Numerical simulations of the attenuation of the fundamental slow magnetoacoustic standing mode in a gravitationally stratified solar coronal arcade

P. Konkol¹, K. Murawski¹, D. Lee², and K. Weide²

¹ Group of Astrophysics and Gravity Theory, Institute of Physics, UMCS, ul. Radziszewskiego 10, 20-031 Lublin, Poland
e-mail: piotrk@kft.umcs.lublin.pl

² ASC/Flash Center, The University of Chicago, 5640 S. Ellis Ave, Chicago, IL 60637, USA

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ABSTRACT

Aims. We aim to explore the influence of thermal conduction on the attenuation of the fundamental standing slow magnetoacoustic mode in a two-dimensional (2D) potential arcade that is embedded in a gravitationally stratified solar corona.

Methods. We numerically solve the time-dependent magnetohydrodynamic equations to find the spatial and temporal signatures of the mode.

Results. We find that this mode is strongly attenuated on a time-scale of about 6 waveperiods.

Conclusions. The effect of non-ideal plasma such as thermal conduction is to enhance the attenuation of slow standing waves. The numerical results are similar to previous observational data and theoretical findings for the one-dimensional plasma.

Key words. magnetohydrodynamics (MHD) – Sun: corona – Sun: oscillations

1. Introduction

Observational findings by SUMER SOHO/EIT and TRACE/EUV have revealed that solar coronal plasma is able to sustain (among other waves) both propagating (e.g., De Moortel et al. 2002a,b; Ofman et al. 1997; Murawski & Zaqarashvili 2010) and standing (e.g., Wang et al. 2002) slow magnetoacoustic (slow henceforth for slow magnetoacoustic) waves. Because slow waves are restricted to propagate along magnetic field lines, they are a valuable tool for probing the coronal plasma and providing information within the framework of coronal seismology (e.g., Roberts 2000; Nakariakov & Verwichte 2005; Ballai 2007; Andries et al. 2009).

Slow waves are found to be strongly attenuated over few waveperiods (Wang et al. 2003). Several mechanisms of attenuation of these waves were proposed: wave leakage into the internal layers of the solar atmosphere (Ofman 2002; Van Doorsselaere et al. 2004; Ogrodowczyk & Murawski 2007), lateral wave leakage as a result of the curvature of magnetic field lines (Roberts 2000; Ogrodowczyk et al. 2007), phase mixing (e.g., Ofman & Aschwanden 2002), resonant absorption (Ruderman & Roberts 2002) and non-ideal magnetohydrodynamic (MHD) effects (Roberts 2000; Nakariakov et al. 2000; De Moortel & Hood 2003; Verwichte et al. 2008).

The intensive numerical investigations of slow standing waves in a straight magnetic field topology showed that without large amplitude slow mode oscillations attenuate rapidly due to shock dissipation (Haynes et al. 2008). Fully nonlinear 1D simulations of slow mode oscillations in the presence of thermal conduction were performed by De Moortel & Hood (2003) and Verwichte et al. (2008), who showed that thermal conduction is an important damping mechanism. In particular, De Moortel & Hood (2003) discussed in detail the influence of thermal

conduction on slow waves. Their numerical results for driven waves revealed that for typical solar coronal conditions thermal conduction appears to be the dominant wave-attenuation mechanism. Selwa et al. (2005) and Ogrodowczyk & Murawski (2007) showed that pressure pulses triggered near a foot-point excite the fundamental mode. The evolution of standing slow waves in a curved magnetic field topology was discussed by Selwa & Murawski (2006), Selwa et al. (2007a,b 2009), Ogrodowczyk et al. (2007) and Selwa & Ofman (2009) to find that loop curvature leads to the reduction of excitation and attenuation times of slow standing modes in a gravity-free arcade loop.

A study of slow wave propagation in gravitationally stratified media is rare. A theoretical 1D model of propagating slow waves in coronal loops by Nakariakov et al. (2000) includes the effects of stratification, nonlinearity, viscosity, resistivity, and thermal conduction. De Moortel & Hood (2004) investigated the effect of gravitational stratification in a straight magnetic field topology as a mechanism of attenuation of propagating slow waves. In another approach, Mendoza-Briceno et al. (2005) found that in a 1D gravitationally stratified atmosphere the attenuation time of slow standing waves was reduced by 10–20% in comparison to a gravity-free case. Selwa & Ofman (2009) explored slow standing waves in a cold curved, gravitationally stratified loop.

