A&A 527, A84 (2011)
DOI: 10.1051/0004-6361/201015951

Astronomy & Astrophysics

Mass and angular momentum loss via decretion disks

J. Krtička[1], S. P. Owocki[2], and G. Meynet[3]

[1] Department of Theoretical Physics and Astrophysics, Masaryk University, 61137 Brno, Czech Republic
e-mail: krticka@physics.muni.cz
[2] Bartol Research Institute, University of Delaware, Newark, DE 19716, USA
[3] Geneva Observatory, 1290 Sauverny, Switzerland

Received 19 October 2010 / Accepted 4 January 2011

ABSTRACT

We examine the nature and role of mass loss via an equatorial decretion disk in massive stars with near-critical rotation induced by evolution of the stellar interior. In contrast to the usual stellar wind mass loss set by exterior driving from the stellar luminosity, such decretion-disk mass loss stems from the angular momentum loss needed to keep the star near and below critical rotation, given the interior evolution and decline in the star's moment of inertia. Because the specific angular momentum in a Keplerian disk increases with the square root of the radius, the decretion mass loss associated with a required level of angular momentum loss depends crucially on the outer radius for viscous coupling of the disk, and can be significantly less than the spherical, wind-like mass loss commonly assumed in evolutionary calculations. We discuss the physical processes that affect the outer disk radius, including thermal disk outflow, and ablation of the disk material via a line-driven wind induced by the star's radiation. We present parameterized scaling laws for taking account of decretion-disk mass loss in stellar evolution codes, including how these are affected by metallicity, or by presence within a close binary and/or a dense cluster. Effects similar to those discussed here should also be present in accretion disks during star formation, and may play an important role in shaping the distribution of rotation speeds on the ZAMS.

Key words. stars: mass-loss – stars: evolution – stars: rotation – hydrodynamics – stars: early-type

1. Introduction

Classical models of stellar evolution focus on the dominant role of various stages of nuclear burning in the stellar core. But in recent years it has become clear that stellar evolution, particularly for more massive stars, can also be profoundly influenced by the loss of mass and angular momentum from the stellar envelope and surface (Maeder & Meynet 2008). In cool, low-mass stars like the sun, mass loss through thermal expansion of a coronal wind occurs at too-low a rate to have a direct effect on its mass evolution; nonetheless the moment arm provided by the coronal magnetic field means the associated wind angular momentum loss can substantially spin down the star's rotation as it ages through its multi-Gyr life on the main sequence. Except in close binary systems, the rotation speeds of cool, low-mass stars are thus found to decline with age, from up to $\sim$100 km s^{-1} near the ZAMS to a few km s^{-1} for middle-age stars like the sun.

By contrast, in hotter, more massive stars the role and nature of mass and angular momentum loss can be much more direct and profound, even over their much shorter, multi-Myr lifetimes. While some specific high-mass stars appear to have been spun down by strongly magnetized stellar winds (e.g. HD 191612, Donati et al. 2006, or HD 37776, Mikulášek et al. 2008), most massive stars are comparitively rapid rotators, with typical speeds more than 100 km s^{-1}, and in many stars, e.g. the Be stars, even approaching the *critical* rotation rate, at which the centrifugal acceleration at the equatorial surface balances Newtonian gravity (Howarth 2004; Townsend et al. 2004; Howarth 2007).

Indeed, models of the MS evolution of rotating massive stars show that, at the surface, the velocity approaches the critical velocity. This results from the transport of angular momentum from the contracting, faster rotating inner convective core to the expanding, slowed down radiative envelope (Meynet et al. 2006). In stars with moderately rapid initial rotation, and with only moderate angular momentum loss from a stellar wind, this spinup from internal evolution can even bring the star to critical rotation (Meynet et al. 2007). Since any further increase in rotation rate is not dynamically allowed, the further contraction of the interior must then be balanced by a net loss of angular momentum through an induced mass loss. In previous evolutionary models, the required level of mass loss has typically been estimated by assuming its removal occurs from spherical shells at the stellar surface (Meynet et al. 2006).

This paper examines the physically more plausible scenario that such mass loss occurs through an *equatorial, viscous decretion disk* (Lee et al. 1991). Such decretion disk models have been extensively applied to analyzing the rapidly (and possibly near-critically) rotating Be stars, which show characteristic Balmer emission thought to originate in geometrically thin, warm, gaseous disks in Keplerian orbit near the equatorial plane of the parent star (Porter & Rivinius 2003; Carciofi & Bjorkman 2008). But until now there hasn't been much consideration of the role such viscous decretion disks might play in the rotational and mass loss *evolution* of massive stars in general.

As detailed below, a key point of the analysis here is that, per unit mass, the angular momentum loss from such a decretion disk can greatly exceed that from a stellar wind outflow. Whereas the angular momentum loss of a nonmagnetized wind is fixed around the transonic point very near the stellar surface, the viscous coupling in a decretion disk can transport angular momentum outward to some outer disk radius $R_{\rm out}$, where the specific angular momentum is a factor $\sqrt{R_{\rm out}/R_{\rm eq}}$ higher than at the equatorial surface. For disks with an extended outer radius $R_{\rm out} \gg R_{\rm eq}$, the angular momentum loss required by the interior evolution can then be achieved with a much lower net mass loss than in the wind-like, spherical ejection assumed in previous evolution models.

For a given angular momentum shedding mandated by interior evolution, quantifying the associated disk mass loss thus requires determining the disk outer radius. For example, in binary systems, this would likely be limited by tidal interactions with the companion, and so scale with the binary separation (Okazaki et al. 2002). But in single stars, the processes limiting this outer radius are less apparent. Here we explore two specific mechanisms that can limit the angular momentum loss and/or outer radius of a disk, namely thermal expansion into supersonic flow at some outer radius, and radiative ablation of the inner disk from the bright central star. For each case, we derive simple scaling rules for the required disk mass loss as a function of assumed stellar and wind parameters, given the level of interior-mandated angular momentum loss.

The organization for the remainder of this paper is as follows: Sect. 2 presents simple analytical relations for how the presence of a disk affects the mass loss at the critical limit. Section 3 develops set of equations governing structure and kinematics of the disk, while Sect. 4 solves these to derive simple scaling for how thermal expansion affects the outer disk radius and disk mass loss. Section 5 discusses the effects of inner-disk ablation by a line-driven disk wind induced from the illumination of an optically thick disk by the centeral star, deriving the associated ablated mass loss and its effect on the net disk angular momentum and mass loss. Section 6 gives a synthesis of the different cases discussed here and offers a specific recipe for incorporating disk mass loss rates into stellar evolution codes. Section 7 discusses some complementary points (e.g. viscous decoupling, tidal effects of nearby stars, reduced metallicity, etc.), while Sect. 8 concludes with a brief summary of the main results obtained in this work.

2. Basic analytic scaling for disk mass loss

Let us begin by deriving some simple analytic expressions for the effect of the disk viscous coupling on the disk mass-loss rate.

