

# A fourth planet orbiting $\nu$ Andromedae

S. Curiel<sup>1</sup>, J. Cantó<sup>1</sup>, L. Georgiev<sup>1</sup>, C. E. Chávez<sup>2</sup>, and A. Poveda<sup>1</sup>

<sup>1</sup> Instituto de Astronomía, UNAM, Apartado Postal 70-264, 04510 México, DF, Mexico  
e-mail: scuriel@astroscu.unam.mx

<sup>2</sup> Instituto de Astronomía, UNAM, sede Ensenada, Km. 103 carretera Tijuana-Ensenada, 22860 Ensenada, Baja California, Mexico

Received 3 September 2010 / Accepted 7 November 2010

## ABSTRACT

We present a 4-planet Keplerian fit for the radial velocity curve of the F8V star  $\nu$  Andromeda, indicating the presence of a fourth planet in the system. We detect an additional fifth coherent signal in the radial velocity curve which we attribute to stellar activity. The discovery of a new planet around  $\nu$  Andromedae makes this system the fifth to contain, at least, four planets. These four planets have minimum masses of 0.69, 1.98, 4.13 and 1.06  $M_{\text{Jup}}$  and orbital periods of 4.62, 241.26, 1276.46 and 3848.9 days, respectively. We have numerically integrated the orbital solution for these four planets and find that the system is stable for at least 10 Myr. The orbit of the fourth planet coincides with an island of stability reported by Rivera & Haghighipour (2007, MNRAS, 374, 599). We find that the characteristics of the new fourth planet are very similar to those of Jupiter and that the planets in this system have very strong interactions with each other. As previously found,  $\nu$  And–b and  $\nu$  And–c are in apsidal alignment, while the orbit of the new planet ( $\nu$  And–e) is close to an external 3:1 resonance with  $\nu$  And–c.

**Key words.** planets and satellites: dynamical evolution and stability – stars: individual:  $\nu$  And – planets and satellites: detection

## 1. Introduction

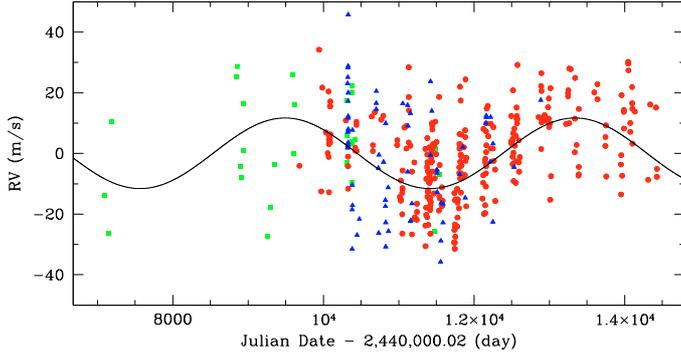
With the discovery of extrasolar planets during the past 15 years, it has now become evident that our solar system is not unique. Similar to our Sun, many stars are believed to be hosts to giant-and/or terrestrial-class planets and smaller objects. Many of the extrasolar single- and multiple-planet-hosting stars show trends in the residuals to their radial velocity fits implying that they may be harbouring additional objects (e.g., Fischer et al. 2001). However, at least one full orbital period must be observed in order to accurately model the Doppler velocity data with a Keplerian orbit. Improvements in precision radial velocity (RV) measurements and continued monitoring allow the detection of lower amplitude planetary signatures, as well as planetary signatures with longer periods.

We searched for planetary systems with large residuals after subtracting 3-orbit models. We found that a system with large residuals was  $\nu$  Andromedae (here after  $\nu$  And). Furthermore, we found that the residuals show a radial velocity trend that suggests the presence of an additional long period orbit in the system. In order to investigate the possible presence of a fourth planet in this planetary system, we use here the improved and updated data set collected at Lick and recently published by Wright et al. (2009), which contains 284 radial velocities taken over 13 years, starting in 1994 November. In addition, we have added 71 radial velocities taken with ELODIE and published by Naef et al. (2004), as well as 30 radial velocities taken at Lick and published by Fischer et al. (2003). The combined data set of 385 radial velocities cover a span of time of about 20 years. The errors of this data set ( $\sim 7.44 \text{ m s}^{-1}$ ) is mainly due to rotational line broadening (e.g., Fischer et al. 2003; Butler et al. 2006; Wright et al. 2009; Naef et al. 2004). There are additional known sources of error associated with astrophysical jitter, the instrument, and the analysis of the data. These sources combine

