

Prospects for accurate distance measurements of pulsars with the Square Kilometre Array: Enabling fundamental physics

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ABSTRACT

Parallax measurements of pulsars allow for accurate measurements of the interstellar electron density and contribute to accurate tests of general relativity using binary systems. The Square Kilometre Array (SKA) will be an ideal instrument for measuring the parallax of pulsars, because it has a very high sensitivity, as well as baselines extending up to several thousands of kilometres. We performed simulations to estimate the number of pulsars for which the parallax can be measured with the SKA and the distance to which a parallax can be measured. We compare two different methods. The first method measures the parallax directly by utilising the long baselines of the SKA to form high angular resolution images. The second method uses the arrival times of the radio signals of pulsars to fit a transformation between time coordinates in the terrestrial frame and the comoving pulsar frame directly yielding the parallax. We find that with the first method a parallax with an accuracy of 20% or less can be measured up to a maximum distance of 13 kpc, which would include 9000 pulsars. By timing pulsars with the most stable arrival times for the radio emission, parallaxes can be measured for about 3600 ms pulsars up to a distance of 9 kpc with an accuracy of 20%.

Key words. parallaxes – pulsars: general – stars: neutron – telescopes – astrometry

1. Introduction

Distance measurements have always played a hugely important role in most aspects of astronomy, on both Galactic and extragalactic scales, but are often very difficult to access observationally. Arguably, the most reliable measurements are naturally obtained from parallax measurements when the annual movement of the Earth around the Sun can be used to detect a variation in the apparent source position. For nearly all sources and applications, this involves imaging the source against the background sky. This can be done at optical wavelengths or, usually more precisely, at radio frequencies using Very Long Baseline Interferometry (VLBI). In the case of radio pulsars however, a second type of parallax can be measured using timing measurements of the radio pulses. Here, distances are retrieved by detecting a variation in pulse arrival times at different positions of the Earth's orbit caused by the curvature of the incoming wavefront. In contrast to an imaging parallax, the precision of timing parallax measurements is highest for low ecliptic latitudes and lowest for the ecliptic pole. Pulsar timing of binary pulsars offers further possibilities to determine the distance from a secular or annual variation in some orbital parameters. In all cases, however, the methods are usually limited to relatively nearby sources, but improvement in telescope baseline lengths and sensitivity promise improvements in precision and hence the accurate measurement of distances.

The most significant advance will be achieved by the Square Kilometre Array (SKA), which is being planned as a multi-purpose radio telescope constructed from many elements, both dishes and aperture arrays, leading to a total collecting area approaching 1 million square metres (e.g. Dewdney et al. 2009).

In the frequency range of 200 MHz to 3 GHz, the sensitivity of the SKA will be around $10\,000\text{ m}^2/\text{K}$, but depending on the design that will be chosen the sensitivity could be slightly higher or lower (see Schilizzi et al. 2007). The core of the SKA will be densely filled with 50% of the collecting area within a 5 km diameter area. The remaining elements will be placed at locations extending up to several thousands of kilometres from the core. The field of view (FoV) of the dishes will be around 0.64 deg^2 at 1.4 GHz for a single-pixel receiver and might be extended up to 20 deg^2 by means of phased array feeds, although for the long baselines only the single-pixel receivers will be available.

Having both a dense core and long baselines allows the SKA to both find radio pulsars and to perform accurate astrometric measurements on them. In particular, the SKA will be able to perform parallax measurements on a great number of pulsars, allowing accurate determination of their distances from the Earth. Most distances to pulsars are currently derived from the measured column density of free electrons between the pulsars and Earth, known as the dispersion measure (DM). Using a model for the interstellar medium and its free electron content, the DM can be converted into a distance. However, the Galactic electron distribution is not accurately known, so that the distance derived from DM values is often uncertain, the errors in the distance estimates being as large as 100% (see e.g. Deller et al. 2009).

The SKA will perform a Galactic census of pulsars (Cordes et al. 2004) that will potentially yield a population of 20 000 or more radio pulsars across the Galaxy (Smits et al. 2009). Parallax measurements of many of these pulsars with the SKA will provide direct measurements of their distances, which will then allow accurate studies of the ionised component of the

interstellar medium, given the measurement of the DM. In this way, the interstellar electron model can be calibrated and will not only provide reliable estimates of the DM distances of pulsars without a parallax measurement in return, but will simultaneously provide a map of the free electron content in the Milky Way that can be combined with HI and HII measurements to unravel the Galactic structure and the distribution of ionised material. Combined with Faraday rotation measurements of the same pulsars, the Galactic magnetic field can also be studied in much greater detail as today, since the precise distance measurements potentially allow us to pin-point field-reversals occurring with some accuracy.

The determination of accurate pulsar distances is also important for those pulsars that are part of binary systems, in particular for those with another compact object, such as the Double pulsar (Burgay et al. 2003; Lyne et al. 2004). Relativistic effects can be used to determine the distance to some of these systems when the validity of general relativity is assumed. In reverse, to perform precision tests of general relativity, kinematic effects have to be removed for which it is often required to know the distance precisely (Lorimer & Kramer 2005; Kramer et al. 2006; Deller et al. 2009).