Assuming a star that rotates as a rigid body, the magnitude of stellar angular momentum J is given by the product of the stellar moment of inertia I and the rotation angular frequency Ω, $J = I\Omega$. During stellar evolution, the time rate of change of angular momentum depends on the changes in moment of inertia and rotation frequency,

$$\dot{J} = \dot{I}\Omega + I\dot{\Omega}. \tag{1}$$

If, for example, the moment of inertia declines at a rate $\dot{I}$, and the change of the angular momentum through any wind, etc. is negligible, i.e. $\dot{J} = 0$, then the star has to spin up at a rate given by

$$\frac{\dot{\Omega}}{\Omega} = -\frac{\dot{I}}{I}. \tag{2}$$

However, once the star reaches the critical rotation frequency $\Omega = \Omega_{\rm crit} \equiv \sqrt{GM/R_{\rm eq}^3}$ (where M is the stellar mass and $R_{\rm eq}$ is the equatorial radius when the star is rotating at the critical limit), this spin-up has to end ($\dot{\Omega} = 0$), requiring instead a shedding of angular momentum given by

$$\dot{J} = \dot{I}\Omega_{\rm crit}. \tag{3}$$

If we assume this occurs purely through mass loss at a rate $\dot{M}$ through a Keplerian decretion disk, the angular momentum loss is set by the outer radius $R_{\rm out}$ of that disk, given by

$$\dot{J}_{\rm K}(R_{\rm out}) = \dot{M}v_{\rm K}(R_{\rm out})R_{\rm out} = \dot{M}\Omega_{\rm crit}R_{\rm eq}^2\sqrt{\frac{R_{\rm out}}{R_{\rm eq}}}, \tag{4}$$

where the Keplerian velocity[1] is $v_{\rm K}(r) = \sqrt{GM/r}$. Setting $\dot{J}_{\rm K}(R_{\rm out})$ equal to the above $\dot{J}$ required by a moment of inertia change $\dot{I}$, we find the required mass loss rate is

$$\dot{M} = \frac{\dot{I}}{R_{\rm eq}^2}\sqrt{\frac{R_{\rm eq}}{R_{\rm out}}}\cdot \tag{5}$$

As $R_{\rm out}$ gets larger, note that the required mass loss rate gets smaller.

Equation (5) can be compared with the case where mass decouples in a spherical shell just at the surface of the star, i.e., where $R_{\rm out} = R_{\rm eq}$. In that case the required mass loss is just

$$\dot{M} = \frac{3}{2}\frac{\dot{I}}{R_{\rm eq}^2}\cdot \tag{6}$$

So when a Keplerian disk is present, the mass loss is reduced by a factor $\frac{3}{2}\sqrt{R_{\rm out}/R_{\rm eq}}$ with respect to the case with no disk. If $R_{\rm out}$ is small, large mass-loss is necessary to shed the required amount of angular momentum to keep the rotation frequency at its critical value. In the opposite case, when $R_{\rm out}$ becomes substantial, only a relatively small mass-loss is necessary to shed the required amount of angular momentum, implying a significant difference in the mass loss evolution.

3. Numerical models

To obtain a detailed disc structure, we solve stationary hydrodynamic equations in cylindrical coordinates integrated over the height above the equatorial plane z (Lightman 1974; Lee et al. 1991; Okazaki 2001; Jones et al. 2008). Assuming axial symmetry, the corresponding variables, i.e., the radial and azimuthal velocities v_r, v_ϕ, and the integrated disk density $\Sigma = \int_{-\infty}^{\infty}\rho\,{\rm d}z$, depend only on radius r. The equation of continuity in such a case is

$$\frac{1}{r}\frac{{\rm d}\left(r\Sigma v_r\right)}{{\rm d}r} = 0\cdot \tag{7}$$

The stationary conservation of the r component of momentum gives

$$v_r\frac{{\rm d}v_r}{{\rm d}r} = \frac{v_\phi^2}{r} + g - \frac{1}{\Sigma}\frac{{\rm d}(a^2\Sigma)}{{\rm d}r} + \frac{3}{2}\frac{a^2}{r}, \tag{8}$$

where $g = -GM/r^2$, $a^2 = kT/(\mu m_{\rm H})$, with the temperature assumed to vary as a power-law in radius, $T = T_0\left(R_{\rm eq}/r\right)^p$, where T_0 and p are free parameters, μ is the mean molecular weight (taking $\mu = 0.62$), and $m_{\rm H}$ is the mass of a hydrogen atom. In the equation of conservation of the ϕ component of momentum we introduce the viscosity term (Shakura & Sunyaev 1973) parametrized via $\tilde{\alpha}$

$$\frac{v_r}{r}\frac{{\rm d}\left(rv_\phi\right)}{{\rm d}r} + \frac{\tilde{\alpha}}{r^2\Sigma}\frac{{\rm d}}{{\rm d}r}\left(a^2r^2\Sigma\right) = 0, \tag{9}$$

[1] When the star is rotating at the critical limit, the critical rotational velocity is equal to the Keplerian velocity at $R_{\rm eq}$, $v_{\rm K}(R_{\rm eq}) = \Omega_{\rm crit}R_{\rm eq}$.

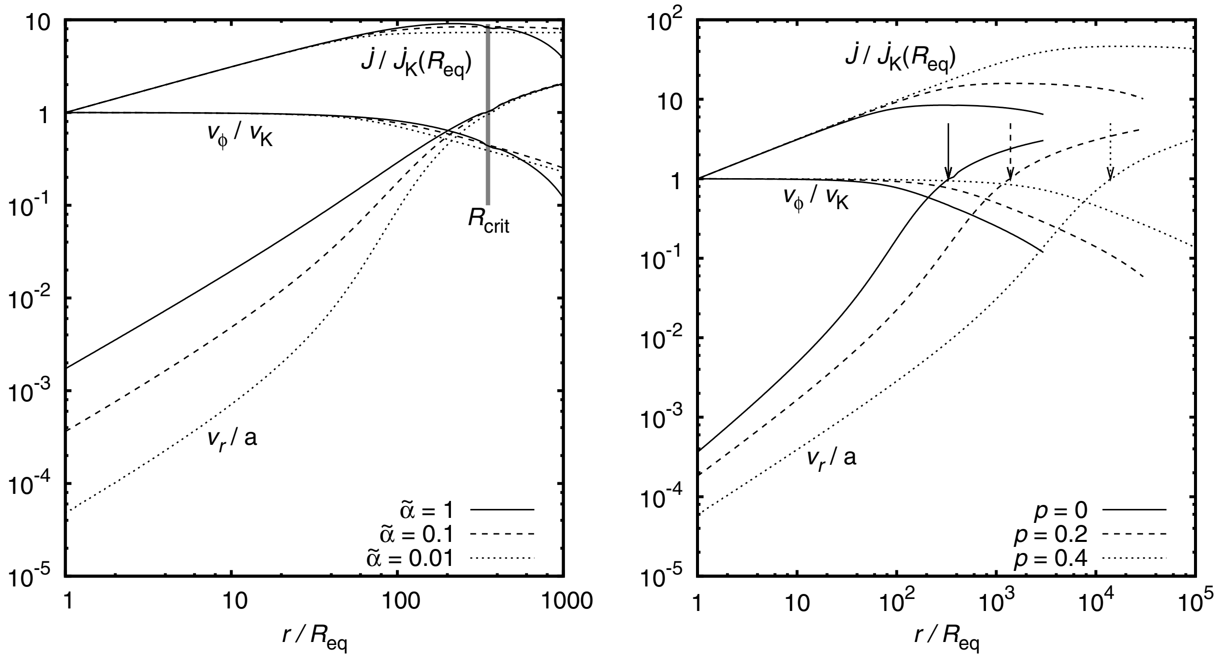

Fig. 1. The dependence of the radial velocity, azimuthal velocity, and the angular momentum loss rate in units of equator release angular momentum loss rate $\dot{J}_{\rm K}(R_{\rm eq})$ on the radius in a viscous disk. *Left*: models of isothermal disk ($p = 0$, $T_0 = \frac{1}{2}T_{\rm eff}$) with different viscosity parameter $\tilde{\alpha}$. *Right*: models with various temperarure profile for fixed $\tilde{\alpha} = 0.1$ and $T_0 = \frac{1}{2}T_{\rm eff}$. Arrows denote the location of critical points.

and the conservation of the θ component of momentum gives the hydrostatic equilibrium density distribution

$$\rho = \rho_0 \exp\left(-\frac{1}{2}\frac{z^2}{H^2}\right), \qquad H = \frac{a}{v_{\rm K}} r. \tag{10}$$

Note that the equatorial density ρ_0 is related to the vertically integrated disk density via $\Sigma = \sqrt{2\pi}\rho_0 H$. Close to the star, detailed energy-balance models (Millar & Marlborough 1998, Carciofi & Bjorkman 2008) show the disk is nearly isothermal with $T_0 = \frac{1}{2}T_{\rm eff}$ and $p = 0$. But to account for the radial decline of the temperature in the outer regions, we also consider here models with power law temperature decline, with $p > 0$.