to give an additional source of noise, collectively termed “jitter”. The expected jitter in  $\nu$  And is between 4.2 and 10  $\text{m s}^{-1}$  (e.g., Saar et al. 1998; Butler et al. 2006). We have adopted a conservative noise jitter of  $\sim 10 \text{ m s}^{-1}$  in the Keplerian fitting of the data. We do not take into account the possible residual velocity offsets due to the changes of CCDs during the time span of the observations, which in the present case seem to be unimportant (e.g., Gregory & Fischer 2010).

We have recently developed a novel Asexual Genetic Algorithm (AGA; Cantó et al. 2009) capable of fitting different kinds of data sets (e.g., Coughlin et al. 2010). We found that this new algorithm can be useful in the fitting of Keplerian orbits, using the radial velocities (RV) of the host star of the planetary systems. In order to test the algorithm, Cantó et al. (2009) fitted 3 and 4 Keplerian orbits in the 55 CANCRI planetary system, using the RV data set published by Fischer et al. (2008). Five parameters ( $P$ ,  $t_0$ ,  $e$ ,  $\omega$ ,  $K$ ) were fitted for each planet, plus a radial velocity reference,  $V_0$ , intrinsic to the telescope and/or to calibration residuals. It was found that the fitted parameters are very similar to those already published (Fisher et al. 2008). This new algorithm can also be used to search for exoplanets in new RV data sets, as well as to search for new planetary components in planetary systems with large residuals.

In the present paper, we report the detection of a fourth planet in this planetary system,  $\nu$  And–e, with a minimum mass of 1.06  $M_{\text{Jup}}$ . In Sect. 2, we briefly recall the main characteristics of the host star  $\nu$  And, and of this planetary system, as reported in the literature. The enlarged set of measurements allows a re-examination of the structure of the  $\nu$  And planetary system, resulting in the discovery of an additional Jupiter-mass planet (Sect. 3). In Sect. 4, we use dynamical stability considerations to verify the robustness of our best-fit solutions. In Sect. 5 we discuss the discovery of a feature in the Doppler spectroscopy of



**Fig. 1.** Residuals of the best 3-planet fit model for the combined data set including 385 radial velocities obtained at Lick and with ELODIE (see text). The red circles corresponds to the 284 radial velocities taken at Lick and published by Wright et al. (2009), the green squares correspond to the additional 30 radial velocities taken at Lick and published by Fischer et al. (2003) and the blue triangles correspond to the 71 radial velocities taken with ELODIE and published by Naef et al. (2004). The solid line corresponds to the solution obtain with the best 4-planet fit model to the combined data set (see Table 1 and Fig. 2).

this planetary system, which we conclude is probably produced by stellar surface inhomogeneities.

## 2. $\nu$ Andromedae

$\nu$  And is a bright F8V star with a mass of  $1.3 M_{\odot}$  and a stellar radius of  $1.56 R_{\odot}$  (e.g., Butler et al. 1999). The distance to this star is estimated to be about 13.47 pc (e.g., Perryman et al. 1997). The estimated age and rotational period of  $\nu$  And are 5 Gyr and 12 days (Baliunas et al. 1997), respectively. Subsequent studies suggest that the rotation period is probably  $\leq 9$  days (Ford et al. 1999) and that its velocity jitter is  $\sim 10 \text{ m s}^{-1}$  (Saar et al. 1998).

The first multiple exoplanet system detected around a main-sequence star was the triple system  $\nu$  And (Butler et al. 1999). Today, over 30 systems comprising more than one planet are known (Wright et al. 2009), including eight triple systems, two quadruple systems ( $\mu$  Arae, Pepe et al. 2007, and GJ581, Mayor et al. 2009), the quintuple system 55 Cancri (Fischer et al. 2008) and HD 10180, which may have up to seven planets detected so far (Lovis et al. 2010).