In this paper, we review the possible ways to determine pulsar distances with the SKA. To compare the technical capabilities with the expected pulsar population, we simulate an SKA pulsar survey, following Smits et al. (2009). Using the results of this simulation we calculate the accuracy with which the SKA can determine the distance to each pulsar by measuring its parallax. We examine two basic methods for performing the astrometry, the first that uses imaging and the long baselines of the SKA, and the second that uses the timing of the pulsar radio signal. Here we distinguish between parallax measurements (applicable to all pulsars) and distance-related effects for binary pulsar parameters.

In Sect. 2, we describe the simulation of the SKA pulsar survey. The two different methods for measuring the parallax of pulsars are explained in Sect. 3, where we also explain how we estimate the capability of the SKA to measure the parallaxes of pulsars. The results are presented in Sect. 4. Finally, we discuss our findings in Sect. 5.

2. Simulation of an SKA pulsar population

To demonstrate the SKA capabilities not only in principle but to also estimate how many pulsars would offer the highest astrometry precision, we gauge the overall SKA performance for pulsar astrometry by simulating the population of pulsars that can be expected to be involved in the experiments.

The computational requirements for data analysis limit the amount of collecting area that can be used in an SKA pulsar survey designed to find new pulsars. According to Smits et al. (2009), at the beginning only the inner 1-km of the SKA core can be used, which contains $\sim 20\%$ of the total collecting area. We performed the same simulation as performed by Lorimer et al. (2006), using their Monte Carlo simulation package. In their study, Lorimer et al. (2006) used the results from recent surveys with the Parkes multi-beam system to derive an underlying population of 30 000 normal pulsars with an optimal set of probability density functions (PDFs) for pulsar period (P), 1400-MHz radio luminosity (L), Galactocentric radius (R), and height above the Galactic plane (z). For our simulations, we adopt the PDFs of model C' in Lorimer et al. (2006). The intrinsic pulse width of each pulsar follows the power law relationship with spin period given in Eq. (5) of Lorimer et al. (2006, see also their Fig. 4). To

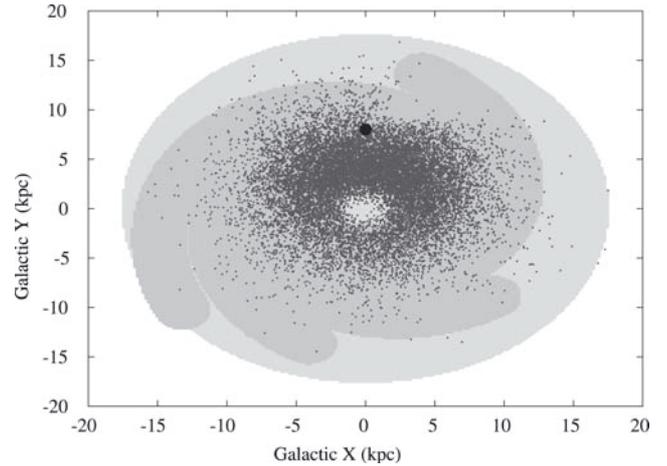


Fig. 1. Distribution of the 15 000 normal pulsars detected in the simulation (see text for details). The centre of the plot corresponds to the centre of the Galaxy. The grey areas mark the inner and outer disks and spiral arms used in the NE2001 electron density model. The black circle indicates the position of the Earth.

compute the expected DM and scatter broadening effects on each pulse, we use the NE2001 electron density model. To scale the scatter-broadening time to arbitrary SKA survey frequencies, we adopt a frequency power-law index of -4 (Löhmer et al. 2001).

We then simulated an all-sky pulsar survey at 800 MHz with a bandwidth of 500 MHz and 20% of the sensitivity of the full SKA (i.e. $2000 \text{ m}^2/\text{K}$). We consider a pulsar to be detected by a given configuration if its flux density exceeds the threshold value, and its observed pulse width is less than the spin period.

Since the core of the SKA will be located near the latitude of -30° , we limited the simulation to only detecting pulsars with a declination of 50° or less. This led to the detection of 15 000 normal pulsars in our simulation. In addition, the SKA would find 6000 MSPs. Figure 1 shows the distribution of these pulsars in the Galaxy. The void at the centre of the figure is due to the extreme scattering of the radio signal at the Galactic centre, which decreases the sensitivity of detection. The degree of scattering near Sgr A* is not yet known but we can expect that frequencies above 10 GHz may be required to detect pulsars very close to the central supermassive black hole (Cordes et al. 2004).

3. Distance measurements

The parallax and hence distance of a stellar object is classically obtained by measuring the celestial coordinates of the object on the sky at different orbital positions of the Earth. Here we examine two methods for measuring the parallax of pulsars with the SKA, imaging parallax and timing parallax, before we also briefly discuss the “orbital” parallax measurements of binary pulsars.

