

# The Earth's variable Chandler wobble

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## ABSTRACT

**Aims.** We investigated the causes of the Earth's Chandler wobble variability over the past 60 years. Our approach is based on integrating of the atmospheric and oceanic angular momentum computed by global circulation models. We directly compared the result of the integration with the Earth's pole coordinate observed by precise astrometric, space, and geodetic techniques. This approach differs from the traditional approach in which the observed polar motion is transformed into a so-called geodetic excitation function, and compared afterwards with the angular momentum of the external geophysical fluid layers.

**Methods.** In the time domain, we integrated the atmospheric angular momentum time series from the National Center for Environmental Prediction/National Center for Atmospheric Research Reanalysis project and the oceanic angular momentum data from the ECCO consortium. We extracted the Chandler wobble from this modeled polar motion by singular spectrum analysis, and compared it with the Chandler wobble extracted from the observed polar motion given by the International Earth Rotation and Reference Systems Service data.

**Results.** We showed that the combination of the atmosphere and the oceans explains most of the observed Chandler wobble variations, and is consistent with results reported in the literature and obtained with the traditional approach. Our approach allows one to appreciate the separate contributions of the atmosphere and the oceans to the various bumps and valleys observed in the Chandler wobble. Though the atmosphere explains the Chandler wobble amplitude variations between 1949 and 1970, the reexcitation of the Chandler wobble that begins in the 1980s, after a minimum around 1970, and that reaches its maximum in the late 1990s is due to the oceans, while the atmospheric contribution remains stable within the same period.

**Key words.** reference systems – Earth

## 1. Introduction

The Chandler wobble (CW) is a free rotational mode of the Earth associated with its dynamical ellipticity. In the absence of forcing, the CW would have a period of 430.3 days (Mathews et al. 2002) fixed mainly by the whole planet's dynamical ellipticity and, more marginally, by mantle and ocean deformabilities, and would lose most of its energy after a few tens of years because of dissipation in the mantle and in the oceans. Observation of the Earth's polar motion, however, reveals a prograde oscillation whose pseudo period can be as far as 20 days from the above value (e.g., Vondrák 1988; Vicente & Wilson 1997). It gains energy at some times (e.g., Danjon & Guinot 1954) so that it never disappears.

Plag (1997) interpreted the CW as a “forced free” mode and proposed a climatic forcing. He associated the temporal characteristics of the fourteen to sixteen months oscillation over the northern hemisphere with frequency or phase shifts of the CW. According to Celaya et al. (1999), Gross (2000), and Brzeziński & Nastula (2002), the CW excitation is accounted for, on average, by the atmosphere and oceans. Following Aoyama et al. (2003), atmospheric and ocean-bottom pressure and winds are comparable sources of excitation, intermittently playing a prominent role. Seitz & Schmidt (2005) conclude that the atmospheric and oceanic bottom pressure fluctuations are the prominent cause of the CW variability. They could account for some features in the band of 400–460 days by wavelet analysis: an amplitude increase between 1980 and 1990 and a decrease after 1993. However, they did not precisely investigate the amplitude and phase variations of the Chandler period and neglected the

wind contribution. Liao et al. (2007) noticed the variability in the wind contribution around the Chandler period. The lack of clear conclusions is partly due to discrepancies up to 0.7 millisecond of arc (mas) between the various atmospheric angular momentum data sets in the Chandler frequency band associated with inaccurately determining the wind term.

Though most of the above studies implement comparisons between the fluid layer angular momentum and the so-called geodetic excitation function derived from the observed pole coordinates, very few studies used the inverse approach that integrates the former and directly compares it with the observed polar motion (see, e.g., Vondrák 1989, 1990). We revisited the CW variability in light of the latter approach, as described in Sect. 2. Data are presented in Sect. 3 and results discussed in Sect. 4.