The lack of very high eccentricities in multiplanet systems may be partially explained by the additional constraint in multiplanet systems of orbital stability, which favors low-eccentricity orbits. Conversely, some single-planet systems may exhibit high eccentricities as a result of a series of ejections of former members from the system. Both factors can be at play simultaneously: Ford et al. (2005) explain the observed eccentricities of the planets in the  $\nu$  And system as the end result of the ejection of a hypothesized fourth planet from the system.

## 3. Keplerian orbital fits

We attempted to fit models consisting of between three and six planets in Keplerian orbits, and found that the data require a 5-orbit model.

The solution of the 3-planet fit model is similar to that published by Wright et al. (2009). The best 3-orbit model fit of the combined data yields residuals with an rms scatter of  $15.61 \text{ m s}^{-1}$  and a  $\chi_{\text{red}}^2 = 1.47$ . The residuals of this 3-orbit model reveal a long period trend (see Fig. 1) indicating the presence of a fourth

planet in the system with a period smaller than the time span observation. The best 4-planet fit of this new data yielded residuals with an rms scatter of  $13.76 \text{ m s}^{-1}$  and a  $\chi_{\text{red}}^2 = 1.11$ , both clearly representing a significant improvement over the 3-planet model (compare also the bottom panel of Fig. 2 with Fig. 1). An F-test shows that the probability of  $\chi_{\text{red}}^2$  dropping that much due to the inclusion of the fourth planet is less than 0.1% by mere fluctuations of noise. Therefore, the existence of a signal of a fourth planet is strongly supported.

The eccentricities of the four planets are 0.02, 0.26, 0.3 and 0.005, respectively for planets b to e. For the four planets of  $\nu$  Andromedae we find:  $m \sin i = 0.69, 1.98, 4.13$  and  $1.06 M_{\text{Jup}}$ , and  $P = 4.62, 241.26, 1276.5$ , and  $3848.9$  d, respectively. In addition, we find that the characteristics of the new fourth planet ( $m_e \sin i \sim 1.06 M_{\text{Jup}}$ ,  $P \sim 10.54$  yr,  $a \sim 5.25$  AU and  $e \sim 0.005$ ) are very similar to those of Jupiter ( $P \sim 11.86$  yr,  $a \sim 5.20$  AU and  $e \sim 0.05$ ). Table 1 lists the resulting orbital elements of these 4 planets, and Fig. 2 displays the corresponding Keplerian orbits.

To verify the robustness of the best-fit solutions, we also performed a step-by-step Lomb-Scargle periodogram (Press et al. 1992) analysis. The top panel in Fig. 3 displays the window function of the RV measurements, showing that the dominant periodicity of the sampling is 1 year. The second panel shows the periodogram of the data. This periodogram is dominated by  $\sim 4.6$  day,  $\sim 243$  day and  $\sim 3.5$  year periodicity, which corresponds to  $\nu$  And b, d and c, respectively. We then fitted the data with three Keplerian orbits with these starting periods, and examined the periodogram of the residuals of this 3-planet solution (Fig. 3c). This second periodogram is dominated by a peak around  $\sim 3800$  day period and a secondary peak around 183 day period. We then fitted the data with four Keplerian orbits including the new period of about 3800 days. We then investigated the periodogram of the residuals of the corresponding fitting of a 4-planet model (see Fig. 3d). This new fit markedly enhances the  $\sim 183$  day peak. This suggests that the 183 day peak is real (not an alias). We then fitted a 5-orbit model to the RV data set using as an initial condition a period of 183 days. The best 5-orbit fit of this new data yielded residuals with an rms scatter of  $12.38 \text{ m s}^{-1}$  and a  $\chi_{\text{red}}^2 = 0.90$ , both representing a significant improvement ( $\sim 10\%$  for the rms scatter and  $\sim 19\%$  in the  $\chi_{\text{red}}^2$ ) over the four-planet model. The estimated period and velocity amplitude of this fifth orbit are 183.4 days and  $10.37 \text{ m s}^{-1}$  (with an eccentricity of about 0.3). The residuals of this 5-orbit solution finally display no statistically significant peaks (Fig. 3e).

