

# Improved determination of $\gamma$ by VLBI (Research Note)

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## ABSTRACT

*Aims.* This study revisits the estimate of the post-Newtonian relativistic parameter  $\gamma$  reported previously. We use (i) improved geophysical and astronomical modeling in the analysis software package, and (ii) a higher number of observations, a large part of which come from a relatively small number of VLBA experiments at 8 GHz.

*Methods.* We analyzed more than seven million group delays measured by very long baseline interferometry between August 1979 and August 2010. The parameter  $\gamma$  was least squares fitted to delays as a global parameter over the entire observational time period.

*Results.* The most complete solution of this study yielded  $\gamma = 0.99992 \pm 0.00012$ , whereas it was  $0.99984 \pm 0.00015$  in our 2009 paper. The item (i), which is recognized as important for geodesy and reference frame realization, provides estimates of  $|\gamma - 1|$  that are smaller than  $10^{-4}$ . As expected, the formal error in  $\gamma$  decreases when additional sessions are processed. In particular, we demonstrate that the inclusion of more than 1.7 million observations from the VLBA (mainly from the RDV and VLBA calibrator survey experiments) in the analysis decreases the formal error in the estimate of  $\gamma$  by about 15% with respect to our previous determination.

**Key words.** astrometry – techniques: interferometric – gravitation

## 1. Introduction

In an earlier paper (Lambert & Le Poncin-Lafitte 2009, hereafter LL09), we estimated the post-Newtonian parameter  $\gamma$  by analyzing group delays recorded by astrometric and geodetic very long baseline interferometry (VLBI) at 8 GHz with an accuracy of  $1.5 \times 10^{-4}$ .

In a parallel study, Fomalont et al. (2009) found agreement with General Relativity of  $3 \times 10^{-4}$  using phase-referenced VLBA observations of 3C 279 and 3C 273 at 15, 23, and 43 GHz. Though the accuracy of their measurement does not compete with other studies, the authors pointed out that the precision of the determination of  $\gamma$  could be improved by scheduling specially designed VLBA experiments.

Since then, a number of efforts in the geodesy and astronomy communities led to the inclusion of substantially improved geophysical and astrometric models in the VLBI data reduction. Moreover, we completed the Paris Observatory VLBI analysis center observational database by adding a large number of new observations that had not been processed in LL09, and by upgrading the analysis software to new models. For these reasons, we thought that it was worthwhile to re-launch the LL09 analyses to appreciate the impact of all the improvements listed above.

