

Effect of the variation of the Higgs vacuum expectation value upon the deuterium binding energy and primordial abundances of D and ^4He

M. E. Mosquera^{1,2} and O. Civitarese²

¹ Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque, 1900 La Plata, Argentina
e-mail: osvaldo.civitarese@fisica.unlp.edu.ar

² Department of Physics, University of La Plata, cc 67, 1900 La Plata, Argentina

Received 19 April 2010 / Accepted 5 July 2010

ABSTRACT

Aims. We calculate the constraints on the time variation of the Higgs vacuum expectation value from Big Bang Nucleosynthesis.

Methods. Starting from the calculation of the deuterium binding-energy, as a function of the pion-mass and using the NN-Reid 93 potential, we calculate the abundances of primordial D and ^4He by modifying Kawano's code. The Higgs vacuum expectation value (v) and the baryon to photon ratio (η_B) enter the calculation as free parameters. By using the observational data of D and ^4He , we set constraints on η_B and on the variation of v , relative to a constant value of Λ_{QCD} .

Results. Results are consistent with null variation in v and ϵ_D for the early universe, within 6σ .

Conclusions. We obtained a linear dependence of ϵ_D upon v and found that the best-fit-value of the variation of v is null within 6σ .

Key words. primordial nucleosynthesis – cosmological parameters – cosmology: theory

1. Introduction

One of the most powerful tools to study the early Universe is the Big Bang nucleosynthesis (BBN). Since BBN is sensible to parameters such as the fine structure constant, the electron mass, the Higgs vacuum expectation value (v), the deuterium binding energy (ϵ_D), among others, it is an important test to set constraints on deviations from the standard cosmology, and on physical theories beyond the standard model (SM). There are some theories which allow fundamental constants to vary over cosmological times scales (Kaluza 1921; Klein 1926; Weinberg 1983; Gleiser & Taylor 1985; Wu & Wang 1986; Barr & Mohapatra 1988; Maeda 1988; Damour & Polyakov 1994; Overduin & Wesson 1997; Youm 2001a,b; Damour et al. 2002a,b; Brax et al. 2003; Palma et al. 2003). The time variation of fundamental constants (e.g. the fine structure constant, the electron mass, the Planck mass), was studied in Campbell & Olive (1995); Bergström et al. (1999); Ichikawa & Kawasaki (2002); Nollett & Lopez (2002); Yoo & Scherrer (2003); Müller et al. (2004); Ichikawa & Kawasaki (2004); Cyburt et al. (2005); Landau et al. (2006); Chamoun et al. (2007); Coc et al. (2007); Mosquera et al. (2008); Landau et al. (2008).

The deuterium binding energy plays a crucial role in the reaction rates involved in the formation of primordial elements during the Big Bang Nucleosynthesis (BBN). All the primordial abundances would be different from the BBN predictions if the deuterium was deeply- or weakly-bound in that epoch (e.g. the abundance of deuterium depends exponentially on ϵ_D). In Flambaum & Shuryak (2002, 2003); Dmitriev & Flambaum (2003); Dmitriev et al. (2004); Berengut et al. (2010) the variation of ϵ_D as function of the quark masses was studied and the authors applied their results to set constraints using data from cosmological epochs. In Flambaum & Wiringa (2007) the

dependence of nuclear binding on hadronic mass was studied. In Yoo & Scherrer (2003) the dependence of the deuterium binding energy on the Higgs vacuum expectation value was considered using the results of Beane & Savage (2003); Epelbaum et al. (2003). In the same work, ϵ_D was represented as a linear function of v and this dependence was used to set constraints on the variation of the Higgs vacuum expectation value during cosmological times. Dent et al. (2007) studied the dependence of the primordial abundances with several parameters such as G_N , neutron decay time, α , m_e , the average nucleon mass, the neutron-proton mass difference and D, T, ^3He , ^4He , ^6Li , ^7Li , and ^7Be binding energies, and found that the deuterium and lithium abundances are strongly dependent on the Higgs vacuum expectation value. However, in Dent et al. (2007), the variations of the binding energies are assumed to obey a linear dependence on the pion mass, as given by Beane & Savage (2003).

In this work, we calculate the dependence of the deuterium binding energy with the pion-mass, using an effective nucleon-nucleon interaction. There exist several nucleon-nucleon effective potentials (Reid 1968; Nagels et al. 1975, 1977; Lacombe et al. 1980; Machleidt et al. 1987; Stoks et al. 1994; Wiringa et al. 1995); for the sake of the present calculation we have chosen the Reid 93 potential (Stoks et al. 1994). Following Berengut et al. (2010), we assume Λ_{QCD} is constant, that is, we measure all dimensions in units of Λ_{QCD} . After determining the dependence of ϵ_D on the dimensionless parameter $N = v/\Lambda_{\text{QCD}}$, we concentrate on the calculation of BBN observables, like the abundances of deuterium (D) and helium (^4He), to determine their sensitivity upon ϵ_D and N . Hereafter, the relative variations $\frac{\delta v}{v_0}$ and $\frac{\delta m_\pi}{(m_\pi)_0}$ might be understood as the relative variations $\frac{\delta N}{N_0}$ and $\frac{\delta M}{M_0}$, where $M = \frac{m_\pi}{\Lambda_{\text{QCD}}}$, respectively. We actually determine BBN abundances, after calculating the D-binding energy, as a function

of v/Λ_{QCD} , through the variation of the pion mass. In this aspect, our attempt differs from the one of [Dent et al. \(2007\)](#), where the variation of the binding energies of the nuclei involved in BBN is taken in a parameter form.