The system of equations Eqs. (7)–(9) has to be supplemented by appropriate boundary conditions. For obtaining the value of v_r at the stellar surface $r = R_{\rm eq}$ we use the fact that at the critical point with radius $R_{\rm crit}$ given by the condition

$$\frac{v_\phi^2}{R_{\rm crit}} - \frac{GM}{R_{\rm crit}^2} + \frac{5}{2}\frac{a^2}{R_{\rm crit}} - \left.\frac{{\rm d}a^2}{{\rm d}r}\right|_{R_{\rm crit}} = 0 \tag{11}$$

we should have that $v_r(R_{\rm crit}) = a$ to ensure the finiteness of the derivatives at this point (Eqs. (7), (8), see also Okazaki 2001). Thus we chose v_r at the surface such that at $R_{\rm crit}$ we have $v_r = a$. Note that the radial disk velocity is supersonic above the critical point. The value of the azimuthal velocity at the stellar surface v_ϕ is equal to the corresponding Keplerian velocity. The system of studied hydrodynamical equations is invariant for the change of the scale $\Sigma' = \gamma\Sigma$ (where γ is constant). Consequently, the equations do not provide any constraint for the mass-loss rate $\dot{M} = 2\pi r v_r \Sigma$, which in our case is obtained from the angular momentum loss required by the evolutionary calculations. This provides the remaining boundary condition for the column density Σ. Here we treat the disk mass-loss rate as a free parameter.

For the numerical solution of the system of equations Eqs. (7)–(9) we approximate the differentiation at selected radial grid and use the Newton-Raphson method (e.g., Krtička 2003). The resulting system of linear equations is solved using the numerical package LAPACK (`http://www.cs.colorado.edu/~lapack`, Anderson et al. 1999).

4. Results of numerical models

The general disk properties do not significantly depend on particular stellar parameters. Nevertheless, to be specific, for a detailed modelling we selected the stellar parameters roughly corresponding to evolved massive first star (Marigo et al. 2001; Ekström et al. 2008b) $T_{\rm eff} = 30\,000$ K, $M = 50\,M_\odot$, $R = 30\,R_\odot$.

The calculated models for different values of $\tilde{\alpha}$ are given in Fig. 1. Close to the star the integration of the momentum equation Eq. (9) using the continuity Eq. (7)

$$r v_\phi + \frac{\tilde{\alpha} a^2 r}{v_r} = {\rm const.} \tag{12}$$

gives linear dependence of the radial velocity on radius in isothermal disks (for $v_r \ll a$), $v_r \sim r$, consequently $\Sigma \sim r^{-2}$ (Okazaki 2001). Finally, from the momentum equation Eq. (8), it follows that close to the star the disk rotates as Keplerian one, i.e. $v_\phi \sim r^{-1/2}$. As a result, the angular momentum loss scales as $\dot{J} \sim r v_\phi \sim r^{1/2}$, in accordance with Eq. (4). As the disk accelerates in radial direction, v_r becomes comparable with a and the term $r v_\phi$ dominates in Eq. (12), consequently the disk is momentum conserving close to the critical point, $r v_\phi$ = const. (see Fig. 1).

In the supersonic region from the momentum equation Eq. (8) follows the logarithmic dependence of the radial velocity on radius, $v_r^2 \sim \ln r$. Consequently, the second term in equation Eq. (12) rises and as a result of this v_ϕ may become even negative. However, this behaviour is a consequence of adopted Shakura-Sunyaev viscosity prescription which predicts non-zero torque even for shear-free disks, and is likely not applicable in the supersonic region.

A maximum angular momentum loss due to the disk is obtained in the case when the disk has its outer edge at the radius where $\dot{J}$ is maximum (see Fig. 1). Note that this value does not significantly depend on the assumed viscosity parameter $\tilde{\alpha}$. An estimate of the maximum angular momentum loss can be obtained assuming that it is equal to the angular momentum loss at the critical point. From the numerical models it follows that the azimuthal velocity at the critical point is roughly equal to the half of the Keplerian velocity (see Fig. 1),

$$v_\phi(R_{\rm crit}) \approx \frac{1}{2} v_{\rm K}(R_{\rm crit}). \tag{13}$$

In this case the critical point condition Eq. (11) yields an estimate of the critical point radius

$$\frac{R_{\rm crit}}{R_{\rm eq}} = \left[\frac{3}{10+4p}\left(\frac{v_{\rm K}(R_{\rm eq})}{a(R_{\rm eq})}\right)^2\right]^{\frac{1}{1-p}} \tag{14}$$

from which the maximum angular momentum loss via the disk follows

$$\dot{J}_{\tilde{\alpha}}(\dot{M}) \approx \frac{1}{2}\left[\frac{3}{10+4p}\left(\frac{v_{\rm K}(R_{\rm eq})}{a(R_{\rm eq})}\right)^2\right]^{\frac{1}{2-2p}} \dot{M} v_{\rm K}(R_{\rm eq}) R_{\rm eq}. \tag{15}$$

In agreement with Fig. 1, comparing the formula Eq. (15) with analytical estimate Eq. (4) the angular momentum loss is roughly given by $\frac{1}{2}\dot{J}_{\rm K}(R_{\rm crit})$, i.e., it is one half of the angular momentum loss of the Keplerian disk truncated at the critical point radius $R_{\rm crit}$. The factor $\frac{1}{2}$ comes from the fact that the disk is not rotating as a Keplerian one at large radii (see Fig. 1). Hence, the minimum disk mass loss rate required for given moment of inertia decline is by a factor of about $\left(v_{\rm K}(R_{\rm eq})/a(R_{\rm eq})\right)^{1/1-p}$ lower than in the case without a disk.

Note also that adding cooling can substantially increase the critical radius and thus the disk angular momentum loss. For example, for $p = 0.4$ the angular momentum loss increases by a factor of 10 compared to the isothermal case (see Fig. 1).

5. Radiative ablation

As the radiative force may drive large amount of mass out of the hot stars via line-driven wind (see Owocki 2004; Puls et al. 2008, for a review) it may also effectively set the outer disk radius. The radiative force may in this case ablate the material from the disk and sustain a radiatively driven outflow (Gayley et al. 1999, 2001). In the following we give an estimate of the disk wind mass-loss rate, which is derived in Appendix.

The disk outflow may in our case resemble the radiation driven winds from luminous accretion disks (Proga et al. 1998; Feldmeier & Shlosman 1999; Feldmeier et al. 1999; Proga et al. 1999). The outflow in these simulations originates from the whole disk surface. Consequently, part of the stellar outflow is carried outwards by the disk and part by the disk wind and the fraction of material carried out by the disk wind increases with radius. The disk will be in this case truncated at the radius where the material is carried away entirely by the wind. As the viscous coupling is likely not maintained in the supersonic wind, only the ablation of the material from the regions close to the star would decrease the effectiveness of braking.