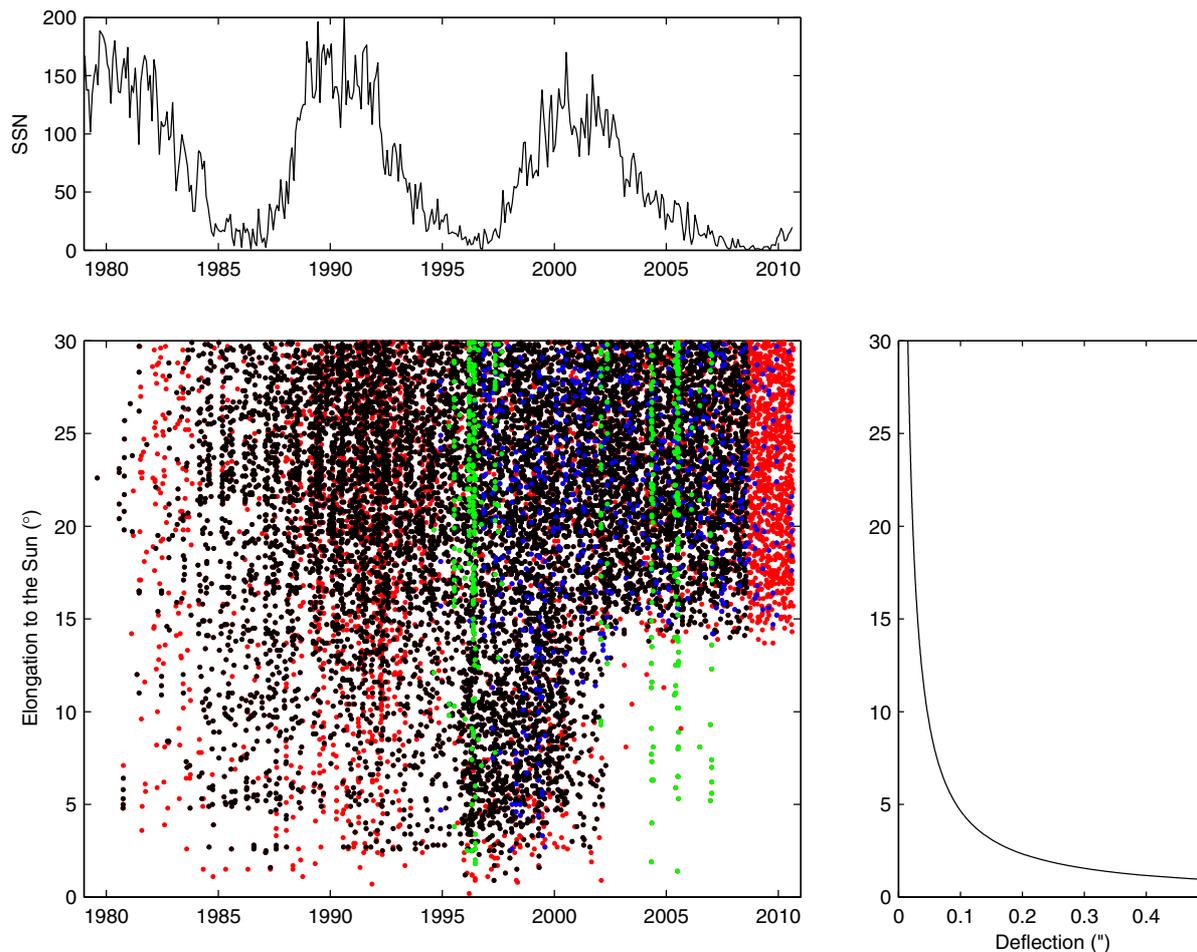
## 2. Analysis configuration

The session list now includes a number of sessions that were not processed in LL09 (Fig. 1). Most of them are generally designed for geodetic application such as Earth orientation parameter monitoring or station coordinate determination. After 2009, most of the additional sessions are the IVS rapid turn around experiments R1 and R4, scheduled every Monday and Thursday, respectively, that are designed to provide twice weekly Earth

orientation parameters and continuity with the NEOS and CORE sessions that were operated before 2002. A relatively small number of additional sessions after 1994 used of the 10-station North American Very Long Baseline Array (VLBA). The observing configuration of the VLBA network allows one to image sources and determine highly accurate station and source positions (see, e.g., Petrov et al. 2009). The VLBA can be used either alone (these sessions will be referred to as VLBA sessions in the following) or together with additional overseas antennas (denoted by VLBA+ sessions in the following) that push baseline lengths to more than 10 000 km. The former category includes the VLBA Calibrator Survey (VCS) programs 1 to 6 (Beasley et al. 2002; Fomalont et al. 2003; Petrov et al. 2005, 2006; Kovalev et al. 2007; Petrov et al. 2008) that were scheduled between 1994 and 2007. They contain observations as close as  $1.4^\circ$  to the Sun, which are indicated as green, vertical bands in Fig. 1. In the latter category, one finds the sessions known as RDV experiments using the VLBA plus up to ten additional geodetic stations located worldwide (in blue in Fig. 1). It appears that VLBA+ sessions stopped observing at less than  $15^\circ$  from the Sun after 2002, like all other routine VLBI experiments. VLBA and VLBA+ sessions usually have a number of observations larger than 10 000 and a postfit rms delay in the range 5–30 ps.

The most complete solution in this study processed 5055 sessions between 3 August 1979 and 30 August 2010, totalling more than 7.3 millions ionosphere-free group delay measurements at 8 GHz. The calculations used the Calc 10.0/Solve 2010.05.21 geodetic VLBI analysis software package<sup>1</sup>, developed and maintained at NASA Goddard Space Flight Center, and were carried out at the VLBI analysis center of the Paris Observatory (Gontier et al. 2006), which is part of

<sup>1</sup> <http://gemini.gsfc.nasa.gov/solve>



**Fig. 1.** The *main plot* displays the observational history of the sources at less than  $30^\circ$  from the Sun (black: observations treated in LL09; red: additional observations of routine experiments not processed in LL09 excluding VLBA+ and VLBA; blue: VLBA+; green: VLBA). The *upper plot* gives the Sun spot number (SSN, Clette et al. 2007). The *right plot* displays the deflection angle predicted by General Relativity.