3.1. Imaging observations

The straightforward method for measuring the parallax of pulsars is to measure the position of the pulsar on the sky over time by means of imaging. Fomalont & Reid (2004) estimate the astrometric capabilities of the SKA by means of interferometry using the stated design goals of this proposed instrument (Schilizzi et al. 2007). They outline some of the calibration techniques for astrometry that could be employed by the SKA and discuss the effects of limited dynamic range on astrometry; they conclude that with multiple calibrator sources available at separations of a few arcminutes the astrometric accuracy that the SKA can

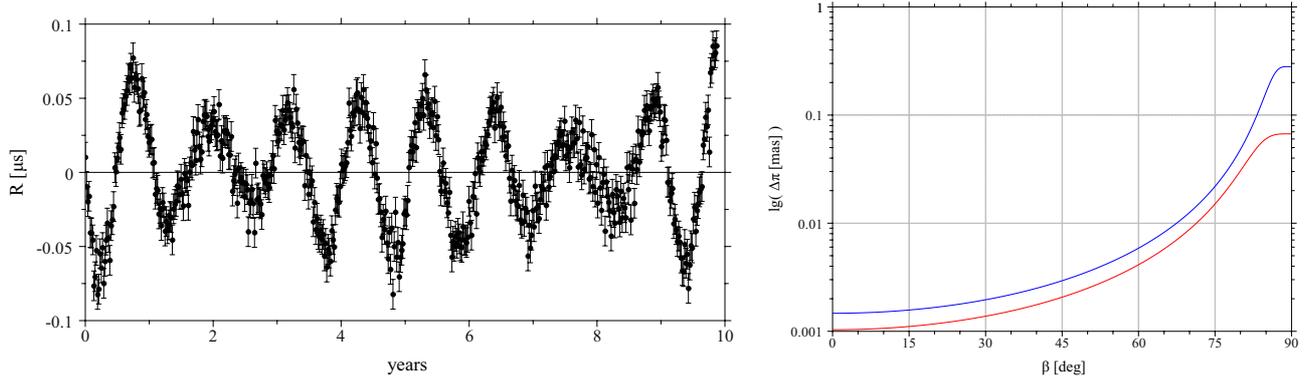


Fig. 2. *Left:* post-fit residuals for a pulsar at a distance of 200 pc located at the pole of the ecliptic after fitting for position and proper motion but not for parallax. Remaining structure in the TOAs is clearly visible. *Right:* error in the timing parallax as a function of the ecliptic latitude β for weekly timing of a pulsar to a TOA with 10 ns error. The blue and red lines correspond to 5 years and 10 years of timing, respectively.

potentially obtain is $\theta_a \sim \theta/1000$, where θ_a denotes the astrometric accuracy, θ is the imaging resolution obtained by the array (typically FWHM of the synthesised beam) and the factor 1000 is the dynamic range. At the frequency of 1.4 GHz and a 3000 km maximum SKA baseline, the potential astrometric accuracy is approximately $15 \mu\text{as}$. However, for many pulsars the limiting factor will be given by the limited SNR of the pulsar detection. We therefore estimate the astrometric accuracy as the maximum of $\theta/1000$ and $\theta/SNR_{\text{pulsar}}$, where SNR_{pulsar} is the SNR of the pulsed flux from the pulsar over an integration time of 12 h. To achieve this SNR requires the correlator to perform de-dispersion and gating over the on-pulse of the pulsar. Therefore, an ephemeris of the pulsar is required that can be obtained from regular timing observations.

As of the end of 2009, parallaxes for approximately 30 pulsars have been obtained using VLBI (for a summary see [Verbiest et al. 2010](#)). Typically about six measurements over a period of two years are required to measure the parallax and proper motion of a single pulsar, typically requiring a total observing time of 72 h of observational time per pulsar (12 h per VLBI observation). Measuring the parallax of multiple pulsars in the FoV of the telescope involves using the correlator to perform de-dispersion and gating for every pulsar in the FoV, which is possible for current VLBI observations using the capabilities of new software correlation systems.

3.2. Timing observations

A second set of methods for performing astrometry of a pulsar involves the accurate timing of the pulsar radio signals, i.e. the measurement of the pulse times-of-arrivals (TOAs) at the telescope. Here we can distinguish between three methods, all relying on very precise TOA determination: *a)* parallax measurements using the Earth orbit, *b)* parallax measurements for binary pulsars using the Earth orbit and that of the pulsar, and *c)* distance estimates of binary pulsars based on the comparison of observed orbital parameters with those predicted by general relativity.

3.2.1. Timing parallax

The ‘‘classical’’ timing parallax measurement utilises the fact that the wave front curvature of a pulsar signal is directly related to the distance of the source. For an infinitely distant source, the wave front is planar and, after taking into account Earth’s orbit, the TOA would essentially be the same at all orbital phases of the Earth. For a finite distance, however, the curvature of the wavefront introduces an annual periodic change in the apparent

direction, hence a six-monthly periodicity in the TOAs due to the changing path-length from the pulsar to the Earth ([Lyne & Smith 2004](#); [Lorimer & Kramer 2005](#)). The apparent change in direction is more easily measurable for low ecliptic latitudes – in contrast to an imaging parallax – as the amplitude of the TOA variation scales with $\cos^2 \beta/d$, where β is the ecliptic latitude and d the distance to the pulsar.

Performing simulations of parallax measurements (see [Fig. 2](#)) deviations from the simple $\cos^2 \beta$ scaling become apparent: it is still possible to measure a parallax with a finite precision even at high ecliptic latitudes. As the Earth’s orbit deviates from a circular shape because of a small eccentricity and deviations caused by other masses, non-linear terms depending on e^2 cause differences in the arrival times that are measurable if the precision is sufficiently high.

