial hydrogen mass fraction is assumed to be

$$X = 0.76 - 3.0Z, \quad (2)$$

(Pols et al. 1998), and then the helium mass fraction is $Y = 1 - X - Z = 0.24 + 2Z$. On the basis of the correlation between X , Y , and Z used here, Pols et al. (1998) accurately reproduced the color-magnitude diagrams (CMD) of some clusters.

In this paper, we skip thermal pulses by taking a longer time-step to reduce the computing time, i.e. we study the average evolution of thermally pulsing AGB models. This treatment of the thermal pulses may miss some information about the structural change of the envelope due to the thermal pulse. However, these treatment will not affect our final results because the situation of $\Delta W = 0$ is fulfilled before the onset of thermal pulse and after the point a simple assumption of $\text{C/O} = 1$ is adopted (see also in Meng et al. 2008).

3. Results

In this paper, we use total carbon abundance to represent the C/O ratio and a core mass to represent the initial WD mass in a similar way to Umeda et al. (1999b). In Fig. 1, we show the abundances of several elements as a mass fraction in the inner core of three representative models with different metallicity at the moment of envelope ejection. In the figure, the H burning shell is located around the mass coordinate of $m = 0.9 M_\odot$, which means that the remnants from the models have a similar mass, i.e. the initial WD mass from the stars is similar. An interesting feature in the figure is that the carbon abundance and the flat part of the carbon abundance profile in the inner core decrease with metallicity (Umeda et al. 1999a; Dominguez et al. 2001). The carbon abundance is mainly derived from the result of the competition between 3α and $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction in the central helium burning phase, while the flat profile is the result of the central helium burning, which occurs in a convective core. An increase in the metallicity leads to an increase in the radiative opacity, and consequently, a decrease in the central temperature at a given phase, and a decrease in the He core mass at the 3α onset. For the central helium-burning phase, this leads to a smaller convective core, hence a smaller region characterized by a flat C/O profile. A low central temperature favors the destruction reaction of ^{12}C ,

Fig. 1. Abundances of several elements in mass fraction in the inner core of three models with different primordial MS masses and different metallicities at the moment of envelope ejection. The solid, dotted, dashed and dot-dashed lines represent hydrogen, helium, carbon, and oxygen abundance, respectively. *Top:* $Z = 0.001$ and $M_i = 3.51$; *middle:* $Z = 0.02$ and $M_i = 4.52$; *bottom:* $Z = 0.06$ and $M_i = 4.32$.

namely the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ with respect the 3α reaction, which is the main cause of the decrease in the central carbon abundance. In addition, the central carbon abundance is also relevant to the helium abundance in the central helium-burning phase, and then to metallicity via $Y = 1 - Z$ at the beginning of central helium burning. For a given temperature, a low helium abundance, i.e. a high metallicity, means slightly lower burning rates for the 3α reaction. However, between the model of $Z = 0.001$ and $Z = 0.02$ in Fig. 1, the difference in the carbon abundance is insignificant,

Fig. 2. The relation between the total carbon abundance of WDs before a supernova explosion and the initial WD mass for SNe Ia with different metallicities. The gray points represent those that might not contribute to SNe Ia.

Fig. 3. The relation between the total carbon abundance of WDs before a supernova explosion, the initial WD mass and metallicity. The solid line is the best-fit linear relation, where the line fits all the points shown in Fig. 2. The lower panel shows the difference between the points and the fitted line.

but the difference between the models of $Z = 0.02$ and $Z = 0.06$ is remarkable. The effect of metallicity on the C/O ratio may then not be neglected when $Z > 0.02$. In addition, the hydrogen abundance decreases and helium abundance increases with metallicity in the figure, which is a natural result of Eq. (2)³.

In Fig. 2⁴, we show the relation between the total carbon mass fraction of WDs before a SN Ia explosion and the initial WD mass for SNe Ia with different metallicities. For the cases of $Z \leq 0.02$, the results here are similar to those in Nomoto et al. (1999, 2003), i.e. the relation between the carbon abundance and the initial WD mass is independent of metallicity, especially for $M_{\text{WD}} \geq 0.8 M_{\odot}$. When $M_{\text{WD}} \leq 0.8 M_{\odot}$, the scatter is enlarged

³ The hydrogen abundance and the helium abundance are not accurately equal to those derived from Eq. (2) because of the first and second dredge-up (Busso et al. 1999).

⁴ Here, the gray points are plotted based on the study in Meng et al. (2009). Please keep in mind that the results for the lower limit of WDs for SNe Ia in Meng et al. (2009) may be higher than that in Meng & Yang (2010a) since some special effects such as mass-stripping effect of an optically thick wind and the effect of thermal unstable disk are considered in Meng & Yang (2010a).