Despite the significant achievements attained in the above mentioned papers there is still a lack of realistic modeling which takes a simultaneous presence of curved magnetic field and gravity into consideration. Although slow propagating waves are an interesting subject to study, we postpone this problem for future studies and limit ourself to slow standing waves. The aim of this paper is to study the influence of thermal conduction on the attenuation of slow standing waves in a gravitationally stratified solar coronal arcade. This way we generalize the model of Verwichte et al. (2008) into a 2D curved geometry and the

model of Selwa & Ofman (2009) by inclusion of the photosphere layer, which results in wave reflection. Selwa & Ofman adopted line-tying boundary conditions, which mimic action of the chromosphere/photosphere layer. We also extend the model of De Moortel & Hood (2003), in which damping of slow waves by thermal conduction was studied in the frame of a 1D homogeneous case. We extend their model to a curved magnetic field geometry and 2D case.

This paper is organized as follows. The numerical model is described in Sect. 2. The numerical results are presented in Sect. 3. We concluded with a short summary of the main results in Sect. 4.

2. The numerical model

We performed numerical simulations in a magnetically structured atmosphere. Henceforth, we neglect radiation and plasma heating, viscosity, and resistivity, but take into account isotropic thermal conduction, which is important for the damping of slow waves (Ofman 2002). As a consequence we use the following MHD equations to describe the coronal plasma:

$$\frac{\partial \varrho}{\partial t} + \nabla \cdot (\varrho \mathbf{V}) = 0, \quad (1)$$

$$\frac{\partial \varrho \mathbf{V}}{\partial t} + \nabla \cdot (\varrho \mathbf{V} \mathbf{V} - \mathbf{B} \mathbf{B}) + \nabla \cdot \mathbf{p}_* = \rho \mathbf{g}, \quad (2)$$

$$\frac{\partial \varrho E}{\partial t} + \nabla \cdot (\mathbf{V}(\varrho E + p_*) - \mathbf{B}(\mathbf{V} \cdot \mathbf{B})) = \varrho \mathbf{g} \cdot \mathbf{V} + \nabla \cdot (\kappa \nabla T), \quad (3)$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{V} \mathbf{B} - \mathbf{B} \mathbf{V}) = 0, \quad (4)$$

$$\nabla \cdot \mathbf{B} = 0, \quad (5)$$

$$p = \frac{k_B}{\hat{m}} \varrho T, \quad (6)$$

with

$$p_* = p + \frac{B^2}{2}, \quad (7)$$

$$E = \frac{1}{2} \mathbf{V}^2 + \epsilon + \frac{1}{2} \frac{B^2}{\varrho}, \quad (8)$$

$$\epsilon = \frac{p}{\varrho(\gamma - 1)}. \quad (9)$$

Here p_* is total (gas plus magnetic) pressure, ϱE is total (kinetic plus internal and magnetic) energy density, $\varrho \epsilon$ is internal energy density, ϱ is the mass density, $\mathbf{V} = [V_x, V_y, 0]$ is the flow velocity, p is the gas pressure, $\mathbf{g} = [0, -g, 0]$ is a constant gravity, $\mathbf{B} = [B_x, B_y, 0]$ is the magnetic field that is normalized as $\mathbf{B} \rightarrow \mathbf{B}/\sqrt{\mu}$, T is plasma temperature, κ is a coefficient of thermal conductivity, $\gamma = 5/3$ is the adiabatic index, k_B is Boltzman's constant and $\hat{m}$ denotes the mean mass. Although heat conduction is strongly prohibited across the field lines and prone to be along the field lines, we use isotropic conduction in this paper for simplicity reasons; anisotropic conduction effects will be studied in the future.

Equations (1)–(5) are solved numerically using the new unsplit staggered mesh (USM) MHD solver (Lee & Deane 2009) in FLASH (Fryxell et al. 2000; Dubey et al. 2009). the FLASH architecture is described in Dubey et al. (2009).

The USM solver implements a high-order Godunov scheme in a directionally unsplit formulation, satisfying the $\nabla \cdot \mathbf{B} = 0$ condition to machine accuracy, using a constrained transport approach (Evans & Hawley 1988) on a uniform grid as well as

the adaptive mesh refinement (AMR) grid. The USM solver can adopt various types of algorithms that provide first, second, and third order accurate Riemann reconstructions, different choices for slope limiters and Riemann solvers. In FLASH, the AMR grid is adopted, using the PARAMESH package (MacNeice et al. 1999).