The paper is organized as follows. In Sect. 2, we discuss the dependence of the deuterium binding energy with the pion-mass. In Sect. 3, we calculate the primordial abundances and obtain constraints on the variation of the deuterium binding energy and on the Higgs vacuum expectation value. Our conclusions are presented in Sect. 4. The details of the formalism, concerning the calculation of various quantities which are needed to computed BBN abundances, are presented in Appendix A.

2. Dependence of the deuterium binding energy with the pion-mass

We are interested in the effects on the deuterium-binding-energy due to the change of the pion-mass; a change which is related to the variation of v . Assuming that the pion-mass acquires different values in different epochs of the Universe, some observables, such as the primordial abundances, might differ from their values predicted by the Standard Model ([Sarkar 1996](#)).

The variation of v produces different effects on the mass of different mesons, namely: light-mesons, like the pion, are effected more drastically than heavier mesons ([Flambaum & Wiringa 2007](#)).

The Reid potential represents the nucleon-nucleon interaction through the one-pion exchange mechanism (OPE) and a combination of central, tensor and spin-orbit functions with cut-off parameters (non-OPE) ([Stoks et al. 1994](#)). The Reid 93 potential is the regularized version of the Reid 68 potential ([Reid 1968](#)). The regularization is made to remove the singularities at the origin, by introducing a dipole form-factor in the Fourier transformation that leads from the momentum-space potential to the configuration-space potential ([Stoks et al. 1994](#)).

The OPE contribution to the Reid 93 potential is then written as¹ ([Stoks et al. 1994](#))

$$V_{\text{OPE}}(r) = -f_{\pi}^2 \left\{ \left(\frac{m_{\pi^0}}{m_s} \right)^2 m_{\pi^0} \left[\phi_{\text{T}}^0(m_{\pi^0}, r) S_{12} + \frac{1}{3} \phi_{\text{C}}^0(m_{\pi^0}, r) (\sigma_1 \sigma_2) \right] + 2 \left(\frac{m_{\pi^{\pm}}}{m_s} \right)^2 m_{\pi^{\pm}} \left[\phi_{\text{T}}^0(m_{\pi^{\pm}}, r) S_{12} + \frac{1}{3} \phi_{\text{C}}^0(m_{\pi^{\pm}}, r) (\sigma_1 \sigma_2) \right] \right\},$$

where m_{π^0} and $m_{\pi^{\pm}}$ are the mass of the neutral and charged pion respectively. The non-OPE contribution are written

$$V_{\text{C}}(r) = \bar{m}_{\pi} \sum_{p=2}^6 \alpha_p p \phi_{\text{C}}^0(p \bar{m}_{\pi}, r),$$

$$V_{\text{T}}(r) = 4 \bar{m}_{\pi} \beta_4 \phi_{\text{T}}^0(4 \bar{m}_{\pi}, r) + 6 \bar{m}_{\pi} \beta_6 \phi_{\text{T}}^0(6 \bar{m}_{\pi}, r),$$

$$V_{\text{LS}}(r) = 3 \bar{m}_{\pi} \gamma_3 \phi_{\text{SO}}^0(3 \bar{m}_{\pi}, r) + 5 \bar{m}_{\pi} \gamma_5 \phi_{\text{SO}}^0(5 \bar{m}_{\pi}, r),$$

where $\bar{m}_{\pi} = (m_{\pi^0} + 2m_{\pi^{\pm}})/3$, $\phi_{\text{C}}^0(m, r)$, $\phi_{\text{T}}^0(m, r)$ and $\phi_{\text{SO}}^0(m, r)$ are the central, tensor and spin-orbit contribution to the potential respectively ([Stoks et al. 1994](#)).

¹ We adopt natural units ($\hbar = c = 1$) through the text, unless indicated.

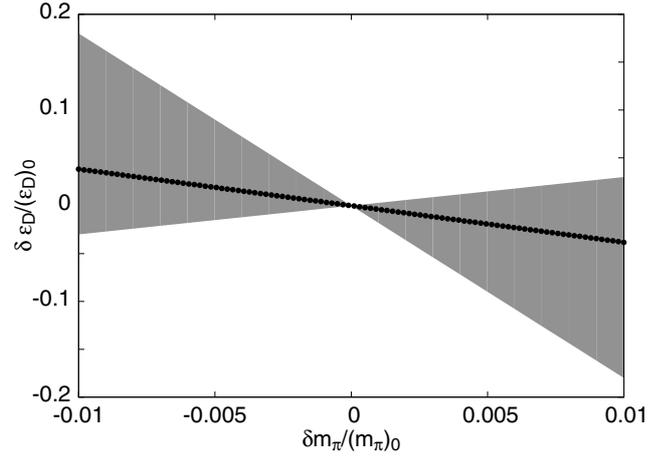


Fig. 1. Dependence of $\frac{\delta \epsilon_{\text{D}}}{(\epsilon_{\text{D}})_0}$ upon the relative change of the pion mass $\frac{\delta m_{\pi}}{(m_{\pi})_0}$, from the work of [Flambaum & Shuryak \(2002\)](#) (grey area) and our calculated value (dotted line).