Fig. 4. The relation between the total carbon abundance of WDs before SNe Ia explosion, the initial WD mass and metallicity, where the points which may not contribute to SNe Ia are cut off. The solid line is the best-fit linear relation. The lower panel shows the difference between the points and the fitted line.

for the cases of $Z \leq 0.02$, which is mainly in the case of low-metallicity (≤ 0.001 , see also Fig. 12 in Umeda et al. 1999a). However, for the low-metallicity cases, the WDs with a mass lower than $0.8 M_{\odot}$ may not contribute to SNe Ia (see Fig. 5 in Meng et al. 2009).

A remarkable feature in Fig. 2 is that when $Z > 0.02$, the relation between the carbon abundance and the initial WD mass deviates significantly from those of $Z \leq 0.02$ and the deviation increases with metallicity, i.e. for a given initial WD mass, a high metallicity leads to a lower carbon abundance as shown in Fig. 1. This is similar to the result found by Timmes et al. (2003), i.e. only when $Z > 0.02$ does the influence of metallicity becomes significant. So, the discovery of Umeda et al. (1999b) and Nomoto et al. (1999, 2003) is a low-metallicity limit of the result found in this paper.

In the three-dimensional space of $(X_C, M_{\text{core}}, Z)$, the relations between the carbon abundance and the initial WD mass with different metallicity are almost in a plane. We take the X_C as a function of $M_{\text{core}} + \alpha Z$ and use the minimum χ^2 method to find the best relation. We found that a linear relation may well represent the relation when $\alpha = 3.5$. The best-fit relation is

$$X_C = 0.5531 - 0.2528(M_{\text{core}} + \alpha Z), \quad (3)$$

where $\chi^2_{\text{min}} = 5.16 \times 10^{-2}$. We also try to use a parabola to fit the relation between X_C and $M_{\text{core}} + \alpha Z$, but the improvement is slight, i.e. $\chi^2_{\text{min}} = 4.84 \times 10^{-2}$. As can be seen from Eq. (3), both a high initial WD mass and a high initial metallicity lead to a lower carbon abundance. In addition, if the metallicity is low enough, its effect on the carbon abundance can be neglected. However, the effect of a high metallicity is significant. For example, for a typical value of $M_i = 0.8 M_{\odot}$ (Meng et al. 2009; Meng & Yang 2010a), the uncertainty in X_C derived for a metallicity of $Z \leq 0.02$ is smaller than 5%, but may be as large as 25% for $Z > 0.02$.

If the points that might not contribute to SNe Ia in Fig. 2 were removed, we may even obtain a tighter linear relation, i.e. $X_C = 0.5824 - 0.2862(M_{\text{core}} + \alpha Z)$ where $\alpha = 3.0$ and $\chi^2_{\text{min}} = 1.46 \times 10^{-2}$ (see Fig. 4).

4. Discussion and conclusions

When one simulates a SN Ia explosion, the carbon abundance (the C/O ratio) and metallicity are always treated as free parameters. However, neither of them has the ability to interpret the variation in the mass of ^{56}Ni ejected by SNe Ia, especially for subluminal SNe Ia (such as 1991bg-like supernovae, Timmes et al. 2003; Röpke et al. 2006; Bravo et al. 2011). However, there is much evidence of a correlation between the maximum luminosity of SNe Ia and metallicity. For example, many groups have found that subluminal SNe Ia occur exclusively in massive galaxies (Neill et al. 2009; Sullivan et al. 2010; González-Gaitán et al. 2011; Lampeitl et al. 2010). When we consider the mass-metallicity relation of galaxies (Tremonti et al. 2004), this result implies that subluminal SNe Ia favor more metal-rich environments. In addition, some 1991T-like SNe Ia have been discovered in metal-poor environments⁵ (Prieto et al. 2008; Badenes et al. 2009; Khan et al. 2011). Hence, metallicity should play a more significant role across a rather wide range of its values than suggested by theory. The conflict between theory and observations should be resolved.

In this paper, we found that when $Z > 0.02$, the carbon abundance before SN Ia explosion is affected not only by the initial WD mass but also the metallicity. For a given initial WD mass, the higher the metallicity, the lower the carbon abundance. The relation between the carbon abundance, the initial WD mass, and the metallicity may be represented by a simple linear relation. Thus, when one simulates the SN Ia explosion, the carbon abundance (the C/O ratio) and metallicity should not be treated as free parameters. Both parameters have indeed an almost equivalent effect on the amount of ^{56}Ni that is produced because both a high metallicity and a low carbon abundance can lead to a lower amount of ^{56}Ni being produced (Nomoto et al. 1999, 2003; Timmes et al. 2003). Hence, our result may explain the variation in the maximum luminosity of SNe Ia, at least qualitatively providing a method to conquer the conflict stated above. However, we recall that it remains unclear whether the combined action of the C/O ratio and metallicity can explain the variation in the maximum luminosity of SNe Ia. *Hence, we encourage further attempts to perform a detailed numerical simulation of this problem based on the results presents here.*