## 2. The polar motion excitation by fluid layers

At low frequency and for a deformable Earth surrounded by fluid layers, the time evolution of the complex pole coordinates  $p = x - iy$  is given by (e.g., Barnes et al. 1983)

$$\underbrace{p + i \frac{\dot{p}}{\sigma_c}}_{\chi_g} = \underbrace{\frac{1.02c}{(C-A)\Omega}}_{\chi_f^{\text{matter}}} + \underbrace{\frac{1.43h}{(C-A)\Omega}}_{\chi_f^{\text{motion}}} = \chi_f \quad (1)$$

where  $\sigma_c = \sigma_0(1+i/2Q)$  is the Chandler frequency,  $c = c_{13} + ic_{23}$  and  $h = h_1 + ih_2$  are the off-diagonal moment of inertia and the equatorial relative angular momentum of the fluid layer, respectively, and  $A$  and  $C$  are the mean equatorial and axial moments of

inertia of the Earth, respectively. The LHS of (1) is referred to as the geodetic excitation  $\chi_g$ , while  $\chi_f$  is referred to as the geophysical (or fluid) excitation. This equation does not account for the existence of fluid outer and inner cores. However, the resonances produced by these internal layers lie in the diurnal frequency band and do not substantially affect seasonal and interannual polar motion. Mathews et al. (2002) predicts a resonance due to the inner core with a theoretical period of  $\sim 2400$  days but with an amplitude expected to be insignificant for our purpose.

The resonant frequency  $\sigma_c$ , or eigenfrequency, can be expressed in terms of the whole Earth's dynamical ellipticity  $e = (C - A)/C$  and the Love number  $k$ , with additional contributions from anelasticity, and ocean loading and currents to the mantle deformability (see, e.g., Sasao et al. 1980; Mathews et al. 1991, 2002). The theoretical computation of  $\sigma_c$  leads to a period of 430.3 days and a quality factor  $Q = 88$  (Mathews et al. 2002). The theoretical period is consistent with other estimates of the observed Chandler period based on analyses of polar motion time series (e.g., Vicente & Wilson 1997). However, there is no consensus on the value of  $Q$ : depending on the time interval and the analysis method, it ranges from 50 (Furuya & Chao 1996; Kuehne et al. 1996) to 180 (Vicente & Wilson 1997; Aoyama et al. 2003). Gross (2000) and Brzeziński & Nastula (2002) find intermediate values of  $\sim 140$ .

The most common approach to explaining the Chandler wobble excitation by external fluid layers consists of computing the LHS of (1) from observed pole coordinates using, e.g., the discrete formula proposed by Wilson (1985) (also Eq. (1) of Vicente & Wilson 1997), and comparing it against the RHS directly given by fluid layer angular momentum time series. Nevertheless, the noise behavior of  $\chi_f$  and  $\chi_g$  around the Chandler frequency and their weak amplitude compared with the seasonal excitation can make this approach difficult. Conversely, a modeled polar motion obtained by time integration of the RHS of (1) can be directly compared with the observed pole coordinates.

The solution of (1) reads

$$p_f(t) = p_0(t_0)e^{i\sigma_c(t-t_0)} - i\sigma_c e^{i\sigma_c t} \int_{t_0}^t \chi_f(t')e^{-i\sigma_c t'} dt', \quad (2)$$

wherein  $t_0$  is the time at which one starts the integration. The first term represents a damped, free wobble of relaxation time  $2Q/\sigma_0 \sim 40$  years for  $Q = 100$ . This progressive damping is not observed, but balanced by the integral expressing the forced polar motion. The latter term mainly contains the effects of the seasonal forcing and an oscillation emerging in the Chandler period.

### 3. The data set

We used the pole coordinates from the IERS EOP C 01 data set, computed at the International Earth Rotation and Reference Systems Service (IERS) Earth Orientation Parameter Product Center<sup>1</sup>. These series are the result of the combination and smoothing of polar motion series obtained from optical astrometry, very long baseline radio interferometry, and various space and geodetic techniques (mainly global navigation and satellite system). Using other Earth orientation parameter series produced by other institutes (e.g., by the IERS Rapid Service/Prediction Center of the United States Naval Observatory, Washington, DC) would lead to the same conclusions since the agreement between the various data sets reaches

<sup>1</sup> <http://hpiers.obspm.fr/eop-pc>

a few mas, which is far below the CW amplitude variations investigated in this paper.

We considered the geophysical forcing made up of (i) atmospheric angular momentum (AAM) as given by the National Center for Environmental Prediction/National Center for Atmospheric Research (NCEP/NCAR) reanalysis project (Kalnay et al. 1996), and (ii) oceanic angular momentum (OAM) output from the ECCO model that employs the MIT global circulation model in a near-global domain (Fukumori et al. 2002). The model is forced by NCEP/NCAR reanalysis products (12-hourly wind stresses, daily adiabatic air-sea fluxes). In some versions of the ECCO run, the temperature and salinity of the model sea surface can be relaxed toward observed values, and sea level anomalies can be assimilated into the model to correct errors associated with inaccuracies in time-varying wind forcing. However, the longest available series over 1949–2002 (kf049f) were produced with no data assimilation. To complete the series up to 2009.5, we therefore concatenated with the series kf079, which is also obtained without data assimilation.