Indeed, by multiplying the pion-mass by a constant factor (which is the same for charged and neutral pion), while keeping the scaling masses m_s and \bar{m}_{π} at a fixed value ([Flambaum & Wiringa 2007](#)), the pion-mass can be varied to affect OPE vertices of the NN potential. Although OPE is not the unique mechanism where the pion-mass appears explicitly, it is the only mechanism accounted for by the Reid 93 potential. Neither the two-pion exchange nor the heavy-meson-exchange mechanisms appear explicitly in this potential.

The effects on the potential due to the change of the pion-mass are noticeable ([Flambaum & Wiringa 2007](#)). Therefore one might expect that both, the binding energy ϵ_{D} and the D ground-state wave function $\phi(r)$ would be affected by changes in m_{π} . The deuteron wave function can be written as a finite set of Yukawa-type functions ([Lacombe et al. 1981](#); [Krutov & Troitsky 2007](#)) because of the functional structure of the potential.

After modifying the Reid potential, to take into account the variation of the pion-mass (as said before affecting only the OPE terms), we calculate the deuteron wave function and the deuteron binding energy for different values of the pion-mass, by solving the corresponding radial Schrödinger equation. With the obtained wave function, for each value of the pion-mass, we have calculated the deuteron binding energy and cast the results as a function of the relative variation $\frac{\delta m_{\pi}}{(m_{\pi})_0}$. If we call $\frac{\delta \epsilon_{\text{D}}}{(\epsilon_{\text{D}})_0}$ the relative variation of the deuteron binding energy (quantities with subindex 0 represent the actual values of the mentioned quantity), we found that the dependence of the variation of the deuteron binding energy on the variation of the pion-mass can be fitted by the straight-line $\frac{\delta \epsilon_{\text{D}}}{(\epsilon_{\text{D}})_0} = -3.65 \frac{\delta m_{\pi}}{(m_{\pi})_0}$. To put this result in perspective, one can compare it with the values reported by [Flambaum & Shuryak \(2002\)](#); [Beane & Savage \(2003\)](#); [Epelbaum et al. \(2003\)](#); [Yoo & Scherrer \(2003\)](#), where the same dependence yields values in the interval $(-18, +3)$. As a consequence of this effect the deuteron binding energy would be dependent on v , since $m_{\pi}^2 \propto v$. A comparison of the previous and our results is shown in Fig. 1.

The effect of these dependencies upon the BBN abundances will be discussed later on (see Sect. 3).

Table 1. Theoretical abundances in the standard model.

Nucleus	Our results
D	2.565×10^{-5}
^4He	0.2468

Table 2. Deuterium observational abundances (Y_D^{obs}).

$Y_D^{\text{obs}} \pm \sigma_D^{\text{obs}}$	Refs.
$(1.60^{+0.25}_{-0.30}) \times 10^{-5}$	Crighton et al. (2004)
$(2.42^{+0.35}_{-0.25}) \times 10^{-5}$	Kirkman et al. (2003)
$(3.30 \pm 0.30) \times 10^{-5}$	Burles & Tytler (1998a)
$(3.98^{+0.59}_{-0.67}) \times 10^{-5}$	Burles & Tytler (1998b)
$(2.54 \pm 0.23) \times 10^{-5}$	O'Meara et al. (2001)
$(2.82^{+0.20}_{-0.19}) \times 10^{-5}$	O'Meara et al. (2006)
$(1.65 \pm 0.35) \times 10^{-5}$	Pettini & Bowen (2001)
$(2.81 \pm 0.20) \times 10^{-5}$	Pettini et al. (2008)
$(3.75 \pm 0.25) \times 10^{-5}$	Levshakov et al. (2002)
$3.6^{+1.9}_{-1.1} \times 10^{-5}$	Ivanchik et al. (2010)

3. Big Bang nucleosynthesis

The standard model of the BBN has only one free-parameter: the baryon to photon ratio η_B , which is determined by the comparison between observed primordial abundances and theoretical calculations, or by the analysis of the cosmic background data (Spergel et al. 2003, 2007). The theoretical abundances are consistent with the observed abundance of deuterium but they are not entirely consistent with the observed abundance of ^4He . In Table 1 we present the theoretical abundances of D and ^4He calculated in the standard model by using Kawano's code (Kawano 1988, 1992). If the Higgs vacuum expectation value ν changes with time, while Λ_{QCD} is fixed, this discrepancy might eventually be reconciled. In order to calculate the primordial abundances of D and ^4He , for variable deuterium binding energy, we modify the numerical code developed by Kawano (1988, 1992), as explained in Appendix A.

To set bounds on the variation of the deuterium binding energy and on the variation of ν we have used the deuterium primordial abundance reported by Burles & Tytler (1998a,b); O'Meara et al. (2001, 2006); Pettini & Bowen (2001); Levshakov et al. (2002); Kirkman et al. (2003); Burles & Tytler (1998a); Pettini et al. (2008); Ivanchik et al. (2010) (see Table 2). Regarding to the ^4He primordial abundance, in the literature, there have been two different methods to determine it that yield quite different results (Izotov et al. 1994, 1997, 2006; Olive et al. 1997; Thuan & Izotov 1998, 2002; Peimbert 2002; Peimbert et al. 2002; Luridiana et al. 2003; Izotov & Thuan 2004). Since 2007, new atomic data were incorporated to the calculations of the ^4He primordial abundance, a quantity that depends on the HeI recombination coefficients. Therefore, new calculations were performed using the new atomic data, resulting into higher values of the ^4He abundance (Izotov et al. 2007; Peimbert et al. 2007; Aver et al. 2010; Izotov & Thuan 2010). In order to study the variation of ϵ_D or ν we only consider the latest ^4He data, reported by Izotov & Thuan (2010), Aver et al. (2010, see Table 3). Regarding the consistency of the data, we have followed the treatment of Yao et al. (2006) and increase the observational error by a factor Θ (see below).