Let us roughly determine the mass-loss rate of such disk wind. The classical Castor et al. (1975, hereafter CAK) stellar wind mass-loss rate estimate

$$\dot{M}_{\rm CAK} = \frac{\alpha}{1-\alpha}\frac{L}{c^2}\left(\Gamma\bar{Q}\right)^{1/\alpha-1}, \tag{16}$$

where $\bar{Q}$ and α are line force parameters (see also Gayley 1995), L is the stellar luminosity, and the Eddington parameter $\Gamma = \kappa_{\rm e}L/(4\pi GMc)$, with $\kappa_{\rm e}$ beeing the Thomson scattering cross-section per unit of mass, can be rewritten in the term of mass flux from a unit surface,

$$\dot{m} = \frac{\alpha}{1-\alpha}\frac{\tilde{F}}{c^2}\left(\frac{\kappa_{\rm e}\tilde{F}\bar{Q}}{c\tilde{g}}\right)^{1/\alpha-1}, \tag{17}$$

where $\tilde{F}$ is the driving flux and $\tilde{g}$ is local gravitational acceleration. The radiative energy impinging the unit of surface parallel to the direction to the star is from geometrical reasons proportinal to FR/r, where R is the polar radius, and F is the radiative flux at radius r. Assuming that the disk is optically thick (see Sect. A.1), and all incident radiation is directed upward, we can roughly estimate

$$\tilde{F} = \frac{R}{r}F. \tag{18}$$

Taking $\tilde{g} = GM/r^2$, the total disk wind mass-loss rate is then given by an integral of the mass-loss rate per unit of the disk surface $\dot{m}$ between the equatorial radius $R_{\rm eq}$ and the outer disk radius $R_{\rm out}$

$$\dot{M}_{\rm dw}(R_{\rm out}) = 2\times 2\pi\int_{R_{\rm eq}}^{R_{\rm out}} \dot{m}r\,{\rm d}r, \tag{19}$$

where factor of 2 in Eq. (19) comes from the fact that the wind originates from both sides of the disk. Inserting the mass flux estimate Eqs. (17) and (18) we derive

$$\dot{M}_{\rm dw}(R_{\rm out}) = \dot{M}_{\rm CAK} P_1\left(\frac{R_{\rm out}}{R}\right), \tag{20}$$

where (using substitution $x = r/R$)

$$P_\ell(x_{\rm out}) = \int_{3/2}^{x_{\rm out}} x^{-1/\alpha-\ell}{\rm d}x = \frac{\left(\frac{3}{2}\right)^{1-\ell-\frac{1}{\alpha}} - x_{\rm out}^{1-\ell-\frac{1}{\alpha}}}{\frac{1}{\alpha}+\ell-1}. \tag{21}$$

Assuming the disk wind is not viscously coupled to the disk, the total angular momentum loss rate via the disk wind is

$$\dot{J}_{\rm dw}(R_{\rm out}) = 2\times 2\pi\int_{R_{\rm eq}}^{R_{\rm out}} \dot{m}v_\phi r^2\,{\rm d}r. \tag{22}$$

As the disk wind originates mainly from the regions close to the star (with $r/R \lesssim 10$, see Fig. A.2), where the azimuthal velocity is roughly equal to the Keplerian one (see Fig. 1), we can assume $v_\phi \approx v_{\rm K}(r)$ in Eq. (22) and consequently the disk wind angular momentum loss rate

$$\dot{J}_{\rm dw}(R_{\rm out}) = R\,v_{\rm K}(R)\,P_{\frac{1}{2}}\left(\frac{R_{\rm out}}{R}\right)\dot{M}_{\rm CAK} \tag{23}$$

is by a factor of $P_{\frac{1}{2}}(R_{\rm out}/R)$ larger than the angular momentum loss due to the CAK wind launched from equator of hypothetical critically rotating spherical star with radius R.

A more detailed calculation (see Appendix A) gives a more complicated form of $P_\ell(x_{\rm out})$ via Eq. (A.23)

$$P_\ell(x_{\rm out}) = \frac{2\pi^{-\frac{1}{\alpha}} 3^{\frac{1}{2\alpha}-\frac{3}{2}}}{\frac{1}{\alpha}+\ell-1}\left[\left(\frac{3}{2}\right)^{1-\ell-\frac{1}{\alpha}} - x_{\rm out}^{1-\ell-\frac{1}{\alpha}}\right], \quad (24)$$

which shall be used in Eqs. (20), (23) instead of Eq. (21).

For an infinite disk ($R_{\rm out} \to \infty$) we derive from Eq. (24) maximum disk wind mass-loss rate

$$\dot{M}_{\rm dw}(\infty) = 2^{1+\frac{1}{\alpha}}\pi^{-\frac{1}{\alpha}}3^{-\frac{1}{2\alpha}-\frac{3}{2}}\alpha \dot{M}_{\rm CAK}, \quad (25)$$

and maximum angular momentum loss rate as

$$\dot{J}_{\rm dw}(\infty) = \frac{2^{\frac{3}{2}+\frac{1}{\alpha}}\pi^{-\frac{1}{\alpha}}3^{-\frac{1}{2\alpha}-1}}{2-\alpha}\alpha R\, \varv_{\rm K}(R)\, \dot{M}_{\rm CAK}. \quad (26)$$

For a typical value of $\alpha \approx 0.6$ (Krtička 2006) the maximal disk wind mass-loss rate is relatively low, just about $1/25$ of the CAK stellar wind mass-loss rate.

6. Mass loss of the star-disk system at the critical limit

The structure of the decretion disk and the radiatively driven wind blowing from its surface depends on the value of the angular momentum loss $\dot{J}$ needed to keep the stellar rotation at or below the critical rate and on the magnitude of the radiative force. If the angular momentum loss is small, then the disk could be blown away by the radiative force already very close to the star. In the opposite case, if the angular momentum loss is large, then the mass carried away by the disk wind is negligible.

In the intermediate case the mass and angular momentum will be carried partly by the disk and partly by the disk wind. When the star has to lose angular momentum at a rate $\dot{J}$ to keep at most the critical rotation, the angular momentum will be carried by the stellar wind ($\dot{J}_{\rm w}$), by the disk wind ($\dot{J}_{\rm dw}$), and by the disk itself ($\dot{J}_{\tilde{\alpha}}$),

$$\dot{J} = \dot{J}_{\rm w} + \dot{J}_{\rm dw} + \dot{J}_{\tilde{\alpha}}(\dot{M}_{\rm d}). \quad (27)$$

For the calculation of total disk mass-loss rate the following procedure could be used.

For a given stellar and line-force parameters ($\bar{Q}$ and α) the maximum disk wind angular momentum loss $\dot{J}_{\rm dw}(\infty)$ corresponding to infinite disk $R_{\rm out} \to \infty$ can be calculated using Eqs. (26). If the net angular momentum loss that should be carried away by the disk outflow $\dot{J} - \dot{J}_{\rm w}$ is lower than the maximum one, $\dot{J} - \dot{J}_{\rm w} < \dot{J}_{\rm dw}(\infty)$, then the disk will be completely ablated by the radiation at the radius $R_{\rm out}$ given (from Eq. (27) for $\dot{J}_{\tilde{\alpha}} = 0$)

$$\dot{J} - \dot{J}_{\rm w} = \dot{J}_{\rm dw}(R_{\rm out}). \quad (28)$$

In this case the outer disk radius $R_{\rm out}$ is equal to the radius above which all material is carried away by the disk wind. The corresponding disk wind mass loss rate $\dot{M}_{\rm dw}(R_{\rm out})$ is then given by Eqs. (20), (24). Note however that the formulae discussed in Sect. 5 are strictly valid only in the optically thick part of the disk (see Sect. A.1).

If the net angular momentum loss rate $\dot{J} - \dot{J}_{\rm w}$ is larger than the maximum one, $\dot{J} - \dot{J}_{\rm w} > \dot{J}_{\rm dw}(\infty)$, then the disk will be only partly ablated by the radiation. The net angular momentum loss $\dot{J} - \dot{J}_{\rm w}$ is in this case the sum of the parts carried by the disk and disk wind,

$$\dot{J} - \dot{J}_{\rm w} = \dot{J}_{\rm dw}(\infty) + \dot{J}_{\tilde{\alpha}}(\dot{M}_{\rm d}), \quad (29)$$

where $\dot{J}_{\tilde{\alpha}}(\dot{M}_{\rm d})$ is given by Eq. (15). Here one can assume a conservative estimate of the isothermal disk with $p = 0$. From Eq. (29) the mass-loss rate carried away purely by the disk $\dot{M}_{\rm d}$ can be calculated, giving the total required mass-loss rate as a sum of parts carried finally by the stellar wind ($\dot{M}_{\rm w}$), disk wind ($\dot{M}_{\rm dw}$), and purely by the disk ($\dot{M}_{\rm d}$) as

$$\dot{M} = \dot{M}_{\rm w} + \dot{M}_{\rm dw}(\infty) + \dot{M}_{\rm d}. \quad (30)$$

The calculation of the functions $\dot{J}_{\rm dw}(r)$ and $\dot{M}_{\rm dw}(r)$ requires the knowledge of the line force parameters $\bar{Q}$ and α. As the NLTE calculation of these parameters for the disk wind environment are not available, one can use their values derived for line driven winds for solar metallicity, i.e., $\bar{Q} \approx 2000$, and $\alpha \approx 0.6$ (Gayley 1995, Puls et al. 2000, Krtička 2006). For the metallicities other than the solar one the scaling $\bar{Q} \sim Z$ can be used (here Z is the mass fraction of heavier elements), which is in a good agreement with the results of NLTE wind models (Vink et al. 2001; Krtička 2006).