Both the new AGA algorithm and the spectral analysis method identify the same 5 coherent signals in the data.

## 4. Dynamical modeling and stability analysis

We have performed long-term numerical integrations to test the stability of the orbital solutions. For these long-term stability tests, we applied direct n-body integrations to the orbital solutions obtained from the 3-, 4- and 5-planet Keplerian orbit fit of the RV data, assuming planar orbits with an edge on orientation. We integrated the orbits for at least 10 Myr using the hybrid integrator in Mercury 6 (Chambers 1999). For the majority of each integration, Mercury uses a mixed-variable symplectic integrator (Wisdom & Holman 1991) with a time step approximately equal to a fiftieth ( $\approx 1/50$ ) of the Keplerian orbital period of the closest planet. During close encounters, Mercury uses a Bulrich-Stoer integrator with an accuracy parameter of  $10^{-12}$ . We identified an unstable system if: 1) two planets collide; 2) a planet is accreted onto the star (astrocentric distance less than

**Table 1.** Fitted orbital solutions for the  $\nu$  And planetary system: 4 Keplerians<sup>a</sup>.

Parameter		$\nu$ And-b	$\nu$ And-d	$\nu$ And-c	$\nu$ And-e
$P$	[days]	4.617033(23)	241.258(64)	1276.46(57)	3848.86(74)
$T$	[JD-2440000]	10005.368(49)	10157.78(84)	11347.48(48)	9535.95(76)
$e$		0.02150(70)	0.2596(79)	0.2987(72)	0.00536(44)
$\omega$	[deg]	324.9(3.8)	241.7(1.6)	258.82(43)	367.3(2.3)
$K$	[m s <sup>-1</sup> ]	70.51(45)	56.26(52)	68.14(45)	11.54(31)
$a$	[AU]	0.05922166(20)	0.827774(15)	2.51329(75)	5.24558(67)
$m \sin i$	[ $M_{\text{Jup}}$ ]	0.6876(44)	1.981(19)	4.132(29)	1.059(28)

**Notes.** <sup>(a)</sup> The  $\chi_{\text{red}}^2$  and rms of the residuals for the four Keplerian orbit fit are 1.11 and 13.76 m s<sup>-1</sup>, respectively. We found that the systemic velocity of the data is  $V_0 = 3.37$  m s<sup>-1</sup>. The data contains 385 measurements taken within a span of time of 7383 days, about twice the estimated period for planet e. The errors in the derived parameters are in brackets and represent the uncertainties in the last two digits of the corresponding figures.

0.005 AU); or 3) a planet is ejected from the system (astrometric distance exceeds 100 AU).

The simulations of the 3- and 4-planet Keplerian fits proved stable for at least 10 Myrs. However, the simulations of the 5-planet Keplerian fit turned out to be unstable in very short times. The fifth planet was ejected after a few thousand years. This can be explained given the proximity of the orbit of this possible planet to  $\nu$  And-d ( $a = 0.69$  and  $0.83$  AU, respectively), the much lower mass of this possible planet compared to the mass of  $\nu$  And-d (about a factor of 7), and the relatively large eccentricity of  $\nu$  And-d and this possible planet ( $e \sim 0.22$  and  $0.3$ , respectively). This suggests that the fifth signal found in the data is not associated to a fifth planet in the system, but instead, it is probably due to a coherent signal from the star (see discuss below).

#### 4.1. Apical Resonance between $\nu$ And-d and $\nu$ And-c

It has been previously reported that  $\nu$  And-d and  $\nu$  And-c are in apical alignment (Chiang et al. 2001). It was found that the difference between their arguments of the pericenter ( $\Delta\omega_{\text{dc}} = \omega_{\text{d}} - \omega_{\text{c}}$ ) librates around zero with an amplitude of  $\sim 30^\circ$  and a period of  $4 \times 10^3$  yrs. This has been associated with a secular resonance (Chiang et al. 2001).