We have computed light nuclei abundances, and performed the statistical analysis using observational data, to obtain the best

Table 3. ^4He observational abundances ($Y_{^4\text{He}}^{\text{obs}}$).

$Y_{^4\text{He}}^{\text{obs}} \pm \sigma_{^4\text{He}}^{\text{obs}}$	Refs.
0.2565 ± 0.0010	Izotov & Thuan (2010)
0.2561 ± 0.0108	Aver et al. (2010)

fit of the deuterium binding energy, the Higgs vacuum expectation value and the baryon to photon ratio. We have considered the following cases:

- i) variation of ϵ_D , by keeping η_B fixed at the WMAP value;
- ii) variation of ϵ_D and η_B ;
- iii) variation of ν and keeping η_B fixed at the WMAP value, and;
- iv) variation of both ν and η_B .

We perform the analysis on the Higgs vacuum expectation value, for three different value of κ ($\frac{\delta\epsilon_D}{(\epsilon_D)_0} = \kappa \frac{\delta m_\pi}{(m_\pi)_0}$) that is: our result ($\kappa = -3.65$), the higher limit used by Yoo & Scherrer (2003) ($\kappa = -10$) and the lower limit of the interval found by Flambaum & Shuryak (2002) ($\kappa = 3$), and considering Λ_{QCD} fixed.

3.1. Constraints on ϵ_D

We have computed the theoretical primordial abundances for different values of the deuterium binding energy, by keeping η_B fixed at the WMAP value $\eta_B = (6.108 \pm 0.219) \times 10^{-10}$ (Spergel et al. 2007). We have found the best-fit-parameter value using a χ^2 -test and the observational data. The results are

$$\begin{aligned} \frac{\delta\epsilon_D}{(\epsilon_D)_0} &= 5.60^{+0.85}_{-0.45} \times 10^{-2}, \\ \frac{\chi_{\min}^2}{N-1} &= 1.79, \end{aligned} \quad (1)$$

where χ_{\min}^2 is the lowest value of χ^2 and N is the number of data ($N = 12$). We found variation of the deuterium binding energy even at the level of six standard deviations (6σ). The result can be explained since an increase in the deuterium binding energy leads to a larger initial abundance of deuterium. The abundance of ^4He is larger since the production of this nuclei starts sooner and the final deuterium abundance is decreased (Yoo & Scherrer 2003).

The next step was to consider the baryon to photon ratio as an extra parameter to be fixed. Therefore, we have computed the theoretical primordial abundances for different values of the deuterium binding energy and of the baryon to photon ratio. Using the data on D and ^4He , we have performed a χ^2 -test to find the best-fit-parameter value

$$\begin{aligned} \frac{\delta\epsilon_D}{(\epsilon_D)_0} &= (7.60 \pm 1.35) \times 10^{-2}, \\ \eta_B &= 4.978^{+0.449}_{-0.360} \times 10^{-10}, \\ \frac{\chi_{\min}^2}{N-2} &= 0.90. \end{aligned} \quad (2)$$

The value of η_B agrees with the value obtained by WMAP (Spergel et al. 2007) within three standard deviation σ . For this case, we found null variation of the deuterium binding energy at the level of 6σ . The result is presented in Fig. 2, for three values of the deviation, that is at one, two and three σ . In the same Figure we show the one-dimensional likelihood, for η_B and $\frac{\delta\epsilon_D}{(\epsilon_D)_0}$.

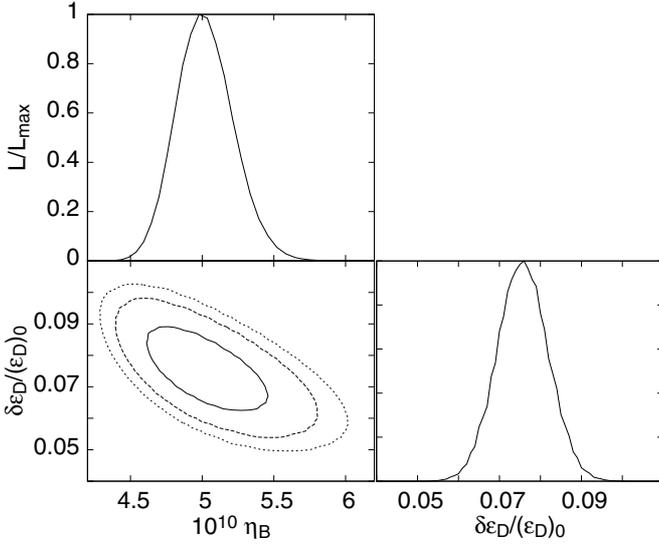


Fig. 2. 1σ , 2σ and 3σ likelihood contours for η_B and $\frac{\delta\epsilon_D}{(\epsilon_D)_0}$, and one-dimensional likelihood, $\left(\frac{L}{L_{\max}}\right)$.

Table 4. Best fit parameter value and 1σ errors constraints on $\frac{\delta v}{v_0}$.