7. Other processes that may influence the outer disk radius

In addition to the radiative force, there may be other processes that may influence the outer disk radius and consequently determine the required mass-loss rate for a given angular momentum loss rate. For example, in binaries the outer disk edge may be naturally truncated due to the presence of the companion. However, the most uncertain part of the proposed model is connected with the mechanism of the viscous transport, which may also influence the outer disk radius.

7.1. Loss of viscous coupling

The magnetorotational instability (Balbus & Hawley 1991) is a promising mechanism to explain the source of anomalous viscosity in accretion disks. As the dynamics of accretion and decretion disks is similar, it is likely to be important also for the angular momentum transfer in decretion disk. However the numerical simulations of magnetorotational instability (e.g. Stone et al. 1996; Hawley & Krolik 2001) concentrate on the inner parts of the disk, whereas the evolution close to the sonic point is, to our knowledge, not very well studied. The stability condition of the positive derivative of the angular frequency (Balbus & Hawley 1991) $\mathrm{d}\Omega^2/\mathrm{d}r \geq 0$ is fulfilled even in the supersonic wind region. The fact that the ratio of the viscous timescale $\tau_{\rm visc} \approx \left(\alpha\Omega\,(H/r)^2\right)^{-1}$ to the growth timescale of the magnetorotational instability $\tau_{\rm MRI} \approx 1/\Omega$ decreases with radius as $\tau_{\rm visc}/\tau_{\rm MRI} \approx (\varv_{\rm K}/a)^2/\alpha$ (Hayasaki & Okazaki 2006) indicates that in the outer parts of the disk where the azimuthal velocity is lower than the thermal speed the magnetorotational instability would not be effective. As this happens at supersonic velocities, this again supports our conclusion that Eq. (15) indeed gives the upper limit for the angular momentum loss.

Moreover, the ratio of the particle kinetic energy to the absolute value of its gravitational potential energy is roughly equal to 1/4 at the critical point. Consequently, for radius few times larger than the critical one the disk material may freely escape the star and the viscous support is no longer needed.

The loss of the viscous coupling may occur even before the radial disk expansion becomes supersonic. In such a case for $\tilde{\alpha} \to 0$ from Eq. (12) follows that the disk starts to be momentum conserving and the location of the point where this occurs sets the outer disk radius $R_{\rm out}$.

Note also that the disk equations were derived assuming that the disk is geometrically thin, i.e., $H \ll r$. However, at the critical point the ratio of the disk scale height to radius $H/r \approx \sqrt{\frac{3}{10}}$ is of the order of unity (assuming isothermal disk). the vertical averaging used for the obtaining of Eqs. (7)–(9) is no longer applicable. On the other hand, because the angular momentum loss reaches a plateau below this point, this effect has not a significant influence on our results.

7.2. Influence of stars in a close neighbourhood

For members of binaries or for stars in a very dense star cluster the disk can be potentially truncated due to the influence of a nearby star.

The nearby star could disrupt the disk by its gravitational interaction with the disk. In this case the outer disk radius is that at which the gravitational field of the nearby star starts to dominate, i.e., the Roche lobe radius in the case of binaries.

If the disrupting star is luminous one, then it may disrupt the disk via the radiative force. This case is analogous to the case of the radiative ablation due to the central star. Consequently, we conclude that this effect would be important only if the nearby star is located within a few radii from the central star.

Finally, the nearby star may heat the disk material increasing the local sound speed, and consequently decreasing the critical radius above which the disk material may leave the star.

Taking all discussed disruption mechanisms together, we conclude that in the case of the nearby star with a similar spectral type the disruption is effective only if the disrupting star is at the distance lower or comparable to the critical radius. If the nearby companion is able to disrupt the disk, the angular momentum loss becomes less efficient, and the star has to lose a larger amount of mass to keep the rotation velocity below the critical one. Consequently, we expect larger disk mass-loss in close binaries and in very dense star clusters.

7.3. The disk build-up and its angular momentum

In the analysis presented here we used an assumption of constant required angular momentum loss rate, which enabled us to use stationary equations. This assumption is reasonable in most phases of the stellar evolution, as the evolutionary timescale is much longer than the typical timescale of the disk build-up, which is of the order of years (Okazaki 2004; Jones et al. 2008). This also means that the transitional processes that occur when the star reaches or leaves the critical limit are more complicated than studied here.

In the course of the stellar evolution, when the surface rotational velocity reaches the critical limit, in a first time the disk appears because it is feeded by the mechanical mass loss. The disc grows and part of it is ablated and part is transported away via viscous coupling until an equilibrium between the required angular momentum loss rate and mass-loss rate is achieved. During this process the disk own angular momentum could be of some importance.

On the other hand, when the star leaves the critical limit, an inner part of the remaining disk is accreted on the star while other parts are expelled into the interstellar medium (Okazaki 2004).

7.4. Implication for stars with disk

The processes discussed here might be relevant also for other stars with disks. For example, the disk radiative ablation might be one of the reasons why the Be phenomenon is typical for B stars only, whereas for more luminous O stars any disk could be destroyed by the radiative force.

Similar effects should also be present in accretion disks during star formation. In more luminous stars the radiative ablation could contribute to the disk photoevaporation (e.g., Adams et al. 2004; Alexander et al. 2006) in dispersing of the disk. Moreover, a similar process of the angular momentum transfer is present also in the accretion disks of these stars, consequently influencing the distribution of the rotational speeds on the ZAMS.

7.5. Implications for first stars

Mechanical mass loss through a decretion disk can be a ubiquitous phenomenon especially for Pop III or very metal poor stars. Indeed as shown by Ekström et al. (2008a) pure hydrogen-helium Pop III stars with masses above 60 $M_\odot$, beginning their evolution on the ZAMS with a surface velocity around 70% of the critical angular velocity, will reach the critical velocity during the MS phase. This arises because of two effects: first angular momentum is transported from the inner regions to the surface during the MS phase; second, the angular momentum accumulates at the surface since it is not removed by stellar winds. Note that in the absence of metals hydrogen and helium are unable to drive a line-driven wind being nearly completely ionized (Krtička & Kubát 2006).

As hydrogen-helium first stars are unable to launch a line-driven wind, we expect the radiative ablation to be inefficient close to the star. On the other hand, at larger distances a nonnegligible fraction of hydrogen could become neutral, enabling the possibility of disk radiative ablation.

The disk wind mass-loss rate in such case could be described as a flow with a very low value of $\bar{Q}$ (corresponding likely just to Lyα line force). A rough estimate of the disk wind mass-loss rate in this case could be obtained inserting instead of $\dot{M}_{\rm CAK}$ the single line mass-loss rate estimate $\dot{M} \approx L/c^2$ (Lucy & Solomon 1970) in Eq. (20). Anyway, in most cases such flow would be likely inefficient, especially because the disk wind mass-loss rate originates close to the star (see Fig. A.2). Consequently, the relation between the mass-loss rate required for a given angular momentum loss rate would be given by the wind-free condition Eq. (15).

7.6. Future work

The most uncertain ingredients of a proposed model are the viscous coupling, the disk temperature distribution and the radiative ablation. To include these processes we applied the same description used in the accretion disk theory and the theory of radiatively driven winds of hot stars. This may not be completely adequate for the description of decretion disk especially at large distances from the star studied here. Consequently, future work should address these problems.