the International VLBI Service for Geodesy and Astrometry (IVS, Schlüter & Behrend 2007). Earth orientation parameters, their rates, and station coordinates were estimated once per session. Loose no-net rotation (NNR) and translation constraints per session were uniformly applied to the positions of all stations excluding Fort Davis (Texas), Pie Town (New Mexico), Fairbanks (Alaska), and the TIGO antenna at Concepción, Chile because of strong non-linear displacements (the latter two sites experienced post-seismic relaxation effects after large earthquakes on the Denali fault in 2003, and between Talca and Concepción in early 2010, respectively). Antenna thermal deformations were mapped using the values of Nothnagel (2009) whereas antenna axis offsets were estimated as global parameters over the full observational time span for a set of 66 stations. The cut-off elevation angle was set to  $5^\circ$ . Zenith wet delays were estimated as a continuous piecewise linear function at 30-min intervals. Troposphere gradients were estimated as 6-hr east and north piecewise functions at all stations except a set of 110 stations with poor observational history.

Station heights were corrected for atmospheric pressure and oceanic tidal loading. The relevant loading quantities were deduced from surface pressure grids from the U. S. NCEP/NCAR reanalysis project atmospheric global circulation model (Kalnay et al. 1996; Petrov & Boy 2004) and from the FES 2004 ocean tide model (Lyard et al. 2004). A critical change with respect to LL09 is the use of atmospheric pressure loading coefficients

at diurnal and semi-diurnal frequencies computed with a non-inverted barometer (NIB) hypothesis. In this hypothesis, oceans do not react to any atmospheric pressure variations and any increment in the atmospheric sea level pressure is fully and instantaneously transmitted to the ocean bottom. The NIB ocean is assumed to be static on time scales around and below a few days (e.g., Willebrand et al. 1980; Wunsch & Stammer 1997). In LL09, we assumed that the ocean reacted to balance atmospheric pressure variations at the sea level.

We used a priori source coordinates of the second realization of the International Celestial Reference Frame (ICRF2, Fey et al. 2010), adopted by the International Astronomical Union (IAU) in August 2009 as the fundamental realization of the International Celestial Reference System (Feissel & Mignard 1997) and a replacement of the ICRF (Ma et al. 1998). The ICRF2 contains the coordinates of 3414 extragalactic sources determined after VLBI observations at 8 GHz during 1979–2009. With respect to the first ICRF, the noise floor ( $40 \mu\text{as}$ ) has improved by a factor of five, and the axis stability ( $10 \mu\text{as}$ ) by a factor of two. Nevertheless, in LL09, we mentioned that the a priori radio source catalogue does not influence the results at a significant level since source coordinates are estimated during the analysis. Therefore, we do not expect the use of the ICRF2 to be the source of the improvement yielded in this paper. As in LL09, we loosely constrained the sources to the ICRF2 catalogue. With respect to LL09, three sources identified

**Table 1.** Characteristics of the solutions and estimates of  $\gamma$ . Details of the various solutions are given in the text.

	No. sessions	No. delays	No. sources	Postfit rms delay (ps)	$\gamma$	$\chi^2/f$
1979–2008 LL09	3937	4 386 112	916	25.0	$0.99984 \pm 0.00015$	0.86
1979–2008 S1	3937	4 400 315	913	24.8	$1.00004 \pm 0.00015$	0.84
1979–2008 S2	4643	4 952 152	1022	24.8	$0.99998 \pm 0.00015$	0.84
1979–2010 S3	4930	5 584 232	1077	24.8	$1.00004 \pm 0.00014$	0.84
1979–2010 S4	5012	6 997 297	1343	23.3	$0.99993 \pm 0.00013$	0.83
1979–2010 S5	5055	7 351 000	3706	23.2	$0.99992 \pm 0.00012$	0.84

as gravitational lenses were removed: 0218+357 and a component 0218+35A (observed in one session each), and PKS 1830–211 (observed in two sessions).

In the new software release, the a priori precession and nutation comply with the IAU 2000/2006 resolutions, which incorporate the nutation model of Mathews et al. (2002) in a way that is consistent with the precession of Capitaine et al. (2003b) and the non-rotating origin-based coordinate transformation between the terrestrial and celestial coordinate systems (Capitaine et al. 2003a).