[Figure 2](#) shows the absolute uncertainty in a parallax measurement for an observing span of 5 years and 10 years, respectively. In both cases, weekly TOAs with 10 ns precision are assumed, which is challenging but not impossible with SKA sensitivity (see [Sect. 5](#)). In these cases, it will be possible to determine the parallax of a pulsar with a precision of $\sim 1 \mu\text{as}$ for pulsars near the ecliptic, and still with a precision of $300 \mu\text{as}$ and $70 \mu\text{as}$, respectively, at the ecliptic pole. In other words, in principle the SKA should allow us to achieve a distance measurement with an uncertainty of less than 5% out to about 30 kpc! If the TOA errors are larger, we can still obtain a precision of 20% out to 30 kpc with 50 ns TOA errors and to 15 kpc with 100 ns TOA errors, respectively. While a timing precision of 100 ns has been achieved already for some pulsars, in reality the timing precision will decrease with distance as the pulsars become fainter. We discuss this further in [Sect. 4](#).

3.2.2. Orbital parallax

If a pulsar happens to be in a binary system, the pulsar orbit will be viewed under slightly different angles from different positions of the Earth’s annual orbit. The result is a periodic change in the observed longitude of periastron, ω , and the projected semi-major axis $x = a \sin i$, where a is the semi-major axis of the pulsar orbit. This effect, known as the *annual orbital parallax* ([Kopeikin 1995](#); [van Straten & Bailes 2003](#)), causes a variation in x and ω given by

$$x(t) = x_0 \left[1 - \frac{\cot i}{d} \mathbf{r}_{\text{SSB}}(t) \cdot \mathbf{J}' \right], \quad (1)$$

$$\omega(t) = \omega_0 - \frac{\csc i}{d} \mathbf{r}_{\text{SSB}} \cdot \mathbf{I}', \quad (2)$$

where $\mathbf{J}' = -\sin \Omega_{\text{asc}} \mathbf{I}_0 + \cos \Omega_{\text{asc}} \mathbf{J}_0$ and $\mathbf{I}' = -\cos \Omega_{\text{asc}} \mathbf{I}_0 + \sin \Omega_{\text{asc}} \mathbf{J}_0$. The vectors \mathbf{I}_0 and \mathbf{J}_0 and the longitude of the ascending node Ω_{asc} , define the orbital orientation in space (see e.g. Fig. 8.3 in Lorimer & Kramer 2005). Since the amplitude of this effect depends on the distance d , a detection of the annual orbital parallax can also lead to an accurate pulsar distance measurement if the orientation of the orbit can be determined, e.g. from aberration and/or spin-precession effects. Furthermore, a significant proper motion in the sky can lead to measurable *secular* changes in x and ω , which can be used to constrain the orientation of the pulsar orbit (Kopeikin 1996). For this, one needs to ensure that the other contributions to these secular changes are either well understood or well below the measurement precision of \dot{x} and $\dot{\omega}$. For instance, the secular variation in x can be the result of various effects that can be summarised as:

$$\left(\frac{\dot{x}}{x}\right)^{\text{obs}} = \left(\frac{\dot{x}}{x}\right)^{\text{PM}} + \left(\frac{\dot{x}}{x}\right)^{\text{GW}} + \frac{d\varepsilon_A}{dt} - \frac{\dot{D}}{D}, \quad (3)$$

where the contributions to the observed semi-major axis derivative are a change in the orbital inclination due to the proper motion, as outlined above, the emission of gravitational waves (GW), a varying aberration, $d\varepsilon_A/dt$, and a changing (radial) velocity contribution, $-\dot{D}/D$.

The last term of Eq. (3) is a combined effect of a changing Doppler shift caused by a relative acceleration in the gravitational field of the Galaxy or a globular cluster and an apparent acceleration due to the proper motion across the sky (Damour & Taylor 1991)

$$-\frac{\dot{D}}{D} = \frac{1}{c} \mathbf{K}_0 \cdot (\mathbf{a}_{\text{PSR}} - \mathbf{a}_{\text{SSB}}) + \frac{V_T^2}{c d}, \quad (4)$$

where \mathbf{K}_0 is the unit vector from the Earth towards the pulsar, and \mathbf{a}_{PSR} and \mathbf{a}_{SSB} are the Galactic accelerations at the location of the binary system and the Solar System barycentre. The last term including the transverse velocity, V_T , is known as the Shklovskii term. Consequently \dot{D}/D depends on the distance to the system, and in principle could provide another way to estimate the distance to the pulsar. In practice, this term is generally below the measurement precision, and anyway difficult to separate from the other contributions. However, the acceleration \dot{D}/D can play an important role in the distance measurement as outlined in the next section.

3.2.3. Using general relativity for distance measurements or kinematic parallax

While the observed \dot{x} can be contaminated by a variety of effects (see Eq. (3)), the observed orbital decay rate of an orbit, \dot{P}_b , is not affected by aberration or proper motion effects. If the intrinsic decay rate is determined purely by the loss of orbital energy due to GW emission, one can predict \dot{P}_b if the pulsar and companion masses are obtained via pulsar timing thanks to the measurement of *post-Keplerian* (PK) parameters. If two PK parameters are inferred, for instance from relativistic orbital precession, gravitational redshift, or a Shapiro delay, the masses can be determined (see e.g. Lorimer & Kramer 2005) and we can compute the value of \dot{P}_b expected for GW emission assuming that general relativity (GR) is correct. In this case (Damour & Deruelle 1985, 1986),

$$\dot{P}_b^{\text{GW}} = -\frac{192\pi}{5} T_{\odot}^{5/3} \left(\frac{P_b}{2\pi}\right)^{-5/3} f(e) \frac{m_p m_c}{(m_p + m_c)^{1/3}}, \quad (5)$$