Including the fourth planet found in our study did not modify significantly the overall behavior of this apical alignment,  $\Delta\omega_{\text{dc}}$  still librates around  $0^\circ$  with an initial amplitude of  $\sim 44^\circ$  and a short period of  $\sim 6 \times 10^3$  yrs. But the amplitude of  $\Delta\omega_{\text{dc}}$  evolves over time due to the resonant interaction between planet c and e, increasing and decreasing in a non-periodical way. Within the 10 Myrs integration time,  $\Delta\omega_{\text{dc}}$  librates around  $0^\circ$  with a maximum amplitude of  $\sim 60^\circ$  and similar short period ( $\sim 6 \times 10^3$  yrs).

As part of the secular interaction the eccentricities of  $\nu$  And-c and d are anti-correlated, that is, when the eccentricity of  $\nu$  And-c is in a minimal, the eccentricity of  $\nu$  And-b is maximum, and vice versa. Additionally it was found that the eccentricities of planets e and c are anti-correlated as well and they have a  $\Delta\omega_{\text{ce}} = \omega_{\text{c}} - \omega_{\text{e}}$  that librates with an amplitude of  $\sim 90^\circ$ . The eccentricity of planet c shows two periodical variations, a long-term variation (of about 6000 yrs) anti-correlated with planet d, and a short term variation (of about 760 yrs) anti-correlated with the eccentricity of planet e.

#### 4.2. $\nu$ And-e and $\nu$ And-c in and external 3:1 resonance

The initial period predicted for the fourth ( $\nu$  And-e) planet is 3848.9 days, and the period of the second planet discovered

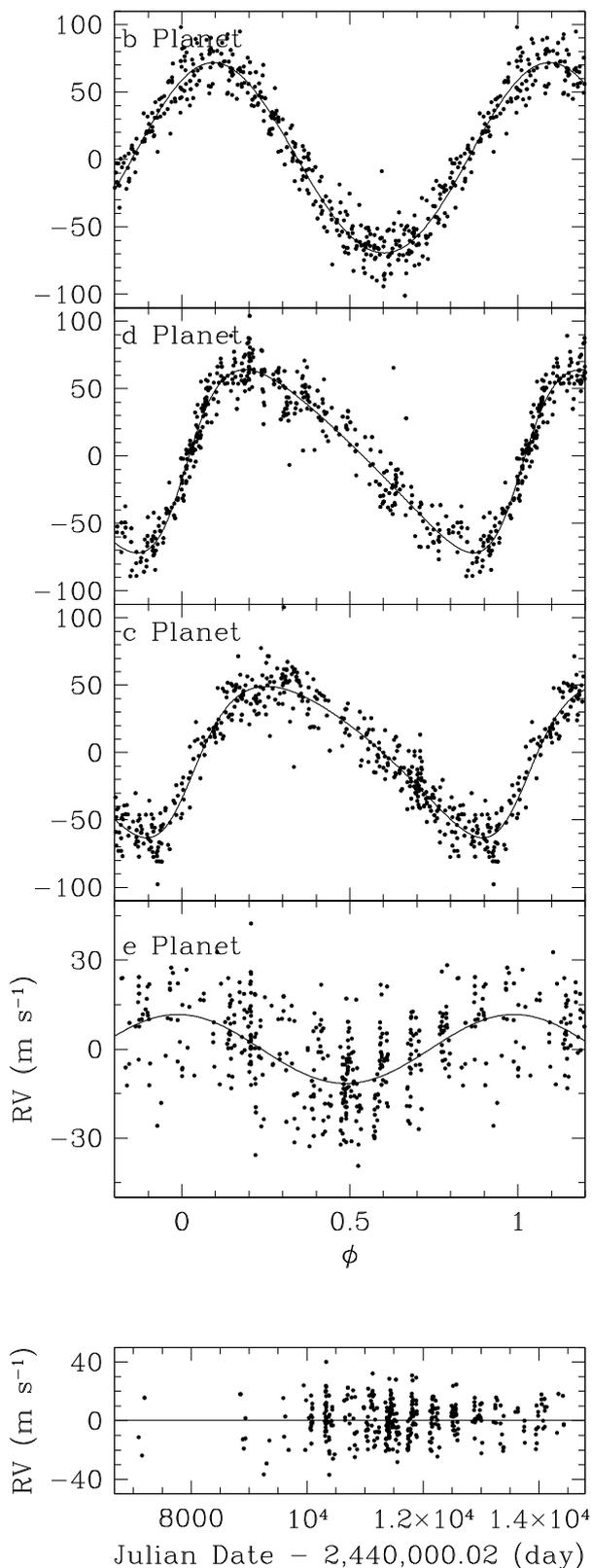
( $\nu$  And-c) is 1276.46. The ratio between these periods is 3.02, suggesting that these two planets are close to an external 3:1 mean motion resonance (hereafter MMR). In addition, the orbit of  $\nu$  And-e coincides with an island of stability, just outside the external 3:1 MMR of  $\nu$  And-c, reported by Rivera & Haghighipour (2007). They point out that just outside the 3:1 MMR, stable particles experience oscillations in their orbital eccentricities, reaching up into the range 0.2–0.3. We observe a similar behavior in  $\nu$  And-e ( $e \lesssim 0.2$ ). Furthermore, we find that, as in the case of the test particles,  $\nu$  And-e is protected from close approaches with  $\nu$  And-c by the  $e-\omega$  mechanism (e.g., Milany & Nobili 1984; Gladman 1993; Lissauer & Rivera 2001). A detailed dynamical analysis of this 4-planet system will be needed to confirm the possible external 3:1 resonance between the two external planets. Such analysis is beyond the scope of this paper.