κ	$\left(\frac{\delta v}{v_0} \pm \sigma\right) \times 10^2$	$\frac{\chi^2_{\min}}{N-1}$
-3.65	3.85 ± 0.45	3.99
-10	-1.89 ± 0.30	4.86
3	1.34 ± 0.14	1.02

3.2. Constraints on v

Next, we have studied the variation of the Higgs vacuum expectation value and of the baryon to photon ratio.

If the Higgs vacuum expectation value varies with time, the effects upon BBN are not only the ones due to the variation of the deuterium binding energy but also those due to the variation of the electron mass m_e ($m_e \propto v$), the neutron-proton mass-difference Δm_{np} and the Fermi constant G_F ($G_F \propto v^{-2}$) (see Appendix A, for details).

We have considered the baryon to photon ratio fixed at the WMAP value, and have computed the light abundances for different values of v . Once again, we performed a χ^2 -test to obtain the best-fit value. The results of our analysis are shown in Table 4, (where $\delta v = v_{\text{BBN}} - v_0$, v_{BBN} is the value of the binding energy during BBN, v_0 is the present value of v) for η_B fixed at the WMAP value ($\eta_B^{\text{WMAP}} = (6.108 \pm 0.219) \times 10^{-10}$) (Spergel et al. 2007), for three different values of κ .

We found variation of v at the level of six standard deviations (6σ), for all the dependencies of the deuterium binding energy with the pion-mass. The first two rows of Table 4 indicate that there is not a good fit for $\kappa = -3.65$ and $\kappa = -10$.

Finally, we have performed the calculation of the primordial abundances and found the best fit of v and η_B simultaneously. The results are given in Table 5, for three different values of κ .

We found null variation of v at 5σ , 4σ and 6σ for $\kappa = -3.65$, $\kappa = -10$ and $\kappa = 3$ respectively. Meanwhile, the value for η_B agrees with the value of WMAP at 4σ , 3σ and 1σ for $\kappa = -3.65$, $\kappa = -10$ and $\kappa = 3$ respectively. However, there is not a good fit if $\kappa = -10$. In Figs. 3 and 4 we present the corresponding likelihood contours for $\kappa = -3.65$ and $\kappa = 3$ respectively.

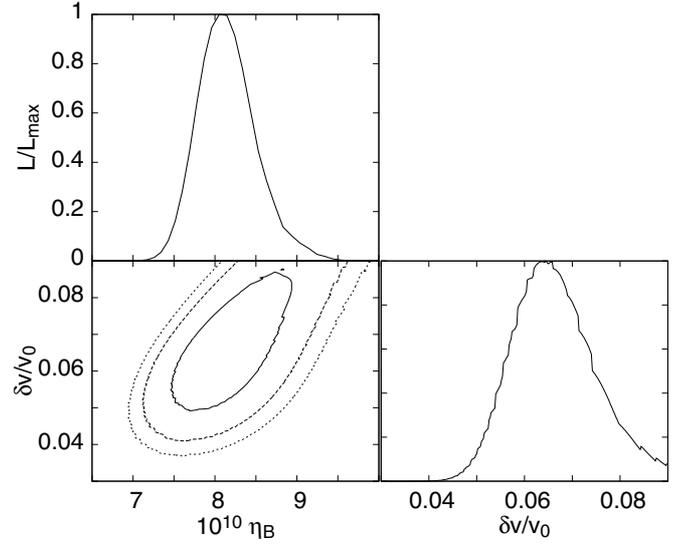


Fig. 3. 1σ , 2σ and 3σ likelihood contours for η_B and $\frac{\delta v}{v_0}$, and one-dimensional likelihood, $\left(\frac{L}{L_{\max}}\right)$, using $\kappa = -3.65$.

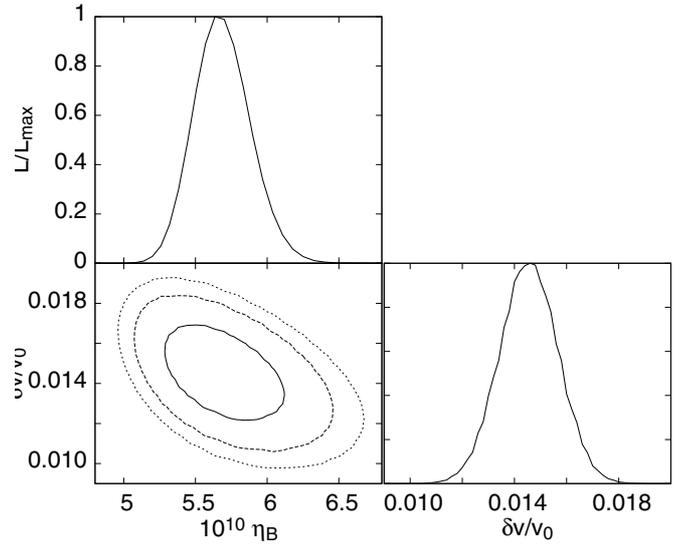


Fig. 4. 1σ , 2σ and 3σ likelihood contours for η_B and $\frac{\delta v}{v_0}$, and one-dimensional likelihood, $\left(\frac{L}{L_{\max}}\right)$, using $\kappa = 3$.

Table 5. Best fit parameter value and 1σ errors constraints on $\frac{\delta v}{v_0}$ and η_B .