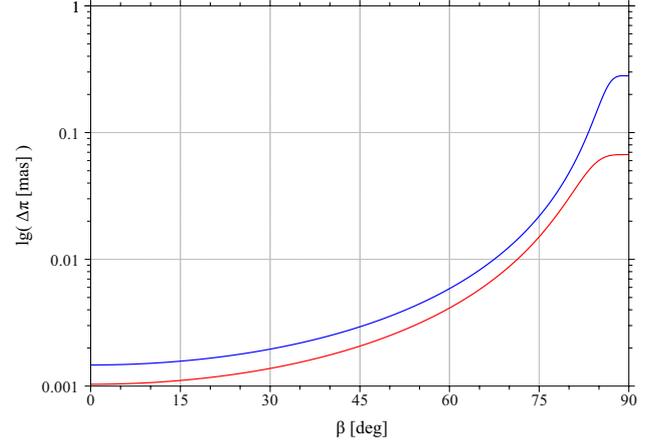


Fig. 3. Decrease in orbital period for a pulsar in a 1-d orbit with a $0.3 M_{\odot}$ companion and a transverse velocity of 100 km s^{-1} . The green curve shows the contribution from gravitational wave damping, while the blue curve is drawn for different time spans of observations, namely 5, 10, and 15 yr, respectively. We assume weekly observations, and a 10 ns TOA uncertainty that increases quadratically with distance to account for the decreasing flux density.

where all masses are expressed in solar units by using the constant $T_{\odot} = (GM_{\odot}/c^3) = 4.925490947 \mu\text{s}$, G is Newton’s gravitational constant, c the speed of light, and

$$f(e) = \frac{(1 + (73/24)e^2 + (37/96)e^4)}{(1 - e^2)^{7/2}}. \quad (6)$$

However, the observed value of \dot{P}_b will often differ from the expected intrinsic value (Eq. (5)) by an additional term that is again given by the relative acceleration, namely

$$\left(\frac{\dot{P}_b}{P_b}\right)^{\text{obs}} = \left(\frac{\dot{P}_b}{P_b}\right)^{\text{GW}} - \left(\frac{\dot{D}}{D}\right) \quad (7)$$

where we have ignored possible contributions due to mass loss or tidal effects in the system (see Lorimer & Kramer 2005).

The term \dot{D}/D can be determined from Eq. (7) by comparing the computed GR value with the observations. Figure 3 compares the relative size of the contributing terms by showing the orbital period change of a pulsar in a 1-d orbit with a $0.3 M_{\odot}$ companion and a typical transverse velocity of 100 km s^{-1} . The red curve shows the kinetic contribution of the Shklovskii term, the green one representing the gravitational wave damping term and the blue line compares the timing precision of three different time spans of observations, namely 5, 10, and 15 years, respectively. Figure 4 compares the resulting absolute uncertainty in distance measurement for the “classical” timing parallax (red) at two different ecliptic latitudes with the results for the “kinematic” parallax (blue) with different orbital periods. It is interesting to see that for the first few years of observations the classical timing parallax yields much higher precision but that in the long-term the kinematic parallax eventually leads to more precise results assuming that the acceleration in the Galactic gravitational potential can be accounted for and that the masses are known with sufficient accuracy via the measurement of PK parameters.

4. Results

While we have demonstrated the SKA capabilities in principle, we now review the applicability of our results to real SKA observations.

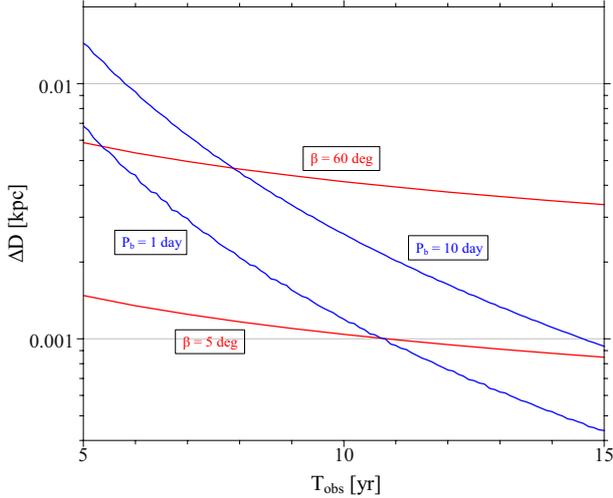


Fig. 4. Precision for distance measurements inferred from the timing of a pulsar at 1 kpc distance as a function of observing time span. The red curve is for the “classical” timing parallax at two different ecliptic latitudes, β . The distance uncertainties achieved with the kinematic parallax are shown in blue for two different orbital periods. We again assume weekly observations with a TOA precision of 10 ns.