The data sets are made available through the IERS Special Bureaus for the Atmosphere<sup>2</sup> (Salstein et al. 1993) and the Oceans<sup>3</sup> (chaired by Gross at JPL, Pasadena). Since we investigated seasonal and interannual time scales, the AAM was used under the inverted barometer hypothesis. All time series spanned 1949.0–2009.5. They were made to be homogeneous by removing the high frequency part of the signal below 10 days with a Vondrák (1977) filter of admittance larger than 99% above 10 days and smaller than 1% below 4 days, and resampling them at 10-day intervals.

### 4. Results and discussion

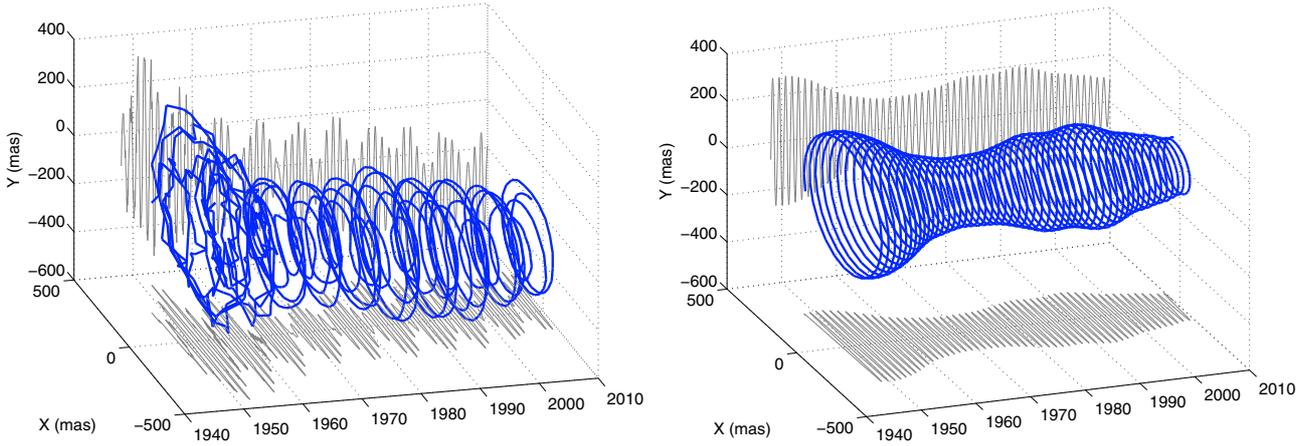
Using (2), we computed the modeled polar motion  $p_f(t)$  from the geophysical excitation by a trapezoidal integration. The value of the observed CW at  $t_0$  was taken as initial condition. Doing so, we assumed that the atmosphere and the oceans are the only sources of the observed CW, thus neglecting the contribution of continental waters. However, based on a predictive model, Celaya et al. (1999) concluded that the hydrological forcing of the CW does not compete with atmosphere and oceans.

The CW component of this modeled polar motion had to be compared with the CW component of the observed, or geodetic, polar motion. The CW was extracted from the observed and modeled polar motions by singular spectrum analysis (SSA, Vautard & Ghil 1989), a technique that allows one to isolate quasi periodic modes without assumptions on their periods. We chose a covariance lag close to 6.4 years in order to separate the CW from the annual oscillation whose amplitude is of the same order of magnitude (see Fig. 1). To figure out the error propagated by the SSA decomposition, we ran the SSA on a large number of replicated pole coordinate time series perturbed by a Gaussian noise of the same variance as the residuals (i.e., signal cleaned up from the first five reconstructed components including CW, annual wobble, and long term irregular variation), allowing one to make a reliable statistics on the stability of the principal components. We found that the relative error is  $\sim 1\%$ .