8. Conclusions

We examine the mechanism of the mass and angular momentum loss via decretion disks associated with near-critical rotation. The disk mass loss is set by the angular momentum needed to keep the stellar rotation at or below the critical rate. We study the potentially important role of viscous coupling in outward angular momentum transport in the decretion disk, emphasizing that the specific angular momentum at the outer edge of the disk can be much larger than at the stellar surface. For a given stellar interior angular momentum excess, the mass loss required from a decretion disk can be significantly less than invoked in previous models assuming a direct, near-surface release.

The efficiency of the angular momentum loss via disk depends on the radius at which the viscous coupling ceases the transport the angular momentum to the outflowing material. When the radiative force is negligible, we argue that this likely happens close to the disk sonic (critical) point setting the most efficient angular momentum loss. In the opposite case, when the radiative force is nonnegligible, there is not a single point beyond which the viscous coupling disappears. The disk is continuously ablated below the sonic point, and the ablated material ceases to be viscously coupled, decreasing the efficiency of angular momentum loss.

We describe the method to include these processes into evolutionary calculations. The procedure provided enables to calculate the mass-loss rate necessary for a required angular momentum loss just from the stellar and line force parameters. We can distinguish three different physical circumstances:

- case A: When the disk wind is able to remove the whole excess of angular momentum (the disk is completely ablated by the wind, see Eq. (28)) then the outer disk radius is given by Eq. (28), and the required mass loss is given by Eq. (20). The limiting case $R_{\rm out} \approx R_{\rm eq}$ would then correspond to the near surface release of the matter without any disk. Note that in a rare case when the analysis leads to $R_{\rm out} > R_{\rm crit}$ the radius $R_{\rm crit}$ should be used as the outer disk radius (case B). The expressions presented in the paper are given in the hypothesis of an optically thick disk and should be appropriately modified for optically thin disks.
- case B: If the raditiave force is not able to remove sufficient angular momentum (the disk is not completely ablated) then part of the excess angular momentum must be carried away by the disk (Eq. (29)). In this case Eqs. (29), (30) can be used to estimate the mass-loss rate. The outer disk edge could be identified with the critical point.
- case C: If the effects of the radiative force are negligible, then the whole excess of angular momentum is carried away by the disk and the the outer disk edge is approximately given by $R_{\rm crit}$ and the required mass-loss rate could be derived from Eq. (15).

Finally, we note that, in absence of strong magnetic field, many of the features discussed here may also be applicable to the case of star-formation accretion disks.

Acknowledgements. This work was supported by grant GA ČR 205/08/0003. This research made use of NASA's Astrophysics Data System.

Appendix A: Disk wind mass-loss rate

A.1. Disk optical depth

In the case when the disk is optically thick in continuum, the disk outflow may be driven not only by the radiation from the stellar surface, but also by the stellar radiation reprocessed by the disk (Gayley et al. 1999). To estimate the optical depth of the disk, let us assume hydrogen and helium to be ionized in the disk. In this case a significant part of the disk optical depth originates due to the light scattering on free electrons (for wavelengths lower than that corresponding to the Balmer or Lyman jump also bound-free transitions may contribute). The transverse optical depth is then roughly given by $\tau = \int \kappa_{\rm e}\rho\,{\rm d}z = \kappa_{\rm e}\Sigma$, where $\kappa_{\rm e}$ is the Thomson scattering cross-section per unit of mass. The disk is optically thick in the vertical direction ($\tau > 1$) if the mass-loss rate is larger than

$$\dot{M} > \frac{2\pi r v_r}{\kappa_{\rm e}} \approx 10^{-12}\,M_\odot\,{\rm year}^{-1}\left(\frac{r}{1\,R_\odot}\right)\left(\frac{v_r}{1\,{\rm m\,s^{-1}}}\right)\cdot \tag{A.1}$$

For a given mass-loss rate the disk is optically thick close to the star, while becoming optically thin at larger distances. For example, for a typical disk mass-loss rate required by the evolutionary calculations $10^{-5}\,M_\odot/{\rm year}^{-1}$ (e.g. Ekström et al. 2008b) the disk is optically thick even at large distances from the star $r \approx 10^3\,R_\odot$ for subsonic radial velocities. Consequently, in realistic situations the disk is likely to be optically thick, at least close to the star, resembling the "pseudophotosphere" of Be stars (e.g. Koubský et al. 1997).

Contrary to very dense hot star winds (where the radiative flux comes from regions below the photosphere), here we expect that the wind from the optically thick disk starts to accelerate above the point where the disk optical depth is unity. Numerical results show that the height of this point is comparable to the disk scale height H for moderate disk mass-loss rates $\dot{M} \lesssim 10^{-5}\,M_\odot/{\rm year}^{-1}$. Consequently, we shall neglect the disk geometrical height in in our analyze here and assume that the disk wind originates from the equatorial plane.

A.2. Disk wind equations

The outflow from the optically thick disk irradiated by the central star can be understood within the framework of the wind driven by external irradiation (Gayley et al. 1999). We study the disk outflow in noninertial frame corotating with the disk. We use the Cartesian coordinates with z axis perpendicular to the disk (see Fig. A.1). The disk wind originates in the disk plane $z = 0$. We assume purely vertical flow with velocity $v_z(z)$ and we neglect a potentially important part of the radiative force due to the Keplerian velocity gradient (Gayley et al. 2001).

The stationary continuity equation

$$\nabla\left(\rho\mathbf{v}\right) = 0, \tag{A.2}$$

takes within our assumptions the form of

$$\frac{\partial}{\partial z}\left(\rho v_z\right) = 0, \quad \text{or} \quad \dot{m} \equiv \rho v_z = \text{const.}, \tag{A.3}$$

where $\dot{m}$ is the disk wind mass-loss rate per unit of disk surface.

The radiative force of the ensemble of lines in the Sobolev approximation is then (Rybicki & Hummer 1978; Cranmer & Owocki 1995; Gayley 1995)

$$\boldsymbol{g}_{\rm rad} = \frac{c^{-2\alpha}}{1-\alpha}\left(\frac{\kappa_{\rm e}\bar{Q}}{c}\right)^{1-\alpha}\oint I(\boldsymbol{n})\left(\frac{\boldsymbol{n}\nabla\left(\boldsymbol{n}\boldsymbol{v}\right)}{\rho}\right)^{\alpha}\boldsymbol{n}\,{\rm d}\Omega, \tag{A.4}$$

where $\bar{Q}$ and α are line force parameters. Ignoring the incoming beam, and simply assuming all the locally normal incident radiation from one hemisphere is directly reflected upward in vertical beam normal to the disk, the intensity is given by

$$\begin{aligned} I(\mu,\phi) &= \delta(\phi)\delta(\mu-1)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}{\rm d}\tilde{\phi}\,\cos\tilde{\phi}\int_{\mu_*}^{1}{\rm d}\tilde{\mu}\,\sqrt{1-\tilde{\mu}^2}\,I_* \\ &= \frac{2}{\pi}\delta(\phi)\delta(\mu-1)F\frac{r^2}{R^2}\int_{\mu_*}^{1}\sqrt{1-\tilde{\mu}^2}{\rm d}\tilde{\mu}, \end{aligned} \tag{A.5}$$

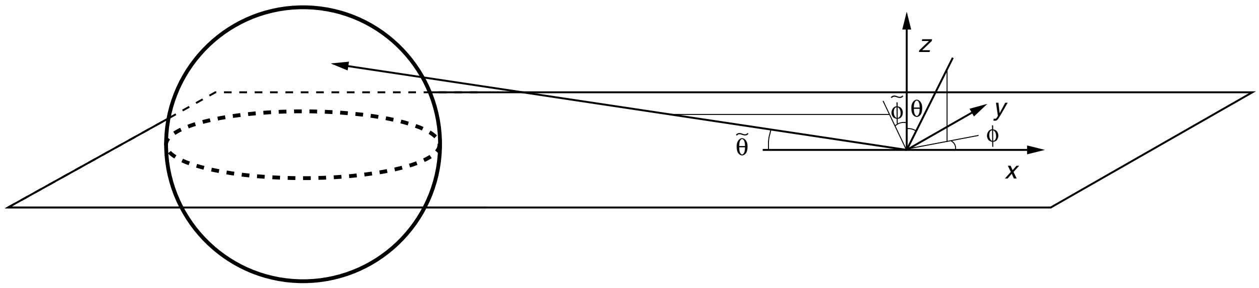

Fig. A.1. The coordinate system for the calculation of the radiative force.