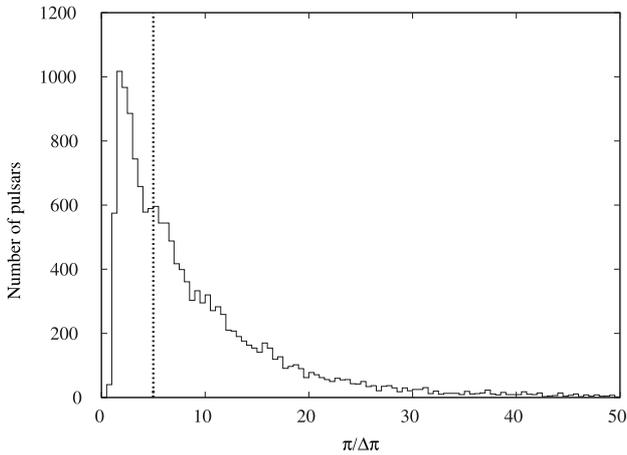


Fig. 5. Histogram of $\pi/\Delta\pi$ for the pulsars detected in the simulation of imaging observations. The vertical dotted line marks the $\pi/\Delta\pi = 5$ cutoff.

4.1. Imaging observations

Figure 5 shows the histogram of the quantity $\pi/\Delta\pi$, where π is the parallax and $\Delta\pi$ is the estimated error in the parallax, for the simulated sample. The histogram corresponds to two underlying distributions for which the parallax is limited by the *SNR* of the pulsar or the dynamic range. The low number of pulsars around $\pi/\Delta\pi \sim 1$ arises from the detection limit of the SKA survey. The main peak corresponds to the large number of pulsars with a low *SNR*. The rise in the distribution at $\pi/\Delta\pi \sim 5$ reflects the peak of the pulsar distance distribution at ~ 6 kpc. Taking a cutoff $\pi/\Delta\pi = 5$, we find that the SKA can potentially measure the parallaxes for ~ 9000 pulsars with an error of 20% or smaller. This includes pulsars out to a distance of 13 kpc. For the Galactic plane, as many as 25 pulsars can be observed in the FoV when using a single-pixel receiver. If we assume that the correlator can deal with all the pulsars in the FoV, it will take 215 days to measure the parallaxes of all the pulsars (9000) in the Galactic plane and 450 days to measure the parallaxes of all the detectable pulsars to a parallax error of 20%.

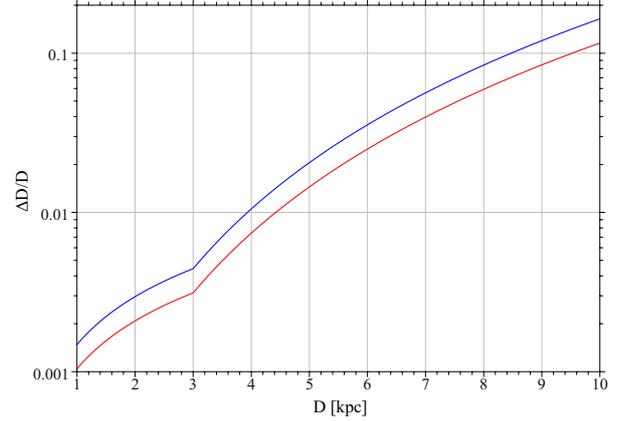


Fig. 6. Fractional precision of distance measurement by timing parallax for weekly observations. The TOA error is assumed to be 10 ns up to 3 kpc beyond which the error scales with distance squared. The blue line indicates five years of observations, the red one ten years, respectively.

4.2. Timing observations

We have shown that timing observations of pulsars can yield a parallax precision that exceeds that of an imaging parallax by at least an order of magnitude in terms of both relative and absolute precision. For longer time spans, the kinematic parallax is of even higher accuracy. For these results, we have typically assumed a TOA uncertainty of 10 ns. Such a precision is only possible for recycled millisecond pulsars (MSPs) and is a factor of 5–10 higher than what has been achieved for the best MSPs to date. Increased sensitivity is expected to help improve the timing precision in a linear way, providing an up to 100 times higher sensitivity with the SKA and ensuring that such timing precision appears easily achievable (Lorimer & Kramer 2005). Uncertain factors are, however, possible intrinsic timing noise of the pulsar clock, propagation effects in variable interstellar weather, and both instrumental and improper calibration effects. Recent and ongoing studies (Verbiest et al. 2009; Kuo et al., in prep.) indicate that despite challenges, a timing precision for MSPs between 10 and 50 ns seems feasible. We expect that about ~ 6000 of the pulsars detected with the SKA will be MSPs (Smits et al. 2009) but not all of those will be suitable for high precision timing. Moreover, as the new pulsars become more distant, the flux density will decrease and hence the timing precision becomes worse. We attempt to take this into account by increasing the TOA errors by a factor of $(d/d_0)^2$, where d_0 determines the distance beyond which the timing precision is dominated by the radio meter equation (rather than other limiting factors). Inspecting the timing precision of the presently known MSPs as a function of known distance, we estimate this transition to be around 3 kpc. This number may be significantly larger for the SKA, so that the present results using classical timing parallax shown in Fig. 6 are probably a conservative estimate. These suggest that distances can be measured with a precision of at least 10% to a distance of about 10 kpc.

In Fig. 7, we compare the distance measurements using classical timing parallax and using kinematic parallax and adjusting the TOA precision for the distance scaling. As before, the classical timing parallax yields the superior results for the first years of observations, but is overtaken by the kinematic parallax after 10 years.