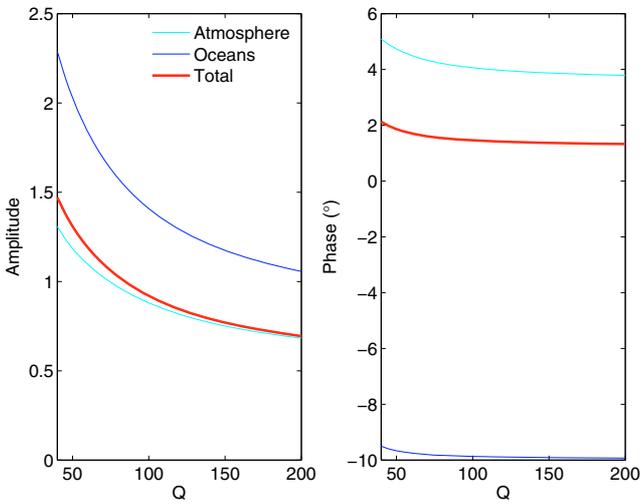
We repeated the integration of the geophysical data sets and the comparison with observed polar motion using different values for  $Q$  and computed the regression coefficient between the observed and modeled Chandler modes (Fig. 2). One sees that  $Q \sim 84$  produces an optimal regression.

<sup>2</sup> <http://www.aer.com/scienceResearch/diag/sb.html>

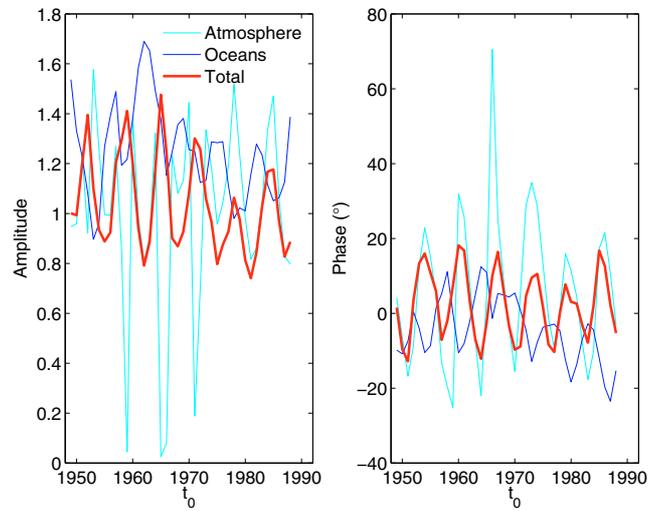
<sup>3</sup> <http://euler.jpl.nasa.gov/sbo>



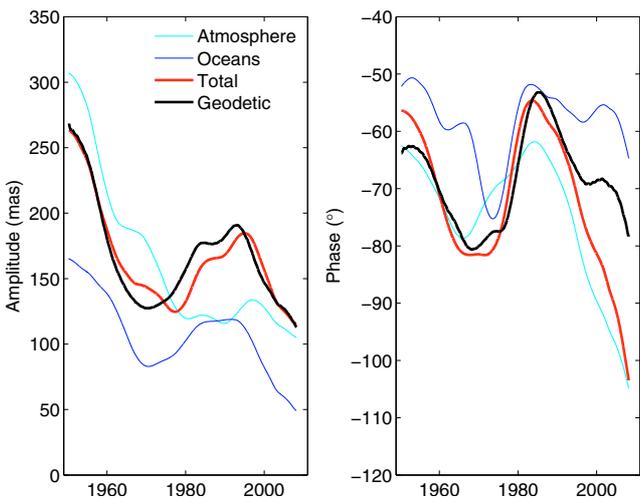
**Fig. 1.** The observed polar motion (*top*) and the Chandler mode (*bottom*) between 1949.0 and 2009.5 from the IERS EOP C 01 data set.



**Fig. 2.** The regression coefficient between the observed and modeled Chandler modes as a function of  $Q$ .



**Fig. 4.** The regression coefficient between the observed and modeled Chandler modes as a function of the starting epoch  $t_0$ .