We use a second-order MUSCL-Hancock method (Toro 1997) with monotized central slope limiter and the HLLD Riemann solver (Miyoshi & Kusano 2005). We set the simulation box as $(0, 50 \text{ Mm}) \times (0, 20 \text{ Mm})$ and impose fixed-in-time boundary conditions for all plasma quantities in the x - and y -directions, while all plasma quantities remain invariant along the z -direction. In our studies we use the AMR grid with a minimum (maximum) level of refinement set to 4 (6), where each block contains 8×8 interior cells. The refinement strategy is based on controlling numerical errors in mass density. This results in an excellent resolution of steep spatial profiles and greatly reduces numerical diffusion at these locations.

2.1. Initial setup

In this section, we detail the initial setup used in this paper. The solar corona is modeled as a low mass density, highly magnetized plasma overlaying a dense photosphere/chromosphere.

2.1.1. The structure of the solar atmosphere

We assume that the solar atmosphere is settled in a two-dimensional and still ($\mathbf{V}_b = 0$) environment. Thus, we assume that the pressure gradient force is balanced by the gravity, that is

$$-\nabla p_b + \varrho_b \mathbf{g} = 0, \quad (10)$$

and the magnetic field is force-free

$$\frac{1}{\mu} (\nabla \times \mathbf{B}_b) \times \mathbf{B}_b = 0. \quad (11)$$

Here the subscript b corresponds to a background quantity.

Using the ideal gas law and the y -component of the hydrostatic pressure balance indicated by Eq. (10), we express the background gas pressure and mass density as

$$p_b(y) = p_0 \exp\left(-\int_{y_r}^y \frac{dy'}{\Lambda(y')}\right), \quad (12)$$

$$\varrho_b(y) = \frac{p_b(y)}{g\Lambda(y)}. \quad (13)$$

Here

$$\Lambda(y) = k_B T(y)/(mg) \quad (14)$$

is the pressure scale-height, and p_0 denotes the gas pressure at the reference level that is chosen at $y_r = 10 \text{ Mm}$.

We adopt a smoothed step-function profile for the plasma temperature

$$T_b(y) = \frac{1}{2} T_c \left[1 + d_t + (1 - d_t) \tanh\left(\frac{y - y_t}{y_w}\right) \right], \quad (15)$$

where $d_t = T_{ch}/T_c$, and T_{ch} denote the chromospheric/photospheric temperature. The symbol T_c corresponds to the temperature of the solar corona that is separated from the bottom layer at $y = y_t = 1.5 \text{ Mm}$ by the transition region of its width $y_w = 200 \text{ km}$. Background gas pressure $p_b(y)$ and mass density $\varrho_b(y)$ profiles that result from Eqs. (12) and (13) with the use of

Fig. 1. Vertical profiles of a background gas pressure (*top panel*) and a background mass density (*bottom*).

Eq. (15) are displayed in Fig. 1. Note that both $p_b(y)$ and $\rho_b(y)$ experience steep gradients at the transition region.

We adopt the magnetic field model was originally described by Priest (1982). In this model, we assume that Eq. (11) is satisfied by a current-free magnetic field

$$\nabla \times \mathbf{B}_b = 0,$$

so that

$$\mathbf{B}_b = \nabla \times (A \hat{\mathbf{z}}). \quad (16)$$

Here A denotes the magnetic flux function

$$A(x, y) = B_0 \Lambda_B \cos(x/\Lambda_B) \exp[-(y - y_r)/\Lambda_B]. \quad (17)$$

The background magnetic field components (B_{bx} , B_{by}) are then given by

$$(B_{bx}, B_{by}) = B_0 [-\cos(x/\Lambda_B), \sin(x/\Lambda_B)] \exp[-(y - y_r)/\Lambda_B], \quad (18)$$

in addition to $B_{bz} = 0$. Here B_0 is the magnetic field at $y = y_r$ and the magnetic scale height is $\Lambda_B = 2\pi/L$, with $L = 25$ Mm. Magnetic field lines corresponding to Eq. (18) are illustrated by solid lines in Fig. 2. Note that we do not directly use Eq. (18) in order to get B_{bx} and B_{by} . We instead take numerical derivatives of $A(x, y)$, according to Eq. (16), to initialize B_{bx} and B_{by} . This is important because a simple approach of using the analytical

Fig. 2. Initial setup of the system. Magnetic lines are represented by solid lines. The mass density is displayed by a color bar.

Fig. 3. Vertical profile of plasma β .

forms defined in Eq. (18) will not give the $\nabla \cdot \mathbf{B} = 0$ condition numerically.