## 5. New planet or spots

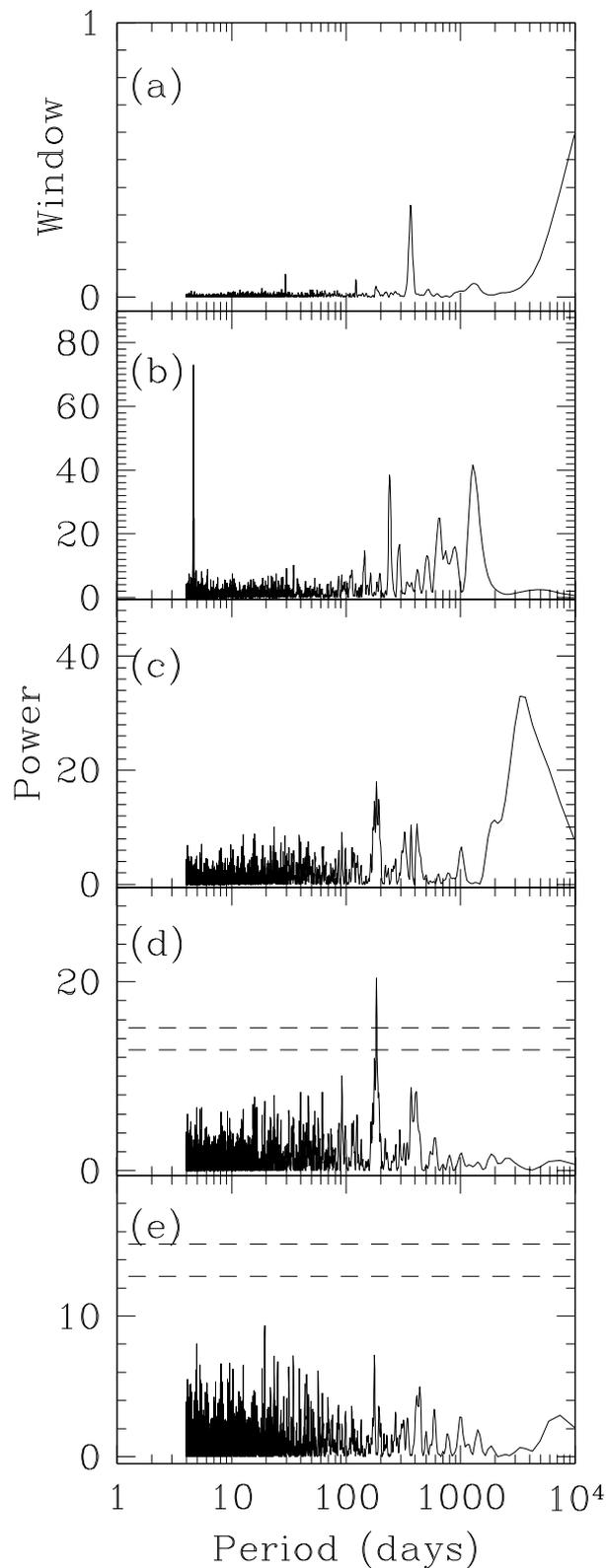
Unfortunately, coherent Doppler shifts do not always correspond to planets. Inhomogeneities of the stellar surface such as spots, plages, flares, or convective patterns can break the even distribution between the red-shifted and the blue-shifted halves of a rotating star and induce apparent radial velocity shifts (e.g., Bonfils et al. 2007). This can introduce noise into the RV measurements, which is usually referred to as “jitter”. In some cases, it does not average out as white noise would, and instead builds up a coherent signal. This apparent Doppler shift can easily mimic a Keplerian orbit (e.g., Queloz et al. 2001; Bonfils et al. 2007). In the case of  $\nu$  And, Butler et al. (1999) found no compelling periodicity in the chromospheric emission from this star. However, they noted that the Ca II 866.2 core periodogram (see their Fig. 8) shows a peak that resides at a period of about 180 days. They argue that although this peak does not appear to be statistically compelling, it constitutes a warning to investigate further the existence of intrinsic periodicities in the star itself with periods of 100–300 days. The results we present here show that the period of the fifth orbit fitted (183.4 days) is very similar to the 180 days period found in the Ca periodogram. This suggests that the fifth Keplerian orbit is probably due to a coherent signal from the star.