Furthermore, the central density at the moment of supernova explosion may also play a role to some extent (Höflich et al. 2010; Krueger et al. 2010), and the carbon abundance, metallicity, and the central density might all contribute to the variation in the maximum luminosity of SNe Ia (Röpke et al. 2005). By making a simple assumption that the carbon abundance is a function of the initial WD mass and the central density is determined by an initial WD mass and its cooling time, Meng et al. (2010) found that WDs with high carbon abundances usually have a lower central density at ignition, while those with the highest central density at ignition generally have a lower carbon abundance. Interestingly, the effects of a high metallicity, a low C/O ratio, and a high central density on the amount of ^{56}Ni are all similar, i.e. they produce a smaller amount of ^{56}Ni and thus a dimmer event (Nomoto et al. 1999, 2003; Timmes et al. 2003; Krueger et al. 2010); hence all of them could explain why elliptical galaxies tend to contain dim SNe Ia (Hamuy 1996). Although it remains unclear how the metallicity affects

the central density, we may hypothesize optimistically that it is unreasonable to consider the carbon abundance, metallicity, and the central density as free parameters when one simulates a SN Ia explosion, and these parameters may all contribute to the production of ^{56}Ni (Röpke et al. 2005), hence help it explain the origin of the Phillips relation. Moreover, these parameters can be inferred from the initial parameters of a progenitor system, i.e. the WD mass, metallicity, orbital period, and secondary mass. For example, the C/O ratio is a function of the initial WD mass and metallicity, while the central density of a WD before a supernova explosion is determined from the accretion rate and its cooling time before the onset of mass transfer, which is related to the initial WD mass, secondary mass, and period. Our study might then provide a bridge linking the progenitor model to the explosion model of SNe Ia.

Acknowledgements. This work was partly supported by Natural Science Foundation of China under grant No. 11003003 and the Project of the Fundamental and Frontier Research of Henan Province under grant No. 102300410223, the Project of Science and Technology from the Ministry of Education (211102) and the China Postdoctoral Science Foundation funded project 20100480222.

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⁵ Even 2003fg-like SNe Ia, which are over-luminous events, tend to always be detected in metal-poor environments (Taubenberger et al. 2011), but the discussion about these SNe Ia is beyond the scope of this paper since these SNe Ia are over Chandrasekhar mass limit (Howell et al. 2006) and this paper is based on the Chandrasekhar mass model.

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Appendix A: Some physical quantities for models

In this paper, we calculated a series of model with various masses and metallicities. Here, we provide some physical quantities of the models such as CO core mass, total carbon abundance $X(\text{C})$, initial main-sequence mass, and metallicity, which may be helpful for resolving open question in this research field. Here, the boundary of the CO core was located at 10% of the maximum helium abundance in the helium shell (see the left dotted line in Fig. 1).

Table A.1. CO core masses (in $M_{\odot}$) for different metallicities (in $Z_{\odot}$, Row 1) and different initial masses (Col. 1, in $M_{\odot}$).