The choice of magnetic field results in the plasma β ,

$$\beta = \frac{p_b(y)}{B_b^2(y)/2\mu}.$$

The plasma β is displayed in Fig. 3. Note that $\beta \approx 0.1$ in the solar photosphere, it declines to a value of 0.025 at the transition region and later grows with altitude, reaching a value of $\beta \approx 0.05$ at $y = 10$ Mm.

2.1.2. Perturbations

Aim to study impulsively excited slow oscillations in the coronal arcade that is described above. We found that these oscillations are triggered most effectively by a simultaneous launching of initial pulses in gas pressure

$$p(x, y, t = 0) = A_p f(x, y), \quad (19)$$

and mass density

$$\rho(x, y, t = 0) = A_\rho f(x, y) \quad (20)$$

with

$$f(x, y) = \exp\left[-\frac{(x - x_0)^2 + (y - y_0)^2}{2w^2}\right] \times \left| \operatorname{erf}\left(\frac{A(x, y) - A_1}{B_0 \Lambda_B}\right) - \operatorname{erf}\left(\frac{A(x, y) - A_2}{B_0 \Lambda_B}\right) \right|. \quad (21)$$

Here A_p and A_g are amplitudes of the initial pulses, w is their width and x_0 and y_0 denote their spatial positions. Unless otherwise stated we choose and hold fixed $A_p = 5 \times 10^{-12} \text{ N m}^{-2}$, $A_g = 1 \times 10^{-2} \text{ kg m}^{-3}$, $x_0 = 8 \text{ Mm}$, $y_0 = 12.4 \text{ Mm}$, and $w = 15 \text{ Mm}$. Additionally, these pulses are launched between two magnetic field lines determined by A_1 and A_2 as

$$A_1 = A(x_1, y_b) = B_0 \Lambda_B \cos(x_1 / \Lambda_B) \exp[-(y_w - y_b) / \Lambda_B], \quad (22)$$

$$A_2 = A(x_2, y_b) = B_0 \Lambda_B \cos(x_2 / \Lambda_B) \exp[-(y_w - y_b) / \Lambda_B]. \quad (23)$$

Unless otherwise stated we set $x_1 = 7.5 \text{ Mm}$, $x_2 = 8.5 \text{ Mm}$ and $y_b = 0.75 \text{ Mm}$. Note that perturbations described by Eqs. (19) and (20) can be considered as a physical process in a coronal plasma where a sudden blast wave produced by flares can affect it.

Figure 3 shows vertical profile of plasma β . As a result of this background and pulse the resulting standing wave will move along the magnetic line s with the mean value of $\langle \beta \rangle = 0.044$.

3. Numerical results

The initial pulses of Eqs. (19) and (20) trigger magnetosonic waves. Fast waves propagate essentially isotropically and after a while they leave the simulation region. As slow waves are guided along magnetic field lines, they remain trapped in the system after experiencing reflections from the dense plasma regions.

Figure 4 displays a velocity component that is parallel to the magnetic field line, $V_{\parallel}$ (solid line), and a mass density ρ (dashed line) at $t = 3550 \text{ s}$. Here s denotes the coordinate along the magnetic field line which corresponds to the magnetic flux function $A(x, y) = (A_1 + A_2)/2$, with $s = 0$ denoting the arcade apex. Note that $V_{\parallel}$ and ρ consist a typical structure for the standing slow mode (e.g., Nakariakov & Verwichte 2005). The asymmetry in these profiles results from the strong initial pulse, which significantly perturbs the background state.

Figure 5 shows the time-signatures that are drawn by collecting wave signals in $V_{\parallel}$ and ρ at the detection point ($x_d = 22 \text{ Mm}$, $y_d = 12.4 \text{ Mm}$). These time-signatures reveal oscillations that decay with time and exhibit the quarter waveperiod phase shift between $V_{\parallel}$ and ρ , which is characteristic for standing slow waves (Nakariakov & Verwichte 2005; Ogrodowczyk et al. 2009; Selwa et al. 2005).

We estimate the waveperiod P and attenuation time τ of the oscillation by fitting the following attenuated sine function:

$$V_{\parallel} \sim \sin\left(\frac{2\pi}{P}t + \phi_0\right) \exp\left(-\frac{t}{\tau}\right), \quad \phi_0 = \text{const.}, \quad (24)$$

to the time-signatures of Fig. 5. In this case, the waveperiod of the fundamental slow mode can be estimated as

$$P \simeq \frac{2l}{c_s} \simeq 400 \text{ s}, \quad (25)$$

where $l \simeq 40 \text{ Mm}$ is the length of the magnetic field line, which corresponds to the flux function $A(x, y) = (A_1 + A_2)/2$ and $c_s = 0.2 \text{ Mm s}^{-1}$ is the average sound speed along this line.