As a major change with respect to LL09, the Niell (1996) mapping functions (NMF) were replaced by the Vienna mapping functions 1 (VMF1, Böhm et al. 2006), where two of the three coefficients of the continued fraction expressing the hydrostatic mapping functions are computed from the 40-yr reanalysis (ERA-40) of the European Center for Medium-Range Weather Forecast (ECMWF) data. Unlike the previously available mapping functions (Niell 1996; Niell 2000; Böhm & Schuh 2004), they depend on the day of the year, and are no longer symmetric with respect to the equator. Tesmer et al. (2007) showed that VMF1 lead to station height repeatability that is 5.7% better than other mapping functions. They also mentioned that, compared with NMF, the effect on the radio source declinations is  $\sim 10 \mu\text{as}$  for declinations below  $-20^\circ$ .

In the next section, we present various solutions designed to test the influence of the changes with respect to LL09 as listed above.

### 3. Analysis and results

Table 1 summarizes the solution information and displays estimates of  $\gamma$  and the reduced  $\chi^2$  of the solution. The first line of Table 1 reproduces the most complete solution of LL09. In solution S1, we considered the same session list as in LL09 to check the effects of the improved geophysical and astronomical modeling and the slight changes in the analysis configuration, as listed in Sect. 2. The number of delays in S1 appears to be higher by less than 0.005% than in LL09 because of the use of different versions for a few sessions. It turns out that the formal errors in  $\gamma$  are expected to be comparable, at about  $1.5 \times 10^{-4}$ . However, it appears that  $\gamma$  is closer to unity:  $|\gamma - 1|$  was  $2 \times 10^{-4}$  in LL09, and less than  $10^{-4}$  in S1.

Taken individually, none of the analysis configuration changes listed in the previous section are able to fully explain the small shift in the central value of  $\gamma$  toward unity. It results from the combined effects of all the geophysical and astronomical model improvements.

Solution S2 included additional VLBI sessions between 1979 and 2008 that had not been processed in LL09 (excluding the VLBA and VLBA+ sessions). It therefore checked the

effect of completing the observational database, but not extending the observing time span. The next three solutions were based on processed sessions until the end of August 2010. Solution S3 included additional VLBI sessions between 2008 and 2010, and checked the ability of the current geodetic VLBI observations (excluding the VLBA and VLBA+ sessions) to constrain  $\gamma$  if the current observing strategy were continued in the future. As for S1, the estimated values of  $|\gamma - 1|$  in S2 and S3 are well below  $10^{-4}$ . Solution S3 processed about 12% more observations than S2. The formal error decreased by 6%. Routine VLBI sessions scheduled every three days on average are only able to decrease the error in  $\gamma$  at a rate of about  $10^{-6}$  per year.

Solution S4 is similar to S3, except that it included an additional 72 VLBA+ sessions (scheduled bi-monthly in average). The error was reduced to  $1.3 \times 10^{-4}$ . Finally, solution S5, which is the most complete solution considered in this study, included both VLBA+ and VLBA sessions. We obtained  $\gamma = 0.99992 \pm 0.00012$ . Although the lengths of the S3 and S5 session lists differ by less than 3%, the addition of the rather sparse VLBA sessions increases the number of delays by 30% and the number of sources by a factor of four. Compared with LL09 and other solutions that used routine VLBI experiments only (S1 to S3), solution S5 has the formal error that is smaller by  $\sim 15\%$ , where  $|\gamma - 1|$  is below  $10^{-4}$ , and shows that the VLBA experiments, although scheduled sparsely, have a good potential for General Relativity tests.

### 4. Concluding remarks

Although we do not challenge the results of Bertotti et al. (2003) from Cassini spacecraft measurements ( $\gamma = 1.00002 \pm 0.00002$ ), the small improvement presented in this study is notable because it illustrates the capability of certain geodetic/astrometric VLBI networks and observing configurations to increase the sensitivity to  $\gamma$ . The VLBI observational data base provides a very flexible way to test General Relativity in the Solar System at the level of  $10^{-4}$  thanks to the public availability of the data and the low CPU time taken by the solutions.

This data base is still increasing with new observations of very good quality thanks to a great, joint effort of worldwide radio astronomical observatories and space agencies. The upcoming VLBI 2010 will be designed in particular to reduce systematic errors, including possible source structure corrections thanks to faster antennas, larger networks, and higher data rates resulting in a  $uv$  coverage that is much better than in the current geodetic experiments (Petrachenko et al. 2008). This new VLBI network will likely lead to improved ground-based tests of General Relativity (Heinkelmann & Schuh 2010). We therefore encourage VLBI observing program committees to schedule

observations of sources close to the Sun as in the VLBA calibrator survey sessions.

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