To gauge how many MSPs would be likely to reach the timing precision assumed above, we compare the obtained precision among a consistent group of known MSPs at a relatively well-determined distance by inspecting the timing results for those

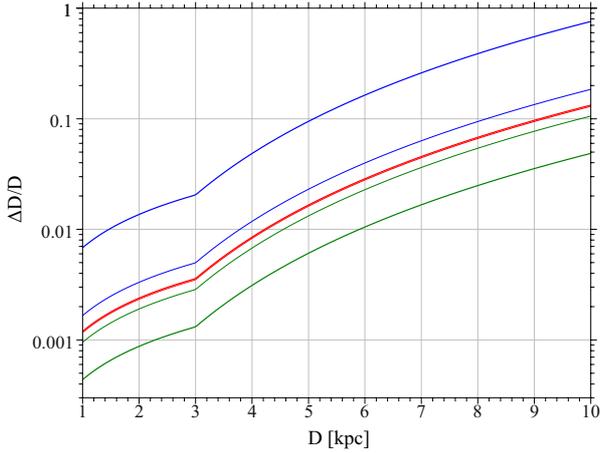


Fig. 7. Fractional precision of a distance measurement for weekly observations with TOA errors of 10 ns up to a distance of 3 kpc. Beyond 3 kpc, we assume the timing precision to become worse by a factor scaling with the square of the distance. Colours indicate the length of the observing span: 5 yr (blue), 10 yr (red), and 15 yr (green), respectively. The thick lines indicate the kinematic distance measurement for a pulsar in a 1 day orbit with an 0.3 solar mass companion. The thin lines indicate the timing parallax for the same pulsar. Note that the 10-yr lines (red) overlap.

Table 1. Error in TOAs from 16 MSPs from the cluster 47 Tucanae and the resulting error in parallax from the timing solution.

MSP	Error in TOA as measured by Parkes (μ s) (Freire et al. 2003)	Estimated error in TOA when measured with the SKA (μ s)	Error in parallax (μ as)
J0024–7204C	11	0.22	38
J0024–7204D	6	0.12	21
J0024–7204E	9	0.18	31
J0024–7204F	10	0.20	34
J0024–7204G	7	0.14	24
J0024–7204H	20	0.40	69
J0024–7204I	14	0.28	88
J0024–7204J	3	0.06	10
J0024–7204L	32	0.64	110
J0024–7204M	17	0.34	58
J0024–7204N	12	0.24	41
J0024–7204O	8	0.16	28
J0024–7204Q	19	0.38	65
J0024–7204S	11	0.22	38
J0024–7204T	49	0.98	168
J0024–7204U	9	0.18	31

16 MSPs in the globular cluster 47 Tucanae that are regularly observed. The other eight MSPs in the cluster are at the current limit of detectability of the Parkes telescope and can only be timed when the interstellar scintillation boosts the flux density above the detection threshold (Freire et al. 2003). Table 1 shows the current and predicted TOA uncertainty as well as an estimate of the accuracy with which the parallax of these 16 MSPs can be measured with the SKA. These estimates are based on a scaling that ensures that the TOAs are improved by a factor of ~ 100 . However, as the full SKA may not be able to be phased up for timing observations, we conservatively estimate that we can only use the inner 5 km, containing 50% of the collecting area, so that we scale the TOA errors by a factor of 50. To estimate the parallax error inferred from timing each of the 6000 new MSPs that will be found by the SKA, we randomly assign a TOA error from the 16 MSPs in 47 Tucanae. The parallax error is then

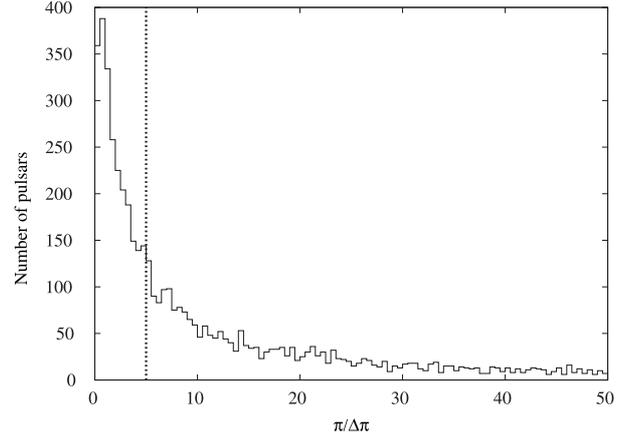


Fig. 8. Histogram of $\pi/\Delta\pi$ for the pulsars detected in the simulation of timing 6000 MSPs. The vertical dotted line indicates the $\pi/\Delta\pi = 5$ cutoff.

corrected for ecliptic latitude and distance following the results shown in Figs. 2 and 7.

Figure 8 shows the histogram for the quantity $\pi/\Delta\pi$, where π is the parallax determined by means of timing MSPs and $\Delta\pi$ is the estimated error in the parallax for the simulated sample. Taking once again a cutoff $\pi/\Delta\pi = 5$, we find that the SKA can potentially measure the parallaxes for ~ 3600 MSPs with an error of 20% or smaller out to a distance of 9 kpc. Timing all these MSPs using the phased array feeds of the SKA with an observation time of 1 h, will take roughly 25 days. To obtain a timing solution requires re-observations of these MSPs once every two weeks over a period of at least one year. The total observation time would then amount to 650 days. If we were to time these MSPs only to achieve a parallax error of 20%, the total observation time would drop to roughly 40 days. In any case, there will be a follow-up timing for each MSP that the SKA will detect to select the most interesting MSPs. Therefore, obtaining parallaxes for many of these MSPs will not require extra observation time.