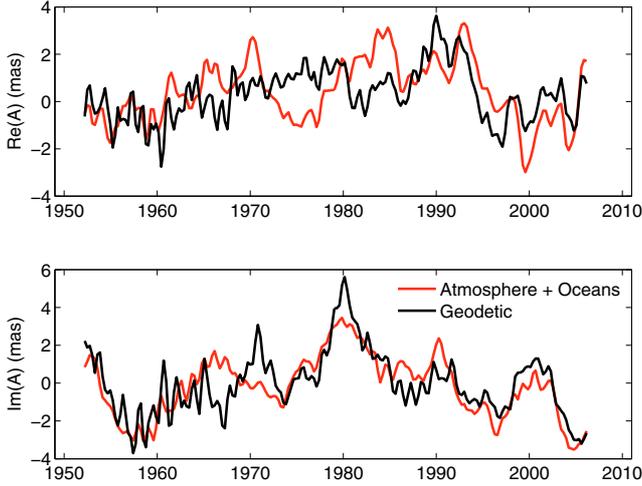
**Fig. 3.** The observed and modeled variations in the amplitude and the phase of the Chandler mode.

The amplitude and phase of the observed and modeled CW obtained with  $Q = 84$  are displayed in Fig. 3. The regression coefficient between the observed and modeled CW

is  $1.001 + 0.027i$ . The atmosphere alone poorly accounts for the CW irregularities. Though it explains the large amplitude before 1960, the minimum around 1970 and the bump in 1980–1990 is mainly due to the oceans, which confirms the results of Gross (2000). The atmospheric contribution remains stable within the same period. Similar remarks can be made for the phase variations before 1995. Around 2000, the phase of the observed CW presents a bump around  $-70^\circ$  that is likely due to the oceans. Nevertheless, the combination of atmosphere and oceans predicts a phase variation that is different by  $\sim 20^\circ$ .

The phase shifts of the CW are equivalent to variations of the observed Chandler period that can reach 33 days. The observed Chandler period, which is the pseudo period of the “forced free” CW, is distinct from the eigenperiod defined by the Earth’s structure properties. It corresponds to the period for which the forcing exhibits maximal energy in the vicinity of the eigenperiod. The eigenperiod cannot change by such an amount on time scales of a few years.

We repeated the integration using different starting epochs  $t_0$ , and we computed the regression coefficient between the observed and modeled Chandler modes (Fig. 4). The combination of the atmosphere with the oceans generally drew the amplitude



**Fig. 5.** The real and imaginary parts of the complex amplitude  $A(t)$  of the observed and modeled excitation functions at the Chandler frequency.

of the regression coefficient towards unity and the phase closer to zero.

As an alternative to the SSA technique, we extracted the Chandler mode from observed or modeled polar motion by least-square adjustments over a 6.4-yr sliding window. This method demands, however, making an assumption about the period of the Chandler mode (fixed to 430.3 days). The results were very close to those presented above. Another alternative consisted of slicing the wavelet spectrum at the Chandler frequency. This method was used in Seitz & Schmidt (2005) over a shorter time period, and it assessed the pressure terms of the atmospheric and oceanic angular momenta only, yielding results that were consistent with ours over the common time period.

To assess the consistency of the integration approach, we finally transformed the observed and modeled CW extracted by SSA into excitation functions using the Wilson (1985) formula. Let

$$p_c(t) = a(t)e^{i\sigma_0(t-t_0)} \quad (3)$$

be the modeled or observed CW, wherein the time variable complex amplitude  $a(t)$  is estimated by least squares over a 6.4-yr sliding window. As previously, the Chandler period is fixed to 430.3 days. Possible, but small, variations in the period are considered in the phase of  $a(t)$ . From Eq. (1), the corresponding excitation is given by

$$\chi_c(t) = A(t)e^{i\sigma_0(t-t_0)}, \quad (4)$$

where

$$A(t) = a(t) \left( 1 - \frac{\sigma_0}{\sigma_c} \right) + \frac{i\dot{a}(t)}{\sigma_c}. \quad (5)$$

Figure 5 displays the real and imaginary parts of the complex amplitude  $A(t)$  deduced from observed and modeled CW. Since the formal error on  $a(t)$  is of the order of 1 mas, one expects the error on  $A(t)$  to be majored by 0.3 mas, which is far smaller than the overall variability of the signals, so the integration approach provides an original way of isolating the CW excitation.

Our results show that the integration of the NCEP/NCAR re-analysis project's atmospheric data, combined with the ECCO oceanic data, explain most of the observed variations in the CW amplitude and phase since the middle of the twentieth century. The originality of this work resides in its using the integration approach, which complements the classical approach in most other works and is based on comparing excitation functions. The integration approach therefore constitutes an interesting alternative method, which has the advantage being more intuitive from the point of view of the astronomer who directly observes the position of the pole.

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