κ	$\left(\frac{\delta v}{v_0} \pm \sigma\right) \times 10^2$	$(\eta_B \pm \sigma) \times 10^{10}$	$\frac{\chi^2_{\min}}{N-2}$
-3.65	$6.44^{+2.26}_{-1.53}$	$8.052^{+0.880}_{-0.623}$	0.91
-10	$-3.14^{+0.82}_{-0.75}$	$4.581^{+0.649}_{-0.523}$	4.33
3	1.46 ± 0.25	$5.636^{+0.514}_{-0.377}$	0.90

4. Conclusion

In the first part of this work we have studied the dependence of the deuterium binding energy as a function of the pion-mass, which is ultimately a function of the Higgs vacuum expectation value. For the analysis, we used the Reid 93 potential to represent the nucleon-nucleon interaction. It is found that the binding energy depends linearly on the pion-mass, and that the calculated value lies in the range obtained by various authors, e. g. Flambaum & Shuryak (2002). Our result for the slope of the

functional dependence of $\frac{\delta\epsilon_D}{(\epsilon_D)_0}$ vs. the variation of m_π (-3.65), may reduce the uncertainties associated to it, since in other works (Beane & Savage 2003; Epelbaum et al. 2003; Yoo & Scherrer 2003) a domain was reported. Next, we have calculated primordial abundances of BBN and focused on the discrepancy between standard BBN estimation for ${}^4\text{He}$ and D and their observational data. We found that, by allowing variations of either ϵ_D or v , this discrepancy is not solve.

Acknowledgements. This work has been partially supported by the National Research Council (CONICET) of Argentina (PIP 5145, PIP 112-200801-00740).

Appendix A: Modifications to Kawano's code

In this Appendix we discuss the dependence on the Higgs vacuum expectation value of the different physical quantities involved in the calculation of primordial abundances.

If during BBN v acquires a value different than the value at the present time, then the electron mass, the Fermi constant, the neutron-proton mass difference and the deuterium binding energy would also take different values (Landau et al. 2008).

A change in the electron mass affects the sum of the electron and positron energy densities, the sum of the electron and positron pressures and the difference of the electron and positron number densities. These quantities are calculated in Kawano's code (Kawano 1988, 1992) as:

$$\rho_{e^-} + \rho_{e^+} = \frac{2}{\pi^2} \frac{(m_e c^2)^4}{(\hbar c)^3} \sum_n (-1)^{n+1} \cosh(n\phi_e) M(nz),$$

$$\frac{p_{e^-} + p_{e^+}}{c^2} = \frac{2}{\pi^2} \frac{(m_e c^2)^4}{(\hbar c)^3} \sum_n \frac{(-1)^{n+1}}{nz} \cosh(n\phi_e) N(nz),$$

$$\frac{\pi^2}{2} \left[\frac{\hbar c^3}{m_e c^2} \right]^3 z^3 (n_{e^-} - n_{e^+}) = z^3 \sum_n (-1)^{n+1} \times \sinh(n\phi_e) L(nz),$$

where $z = \frac{m_e c^2}{k_B T_\gamma}$, ϕ_e is the electron chemical potential and $L(z)$, $M(z)$ and $N(z)$ are combinations of the modified Bessel function $K_i(z)$ (Kawano 1988, 1992). In order to include the variation in m_e we replace, in all the equations, m_e by $(m_e)_0 \left(1 + \frac{\delta m_e}{(m_e)_0}\right)$, and consider $\frac{\delta m_e}{(m_e)_0} = \frac{\delta v}{v_0}$.

The $n \leftrightarrow p$ reaction rates and the weak decay rates of heavy nuclei are also modified if the electron mass varies with time. The $n \leftrightarrow p$ reaction rate is calculated by

$$\lambda_{n \rightarrow p} = \lambda_0 \int_{m_e c^2}^{\infty} dE_e \frac{E_e p_e}{1 + e^{E_e/k_B T_\gamma}} \times \frac{(E_e + \Delta m_{np} c^2)^2}{1 + e^{-(E_e + \Delta m_{np} c^2)/k_B T_\nu - \xi_l}} + \lambda_0 \int_{m_e c^2}^{\infty} dE_e \frac{E_e p_e}{1 + e^{-E_e/k_B T_\gamma}} \times \frac{(E_e - \Delta m_{np} c^2)^2}{1 + e^{(E_e - \Delta m_{np} c^2)/k_B T_\nu + \xi_l}}, \quad (\text{A.1})$$

where λ_0 is a normalization constant proportional to $G_F^2 E_e$ and p_e are the electron energy and momentum respectively ($E_e = \sqrt{p_e^2 c^2 + m_e^2 c^4}$), T_γ and T_ν are the photon and neutrino temperature and ξ_l is the ratio between the neutrino chemical

potential and the neutrino temperature. This normalization constant is obtained at a very low temperature and for no variation of v .

The Fermi constant is proportional to v^{-2} (Dixit & Sher 1988), affecting the $n \leftrightarrow p$ reaction rate, since $\lambda_0 \sim G_F^2$.