## 6. Summary

Fitting a model that includes the three previously identified Jovian-mass companions to the radial velocity data obtained at the Lick Telescope and with ELODIE results in residuals that



**Fig. 2.** Quadruple-Keplerian orbit fit to the velocities for  $\nu$  And. The velocities and fits for each of the four planets are shown separately for clarity by subtracting the effects of the other three planets. The panels contain inner planet b (*top*), second planet d (*second panel*), third planet c (*third panel*) and outer planet e (*fourth panel*). This figure also shows the residuals after subtracting the four planets (*bottom*). For a comparison with a 3-planet fit model see Fig. 1. The data taken at the Lick observatory and with ELODIE have a mean velocity uncertainty of  $\sim 7.44$  m s $^{-1}$ .



**Fig. 3.** **a)** Window Function for the data taken with the Lick observatory. **b)** Periodogram of the data showing the periods of the 3 planets previously known ( $\nu$  And–b, c and d). **c)** Periodogram of the residuals after subtracting planets b, c and d. This periodogram shows a strong peak at a period of about 3800 days, which corresponds to planet e (see main text). **d)** Periodogram of the residuals after subtracting 4 orbits. This periodogram shows a strong peak (with a FAP  $\sim 0.000005$ ) at a period of about 183 days. **e)** Periodogram of the residuals after subtracting 5 orbits. This periodogram shows no significant peak. FAP thresholds of 1% and 0.1% are indicated as dashed lines.

contain significant power at a period of 3848.9 days, which appear to be Keplerian in nature. Including a fourth companion with this period in a 4-planet Keplerian model results in a significant improvement in the quality of the fit. A dynamical study to this four-planet system shows that the orbits of the four planets are stable for at least 10 Myr. Thus  $\nu$  And is orbited by at least 4 giant planets, the outermost of which ( $\nu$  And–*e*) has nearly completed two orbits since this star started to be monitored with the Lick Observatory. This makes  $\nu$  And the fifth system known to host at least 4 planets. This also makes  $\nu$  And the first system known to host at least 4 Jupiter-type planets.