4.3. Pulsar proper motions

Astrometry with the SKA will permit us to obtain accurate measurements of the proper motion of pulsars. These are extremely valuable for constraining the origin of pulsar velocities, which is currently known with an accuracy of only a few to tens of mas per year for normal pulsars and often < 1 mas per year for MSPs (Lorimer & Kramer 2005). Using imaging observations with the SKA allows a potential accuracy of 15μ as per year. From our simulations, we find that for 60% of all pulsars the proper motion can be measured with an accuracy of $15\text{--}25 \mu$ as per year. Furthermore, we find that using timing observations, the SKA can achieve a proper motion accuracy of $< 5 \mu$ as per year for almost 1000 MSPs and an accuracy of $< 30 \mu$ as per year for more than 3000 MSPs.

5. Discussion

We have investigated two methods to measure the parallax of radio pulsars using the SKA. The imaging parallax method utilises the long baselines of the SKA to keep track of the position of the pulsar on the sky during different orbital positions of the Earth. The timing method uses the arrival times of the radio signals of pulsars to derive a “classical” parallax, an “orbital-annual” parallax, or a “kinematic” parallax. With imaging parallax, it is possible to determine parallaxes with an error of 20% or smaller

for 9000 pulsars out to a distance of 13 kpc. Measuring the parallaxes of all the pulsars in the Galactic plane alone will require 215 full days of observing. Furthermore, to obtain the required astrometric accuracy of $15 \mu\text{as}$ requires several calibrators to be within a few arc-minutes of the source. With the timing parallax method, parallaxes can be measured out to 9 kpc for about 3600 MSPs, with 20% or higher accuracy. This would require a minimum of roughly 40 days of observation time. However, there will have to be follow-up timing in any case for each MSP that will be detected by the SKA to select the most interesting MSPs. Therefore, obtaining parallaxes for many of these MSPs will not require extra observation time.

When we consider the increased timing precision with the SKA and the recent realization that the apparent “timing noise” in non-millisecond pulsars is non-random and of magnetospheric origin, possibly allowing to correct for it (Lyne et al. 2010), it seems likely that parallax measurements will even be possible for normal, non-recycled pulsars. In this case, the sample of available pulsars could easily exceed the number of 10 000 but this is difficult to estimate at this stage.

The imaging and timing parallax methods are complementary, with timing parallax providing high quality measurements for a relatively small number of objects but to large distances, whereas imaging parallax can provide good results for a large number of closer objects. Timing parallax is limited to millisecond pulsars with stable timing characteristics, whereas an imaging parallax depends only on the observed strength of the radio emission, not the rotation characteristics of the pulsar. Imaging parallax measurements can indeed help us significantly with precision tests of general relativity to correct for the same kinematic effects that we can otherwise use for precise distance measurements. Moreover, obtaining imaging information immediately after the discovery of a new pulsar will help us to obtain positional information that will help the typical “solving procedure” of a pulsar, i.e. the first determination of a coherent timing solution. In general, solving pulsars takes several weeks to months of intensive observing, but the positional information from imaging eradicates the correlations between position and period derivative and allows us to reduce the cadence of the observations significantly.

From the above results, it is clear that the SKA will become a superb astrometry instrument, the data of which feeds directly back into fundamental astrophysical questions. For instance, the distance to about 9000 pulsars can be measured with an error of 20% or smaller. The dispersion measure from each pulsar can then be used to accurately calculate the electron density between the Earth and each of these pulsars. This will result in a map of the distribution of the ionised gas of high accuracy out to a distance of 13 kpc. By timing the most stable MSPs that the SKA will discover, the ionised gas can be mapped to a distance of 9 kpc. By imaging pulsars, we can locate them with a precision that is comparable with that of GAIA, which is able to measure distances out to 10 kpc with a precision of 10%. This is also achievable by timing MSPs and may be able to be, for selected sources, improved upon. This is exciting as we will be able to get independent distance measurements from GAIA and the SKA for pulsars that have an optical companion or are located, e.g. in a globular cluster.

The simulations presented in this paper are not corrected for the Lutz-Kelker bias (Lutz & Kelker 1973). This bias is introduced when the parallax measurement does not take into account the larger volume of space that is sampled at smaller parallax values and leads to a systematic overestimate of the parallax, thus to an underestimate of the distance to the star. This bias is

strongly dependent on the accuracy of the parallax measurement, becoming smaller as the measurement becomes more accurate. Verbiest et al. (2010) have studied this bias for the specific case of the parallax measurements of neutron stars, incorporating the bias introduced by the intrinsic radio luminosity function and a realistic Galactic population model for neutron stars. They found a significant bias for measurements with a 50% error. In the present paper, we consider parallax accuracies of 20% and higher. We find that in our simulations the Lutz-Kelker bias is always smaller than one standard deviation and can be neglected for most pulsars, hence does not affect our conclusions. Nevertheless, since the bias in a pulsar parallax measurement depends on the luminosity of the pulsar and the position on the sky, it is advisable to correct for it in every pulsar parallax measurement¹.

Overall, our result shows that the SKA requires both a concentration of collecting area in the core, to search for and detect the Galactic pulsar population and then measure accurate timing parallaxes, and a distribution of collecting area on long baselines, to achieve the angular resolution required to perform imaging parallax measurements. Thus, the current design specifications for the SKA are adequate for achieving both of these objectives.

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¹ The Lutz-Kelker bias for pulsars can be determined online: <http://psrpop.phys.wvu.edu/LKbias>