The neutron-proton mass difference changes by (Christiansen et al. 1991)

$$\frac{\delta \Delta m_{np}}{(\Delta m_{np})_0} = 1.587 \frac{\delta v}{v_0}, \quad (\text{A.2})$$

affecting $n \leftrightarrow p$ reaction rates (see Eq. (A.1)), Q -values of several reaction rates (e.g. ${}^3\text{He}(n, p){}^3\text{H}$, ${}^7\text{Be}(n, p){}^7\text{Li}$) and the initial neutrons and protons abundances:

$$Y_n = \frac{1}{1 + e^{\Delta m_{np} c^2 / k_B T_9 + \xi}}, \quad Y_p = \frac{1}{1 + e^{-\Delta m_{np} c^2 / k_B T_9 - \xi}}, \quad (\text{A.3})$$

where T_9 is the temperature in units of 10^9 K. In order to include these effects we replace Δm_{np} by $\Delta m_{np} \left(1 + \frac{\delta \Delta m_{np}}{(\Delta m_{np})_0}\right)$. We have also modified the masses of the light nuclei (Flambaum & Wirlinga 2007) affecting the Q -values and the reverse coefficient of the reactions that involve neutrons.

The deuterium binding energy must be corrected by

$$\frac{\delta \epsilon_D}{(\epsilon_D)_0} = \frac{\kappa}{2} \frac{\delta v}{v_0}, \quad (\text{A.4})$$

where κ is a model dependent constant. In the present work this constant is found to be $\kappa = -3.65$. This correction affects the initial value of the deuterium abundance

$$Y_d = \frac{Y_n Y_p e^{\epsilon_D / k_B T_9}}{0.471 \times 10^{-10} (k_B T_9)^{3/2}}, \quad (\text{A.5})$$

where ϵ_D is in MeV, and the Q -values of several reactions, such as $d(\gamma, n)p$ from its reverse reaction. Once again we replace ϵ_D by $\epsilon_D \left(1 + \frac{\delta \epsilon_D}{(\epsilon_D)_0}\right)$ in order to modify the code.

References

- Aver, E., Olive, K. A., & Skillman, E. D. 2010, *J. Cosmol. Astropart. Phys.*, 5, 3
Barr, S. M., & Mohapatra, P. K. 1988, *Phys. Rev. D*, 38, 3011
Beane, S. R., & Savage, M. J. 2003, *Nucl. Phys. A*, 713, 148
Berengut, J. C., Flambaum, V. V., & Dmitriev, V. F. 2010, *Phys. Lett. B*, 683, 114
Bergström, L., Iguri, S., & Rubinstein, H. 1999, *Phys. Rev. D*, 60, 45005
Brax, P., van de Bruck, C., Davis, A.-C., & Rhodes, C. S. 2003, *Astrophys. Space Sci.*, 283, 627
Burles, S., & Tytler, D. 1998a, *ApJ*, 499, 699
Burles, S., & Tytler, D. 1998b, *ApJ*, 507, 732
Campbell, B. A., & Olive, K. A. 1995, *Phys. Lett. B*, 345, 429
Chamoun, N., Landau, S. J., Mosquera, M. E., & Vucetich, H. 2007, *J. Phys. G Nucl. Phys.*, 34, 163
Christiansen, H. R., Epele, L. N., Fanchiotti, H., & García Canal, C. A. 1991, *Phys. Lett. B*, 267, 164
Coc, A., Nunes, N. J., Olive, K. A., Uzan, J.-P., & Vangioni, E. 2007, *Phys. Rev. D*, 76, 023511
Crighton, N. H. M., Webb, J. K., Ortiz-Gil, A., & Fernández-Soto, A. 2004, *MNRAS*, 355, 1042
Cyburt, R. H., Fields, B. D., Olive, K. A., & Skillman, E. 2005, *Astropart. Phys.*, 23, 313
Damour, T., & Polyakov, A. M. 1994, *Nucl. Phys. B*, 95, 10347
Damour, T., Piazza, F., & Veneziano, G. 2002a, *Phys. Rev. Lett.*, 89, 081601
Damour, T., Piazza, F., & Veneziano, G. 2002b, *Phys. Rev. D*, 66, 046007
Dent, T., Stern, S., & Wetterich, C. 2007, *Phys. Rev. D*, 76, 063513