	0.005	0.015	0.05	0.2	0.5	1.0	1.5	2.0	2.5	3.0	4.0	5.0
1.00	0.6683	0.6490	0.6196	0.5786	0.5531	0.5366	0.5283	0.5272	0.5240	0.5292	0.5698	0.6358
1.30	0.7033	0.6798	0.6433	0.5981	0.5698	0.5533	0.5438	0.5406	0.5403	0.5435	0.5844	0.6438
1.50	0.7266	0.7007	0.6602	0.6112	0.5812	0.5634	0.5538	0.5503	0.5477	0.5609	0.5877	0.6450
1.80	0.7562	0.7281	0.6846	0.6308	0.5985	0.5784	0.5687	0.5621	0.5659	0.5687	0.6036	0.6481
2.00	0.7746	0.7488	0.7038	0.6437	0.6094	0.5881	0.5788	0.5749	0.5724	0.5774	0.6242	0.6698
2.29	0.7725	0.7863	0.7454	0.6680	0.6284	0.6051	0.5953	0.5938	0.5943	0.6060	0.6698	0.6736
2.51	0.8258	0.7898	0.7637	0.6939	0.6449	0.6196	0.6109	0.6093	0.6223	0.6378	0.6764	0.6973
2.79	0.8319	0.8208	0.7961	0.7515	0.6761	0.6460	0.6373	0.6410	0.6648	0.6747	0.6856	0.7737
2.99	0.8522	0.8420	0.8178	0.7640	0.7093	0.6718	0.6657	0.6729	0.6872	0.6894	0.7027	0.7777
3.31	0.8884	0.8804	0.8570	0.7971	0.7921	0.7323	0.7120	0.7120	0.7115	0.7136	0.7377	0.7913
3.51	0.9112	0.9059	0.8837	0.8187	0.7674	0.7411	0.7308	0.7292	0.7276	0.7289	0.7573	0.8208
3.80	0.9500	0.9461	0.9258	0.8563	0.7978	0.7647	0.7550	0.7512	0.7526	0.7564	0.7899	0.8765
3.98	0.9748	0.9740	0.9543	0.8801	0.8176	0.7805	0.7699	0.7652	0.7711	0.7761	0.8145	0.9095
4.32	1.0252	1.0274	1.0159	0.9313	0.8620	0.8134	0.7983	0.7977	0.7992	0.8124	0.8619	0.9869
4.52	1.0561	–	1.0669	0.9661	0.8898	0.8367	0.8166	0.8167	0.8199	0.8351	0.8971	–
5.01	–	–	–	1.0816	0.9733	0.9013	0.8725	0.8727	0.8833	0.9049	0.9983	–
5.50	–	–	–	–	1.0913	0.9869	0.9391	0.9431	0.9582	0.9956	–	–
6.03	–	–	–	–	–	–	1.0448	1.0476	1.0710	–	–	–

Notes. The bars represent models whose final fates are ONeMg white dwarfs.

Table A.2. Total carbon mass fraction of WDs before SNe Ia explosion for different metallicities (in $Z_{\odot}$, Row 1) and different initial masses (Col. 1, in $M_{\odot}$).

	0.005	0.015	0.05	0.2	0.5	1.0	1.5	2.0	2.5	3.0	4.0	5.0
1.00	0.3971	0.3964	0.3901	0.3886	0.3828	0.3775	0.3735	0.3560	0.3589	0.3543	0.3298	0.3048
1.30	0.3949	0.3943	0.3898	0.3883	0.3825	0.3771	0.3726	0.3566	0.3578	0.3516	0.3294	0.2917
1.50	0.3930	0.3923	0.3906	0.3881	0.3823	0.3770	0.3721	0.3563	0.3561	0.3364	0.3357	0.2917
1.80	0.3879	0.3861	0.3878	0.3876	0.3821	0.3764	0.3714	0.3607	0.3480	0.3489	0.3337	0.2743
2.00	0.3835	0.3820	0.3829	0.3871	0.3820	0.3761	0.3711	0.3550	0.3540	0.3474	0.3298	0.2638
2.29	0.3653	0.3696	0.3741	0.3845	0.3821	0.3760	0.3704	0.3464	0.3501	0.3435	0.3028	0.2635
2.51	0.3554	0.3520	0.3582	0.3795	0.3808	0.3757	0.3702	0.3547	0.3478	0.3386	0.2848	0.2598
2.79	0.3417	0.3395	0.3408	0.3695	0.3765	0.3735	0.3680	0.3544	0.3443	0.3172	0.2880	0.2342
2.99	0.3335	0.3327	0.3350	0.3493	0.3716	0.3702	0.3645	0.3520	0.3395	0.3243	0.2823	0.2444
3.31	0.3184	0.3207	0.3235	0.3334	0.3512	0.3625	0.3541	0.3403	0.3221	0.3060	0.2804	0.2267
3.51	0.3080	0.3110	0.3140	0.3275	0.3393	0.3472	0.3378	0.3274	0.3150	0.3040	0.2735	0.2144
3.80	0.2925	0.2958	0.2984	0.3165	0.3303	0.3337	0.3262	0.3193	0.3094	0.2978	0.2625	0.1977
3.98	0.2836	0.2870	0.2886	0.3078	0.3250	0.3294	0.3231	0.3156	0.3045	0.2939	0.2562	0.1893
4.32	0.2651	0.2700	0.2714	0.2899	0.3111	0.3212	0.3151	0.3078	0.2964	0.2836	0.2432	0.1605
4.52	0.2526	–	0.2601	0.2794	0.3009	0.3153	0.3105	0.3026	0.2907	0.2770	0.2282	–
5.01	–	–	–	0.2512	0.2753	0.2942	0.2928	0.2842	0.2706	0.2530	0.1952	–
5.50	–	–	–	–	0.2460	0.2712	0.2693	0.2647	0.2477	0.2281	–	–
6.03	–	–	–	–	–	–	0.2416	0.2407	0.2258	–	–	–

Notes. The bars represent models whose final fates are ONeMg white dwarfs.