The nearly circular orbit of the fourth, outermost planet of  $\nu$  And has a derived orbital semi-major axis of about 5.25 AU and an  $m_e \sin i$  of  $1.06 M_{\text{Jup}}$ , making this planet a *strikingly close analog of Jupiter in our own solar system*. This new planet *e* appears to be in an external 3:1 resonance with planet *c* and it is in apsidal alignment with this planet. The orbit of  $\nu$  And–*e* coincides with an island of stability reported by Rivera & Haghighipour (2007). The previously reported apsidal alignment between planet *d* and *c* persist in our integrations including the fourth planet.

The residuals of the 4-planet Keplerian fit contains significant power at a period of 183 days. Including a fifth companion with this period in the multi-planet Keplerian model results in a significant improvement in the quality of the fit. However, the orbit of this fifth component is dynamically unstable in a very short period of time due to its proximity to  $\nu$  And–*d*. We argue that a better explanation for this periodic signal in the data is that it is due to a coherent signal from the star.

*Acknowledgements.* We thank G. Torres for valuable discussions. We also thank the anonymous referee for a careful reading of the manuscript and constructive comments which improved this paper. S.C. acknowledges support from

CONACyT grant 60581. J.C. and L.G. acknowledges support from CONACyT grants 61547 and 83016, respectively. C.C. thanks CONACyT postdoctoral program for its financial support. C.C. acknowledges support from DGAPA/PAPIIT grant IN 112210 and CONACyT grant 60581.

## References

- Baliunas, S. L., Henry, G. W., Donahue, R. A., Fekel, F. C., & Soon, W. L. 1997, *ApJ*, 474, L119
- Beaugé, C., Ferraz-Mello, S., & Michtchenko, T. A. 2003, *ApJ*, 593, 1124
- Bonfils, X., Mayor, M., Delfosse, X., et al. 2007, *A&A*, 474, 293
- Butler, R. P., Marcy, G. W., Fischer, D. A., et al. 1999, *ApJ*, 526, 916
- Cantó, J., Curiel, S., & Martínez-Gómez, E. 2009, *A&A*, 501, 1259
- Chambers, J. E. 1999, *MNRAS*, 304, 793
- Chiang, E. I., Tabachnik, S., & Tremaine, S. 2001, *AJ*, 122, 1607
- Coughlin, J. L., López-Morales, M., Harrison, T. E., Ule, N., & Hoffman, D. I. 2010 [arXiv: 1007.4295c]
- Fischer, D. A., Marcy, G. W., Butler, R. P., et al. 2003, *ApJ*, 586, 1394
- Fischer, D. A., Marcy, G. W., Butler, R. P., et al. 2008, *ApJ*, 675, 790
- Ford, E. B., Rasio, F. A., & Sills, A. 1999, *ApJ*, 514, 411
- Ford, D. A., Lystad, V., & Rasio, F. A. 2005, *Nature*, 434, 873
- Gladman, B. 1993, *Icarus*, 106, 247
- Gregory, P. C., & Fischer, D. A. 2010, *MNRAS*, 403, 731
- Lissauer, J. J., & Rivera, E. J. 2001, *ApJ*, 554, 1141
- Lovis, C., et al. 2010, *A&A*, submitted
- Mayor, M., Bonfils, X., Forveille, T., et al. 2009, *A&A*, 507, 487
- Milani, A., & Nobili, A. M. 1984, *Celest. Mech.*, 34, 343
- Naef, D., Mayor, M., Beuzit, J. L., et al. 2004, *A&A*, 414, 351
- Pepe, F., Correia, A. C. M., Mayor, M., et al. 2007, *A&A*, 462, 769
- Perryman, M. A. C., Lindegren, L., Kovalevsky, J., et al. 1997, *A&A*, 323, L49
- Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, *Numerical recipes in C. The art of scientific computing*, 2nd edn (Cambridge: University Press)
- Queloz, D., Henry, G. W., Sivan, J. P., et al. 2001, *A&A*, 379, 279
- Rivera, E., & Haghighipour, N. 2007, *MNRAS*, 374, 599
- Saar, S. H., Butler, R. P., & Marcy, G. W. 1998, *ApJ*, 403, L153
- Wisdom, J., & Holman, M. 1991, *AJ*, 102, 1528
- Wright, J. T., Upadhyay, S., Marcy, G. W., et al. 2009, *ApJ*, 693, 1084