Fig. 4. Velocity (solid line) and mass density (dashed line) profiles along the magnetic line.

Fig. 5. Time-signatures of a parallel velocity $V_{\parallel}$ (solid line) and perturbed mass density (dashed line), collected at the detection point $x_d = 22 \text{ Mm}$, $y_d = 12.4 \text{ Mm}$.

Figure 6 illustrates the attenuation time (top panel) and waveperiod (bottom panel). The solid lines result from the dispersion relation of De Moortel & Hood (2003). From their dispersion relation for a real value of k we obtain a complex wavefrequency $\omega = \text{Re}(\omega) + i \text{Im}(\omega)$. Hence we get a wave period P_{DH} and attenuation time τ_{DH}

$$P_{\text{DH}} = \frac{2\pi}{\text{Re}(\omega)}, \quad \tau_{\text{DH}} = -\frac{1}{\text{Im}(\omega)}. \quad (26)$$

The index DH corresponds to the theoretical data of De Moortel & Hood (2003). Figure 7 compares $\tau_{\text{DH}}/P_{\text{DH}}$ ratio (upper panel) and τ/P taken from numerical simulations of full MHD equations with more complex background state (lower panel). In both cases there is some minimum located at about the same place.

Note that De Moortel & Hood (2003) deal with the homogeneous plasma and a straight magnetic field case, while we treat a more complex system in which the fundamental mode is attenuated by thermal conduction as well as by the curved magnetic field (Ogrodowczyk & Murawski 2007; Ogrodowczyk et al. 2009).

Fig. 6. Attenuation time τ (upper panel) and waveperiod P (lower panel) vs. κk . Solid lines (dots) correspond to the analytical data of De Moortel & Hood (2003) (the numerical solutions of the full set of MHD equations).

In numerical solutions waves are attenuated much stronger than in the theoretical model of De Moortel & Hood (2003). This is mainly due to the curved magnetic field, gravity, and non-ideal reflections from the denser part of the atmosphere. Those effects are always present, so we have some finite attenuation even for $\kappa = 0$.

The shorter waveperiod in the numerical simulations results from the action of gravity. For a strictly vertical oscillation the dispersion relation can be written as

$$\omega^2 = k^2 c_s^2 + \Omega_{\text{cut}}^2,$$

where Ω_{cut} is the cutoff frequency. Hence we infer that in the presence of gravity the wave frequency ω (P) is higher (lower) than in the gravity-free case.

In order to show that these effects are present we ran a few cases for different values of the sound speed, c_s . Note that according to Eq. (14) a higher value of c_s corresponds to higher Λ , which in turn results in a less inhomogeneous medium. Figure 8 illustrates a dependence of τ/P on c_s . As τ/P grows with c_s we infer that for a larger c_s waves are less attenuated, which partially explains difference between our results of Fig. 6 for $\kappa = 0$ and those of De Moortel & Hood (2003).

Fig. 7. Ratio of attenuation time τ and waveperiod P vs. the product of the normalized thermal conductivity κ and a wavenumber k for the De Moortel & Hood (2003) (upper panel) and numerical solutions of the full set of MHD equations (lower panel).

Figure 9 illustrates a dependence of the ratio of attenuation time τ and waveperiod P on the initial pulse strength A_p . According to our expectations τ/P declines with A_p ; for a higher value of A_p nonlinear effects become more important, which leads to wave steepening. As a result thermal conduction is more effective for steep profiles, which explains the fall, off of τ/P with A_p (Verwichte et al. 2008).

4. Summary

We developed a two-dimensional model of a coronal arcade to explore the attenuation of the fundamental slow magnetoacoustic standing mode in the presence of gravity and thermal conduction.

Our findings can be summarized as follows: the fundamental mode is excited impulsively by localized pulses in gas pressure and mass density that are initially launched in two nearby regions close to the chromosphere/photosphere. The obtained values of attenuation time depart from the analytical data for a homogeneous plasma studied by De Moortel & Hood (2003). This departure results from the adopted model, which implements wave energy leakage as a consequence of curved magnetic field

Fig. 8. Ratio of the attenuation time τ and period P vs. c_s .

Fig. 9. Ratio of attenuation time τ and waveperiod P vs. initial pulse strength A_p .

lines and a presence of dense chromosphere/photosphere layer. However, what remains similar in both cases is the existence of some minimal value of damping time for a given wavelength and

thermal conductivity. This minimum remains at about the same place.

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