where $I_* = (r/R)^2 F/\pi$ is the emergent intensity from the stellar photosphere, F is the radiative flux at radius r, R is the stellar radius, $\tilde{\mu} = \cos\tilde{\theta}$, $\tilde{\phi}$ are spherical coordinates with origin at the stellar centre, $\mu_* = \sqrt{1 - R^2/r^2}$, and μ, and ϕ are the direction cosine and azimuthal angle measured from the disk plane (see Fig. A.1). The z-component of the radiative force in this case is (Eq. (A.4), Gayley et al. 1999)

$$g_{\rm rad} = C \left| v_z \frac{\partial v_z}{\partial z} \right|^{\alpha} f_z, \tag{A.6}$$

where

$$f_z = \frac{2}{\pi} F \frac{r^2}{R^2} \int_{\mu_*}^{1} \sqrt{1-\tilde{\mu}^2}\, {\rm d}\tilde{\mu} = \frac{F}{\pi}\frac{r^2}{R^2} \arccos(\mu_*) - \frac{F}{\pi}\frac{r}{R}\mu_*, \tag{A.7}$$

and

$$C = \frac{1}{1-\alpha} \left(\frac{\kappa_{\rm e} \bar{Q}}{c} \right)^{1-\alpha} \left(\dot{m} c^2 \right)^{-\alpha} \cdot \tag{A.8}$$

The z-component of the momentum equation including the gravity term and neglecting the gas pressure term is

$$v_z \frac{\partial v_z}{\partial z} = C \left| v_z \frac{\partial v_z}{\partial z} \right|^{\alpha} f_z - \frac{GMz}{(r^2+z^2)^{3/2}} \cdot \tag{A.9}$$

The vertical momentum equation Eq. (A.9) can be solved using the transformations

$$w = \frac{r}{2GM} v_z^2, \tag{A.10a}$$

$$\zeta = \frac{z}{r}, \tag{A.10b}$$

$$K = C f_z \left(\frac{GM}{r^2} \right)^{\alpha - 1}, \tag{A.10c}$$

yielding

$$w' = K w'^{\alpha} - \frac{\zeta}{(1+\zeta^2)^{3/2}}, \tag{A.11}$$

where the prime denotes the derivative with respect to ζ. This equation has a critical point

$$1 - \alpha K_{\rm c} w_{\rm c}'^{\alpha-1} = 0, \tag{A.12}$$

where the subscript c denotes the critical point values, from which using Eq. (A.11) we derive

$$w_{\rm c}' = \frac{\alpha}{1-\alpha} \frac{\zeta_{\rm c}}{\left(1+\zeta_{\rm c}^2\right)^{3/2}} \cdot \tag{A.13}$$

The location of the critical point above the disk plane can be derived from the regularity condition (CAK), which yields that the critical point occurs at the point of maximum of z component of the gravity acceleration at a given streamline,

$$\zeta_{\rm c} = \frac{1}{\sqrt{2}} \cdot \tag{A.14}$$

Hence, the point of the maximum acceleration acts as the throat of the nozzle flow (Feldmeier & Shlosman 1999).

The total disk wind mass-loss rate is then given by an integral of the mass-loss rate per unit of the disk surface $\dot{m}$ between the equatorial radius $R_{\rm eq}$ and the outer disk radius $R_{\rm out}$

$$\dot{M}_{\rm dw}(R_{\rm out}) = 2 \times 2\pi \int_{R_{\rm eq}}^{R_{\rm out}} \dot{m} r\, {\rm d}r, \tag{A.15}$$

where from Eqs. (A.8), (A.10), (A.12)

$$\dot{m} = \frac{1}{c^2} \left(\frac{\kappa_{\rm e}\bar{Q}}{c w_{\rm c}'} \right)^{\frac{1-\alpha}{\alpha}} \left(\frac{f_z \alpha}{1-\alpha} \right)^{\frac{1}{\alpha}} \left(\frac{GM}{r^2} \right)^{\frac{\alpha-1}{\alpha}} \cdot \tag{A.16}$$

The factor of 2 in Eq. (A.15) comes from the fact that the wind originates from both sides of the disk. Consequently, the total disk wind mass-loss rate is

$$\dot{M}_{\rm dw}(R_{\rm out}) = \frac{\alpha}{1-\alpha} \frac{L}{c^2} \left(\Gamma \bar{Q} \right)^{\frac{1-\alpha}{\alpha}} P_1 \left(\frac{R_{\rm out}}{R} \right), \tag{A.17}$$

where the Eddington parameter $\Gamma = \kappa_{\rm e} L/(4\pi GMc)$, and

$$P_\ell(x_{\rm out}) = \left(\frac{\alpha}{1-\alpha} \right)^{\frac{1-\alpha}{\alpha}} \int_{3/2}^{x_{\rm out}} w_{\rm c}'^{\frac{\alpha-1}{\alpha}} \left(\frac{f_z}{F} \right)^{\frac{1}{\alpha}} \frac{{\rm d}x}{x^\ell}, \tag{A.18}$$

and F is the flux at radius r. Comparing with the CAK mass-loss rate estimate Eq. (16) we have

$$\dot{M}_{\rm dw}(R_{\rm out}) = P_1 \left(\frac{R_{\rm out}}{R} \right) \dot{M}_{\rm CAK}. \tag{A.19}$$

Assuming the disk wind is not viscously coupled to the disk, the total angular momentum loss via the disk wind is

$$\dot{J}_{\rm dw}(R_{\rm out}) = 2 \times 2\pi \int_{R_{\rm eq}}^{R_{\rm out}} r^2 v_\phi \dot{m}\, {\rm d}r, \tag{A.20}$$

where $\dot{m}$ is given by Eq. (A.16). As the disk wind originates mainly from the regions close to the star (with $r/R \lesssim 10$, see Fig. A.2), where the azimuthal velocity is roughly equal to the Keplerian one (see Fig. 1), we can assume $v_\phi \approx v_{\rm K}(r)$ in Eq. (A.20) and consequently

$$\dot{J}_{\rm dw}(R_{\rm out}) = \frac{\alpha}{1-\alpha} \frac{L}{c^2} \left(\Gamma \bar{Q} \right)^{\frac{1-\alpha}{\alpha}} R\, v_{\rm K}(R)\, P_{\frac{1}{2}} \left(\frac{R_{\rm out}}{R} \right) \cdot \tag{A.21}$$

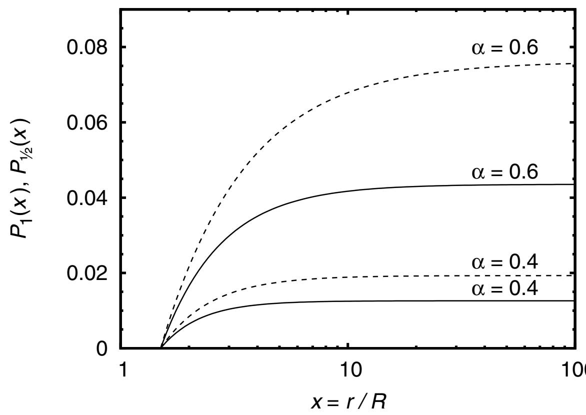

Fig. A.2. The behaviour of $P_1(x)$ (solid line), and $P_{\frac{1}{2}}(x)$ (dashed line, Eq. (A.18)) for different values of CAK parameter α.