- Dixit, V. V., & Sher, M. 1988, *Phys. Rev. D*, 37, 1097
- Dmitriev, V. F., & Flambaum, V. V. 2003, *Phys. Rev. D*, 67, 063513
- Dmitriev, V. F., Flambaum, V. V., & Webb, J. K. 2004, *Phys. Rev. D*, 69, 063506
- Epelbaum, E., Meißner, U., & Glöckle, W. 2003, *Nucl. Phys. A*, 714, 535
- Flambaum, V. V., & Shuryak, E. V. 2002, *Phys. Rev. D*, 65, 103503
- Flambaum, V. V., & Shuryak, E. V. 2003, *Phys. Rev. D*, 67, 083507
- Flambaum, V. V., & Wiringa, R. B. 2007, *Phys. Rev. C*, 76, 054002
- Gleiser, M., & Taylor, J. G. 1985, *Phys. Rev. D*, 31, 1904
- Ichikawa, K., & Kawasaki, M. 2002, *Phys. Rev. D*, 65, 123511
- Ichikawa, K., & Kawasaki, M. 2004, *Phys. Rev. D*, 69, 123506
- Ivanchik, A. V., Petitjean, P., Balashev, S. A., et al. 2010, *MNRAS*, 404, 1583
- Izotov, Y. I., & Thuan, T. X. 2004, *ApJ*, 602, 200
- Izotov, Y. I., & Thuan, T. X. 2010, *ApJ*, 710, L67
- Izotov, Y. I., Thuan, T. X., & Lipovetsky, V. A. 1994, *ApJ*, 435, 647
- Izotov, Y. I., Thuan, T. X., & Lipovetsky, V. A. 1997, *ApJSS*, 108, 1
- Izotov, Y. I., Schaerer, D., Blecha, A., et al. 2006, *A&A*, 459, 71
- Izotov, Y. I., Thuan, T. X., & Stasińska, G. 2007, *ApJ*, 662, 15
- Kaluza, T. 1921, *Sitzungber. Preuss. Akad. Wiss. K*, 1, 966
- Kawano, L. 1988, Fermilab Report No. fERMILAB-PUB-88-034-A
- Kawano, L. 1992, Fermilab Report No. fERMILAB-PUB-92-004-A
- Kirkman, D., Tytler, D., Suzuki, N., O'Meara, J. M., & Lubin, D. 2003, *ApJS*, 149, 1
- Klein, O. 1926, *Z. Phys.*, 37, 895
- Krutov, A. F., & Troitsky, V. E. 2007, *Phys. Rev. C*, 76, 017001
- Lacombe, M., Loiseau, B., Richard, J. M., et al. 1980, *Phys. Rev. C*, 21, 861
- Lacombe, M., Loiseau, B., Mau, R. V., et al. 1981, *Phys. Lett. B*, 101, 139
- Landau, S. J., Mosquera, M. E., & Vucetich, H. 2006, *ApJ*, 637, 38
- Landau, S. J., Mosquera, M. E., Scóccola, C. G., & Vucetich, H. 2008, *Phys. Rev. D*, 78, 083527
- Levshakov, S. A., Dessauges-Zavadsky, M., D'Odorico, S., & Molaro, P. 2002, *ApJ*, 565, 696
- Luridiana, V., Peimbert, A., Peimbert, M., & Cervi no, M. 2003, *ApJ*, 592, 846
- Müller, C. M., Schäfer, G., & Wetterich, C. 2004, *Phys. Rev. D*, 70, 083504
- Machleidt, R., Holinde, K., & Elster, C. 1987, *Phys. Rep.*, 149, 1
- Maeda, K. 1988, *Mod. Phys. Lett. A*, 31, 243
- Mosquera, M. E., Scoccola, C., Landau, S., & Vucetich, H. 2008, *A&A*, 478, 675
- Nagels, M. M., Rijken, T. A., & de Swart, J. J. 1975, *Phys. Rev. D*, 12, 744
- Nagels, M. M., Rijken, T. A., & de Swart, J. J. 1977, *Phys. Rev. D*, 15, 2547
- Nollett, K. M., & Lopez, R. E. 2002, *Phys. Rev. D*, 66, 063507
- Olive, K. A., Steigman, G., & Skillman, E. D. 1997, *ApJ*, 483, 788
- O'Meara, J. M., Tytler, D., Kirkman, D., et al. 2001, *ApJ*, 552, 718
- O'Meara, J. M., Burles, S., Prochaska, J. X., et al. 2006, *ApJ*, 649, L61
- Overduin, J. M., & Wesson, P. S. 1997, *Phys. Rep.*, 283, 303
- Palma, G. A., Brax, P., Davis, A. C., & van de Bruck, C. 2003, *Phys. Rev. D*, 68, 123519
- Peimbert, A., Peimbert, M., & Luridiana, V. 2002, *ApJ*, 565, 668
- Peimbert, M. 2002, *Rev. Mex. Astron. Astrofis. Conf. Ser.*, ed. W. J. Henney, J. Franco, & M. Martos, 12, 275
- Peimbert, M., Luridiana, V., & Peimbert, A. 2007, *ApJ*, 666, 636
- Pettini, M., & Bowen, D. V. 2001, *ApJ*, 560, 41
- Pettini, M., Zych, B. J., Murphy, M. T., Lewis, A., & Steidel, C. C. 2008, *MNRAS*, 391, 1499
- Reid, Jr., R. V. 1968, *Ann. Phys.*, 50, 411
- Sarkar, S. 1996, *Rep. Progr. Phys.*, 59, 1493
- Spergel, D. N., Verde, L., Peiris, H. V., et al. 2003, *ApJS*, 148, 175
- Spergel, D. N., Bean, R., Doré, O., et al. 2007, *ApJS*, 170, 377
- Stoks, V. G. J., Klomp, R. A. M., Terheggen, C. P. F., & de Swart, J. J. 1994, *Phys. Rev. C*, 49, 2950
- Thuan, T. X., & Izotov, Y. I. 1998, *Space Sci. Rev.*, 84, 83
- Thuan, T. X., & Izotov, Y. I. 2002, *Space Sci. Rev.*, 100, 263
- Weinberg, S. 1983, *Phys. Lett. B*, 125, 265
- Wiringa, R. B., Stoks, V. G. J., & Schiavilla, R. 1995, *Phys. Rev. C*, 51, 38
- Wu, Y., & Wang, Z. 1986, *Phys. Rev. Lett.*, 57, 1978
- Yao, W. M., et al. 2006, *J. Phys. G*, 33, 1
- Yoo, J. J., & Scherrer, R. J. 2003, *Phys. Rev. D*, 67, 043517
- Youm, D. 2001a, *Phys. Rev. D*, 63, 125011
- Youm, D. 2001b, *Phys. Rev. D*, 64, 085011