Again, using the CAK mass-loss rate estimate Eq. (16) the disk wind angular momentum loss

$$\dot{J}_{\rm dw}(R_{\rm out}) = R\, \upsilon_{\rm K}(R)\, P_{\frac{1}{2}}\left(\frac{R_{\rm out}}{R}\right)\dot{M}_{\rm CAK} \tag{A.22}$$

is by a factor of $P_{\frac{1}{2}}(R_{\rm out}/R)$ larger than the angular momentum loss due to the CAK wind launched from equator of hypothetical critically rotating spherical star with radius R.

Expansion of Eq. (A.7) about $R/r = 0$ shows that far away from the star, $f_z/F \approx 2R/(3\pi r)$. The comparison between a precise formula Eq. (A.7) and its asymptotic form shows apart from a small region $1 < x < 3/2$, the agreement is actually quite good. If we use this asymptotic form over the full range from $x = 3/2$ to $x_{\rm out}$ in Eq. (A.18) we find (using Eqs. (A.13), (A.14))

$$P_\ell(x_{\rm out}) = \frac{2\pi^{-\frac{1}{\alpha}}3^{\frac{1}{2\alpha}-\frac{3}{2}}}{\frac{1}{\alpha}+\ell-1}\left[\left(\frac{3}{2}\right)^{1-\ell-\frac{1}{\alpha}} - x_{\rm out}^{1-\ell-\frac{1}{\alpha}}\right]. \tag{A.23}$$

Our analytical results (see Fig. A.2) are in agreement with numerical calculations of Proga et al. (1998) that show the disk mass loss is dominated by material arising from the inner region of the disk ($r < 10\,R$). Consequently, if the radiative force is not strong enough to disrupt the disk close to the star, it is unlikely that it would be able to do so in the outer parts of the disk.

The modern hot star wind models give slightly lower estimate of the mass-loss rate than the CAK formula Eq. (16) due to inclusion of finite disk correction. However, because formula Eq. (A.23) for a typical value of $\alpha \approx 0.6$ (Puls et al. 2000; Krtička 2006) gives for $x_{\rm out} \to \infty$ the value of $P_1 \approx 0.04$, the disk wind mass-loss rate is significantly lower than the stellar wind mass-loss rate. Similarly, the value of $P_{1/2} = 0.08$ indicates that the angular momentum loss from the disk wind is also lower than the angular momentum loss due to the stellar wind.

References

Adams, F. C., Hollenbach, D., Laughlin, G., & Gorti, U. 2004, ApJ, 611, 360
Alexander, R. D., Clarke, C. J., & Pringle, J. E. 2006, MNRAS, 369, 216
Anderson, E., Bai, Z., Bischof, C., et al. 1999, LAPACK Users' Guide, (Philadelphia: SIAM)
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
Bowen, G. H. 1988, ApJ, 329, 299
Carciofi, A. C., & Bjorkman, J. E. 2008, ApJ, 684, 1374
Castor J. I., Abbott D. C., & Klein R. I. 1975, ApJ, 195, 157 (CAK)
Cranmer, S. R., & Owocki, S. P. 1995, ApJ, 440, 308
Donati, J.-F., Howarth, I. D., Bouret, J.-C. et al. 2006, MNRAS, 365, L6
Ekström, S., Meynet, G., Chiappini, C., Hirschi, R., & Maeder, A. 2008a, A&A, 489, 685
Ekström, S., Meynet, G., Maeder, A., & Barblan, F. 2008b, A&A, 478, 467
Feldmeier, A., & Shlosman, I. 1999, ApJ, 526, 344
Feldmeier, A., Shlosman, I., & Vitello, P. 1999, ApJ, 526, 357
Gayley, K. G. 1995, ApJ, 454, 410
Gayley, K. G., Owocki, S. P., & Cranmer, S. R. 1999, ApJ, 513, 442
Gayley, K. G., Ignace, R., & Owocki, S. P. 2001, ApJ, 558, 802
Hawley, J. F., & Krolik, J. H. 2001, ApJ, 548, 348
Hayasaki, K., & Okazaki, A. T. 2006, MNRAS, 372, 1140
Howarth, I. D. 2004, in Stellar Rotation, ed. A. Maeder, & P. Eenens (San Francisco: ASP), IAUS, 215, 33
Howarth, I. D. 2007, Active OB-Stars: Laboratories for Stellar & Circumstellar Physics, ed. S. Štefl, S. P. Owocki, & A. T. Okazaki (San Francisco: ASP), 15
Jones, C. E., Sigut, T. A. A., & Porter, J. M. 2008, MNRAS, 386, 1922
Kautsky, J., & Elhay, S. 1982, Numer. Math., 40, 407
Korčáková, D., & Kubát, J. 2005, A&A, 440, 715
Koubský, P., Harmanec, P., Kubát, J., et al. 1997, A&A, 328, 551
Krtička, J. 2003, in Stellar Atmosphere Modelling, ed. I. Hubeny, D. Mihalas, & K. Werner (San Francisco: ASP), ASP Conf., 288, 259
Krtička, J. 2006, MNRAS, 367, 1282
Krtička, J., & Kubát, J. 2006, A&A, 446, 1039
Lee, U., Osaki, Y., & Saio, H. 1991, MNRAS, 250, 432
Lightman, A. P. 1974, ApJ, 194, 419
Lucy, L. B., & Solomon, P. M. 1970, ApJ, 159, 879
Maeder, A., & Meynet, G. 2008, Rev. Mex. Astron. Astrofis. Conf. Ser., 33, 38
Marigo, P., Girardi, L., Chiosi, C., & Wood, P. R. 2001, A&A, 371, 152
Meynet, G., Ekström, S., & Maeder, A. 2006, A&A, 447, 623
Meynet, G., Ekström, S., Maeder, A., & Barblan, F. 2007, Active OB-Stars: Laboratories for Stellar & Circumstellar Physics, ed. S. Štefl, S. P. Owocki, & A. T. Okazaki (San Francisco: ASP)325
Mikulášek, Z., Krtička, J., Henry, G. W., et al. 2008, A&A, 485, 585
Millar, C. E., & Marlborough, J. M. 1998, ApJ, 494, 715
Okazaki, A. T. 2001, PASJ, 53, 119
Okazaki, A. T. 2004, in Stellar Rotation, ed. A. Maeder, & P. Eenens (San Francisco: ASP), IAUS, 215, 529
Okazaki, A. T., Bate, M. R., Ogilvie, G. I., & Pringle, J. E. 2002, The Physics of Cataclysmic Variables and Related Objects, ed. B. T. Gänsicke, K. Beuermann, & K. Reinsch (San Francisco: ASP), 519
Owocki, S. P. 2004, in Evolution of Massive Stars, Mass Loss and Winds, ed. M. Heydari-Malayeri, Ph. Stee, & J.-P. Zahn (Les Ulis Cedex: EDP), EAS Publ. Ser., 13, 163
Owocki, S. P., Cranmer, S. R., & Gayley, K. G. 1996, ApJ, 472, L115
Petrenz P., & Puls J. 2000, A&A, 358, 956
Porter, J. M., & Rivinius, T. 2003, PASP, 115, 1153
Proga, D., Stone, J. M., & Drew, J. E. 1998, MNRAS, 295, 595
Proga, D., Stone, J. M., & Drew, J. E. 1999, MNRAS, 310, 476
Puls, J., Springmann, U., & Lennon, M. 2000, A&AS, 141, 23
Puls, J., Vink, J. S., & Najarro, F. 2008, A&ARv, 16, 209
Rybicki, G. B., & Hummer, D. G. 1978, ApJ, 219, 645
Townsend, R. H. D., Owocki, S. P., & Howarth, I. D. 2004, MNRAS, 350, 189
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Stone, J. M., Hawley, J. F., Gammie, C. F., & Balbus, S. A. 1996, ApJ, 463, 656
Vink, J. S., de Koter, A., & Lamers, H. J. G. L. M. 2001, A&A, 369, 574
Woitke, P. 2006, A&A, 452, 537